A Thesis for the Degree of Ph.D. in Engineering

Proposal and Verification of Improvement Measures to Raise Operational Efficiency in Distribution Centers

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Abstract

The role of distribution centers has expanded these days from a burden that does not produce any profit but cost to a place that adds value to supply chain management. Since each sector such as factories has faced its limit of productivity, the efficiency in supply chain has been reconsidered. Also, due to the development of the Internet, information may now be transferred quickly, and distribution centers have become the origin of competition. Under these backgrounds in recent logistics environments, this research investigated measures to improve operational efficiency in distribution centers and verified their performances.

In this research, improving work efficiency in distribution centers has been considered by categorizing into two types: transfer-type and inventory-type. In the former, items received from multiple producers are stored temporarily and redistributed towards next destinations. In the latter, items are stored as inventories beforehand, and they are picked up and shipped to them according to the order of customers. Measures to improve work efficiency in both types of distribution centers are suggested and the travel distance of items is mainly evaluated using mathematical models.

Regarding the efficiency improvement in transfer-type distribution centers, arrival and shipping dates of items are utilized to determine the storage location of items. Under both situations where arrival and shipping dates of items in the whole planning horizon are known beforehand and where they are uncertain are discussed. In the former, properties obtained with greedy policies to determine storage locations of items have also been proved and investigated. In the latter, a greedy policy which utilizes arrival dates of items is improved to determine the storage locations

Regarding the efficiency improvement in inventory-type distribution centers, changing floor layout and implementing class based storage are considered. ①Inserting a diagonal aisle to access more directly to the destination, ②replacing the entrance/exit to shorten travel distance of items, and ③introducing class based storage locations based on the demand of items are suggested, and the performances are compared. Then implementing them at the same time has been discussed for further improvements.

Under the situation that improvements in distribution centers have stronger effects on supply chain management these days, this study has evaluated the improvement measures of distribution centers using mathematical models. The results may give suggestions when selecting which measures to introduce in distribution centers.

Chapter 1. Introduction

1.1 Role of distribution centers

According to the Japanese law by Ministry of Land, Infrastructure, Transport and Tourism, distribution centers are to custody items that have been received, and items may be anything such as materials, products, refrigerated, frozen, and even hazardous items. Various items are kept safe in distribution centers, and they are vital for our lives and economic activities. Distribution centers play critical roles in supply chain as they connect material flows between suppliers and customers [1].

Before, distribution centers had been thought as “cost centers that do not provide any profit” which only keeps items inside as inventories. However, since the improvement in each sector has faced its limit as competition in the market increased and the speed of information transmission increased, the role of distribution centers has changed. They have become a source of competitive advantage in supply chain management as to outperform competitors on customer service, lead times, and costs [2]. Supply chain management is the relationship of all parties that are involved through upstream and downstream linkages in different phases that produces value as products and services delivered to the ultimate customer [3], and its management of whole supply chain in textile industry has been discussed by [4] and in food industry by [5]. The role of distribution center has extended from a passive role of storing items temporarily to also an active role of providing value-added services [6].

There are about 500,000 distribution centers only in the United States [7]. Warehousing contributed about 20% of the logistics cost of companies that have been surveyed in 2003 [8]. Recently, the rapid growth in the warehousing employment and mega distribution centers which serve mega markets has occurred [9], and operations have increased in complexity because of broader product varieties and increased demand volatility [10]. Since it has been found out that it is important to focus on distribution centers in order to improve the function of whole supply chain [11] and customer services [12], many researchers have introduced mathematical models and technological solutions to improve the efficiency [13].

In this research, improvement measures to raise operational efficiency in distribution centers are proposed and verified to add value to products, which provides higher service level to consumers, as well as to store them for a certain period of time.

1.2 Classification of distribution centers

There are several ways to classify distribution centers. In this research, distribution centers are classified into two different types because they have different functions and different measures have to be involved to make improvements. One is transfer-type distribution center, which is implemented under business-to-business situation, in which items received from producers arrive on arrival dates, and they are stored temporarily until shipped by their orders. The other is inventory-type distribution center, which may be utilized under business-to-customer situation, in which items are stored as inventories, and after items are ordered by unspecified customers, they are picked by workers and shipped to them.

1.2.1 Transfer-type distribution centers

As it is shown in **Fig. 1-1**, a transfer-type distribution center has a function to collect items from multiple producers, and re-distribute them to multiple retailers, for example. It is suitable in cases where one producer is willing to ship items to multiple retailers, one retailer needs items from multiple producers, or fresh items are need to be shipped immediately because they cannot be stored as inventories. Since items arrive and they are stored temporarily until the shipping date, it may be efficient to utilize the dates to determine storage locations.

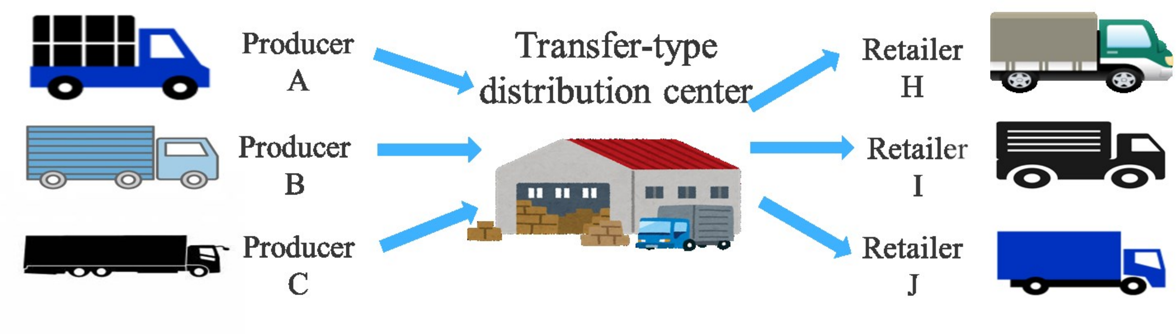


Fig.1-1 Transfer-type distribution center

1.2.2 Inventory-type distribution centers

As shown in **Fig. 1-2**, an inventory-type distribution center has a function to pick items from inventories and ship them out by the order of customers. It is suitable for cases such as third party logistics where they function as logistics of producers and e-commerce which only has virtual stores on the internet.

In inventory-type distribution centers, order picking which is to collect items ordered by customers from racks [14] is employed. Order picking is the major activity in distribution centers which includes the process of scheduling customer orders, releasing orders from customers to the floor, picking items from storage locations and retrieving the items that have been picked, which are about 55% of the total warehousing cost [15]. Order picking is ordinary taken as the most expensive operation in distribution centers [16]. Time, cost and productivity dimensions are the main measurements of the performance of distribution centers [17]. Improving the efficiency in order picking would contribute to save time and cost and raise productivity.

In distribution centers, both human beings and automated machines are employed for order picking [18] [19], and it could be both labor and capital intensive [16]. There are distribution centers that are completely operated by machines or robots. Storage assignment with rack-moving robots, which lift racks and move them to pickers, has been investigated [20], and a wheeled robot, which follows free trajectory controlled by artificial and fuzzy logic modules for object transportation in a distribution center, has been presented by [21]. However, it is quite difficult to introduce robots into distribution

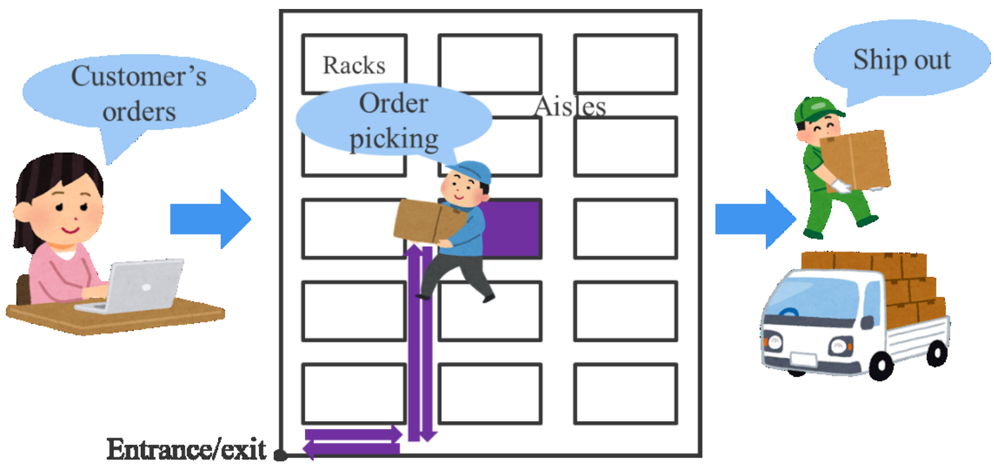


Fig.1-2 Inventory-type distribution center

centers that already exist because of the specifications, and it is sometimes dangerous for robots carry fragile or hazardous items, so there are still distribution centers mainly employed by human beings on the other hand. For example, there are order picking methods of pickers-to-parts and parts-to-pickers systems. In picker-to-part systems, order pickers travel along the warehouse and retrieve the items that have been requested, and in parts-to-picker systems, the requested items are handled and transported to pickers by automatic storage and retrieval systems [22]. Those operational differences would be whether human beings or automated machines are carrying them, and the time it takes for the operation, but the basic movements of items in the distribution center would be similar. In this paper, the case of picker-to-part system is assumed, but it may be applied in automated distribution centers as well.

Chapter 2. Previous researches

2.1 Previous researches of transfer-type distribution centers

In transfer-type distribution centers, it is assumed that items arrive from producers and stay in the distribution center until they are shipped by the order of the producers. In order to improve the efficiency, storage location assignment problem (SLAP) based on arrival and shipping dates of items may be considered. The objective of SLAP is to determine which location to store the incoming items to save the total operational cost for collecting and moving them to unloading area.

A review of papers dealing with SLAP, which has been published up to 2004, has been presented by [23]. Similarly, the papers between 2005 to 2017 have been investigated by [24].

Until the era of digitalization, it had been difficult to obtain accurate information of arrival and shipping dates of items beforehand. Instead, the researchers have estimated storage duration of stay (DOS) of items to utilize them to decide storage locations of items in distribution centers. The optimal storage location policy to store the items based on their DOS under the case that the number of incoming items is equal to that of departing items for every period in the planning horizon has been shown [25]. Using the result obtained in the ideal condition that the incoming and departing items are balanced as a bound, the heuristic method to find near-optimal storage location has been found where the incoming and departing items are not balanced [25]. In [26] and [27], information of DOS of items has been utilized to classify the items by their lengths. For example, items that stay in for relatively short period of time are located near the entrance/exit. A framework to obtain near-optimal solution of SLAP under uncertain information of DOS has been introduced by statistically estimating the DOS by [28].

These days, due to digital transformation, there are more opportunities that information of arrival and shipping dates of items are available beforehand. In this research, how information of arrival and shipping dates of items may be utilized to determine storage location items has been proposed and verified. By [29], information of shipping dates of items is utilized to determine the storage location of items, and it has been evaluated from the point of view of the travel distance of items. In this research, information of arrival dates of items, which are always obtained at the time of arrival, is also utilized to determine the storage location of items, and the performance of greedy policies is verified from other points of view as well as the travel distance of items.

2.2 Previous researches of inventory-type distribution centers

In inventory-type distribution centers, it is assumed that items are already stored as inventories in the distribution center and they are picked and shipped by the order of consumers. In order to improve the efficiency in this case, and replenishment of items from inventories, the movement of order pickers, configuration design of layout, storing inventories based on the demand of items in distribution centers have been discussed in previous papers.

2.2.1 Replenishment of items from inventories

In operation of inventory-type distribution centers, since stock-outs need to be reduced in order to improve the efficiency, replenishment has been considered. Different storage location and replenishment procedures to reduce the number of stock-outs have been presented and analyzed by [30], and the problem of replenishment cycles of multiple items in order to minimize the required storage space has been discussed by [31]. The Entry-Item-Quantity method as well as the total replenishment cost optimization model have been employed to propose multi picking strategies by [32].

In some inventory-type distribution centers, the space is partitioned into stock and picking areas [33]. It helps to shorten the travel distance of order picking by reducing the space of the latter, but internal replenishment is needed from the former. The problem of the size of the picking area relative to the stock area has been addressed by [34]. A heuristic procedure to determine how much space of picking area is provided to each item under the situation that all kinds of items are stored in storage area with an infinite capacity and those could be directly picked as well has been solved [35]. [36] has investigated the model by [35] and compared the solution with standard stocking strategies commonly implemented in practice. Based on the same model by [35], a branch-and-bound algorithm to maximize the benefit of picking area by trading off the relevant costs of order picking and internal replenishment has been proposed by [37]. The issue of deciding quantities of items stored in the picking area with given size under the situation that the quantity of each item which can be placed in the whole warehouse is given, has been addressed by [38].

2.2.2 Movement of order pickers

(a) Single command operation

Under single command operation (SCO), order pickers only pick one item per travel. SCO is conducted usually when picking items that are large and heavy, but in some cases, when several items are ordered, each item is picked under SCO, and items are sorted at the entrance/exit. In addition, in some distribution centers, racks are carried to pickers by robot [39] and this movement of robots may also be considered as SCO.

(b) Multi command operation

Under multi command operation (MCO), more than one item is picked per travel. In this case, routing is one thing to be considered to shorten the travel distance of items.

There have been a plenty of researches on routing as by [40]. Traveling salesman problem has been applied to the problem of optimal routing of picking inside distribution centers [41], and the polynomial algorithm by [41] has been extended by [42] and [43]. A routing algorithm for the minimization of the travel distance of pickers in distribution centers using ant colony optimization has been discussed by [44]. The performances of multi-objective ant colony optimization algorithms when applied to traveling salesman problem have been investigated by [45]. The effects of human behavior on the efficiency of routing policies in order picking have been focused on and it has been concluded that even if pickers deviate from the optimal routing policy, it is preferred to be implemented because it still shows improvements in efficiency [46].

In general, traveling salesman problem requires a lot of calculations and it is complicated. Routing policy heuristics are simpler to apply in practice compared to route optimization of order pickers to collect items in distribution centers [23]. [47] has attempted to obtain near-optimal solution of traveling salesman problem using a genetic algorithm. Traveling salesman problem has not always been the base of routing policies in previous papers. Approximations for expected travel distance for traversal, midpoint and largest gap strategies have been developed by [48]. Under those policies, pickers travel along the aisles picking multiple items in distribution centers [48]. Several heuristics for routing policies have been evaluated and compared with the optimal by [49]. The combination of routing and volume-based storage policies which assign items with the highest demand closest to the entrance/exit under different operating conditions of pick list size and demand skewedness have been examined by [50].

Congestion may become a problem especially under MCO, in which order pickers tend to travel longer compared to the case under SCO. [51] has stated that in researches of order picking, most of them focus only on one person picking, but there are some that consider how congestion may be mitigated when multiple persons are picking in distribution centers. The efficiency in order picking system under congestion situations when different storage policies and picking strategies were conducted has been analyzed by [52]. Simulated annealing based sampling to solve the problem of storage location assignment has been used to analyze warehouse layout network in order to reduce traffic congestion while increasing the speed of order picking [53]. An ant colony optimization has been utilized to develop a routing method to control congestion when there are two pickers [54], and an agent-based modeling approach has been discussed to analyze how congestion affects the cost and performance of distribution centers [55]. A model for the determination of storage location assignment considering a trade-off between the travel distance and the block-caused delay has been proposed [56], and the research has been extended by [57].

2.2.3 Configuration design of inventory-type distribution centers

In this research, layout of facilities such as aisles and racks and location of the entrance/exit are taken as configuration design of distribution centers. Japan Industrial Standards Committee defines layout as to locate buildings, facilities and apparatuses in order to transport, process and move reasonably in a system. There are plenty of researches about design of distribution centers as discussed in literature reviews [58] [59] [60]. In researches, layouts of distribution centers in order to shorten the travel distance or time of pickers have been considered.

(a) Traditional layout

Traditional layout shown in **Fig. 2-1** has conventionally been adopted in warehouses is set as a benchmark. As **Fig. 2-1**, in some inventory-type distribution centers, middle cross aisles are inserted, which may shorten travel distance of order picking under MCO. For example, the efficiency of inserting middle cross aisles in Traditional layout has been analyzed under dual command operation [61]. On the other hand, under SCO,

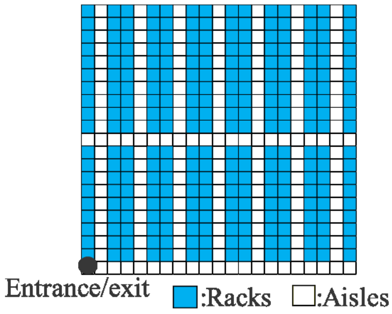


Fig. 2-1 Traditional layout

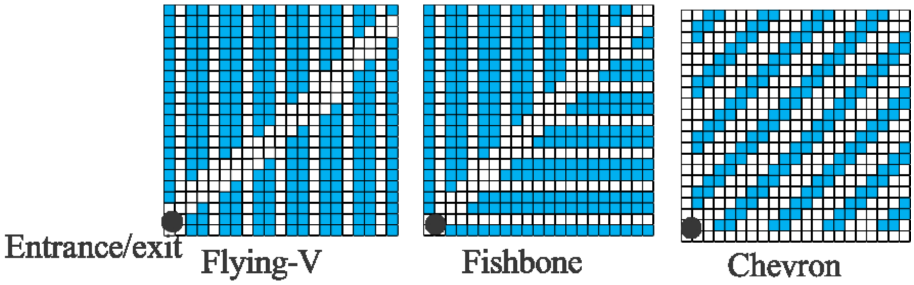


Fig. 2-2 Inserting diagonal aisles and changing the directions of racks and aisles

this pushes racks further away from the entrance/exit resulting in longer travel distance.

(b) Aisle layout other than Traditional layouts

In previous researches, improving the efficiency in distribution centers by changing from Traditional layout has been investigated. As in **Fig. 2-2**, changes are made by inserting diagonal aisles, and replacing or introducing multiple entrance/exit points. As the left figure in **Fig. 2-2**, Flying-V layout has been suggested as a different type of layout from Traditional layout, in which a diagonal aisle is inserted, by [62]. In addition to that, Fishbone layout has been suggested by changing directions of racks and aisles as the middle figure in **Fig. 2-2** by [62] and [63]. The efficiency of Fishbone layout has been investigated under SCO [62] and MCO [63]. It has been found out that Fishbone layout performs better in terms of the travel distance compared to Traditional layout especially under SCO. A detailed design of Fishbone layout has been provided and an optimization model using a genetic algorithm to minimize the total operational cost of distribution centers has been presented by [64].

The travel distances in different layouts with multiple entrance/exit points and the case with a single entrance/exit point have been compared and evaluated by [65] and [66].

As the right figure in **Fig. 2-2**, Chevron layout, which has closer performance to Fishbone layout under SCO, has been introduced by [61], and Leaf and Butterfly layouts have been proposed as extensions by [67].

2.2.4 Storage location of items based on the demand

In some inventory-type distribution centers, dedicated storage policy where the location of each item is fixed is implemented. The travel distance of items may be shortened, for example by placing items with higher demand rate in racks closer to the entrance/exit. In other distribution centers, shared storage location system where items may be located in any unoccupied space within the range is introduced. When considering the efficiency of order picking, dedicated storage policy saves travel distance, but completely random storage policy, which is a type of shared storage location system, saves space [68]. This is because especially under random storage policy, in which any item may be put in the closest open location from the entrance/exit, there is a chance that pickers may have to travel longer when items with higher demand are placed far from the entrance/exit.

(a) Dedicated storage system

Dedicated storage system is where each item has its own storage location. There are several slotting methods to determine dedicated storage of items, and these are popularity, turnover, volume, pick density, and cube-per-order index (CPO) [16]. Popularity is the number of times a picker travels to pick up one item to its storage location. The travel distance in distribution centers under turnover based storage location policy has been estimated by [69]. Turnover based storage is to place items according to its demand. The travel distance of items in distribution centers where turnover based assignment, in which items with the highest demand rate are located closest to entrance/exit, is employed in both situations that racks are placed vertically and horizontally has been considered by [70]. The effect on travel distance when diagonal aisles are inserted in the case of turnover based storage has been investigated by [71], and the efficiency in layouts that have diagonal cross aisles when the items are stored both randomly and turn-over based have been compared to that of Traditional layout [11]. Volume is the demand of an item multiplied by the volume of an item, and pick density is the ratio of the popularity of an item to the volume of the item [72]. Factors including pick density that have effects on the performance of order picking have been considered by [73]. Among studies on dedicated storage policy, CPO is the most widely studied, and where to store each item is determined by how the items are ranked and how they are assigned to storage locations [72]. CPO is the ratio of the item's storage space requirement to its popularity [16]. The management of storage location of products in a dedicated storage warehouse under SCO has been considered by [74] and [75], in which items were sorted by increasing CPO and assigned sequentially from nearest location.

(b) Class based storage system

Fig.2-3 shows examples of distribution centers which are divided into classes. As the first research of class based storage system, the effective storage allocation of items to shorten the travel distance of items under the assumption that storage locations allow different items to be stored successively in the planning horizon has been considered by [76]. It may be said it is efficient to prepare dedicated storage location for each product, but it is sometimes difficult in practice because demand rate has uncertainty and volatility, and it is hard to manage every item in the distribution center. Class based storage policy implies that the storage locations are divided into a number of classes and items are allocated to classes based on their turnover [77]. This contributes to save both costs of order picking and space of distribution centers.

Class based storage with only one class is random storage, and the case when each item is considered as one class is called turnover based storage [78] which has been discussed in Subsection 2.2.4 (a).

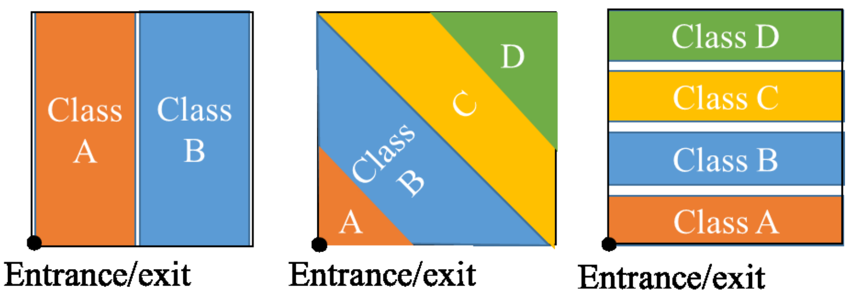


Fig. 2-3 Images of class-based storage locations

Numerical simulation is conducted using real data and shows the effectiveness of class based storage by [79]. Performances of turnover based dedicated storage policy and three shared storage policies (DOS, closest-open-location and 2-class storage) in the single command storage system with imbalanced input and output items have been compared by [25], and performance implications of class based storage with random and volume-based storage have been compared by [80] and [81]. Inserting middle cross aisles in Traditional layout and involving class based storage are discussed simultaneously by [82].

How items may be divided into classes is also an important issue. A probabilistic model to estimate the average travel distance has been proposed and a mathematical formulation for the storage zone optimization problem of class based storage has been presented by [81]. A measurement to divide items into classes has been introduced and its effectiveness has been investigated by [83], and classifying items in distribution centers with analytic hierarchy process (AHP) and data envelopment analysis (DEA) is proposed [84]. As another way to obtain solutions, branch and bound algorithm to solve the class based storage implementation decision based on CPO has been proposed by [85]. A solution procedure to determine the optimal boundaries for class based storage has been developed by [86].

In previous studies, measures to improve efficiency in inventory-type distribution centers are investigated individually. Also, the sizes of racks and aisles have been given when simulation was conducted to evaluate the performance of distribution centers (e.g. [7], [61], [62], [66], [82]). In that case, they have an impact on the result and the storage capacity when the layout is changed. This research makes it possible to compare the improved efficiency of measures and evaluate them comprehensively.

Chapter 3. Aim of this research

The aim of this research is to propose measures to improve efficiency in transfer-type and inventory-type distribution centers including measures that have been implemented in practice. Mathematical models are utilized to verify performance of proposed measures from the point of view of travel distance. The goal of this research is to give suggestions that help to improve operational efficiency in transfer-type and inventory-type distribution centers.

3.1 Improvement of efficiency in transfer-type distribution centers

In transfer-type distribution centers, items arrive on arrival dates, and they are shipped on shipping dates. Before the era of digitalization, it had been difficult to obtain the information of arrival and shipping dates of items beforehand. In order to determine the placements of items in distribution centers, estimated duration of stay of items had been utilized. However, due to digital transformation these days, we now have chances to obtain information of items beforehand. The aim of research is to investigate how arrival and shipping dates of items are utilized to determine effective storage locations of items in distribution centers. Locations of items have been determined by the order of shipping dates, and the performance was evaluated by travel distances of items in [29]. However, in this research, arrival dates of items, which we always know at the time of arrival, are also utilized to determine storage locations, and performances are evaluated by travel distances of items as well as different points of view.

3.2 Improvement of efficiency in inventory-type distribution centers

In inventory-type distribution centers, items are stored as inventories and they are picked and shipped by the order of customers, for example in the case of e-commerce. In previous researches, in order to reduce the travel distance of items, improvements such as changing floor layout and class based storage of items according to the demand have been implemented individually, and the performances of the measures were compared to the original Traditional layout. In order to do so, numerical simulations have been conducted, but since capacities to store items depend on the sizes of aisles and racks, comparisons between them have not been appropriate. In this research, in order to make it possible to compare performances of different measures, mutual assumptions are set by mathematical models. The aim of research is not only to obtain solutions, but also give suggestions when selecting which measures to implement to improve efficiency in distribution centers by making the performances of different measures apparent.

Chapter 4. Storage location assignment problem in transfer-type distribution centers

In general, in transfer-type distribution centers, items are stored until they are shipped by the order of the owners of items. Items that arrive in the distribution center are possibly stored temporarily together with other items that are shipped to the same place, so it may be efficient to determine storage location of items using the arrival and shipping dates. In this chapter, Storage location assignment problem (SLAP) utilizing arrival and shipping dates of items is solved to determine the storage allocations of items in the distribution center in order to improve operations by shortening the travel distance between the pick-and deposit point (PDP) and racks.

In the first half of this chapter, after defining the problem, the performances of arrival date (AD), shipping date (SD), and duration of stay (DOS) policies are evaluated under situation that arrival and shipping dates of all items are known beforehand. Then, four properties of the above greedy policies are clarified and proved, which have not been done previously (except property 2). The properties are then utilized to efficiently obtain the optimal solutions of the SLAP. In the second half of the chapter, under the case the information on the arrival and shipping dates of the items are incomplete, which is closer to reality, improved greedy policies are proposed to obtain near-optimal solutions that are relatively easy to implement in practice.

4.1 Problem definition

4.1.1 Assumptions

In this chapter, the problem of determining the storage location of product items in a distribution center under a shared storage policy was addressed. The following assumptions were made.

1) Items are carried into the distribution center according to their arrival dates and are carried out according to their shipping dates. An "item" is a product stock-keeping unit.
2) In the distribution center, single command operation (SCO) is executed; that is, items are always handled individually without being combined.
3) Items arrive at the pick-and-deposit point (PDP) of the distribution center, are moved to the storage sections immediately and stored items are shipped from the PDP.
4) The storage racks in the distribution center are divided into storage sections and only one item may be stored in each section. Multiple items cannot be stored in the same sections simultaneously.
5) The distribution center has enough sections to store all items that have arrived.
6) Obtained solutions are evaluated using the travel distance to move all of the items in the planning horizon.

4.1.2 Mathematical formulation

Under the assumptions explained above, the problem can be formulated as follows.

I_i : Arrival date of the i^{th} ($i = 1, 2, \cdots$) item

O_i : Shipping date of the i^{th} ($i = 1, 2, \cdots$) item

d_j : Distance between the PDP and the j^{th} ($j = 1, 2, \cdots$) section.

$$x_{ij} = \begin{cases} 1: the\ i^{th}\ item\ is\ stored\ in\ the\ j^{th}\ section \\ 0: otherwise \end{cases}$$

Here, $m \le n$ is assumed to hold a sufficient number of storage sections. The distance d_j satisfies Eq. (1) to simplify the following explanation.

$$d_1 \leq d_2 \leq \cdots \leq d_n \tag{1}$$

The objective function *TD*(*X*) is described by Eq. (2).

$$TD(X) = \sum_{i=1}^{m} \sum_{j=1}^{n} d_j x_{ij} \rightarrow min \tag{2}$$

The objective of solving the problem is to determine the optimal storage section of items to minimize the travel distance in the planning horizon under the following conditions.

$$\sum_{j=1}^{n} x_{ij} = 1 \ \ (i = 1,2,\dots,m) \tag{3}$$

$$x_{sj} + x_{tj} \leq 2 - a_{st} \ (j = 1,2,\dots,n) \ \ for \ \ any \ \ s,t \ \ (s \neq t, s,t = 1,2,\dots,m) \tag{4}$$

Where

$$a_{st} = \begin{cases} 1: the \ \ s^{th} \ \ and \ \ j^{th} \ \ items \ \ overlap \\ 0: otherwise \end{cases}$$

Eq. (3) indicates that every item must be stored in one section somewhere. Eq. (4) means that multiple items that have overlapping storage periods cannot be stored in the same section.

4.1.3 Minimal cost vertex coloring problem

In the coloring problem, there exists a problem of determining colors of the vertices in the graph to minimize the total cost, which refers to as the minimal cost vertex coloring problem. In this problem, the colors of the vertices are determined so that the vertices connected with the edge are not painted with the same color. Minimizing the total cost is the objective of this problem when the cost is given to the colors.

Applying this vertex coloring problem to the SLAP, the vertex corresponds to the item and the color corresponds to the storage section. The vertices connected with the edge represents the overlap of the storage periods of the items. It can be understood that the problem of this research is to determine the colors (sections) of the vertices (items) so as to minimize the total cost (travel distance).

Determining the chromatic number of the general graph in the coloring vertex problem is NP-hard. Similarly, the minimal cost vertex coloring problem is known to be NP-hard. The chromatic number of the general graph satisfies the following Eq. (5) [82]. Here, $\chi(G)$ represents the chromatic number of the graph *G,* and $\deg(v)$ represents the number of edges connected with the vertex *v*.

$$\chi(G) \leq \left\{\max_{V} \ \deg(v)\right\} + 1 \tag{5}$$

In addition, the chromatic number of the complete graph with *K* vertices is given by the following Eq. (6). That is, it is known to be *K*.

$$\chi(G_k) = K \tag{6}$$

In the SLAP, the chromatic number $\chi(G)$ means the minimal number of sections to store all items in the planning horizon. The relationship between this value and the storage location policies proposed is discussed later.

4.2. Greedy policies and their features

In this section, the AD and SD storage policies which greedily determine where to store the items according to arrival and shipping dates of items are introduced.

4.2.1 AD and SD policies

1) AD policy

The AD policy decides the storage location of items based on the date the item arrives. Items are stored in the section as close as possible to the PDP in the order of arrival starting with the earliest. The storage location of the deterministic problem, in which arrival and shipping dates of all items in the planning horizon are known, is determined through the following procedure. The storage location determined by this procedure will be referred to as the "AD storage location".

Step 1. Sort all items in the planning horizon in the order starting with the earliest arrival date I_i.

Step 2. Store the item in order in the nearest section from the PDP. If there are items with the same arrival dates, the one with the earlier shipping date occupies the nearest section preferentially.

Fig. 4-1 shows the arrival and shipping schedule of five items. Under the AD policy, the storage locations are determined as shown in **Fig. 4-2**. The following, Eq. (7), shows the travel distance TD_1 of the storage location under the assumption that $d_j = j,\ j = 1, 2, \cdots$. Eq. (7) is given as the sum of the number of items stored in each section multiplied by the travel distance from the PDP. In **Fig. 4-2**, there are two items in section 1, two in section 2, and one in section 3. Because the travel distance between the PDP and each section is given by d_j, assuming $d_j = j,\ j = 1, 2, \cdots$, the travel distance TD_1 is calculated as follows.

$$TD_1 = 1 \times 2 + 2 \times 2 + 3 \times 1 = 9 \qquad (7)$$

The required number of storage sections to store all of the items in the planning horizon (i.e., the maximum value of $S(t), t = 1, 2, \cdots$) is 3.

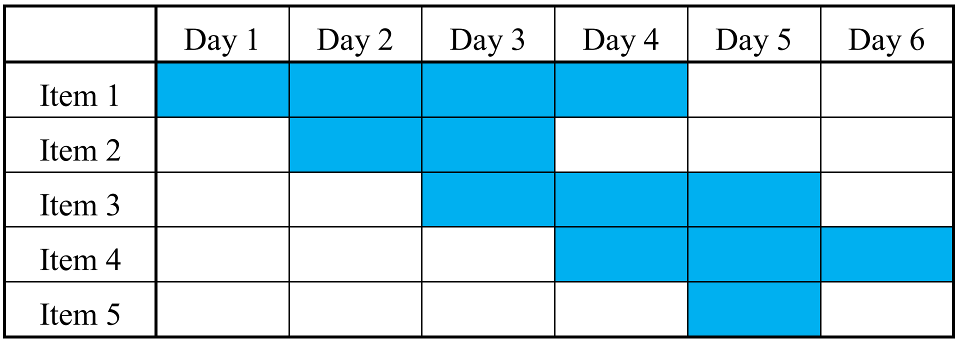


Fig. 4-1 Example 1: Arrival and shipping schedule of items

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6
Sect. 1	Item 1				Item 5	
Sect. 2		Item 2		Item 4		
Sect. 3			Item 3			
S(t)	1	2	3	3	3	1

Fig. 4-2 The AD storage location of Example 1

The AD storage location does not necessarily guarantee optimality for shortening the travel distance of items. However, the AD policy satisfies the following property.

Property 1:

The AD policy minimizes the number of storage sections required to store all items.

Proof of property 1

The storage sections of two arbitrary items are exchanged repeatedly to convert a storage location that does not follow the AD policy into an AD storage location.

Any two items that do not follow the AD policy in the storage location are the focus. In **Fig. 4-3**, when section F is closer to the PDP than section G; that is, $F < G$, item F_l in section F and item G_m in section G are stored in a manner contrary to that in the AD policy.

When the sections of items F_l and G_m are exchanged, the sections of the subsequent items $F_{l+1}, F_{l+2}, \cdots$ and $G_{m+1}, G_{m+2}, \cdots$ are simultaneously exchanged.

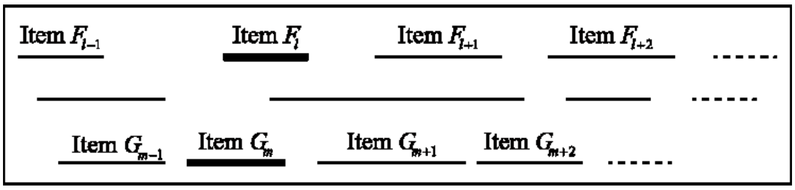


Fig. 4-3 The storage location which does not follow the AD and SD policies

Items $F_l, F_{l+1}, F_{l+2}, \cdots$ can always move to section G because $I_{F_l} \geq I_{G_m}$. However, regarding the movement of items $G_m, G_{m+1}, G_{m+2}, \cdots$,

1) $O_{F_{l-1}} \leq I_{G_m}$

In the case of $O_{F_{l-1}} \leq I_{G_m}$, items $G_m, G_{m+1}, G_{m+2}, \cdots$ can move to section F.

2) $O_{F_{l-1}} > I_{G_m}$

In the case of $O_{F_{l-1}} > I_{G_m}$, there is no feasible solution in which item F_{l-1} and item G_m are stored in the same section.

From this discussion, one can obtain the AD storage location from the storage location that does not follow the AD policy by repeatedly exchanging the storage sections of items $F_l, F_{l+1}, F_{l+2}, \cdots$ and items $G_m, G_{m+1}, G_{m+2}, \cdots$. In addition, the number of sections required to store all items does not increase during this process—it may be possible to reduce the number of storage sections required by packing the item(s) into the closer section after exchanging the storage sections.

As a result, assuming that there is a storage location that minimizes the number of storage sections required and that it does not follow the AD policy, one can convert to the AD storage location without increasing the number of storage sections.

Therefore, property 1 means that the AD policy can minimize the number of storage sections used to store all items in the planning horizon. Although this property holds within the finite-period problem, the AD policy could also work to reduce the number of storage sections for-infinite-period problems.

In general, the required number of storage sections in the AD storage location corresponds to the maximum number of items with overlapping storage periods in **Fig. 4-1**. Here, this reason is confirmed using the graph theory.

When the storage periods of K items are overlapped, this relationship is represented as a complete graph with K vertices. The chromatic number of this complete graph is K from Eq. (6). Since, the overlaps of the items are changed within the planning horizon, the SLAP in this research has a structure, in which complete graphs with $K_1, K_2, \cdots$ vertices are connected, where $K_t(t = 1,2,\cdots)$ indicates the number of overlapped items at time *t* in **Fig. 4-1**. Accordingly, the chromatic number of this graph, i.e., the number of required storage sections of this SLAP is given by Eq. (8).

$$\chi(G) = max\left(K_1, K_2, \cdots\right) \tag{8}$$

2) SD policy

The SD policy determines the storage locations of items based on their shipping dates. The earlier the shipping dates, the closer the sections where they are stored to the PDP. Items are stored in the section as close as possible to the PDP in the order of the shipping dates, starting with the earliest. The storage location of deterministic problem is determined through the following procedure.

Step 1. Sort all items in the planning horizon in the order starting with the earliest shipping date O_i.

Step 2. Store the items in order in the nearest section from the PDP. If there are items with the same shipping dates, the item with the earlier arrival date is stored in the nearer section preferentially.

Fig. 4-4 shows the SD storage location of five items in **Fig. 4-1**. The SD policy determines the storage locations of items based on their shipping dates. The earlier the shipping dates, the closer the sections where they are stored to the PDP. Items are stored in the section as close as possible to the PDP in the order of their shipping dates, starting with the earliest. **Fig. 4-4** shows the SD storage locations of the five items in **Fig. 4-1**. Assuming $d_j = j,\ j = 1,2,\cdots$, the travel distance TD_2 is calculated as 11. The number of storage sections required is four.

$$TD_2 = 1 \times 2 + 2 \times 1 + 3 \times 1 + 4 \times 1 = 11 \tag{9}$$

In this numerical example, the SD policy increased both the required number of storage sections and the travel distance compared to the AD policy. In general, the SD policy satisfies the following two properties.

<table>
<tr><th></th><th>Day 1</th><th>Day 2</th><th>Day 3</th><th>Day 4</th><th>Day 5</th><th>Day 6</th></tr>
<tr><td>Sect. 1</td><td></td><td colspan="2">Item 2</td><td></td><td>Item 5</td><td></td></tr>
<tr><td>Sect. 2</td><td colspan="4">Item 1</td><td></td><td></td></tr>
<tr><td>Sect. 3</td><td></td><td></td><td colspan="3">Item 3</td><td></td></tr>
<tr><td>Sect. 4</td><td></td><td></td><td></td><td colspan="3">Item 4</td></tr>
<tr><td>S(t)</td><td>1</td><td>2</td><td>3</td><td>3</td><td>3</td><td>1</td></tr>
</table>

Fig. 4-4 The SD storage location of Example 1

Property 2:

The SD policy maximizes the number of stored items in the nearest storage section from the PDP (section 1).

Property 2 has been discussed in relation to the conventional interval scheduling problem. Its proof has been provided by [85]. Under the SD policy, items are assigned to the nearest vacant section to the PDP in the order of shipping dates starting from the earliest. This means that in section 1, items that have the earliest shipping dates are stored. In other words, the items with forward-shipping dates are assigned to section 1 as much as possible. As a result, section 1 may be most frequently occupied, and the number of items stored in section 1 is maximized.

Property 3:

The SD policy maximizes the required number of storage sections to store all of items among all of the top alignment solutions. Here, 'top alignment' means that items are stored in sections as close as possible from the PDP when items are assigned to sections.

Proof of property 3

The intent here is to convert a storage location that does not follow the SD policy into the SD storage location by repeatedly exchanging the storage sections of two arbitrary items. However, this time, only the sections of the two target items are changed, and the sections of the subsequent items are not changed.

In **Fig. 4-3**, the storage locations of items F_l and G_m do not follow the SD policy because $O_{F_l} > O_{G_m}$ holds. Changing the storage section of item G_m from *G* to *F* results in the following.

1) $O_{F_{l-1}} \le I_{G_m}$

In the case of $O_{F_{l-1}} \le I_{G_m}$, item G_m can move to section *F*.

2) $O_{F_{l-1}} > I_{G_m}$

In the case of $O_{F_{l-1}} > I_{G_m}$, there is no feasible solution in which items F_{l-1} and G_m are stored in the same section.

Next, changing the storage section of item F_l from section *F* to *G* after moving item G_m to section *F*; that is, when the above condition 1) holds,

3) $O_{G_{m-1}} \le I_{F_l}$ and $O_{F_l} \le I_{G_{m+1}}$

If $O_{G_{m-1}} \le I_{F_l}$ and $O_{F_l} \le I_{G_{m+1}}$ hold, item F_l can move to section *G*, and the number of storage sections required remains the same.

4) $O_{G_{m-1}} > I_{F_l}$ or $O_{F_l} > I_{G_{m+1}}$

If $O_{G_{m-1}} > I_{F_l}$ or $O_{F_l} > I_{G_{m+1}}$ holds, item F_l cannot be moved to section *G*. In this case, item F_l is stored in the newly added storage section. As a result, the required number of storage sections increases by one.

From this discussion, assuming that there is a top alignment storage location that maximizes the required number of storage sections, and that it does not follow the SD policy, one can convert to the SD storage location without decreasing the number of storage sections.

Properties 2 and 3 will be used to obtain the optimal solution in the next section together with property 1. As shown in property 2, the SD policy tends to increase the frequency of using the nearest storage section. With this feature, the SD policy intends to shorten the travel distance although the required number of storage sections is maximized.

4.2.2 Performance evaluation of greedy policies

The performances of the AD and SD greedy policies were evaluated using numerical simulations. The following two policies focusing on the duration of stay of items were added to investigate the features of the AD and SD policies.

1) DOS1 policy

The DOS1 policy determines the storage location of items depending on how long they are stored in the distribution center. Items are stored in the section as close as possible to the PDP in the order of their length of storage duration, starting with the shortest period. When there are items with the same storage duration, they are placed in the order of arrival dates from the nearer section to the PDP. **Fig. 4-5** shows the DOS1 storage locations of the five items in **Fig. 4-1**. Assuming $d_j = j,\ j = 1, 2, \cdots$, the travel distance TD_3 is calculated as 11 as in Eq. (10). The required number of storage sections is four.

$$TD_3 = 1 \times 2 + 2 \times 1 + 3 \times 1 + 4 \times 1 = 11 \quad (10)$$

2) DOS2 policy

The DOS2 policy also determines the storage location of items depending on the period of time they are stored in the distribution center. However, in this case, items are stored in the closest vacant section from the PDP in the order of their length of storage duration starting with the longest period. When there are items with the same storage duration, they are placed in the order of arrival dates from the nearest section to the PDP. **Fig. 4-6** shows the DOS2 storage locations of the five items in **Fig.**

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6
Sect. 1		Item 2	Item 2		Item 5	
Sect. 2			Item3	Item3	Item3	
Sect. 3				Item4	Item4	Item4
Sect. 4	Item1	Item1	Item1	Item1		
S(t)	1	2	3	3	3	1

Fig. 4-5 The DOS1 storage location of Example 1

<table>
<tr><th></th><th>Day 1</th><th>Day 2</th><th>Day 3</th><th>Day 4</th><th>Day 5</th><th>Day 6</th></tr>
<tr><td>Sect. 1</td><td colspan="4">Item 1</td><td>Item 5</td><td></td></tr>
<tr><td>Sect. 2</td><td></td><td></td><td colspan="3">Item 3</td><td></td></tr>
<tr><td>Sect. 3</td><td></td><td colspan="2">Item 2</td><td colspan="3">Item 4</td></tr>
<tr><td>S(t)</td><td>1</td><td>2</td><td>3</td><td>3</td><td>3</td><td>1</td></tr>
</table>

Fig. 4-6 The DOS2 storage location of Example 1

4-1. Assuming $d_j = j,\ j = 1, 2, \cdots$, the travel distance TD_4 is calculated as 10 as in Eq. (11). The required number of storage sections is three.

$$TD_4 = 1 \times 2 + 2 \times 1 + 3 \times 2 = 10 \tag{11}$$

To compare the four greedy policies (AD, SD, DOS1, and DOS2), computer simulations were performed. The arrival and shipping dates of 60 items were generated according to the following probability distribution function. In Eq. (12), Δ_1 is the arrival interval between the i^{th} and $i+1^{th}$ items, which is assumed to follow an exponential distribution with a mean of λ_1.

$$I_{i+1} = I_i + \Delta_1 \ \ where \ \ \Delta_1 \sim Exp(\lambda_1) \tag{12}$$

Three types of distributions are compared to see how the variance of storage durations may have effects on the performances. As in Eq. (13), the shipping date O_i of the item is calculated by adding Δ_2, which indicates the storage duration of the item, to the arrival date. It is assumed that it follows an exponential distribution with a mean of λ_2.

$$O_i = I_i + \Delta_2 \ \ where \ \ \Delta_2 \sim Exp(\lambda_2) \tag{13}$$

Eq. (14) represents another case that the shipping date O_i of the item is calculated by adding Δ_3 to the arrival date. It is assumed that it follows a lognormal distribution.

$$O_i = I_i + \Delta_3 \;\; where \;\; \Delta_3 \sim \Lambda(\mu, \sigma^2) \tag{14}$$

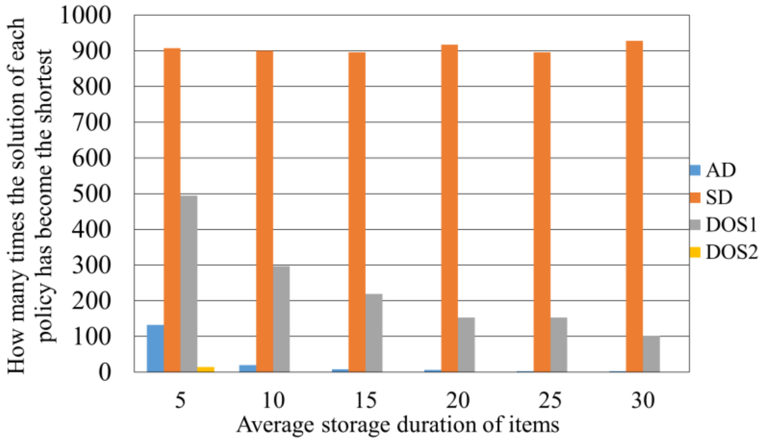
In the investigation, the mean of Δ_2 and Δ_3 increases by 5 in the range of 5-30, and the mean of the arrival interval Δ_1 is always set as 10. Because the average storage duration becomes longer when the average arrival interval is constant, the storage periods of items are more likely to overlap as the storage duration increases. The variance of Δ_3 is set as 25 and 900. 25 and 900 have been chosen as the variances because when the storage duration of items follows an exponential distribution with an average of 5 days, the variance is 25, and when it follows that of 30 days, the variance is 900. How variance has an effect on the solutions has been investigated.

In addition, d_j, which is the distance between the j^{th} section and the PDP, is given by Eq. (15).

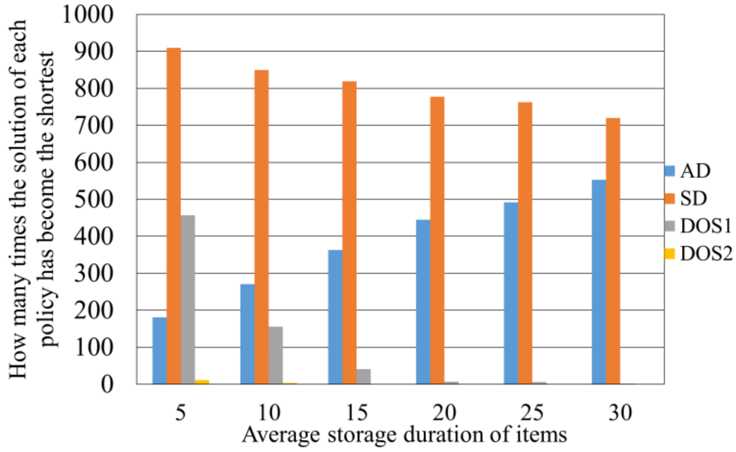
$$d_j = \sqrt{j} \tag{15}$$

Fig. 4-7 shows the number of times each storage policy obtained the solution with the shortest travel distance among the four storage policies during 1,000 trials. The solution with the shortest travel distance of the four storage policies obtained here does not indicate the optimal solution among all feasible solutions. A comparison between the solutions using the four storage policies and the optimal solution was conducted later (see **Fig. 4-8**). Both in **Figs. 4-7** and **4-8**, a) the average storage duration is set as $\Delta_2 \sim Exp(\lambda_2)$, b) the variance of $\Delta_3 \sim \Lambda(\mu, \sigma^2)$ is set as 25, and c) the variance of $\Delta_3 \sim \Lambda(\mu, \sigma^2)$ is set as 900.

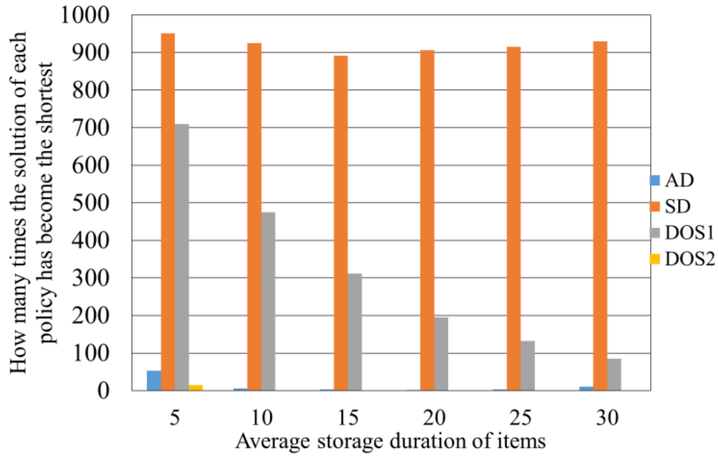
The sum of the number of times in the numerical experiments at a certain storage duration may exceed 1,000 because there may be several solutions with the shortest distance. For the problem of reducing the travel distance, the SD policy is the most effective regardless of the storage duration and the variance. In **Fig. 4-7** a) c), as the storage duration increases, in all policies except the SD policy, the number of times that the solution becomes the minimum is reduced. When the average storage duration is short, the variance is small as well, and since the average arrival interval is 10 days, it is likely that when an item arrives, the nearest section to the



a) The average storage duration is set as $\Delta_2 \sim Exp(\lambda_2)$



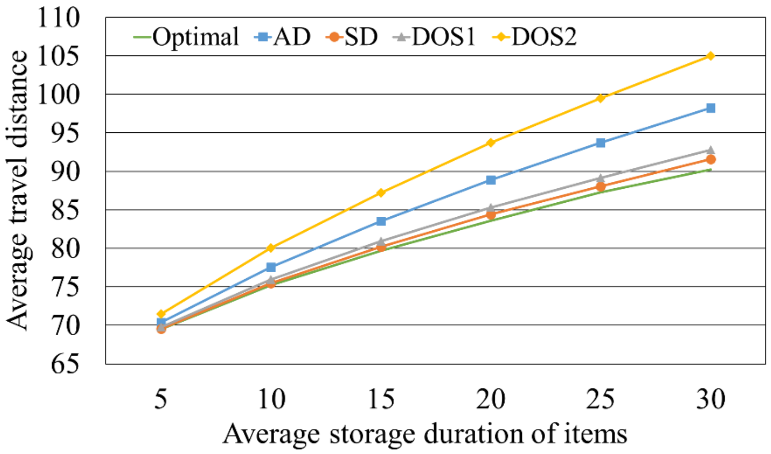
b) The variance of $\Delta_3 \sim \Lambda(\mu, \sigma^2)$ is set as 25



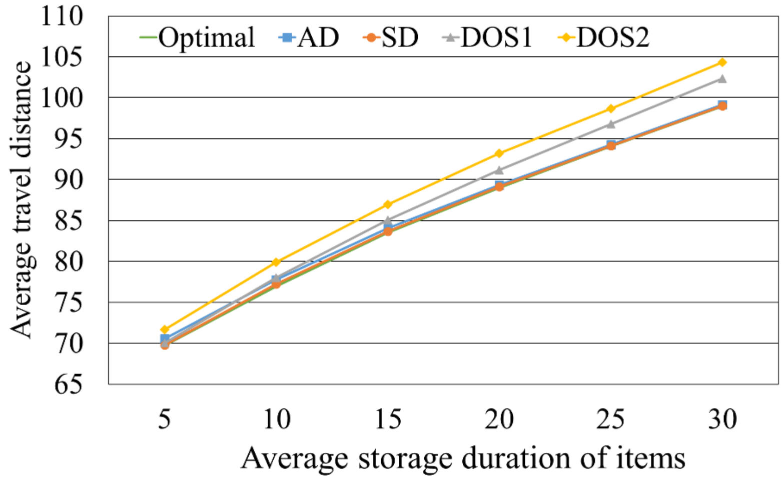
c) The variance of $\Delta_3 \sim \Lambda(\mu, \sigma^2)$ is set as 900

Fig. 4-7 Comparison of the four storage policies

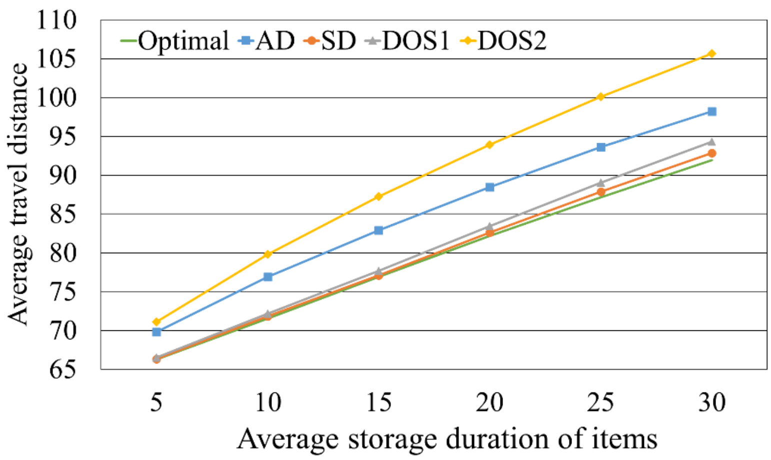
(The number of times each storage policy has given the shortest travel distance among 1,000 trials)



a) The average storage duration is set as $\Delta_2 \sim Exp(\lambda_2)$



b) The variance of $\Delta_3 \sim \Lambda(\mu, \sigma^2)$ is set as 25



c) The variance of $\Delta_3 \sim \Lambda(\mu, \sigma^2)$ is set as 900

Fig. 4-8 Average travel distances of items using the four storage policies

PDP is not occupied, so the solution by any policy is the same. In **Fig. 4-7** b), as the storage duration increases, the performances of the AD and SD policies become closer.

Furthermore, the performance was evaluated by making a comparison with the optimal travel distance, which was obtained using the method explained in Section 4.4. **Fig. 4-8** shows the average travel distances determined using the four storage policies based on 1,000 trials. Similar to the results shown in **Fig. 4-7**, the SD policy was able to find the closest solution to the optimal solution regardless of the storage duration and the variance. In **Fig. 4-8**, the second closest to the optimal solution was the DOS1 policy in a) and c), while in b) the second closest to the optimal solution was the AD policy. In **Fig. 4-8** a), the differences from the optimal values tended to increase as the storage duration increased, but compared to the other policies, the SD policy did not differ much from the optimal solution. In **Fig. 4-8** b) c), the average travel distances of items by the AD policies are close to that in **Fig. 4-8** a). On the other hand, in **Fig. 4-8** b), the average travel distances of items by the SD and DOS1 policies and the optimal solution are longer compared to the case in **Fig. 4-8** a).

Both results drawn from **Figs. 4-7** and **4-8** show that the variances did not have effects on the AD policy. Variances had influences on the travel distances of items by the SD and DOS1 policies. When the variance is relatively large, since the differences among storage durations of items would be relatively large as well, they are unlikely to be overlapped, so the number of sections needed decreases and the travel distance becomes shorter. On the other hand, when the variance is relatively small, and the storage durations of items are more likely to be overlapped especially when the average storage duration is longer. As a result, the number of sections needed increases and the travel distance becomes longer. Also, it can be seen in **Figs. 4-7** b) and **4-8** b) that the performances of the AD and SD policies are close because the variance of storage durations is small, so the solution by the AD and SD policies are more likely to be the same.

From **Figs. 4-7** and **4-8**, it can be concluded that the SD policy is superior to other policies. In **Figs. 4-7** a) c) and **4-8** a) c), the DOS 1 policy showed better performance than the AD policy, and vice versa in **Figs. 4-7** b) and **4-8** b). The SD and DOS1 policies use the information of the shipping dates of items directly or indirectly. The DOS2 policy cannot effectively use the information of the arrival and shipping dates.

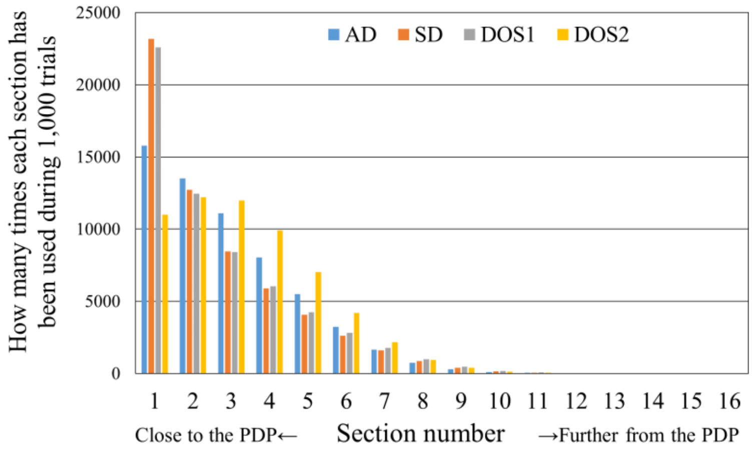
These results suggest the following: 1) Because shipping date information contributes to efficient storage location management in the distribution center, obtaining/estimating the shipping dates should be encouraged as much as possible. 2) When the shipping dates can be obtained beforehand, it is important to utilize it as information related to shipping dates not storage duration.

With regard to 2) in particular, in actual distribution centers, there are some cases where the storage location of items is managed based on the storage durations of items. If the information of storage duration is available, both the SD and DOS1 policies can be adopted. This is because the shipping dates of the items can be calculated by adding the storage durations to their arrival dates. It is suggested that distribution centers adopting the DOS1 policy can reduce the travel distance by switching to the SD policy.

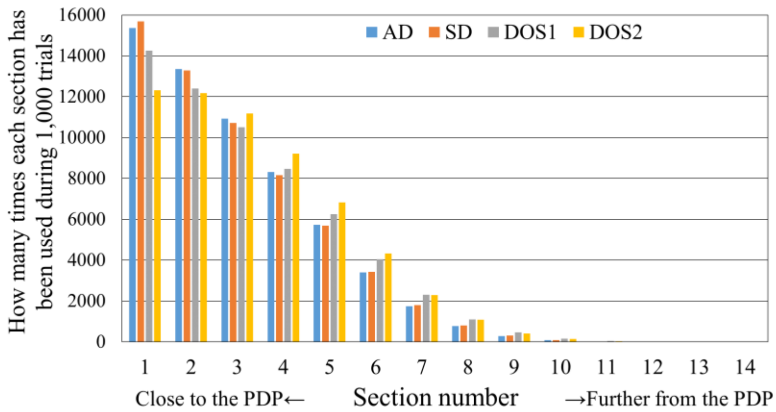
4.3.3 Additional consideration of the SD policy

According to property 2, the SD policy maximizes the number of stored items in the nearest storage section from the PDP (section 1), and according to property 3, the SD policy maximizes the required number of storage sections to store all of items among all of the top alignment solutions. Also, according to performance evaluation in Subsection 4.3.2, the SD policy showed the best performance among the AD, SD, DOS1 and DOS2 policies. In order to present how these may happen simultaneously, another computer simulation has been performed. The arrival and shipping dates of 60 items were generated according to the probability distribution functions in Eqs. (12), (13) and (14). In the investigation, the mean of Δ_2 and Δ_3 is always set as 30, and the mean of the arrival interval Δ_1 is always set as 10. **Fig. 4-9** shows the sum of items which have been kept in each section when storage locations were determined by the AD, SD, DOS1 and DOS2 policies during 1,000 trials. In **Fig. 4-9** a), the storage duration of items follows an exponential distribution with an average of 30 days, in b), the storage duration of items follows a lognormal distribution with an average of 30 days and a variance of 25, and in c), the storage duration of items follows a lognormal distribution with an average of 30 days and a variance of 900.

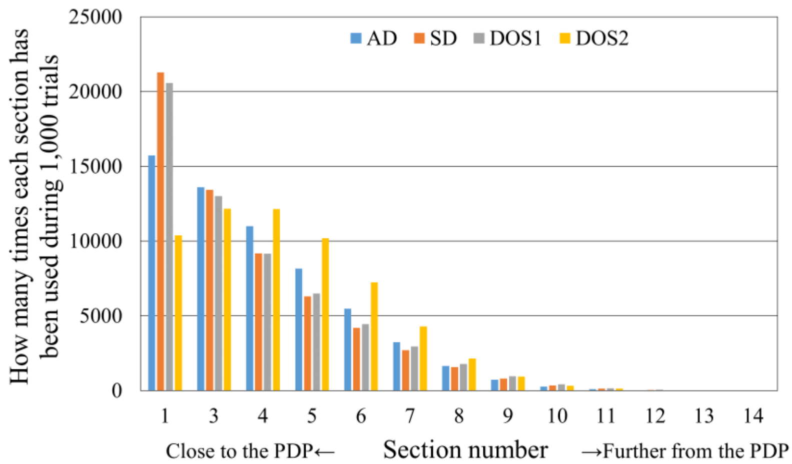
The results show that as it has been proved, storage locations determined by



a) The storage duration of items follows an exponential distribution with an average of 30 days



b) The storage duration of items follows a lognormal distribution with an average of 30 days and a variance of 25



c) The storage duration of items follows a lognormal distribution with an average of 30 days and a variance of 900

Fig. 4-9 Sum of items kept in each section during 1,000 trials the SD policy utilizes section 1 the most, and the number of sections used is equal to or more than that in other policies. In addition, the number of items kept in each section other than section 1 is less than that by other policies. These are the explanations of why the travel distance by the SD policy is often close to the optimal solution compared to that by other policies.

In addition, as the result in Subsection 4.3.2, in **Fig. 4-9** b), how many times each section has been used during 1,000 trials by the AD policy is close to that by the SD policy because the solutions are likely to be similar when the variance of storage duration is relatively small.

4.3 Utilization of the AD and SD policies to obtain the optimal solution

In this section, the use of the AD and SD policies to obtain the optimal solution for the SLAP is discussed. It will be shown that properties 1, 2 and 3 explained in Section 4.1 can be used to increase the efficiency of searching the optimal solution. In addition, the preliminaries for handling large-scale problems efficiently are explained.

4.3.1 Preliminaries

Before going into the discussion of searching for a optimal solution, methods to handle large-scale problems efficiently are prepared.

1) Dividing into independent problems

If the storage location assignment of items belonging to one planning horizon does not affect the storage location assignment in the subsequent planning horizon, each storage location assignment problem can be treated independently. Utilizing this property, a large-scale problem can be divided into independent problems through the following procedure (see **Fig. 4-10**).

Step 1. Find the time t that does not split the storage periods of any items within the planning horizon [0, T].

Step 2. Divide the original problem into the problems with the planning horizon of [0, t] and that of [t, T].

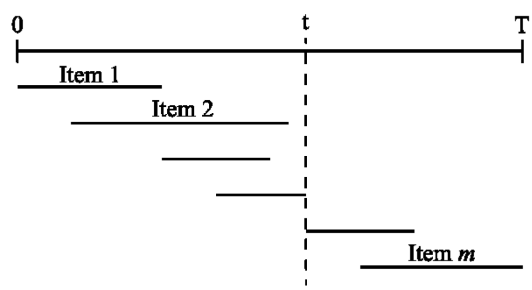


Fig. 4-10 Division into independent problems

2) In case of satisfying the first-in, first-out condition

If the arrival and shipping dates of items satisfy the condition of FIFO (First-In, First-Out), the AD and SD storage locations of the problem must be the same. Furthermore, the following property holds.

Property 4:

When the arrival and shipping dates of items satisfy the FIFO condition, the AD (or SD) storage location minimizes the travel distance of the items, which is optimal.

Proof of property 4

If the arrival and shipping dates of items satisfy the condition of FIFO, the AD and SD storage locations of the problem are the same. Then, from properties 1 and 3, the number of reqired sections of the optimal solution is that of the AD storage location. In the proof of property 1, it has been confirmed that a storage location which does not follow the AD policy can be converted to the AD storgae location without increasing the number of storage sections required by exchanging the storage sections of items $F_l, F_{l+1}, F_{l+2}, \cdots$ and items $G_m, G_{m+1}, G_{m+2}, \cdots$ repeatedly, where $I_{F_l} > I_{G_m}, F < G$. At this time, it must hold that $O_{F_l} \geq O_{G_m}$, if the arrival and shipping dates of items F_l and G_m satisfy the condition of FIFO.

Representing the number of items $F_{l+1}, F_{l+2}, \cdots$ by $I(F_{l+1})$ and items $G_{m+1}, G_{m+2}, \cdots$ by $I(G_{m+1})$ respectively, from $O_{F_l} \geq O_{G_m}$, it holds that $I(F_{l+1}) \leq I(G_{m+1})$ in the top alignment storage location. The travel distance accordingly does not increase; that is, it remains equal or decreases in the process of exchanging the storage sections of items $F_l, F_{l+1}, F_{l+2}, \cdots$ and items $G_m, G_{m+1}, G_{m+2}, \cdots$.

As a result, assuming that there is a storage location which minimizes the travel distance, and that it does not follow the AD policy, it can converted to AD storage location 1) remaining that the number of the storage sections required is that of the AD storage location, and 2) without increasing the travel distance, if the arrival and shipping dates of items satisfy the condition of FIFO.

In actual distribution centers, various improvement activities for the realization of FIFO management are being carried out to ensure product quality. At the same time, property 4 means that improvement for FIFO management leads to increase the efficiency of operations of distribution centers in terms of the travel distance. Meanwhile, it can be said that FIFO management can also minimize the number of sections occupied, i.e., space to store items from property 1.

Regarding property 4, just because the AD and SD storage locations of items are the same does not always hold that all the items satisfy the FIFO condition. **Figs. 4-11** and **4-12** show a counter example. Although both the AD and SD storage locations of the items are shown in **Fig. 4-12**, items 2 and 3 do not satisfy the FIFO condition. However, if items 3 and 5 are considered as one item, whose arrival and shipping dates are at the beginning of day 3 and at the end of day 7 respectively, the numerical example shown in **Fig. 4-11** satisfies the FIFO condition. In this numerical example, the storage locations determined based on the AD and SD policies become the same as a result. It can generally be transformed to the problem which satisfies the FIFO condition by combining items, when the AD and SD storage locations of a problem are the same.

From these discussions and property 4, the optimal solution can immediately be obtained through the following procedure.

Step 1. Determine the two types of storage locations under the AD and SD policies.

Step 2. If the AD and SD storage locations are the same, the AD storage locations are adopted as the optimal solution of the problem.

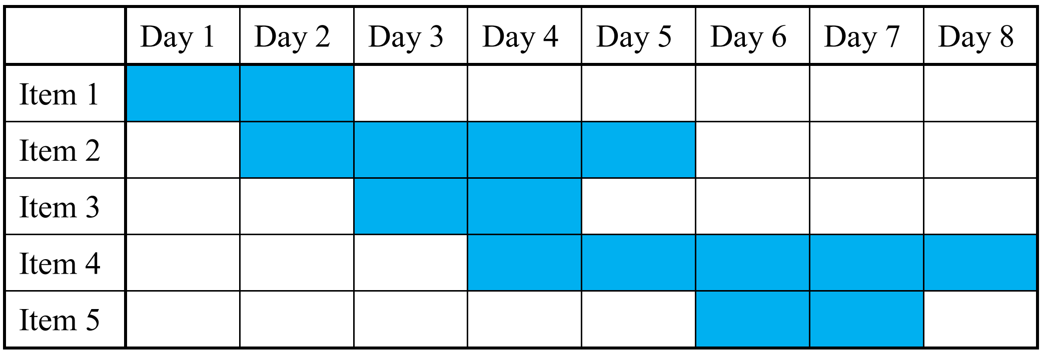


Fig. 4-11 An example which does not satisfy the FIFO

<table>
<tr><th></th><th>Day 1</th><th>Day 2</th><th>Day 3</th><th>Day 4</th><th>Day 5</th><th>Day 6</th><th>Day 7</th><th>Day 8</th></tr>
<tr><td>Sect. 1</td><td colspan="2">Item 1</td><td colspan="2">Item 3</td><td></td><td colspan="2">Item 5</td><td></td></tr>
<tr><td>Sect. 2</td><td></td><td colspan="4">Item 2</td><td></td><td></td><td></td></tr>
<tr><td>Sect. 3</td><td></td><td></td><td></td><td colspan="5">Item 4</td></tr>
</table>

Fig. 4-12 The AD and SD storage location of items in **Fig. 4-11**

4.3.2 Using properties of the AD and SD policies

After the two preliminaries mentioned in Subsection 4.4.1, utilizing properties of the AD and SD policies is discussed to obtain the optimal storage location of items efficiently. Here, three methods are introduced.

A) Setting the upper bound for the number of items stored in the nearest storage section (section 1) from the PDP.

From property 2, the maximum number of items that can be stored in section 1 is known from the SD storage location. This value can be used as an upper bound of the number of items stored in section 1. After denoting this value as c, the following inequality is added to the constraint conditions shown in Subsection 4.2.2.

$$\sum_{i=1}^{m} x_{i1} \leq c \qquad (16)$$

B) Setting the upper bound for the number of used storage sections

From property 3, the maximum number of storage sections to store all of the items among all top alignment solutions can be obtained. This value can be used for the upper bound of the number of required storage sections. Property 3 can reduce the value of n in the problem formulation shown in Subsection 4.2.2. As a result, this upper value limits the number of equations represented by Eqs. (3) and (4). Especially, Eq. (4) indicates that items s and t whose storage periods are overlapped are not located in any section j (j=1, 2, $\cdots$, n).

C) Using the AD or SD storage locations as a provisional solution

From the numerical simulation results shown in Subsection 4.3.2, the SD policy could obtain the solution with the shortest travel distance among the four greedy policies with high accuracy (from **Fig. 4-7**). It has also been confirmed that the AD policy frequently obtains the optimal solution when the SD policy cannot obtain it as shown especially in the case of **Fig. 4-7** b). From these results, either the AD or SD storage location is adopted as a provisional solution with a smaller travel distance.

To investigate the performance of A), B) and C), the computer simulation has been carried out without introducing A), B) and C) and the results were used as a benchmark. The function 'intlinprog' of MATLAB 2019a was used in a computer environment powered by an Intel® Core™i7-8700K CPU 3.70GHz. The searching time of the optimal solution was measured adding the methods A), B) and C) individually to the benchmark. The conditions in the numerical experiments has been set as follows.

- The arrival intervals of items to the distribution center follow an exponential distribution with an average of 5 days.
- The storage duration of items follows an exponential distribution with an average of 20 days.
- 1,000 trials were conducted.

Fig. 4-13 shows the histogram of the searching time among 1,000 trials of the benchmark. The number of items was set to 60. It was found out that there were trials, in which the searching time was extremely long, although the frequency of occurrence was extremely low. In using A), B) and C), similar results were also observed. These data have been recognized as outliers and interquartile range *IQR* has been used to cancel their effects. The trials whose searching time exceeded ($\textit{the third quartile} + 1.5 \times \textit{IQR}$) were eliminated. The trials whose searching time did not exceed ($\textit{the first quartile} - 1.5 \times \textit{IQR}$) were also to be eliminated, but there was none in the simulations.

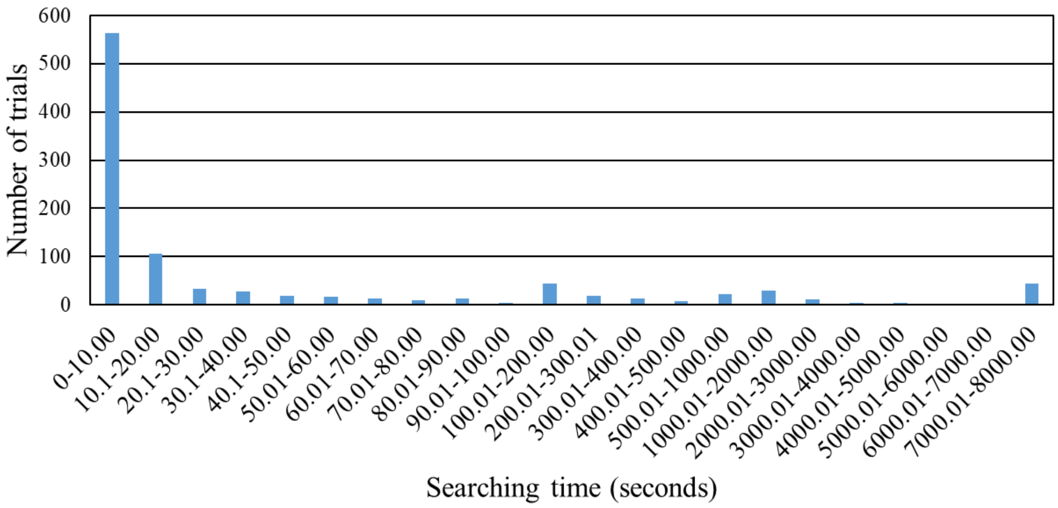


Fig. 4-13 Histogram of searching time among 1,000 trials in the benchmark

Fig. 4-14 shows the average searching time of the three individual methods compared with the bencmark after eliminating outliers. While the number of items increases from 30 to 60, B) found the optimal location most efficiently because this method severely limits the upper bound of the required number of sections represented by Eqs. (3) and (4).

Method B) is considered to have contributed strongly to the reduction of the search space. The other two do not perform as well as the case in B) because they add another constraint condition to the problem formulation shown in Subsection 4.2.2; thus, the searching time is not reduced as much. However, when the number of items was 60, the searching times of A) and C) also have a certain effect on shortening the solution search time.

As shown in **Fig. 4-14**, B) was able to search the optimal solution for the problem of 60 items in about 4 seconds on average. Its searching time was less than one-third of the benchmark. It is believed that the effect of reducing the solution searching time by introducing B) becomes more prominent as the number of items increases.

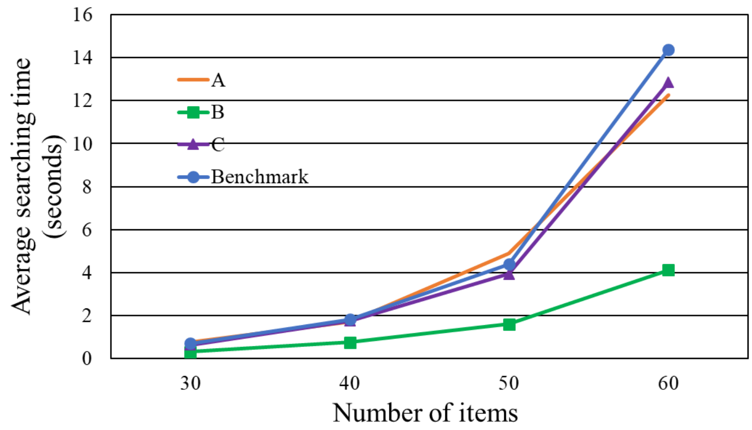


Fig. 4-14 Searching time of three methods compared to the benchmark

Whether there are differences between the efficiency of benchmark and that of A), B), and C) when the number of items is 60 was tested. The result is shown in **Table 4-1**. Assuming the samples have unequal variances, one-sided t-test has been done. One-sided t-test has been chosen because there were outliers only on the one side in all of the methods. From **Table 4-1**, it can be seen only the samples of B) has a significant difference from those of benchmark in 95% confidence interval, because its P value is smaller than 0.05 and absolute value of t-statistic is larger than its t critical one tall value. On the other hand, the samples of A) and C) do not have significant differences from those of benchmark because the P values are larger than 0.05 and the absolute values of t-statistic are smaller than the t critical one tall values. Therefore, it can also be said that among three methods, B) was statistically the most efficient.

Table 4-1 Statistical test of the search performance of three methods

	A	B	C	Benchmark
Mean	458.50	283.85	520.96	472.08
Variances	2.43×10^{6}	1.53×10^{6}	2.85×10^{6}	2.42×10^{6}
Degrees of freedom	1988	1901	1985	-
t-statistic	-0.20	-3.00	0.67	-
P(T<=t) one tall	0.42	1.38×10^{-3}	0.25	-
t critical one tall	1.65	1.65	1.65	-

4.4 Cases when storage schedule is not known completely

So far, deterministic problems, in which the arrival and shipping dates of all the items in the planning horizon were known in advance, have been discussed. However, it may also be noticed that in practice, there are cases where the storage schedule of items is uncertain until just before their arrival or shipping dates. As one of the problems with uncertainty in the storage schedule of items, the problem where the arrival and shipping dates of the items become gradually known over time is addressed. To cope with this situation, new greedy policies are proposed to determine the storage location of items.

It is assumed that the arrival and shipping dates of the items for two days (the day of arrival and the following day) are gradually revealed; that is, it is assumed that the arrival and shipping dates for only the items that arrive on the day of arrival (day t) and the next day (day t+1) are known. In **Fig. 4-15**, assuming today is day 1, the arrival and shipping dates of only items 1, 2 and 3 are known. Since the whole storage schedule in the planning horizon is unknown, the SD and DOS policies may not be applied. The AD policy however can be used to determine the storage location in time series. Therefore, it is intended to apply the AD policy to the problem. Because its performance was not always high, as shown in Subsection 4.3.2, two modified AD policies were applied to the problem with inadequate storage schedule information.

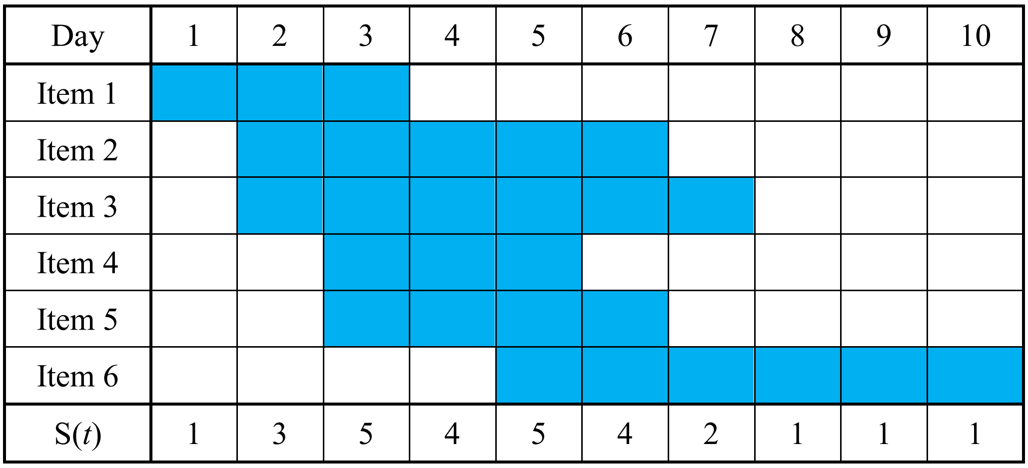


Fig. 4-15 Example 2: Arrival and shipping schedule of items

4.4.1 Two improved AD policies

For improved AD policies, the threshold of the storage duration of items, which is denoted by *H,* is introduced. When the storage duration is *H* or greater, the item is stored farther from the section originally decided under the AD policy.

<table>
<tr><th>Day</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>9</th><th>10</th></tr>
<tr><td>Sect. 1</td><td colspan="3">Item 1</td><td></td><td colspan="6">Item 6</td></tr>
<tr><td>Sect. 2</td><td></td><td colspan="5">Item 2</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Sect. 3</td><td></td><td colspan="6">Item 3</td><td></td><td></td><td></td></tr>
<tr><td>Sect. 4</td><td></td><td></td><td colspan="3">Item 4</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Sect. 5</td><td></td><td></td><td colspan="4">Item 5</td><td></td><td></td><td></td><td></td></tr>
</table>

a) AD storage location

<table>
<tr><th>Day</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>9</th><th>10</th></tr>
<tr><td>Sect. 1</td><td colspan="3">Item 1</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Sect. 2</td><td></td><td></td><td colspan="3">Item 4</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Sect. 3</td><td></td><td colspan="5">Item 2</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Sect. 4</td><td></td><td colspan="6">Item 3</td><td></td><td></td><td></td></tr>
<tr><td>Sect. 5</td><td></td><td></td><td></td><td></td><td colspan="6">Item 6</td></tr>
<tr><td>Sect. 6</td><td></td><td></td><td colspan="4">Item 5</td><td></td><td></td><td></td><td></td></tr>
</table>

b) AD1 storage location (*H*=4)

<table>
<tr><th>Day</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>9</th><th>10</th></tr>
<tr><td>Sect. 1</td><td colspan="3">Item 1</td><td></td><td colspan="6">Item6</td></tr>
<tr><td>Sect. 2</td><td></td><td></td><td colspan="3">Item 4</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Sect. 3</td><td></td><td></td><td colspan="4">Item 5</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Sect. 4</td><td></td><td colspan="5">Item 2</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Sect. 5</td><td></td><td colspan="6">Item 3</td><td></td><td></td><td></td></tr>
</table>

c) AD2 storage location (*H*=4)

Fig. 4-16 Three kinds of AD storage locations of Example 2

1) AD1 policy

The first improved policy, which is referred to as the AD1 policy, basically follows the AD policy. However, when the storage duration $O_i - I_i$ of item *i* is *H* or greater, item *i* is stored in the second nearest free section from the PDP, instead of the nearest section.

When *H* is set to 4 days, the AD1 storage location of Example 2 shown in **Fig. 4-15** is determined as shown in **Fig. 4-16** b). Item 2 whose storage duration exceeds *H* (=4) is stored in section 3 instead of section 2. The required number of storage sections in this storage location is six. However, the required number of storage sections in the AD storage location is five as shown in **Fig. 4-16** a). In general, the additional number of storage sections occupied under the AD1 policy is a maximum of one more when compared to that under the AD policy, since items are stored in the original section determined under the AD policy or in the section one distance away.

2) AD2 policy

The second improved policy, which is referred to as the AD2 policy, determines the storage sections of items within the used sections for two days (days *t* and *t*+1). When the number of storage sections used to store items at time *t* is denoted by $S(t)$ (see Subsection 4.3.1 and the bottom row in **Fig. 4-15**), the required number of storage sections to store the items for two days or less is given by Eq. (17).

$$R(t) = max\{S(t), S(t+1)\}, t = 1,2,\ldots,T-1 \quad (17)$$

The AD2 policy determines the storage location of the items arriving on day *t* in the R(*t*) storage sections. The rules for determining the storage location are as follows.

I) Case of $O_i - I_i < H$

The items arriving on day *t* whose storage durations are less than *H* are stored in the nearest free section following the AD policy. Here, when multiple items arrive on day *t*, they are stored in the closer sections in ascending order of their storage durations.

II) Case of $O_i - I_i \geq H$

The items arriving on day *t*, whose storage durations are greater than *H* are stored in the farthest free sections from the PDP in the order of longest storage duration within the $R(t)$ sections.

When *H* is set to 4 days, **Fig. 4-16** c) is obtained under the AD2 storage location. On day 1, since the storage duration of item 1 is less than *H*, item 1 is stored in section 1. On day 2, both storage durations of items 2 and 3 are greater than *H*. Since it holds that $\mathrm{R}(2) = 5$ from Eq. (17), items 2 and 3 are stored in sections 4 and 5 respectively instead of sections 2 and 3. In this storage location, the required number of storage sections is five.

In general, the maximum number of storage sections used for two days (days *t* and *t*+1) under the AD2 policy is always equal to that under the AD policy. Therefore, the required number of storage sections for all items under the AD2 policy corresponds to that under the AD policy. From property 1, the AD2 policy intends to reduce the travel distance of items remaining such that the number of storage sections is the minimum in the deterministic problem.

4.4.2 Performance evaluations of the improved policies

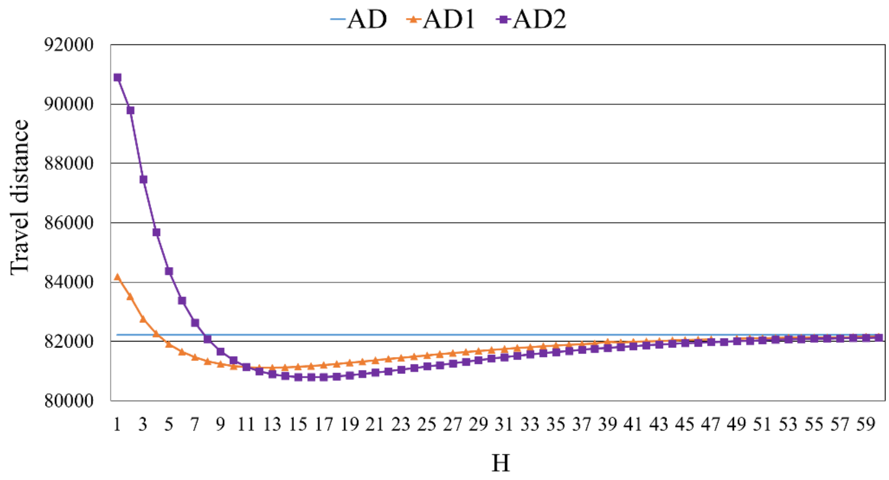
Numerical experiments of the AD1 and AD2 policies have been conducted to investigate how they perform compared to the AD policy. It is assumed that items arrive at the distribution center every day for 1500 days. Performance verification was carried out while changing the number of items that arrive per day and their storage durations.

In **Figs. 4-17**, **4-18**, **4-19** and **4-20**, the travel distances in the AD, AD1 and AD2 storage locations are plotted by changing the threshold *H*. The daily arrival number of items is set as 10, 25, 50 and 100. The storage durations of items follow exponential distributions with averages of a) 10 days, b) 20 days, and c) 30 days. The travel distance of the AD policy is stable because *H* does not have an effect. In addition, when *H* becomes large enough, the travel distances of AD AD1 and AD2 storage locations are sufficiently close because the storage duration of all the items meets the condition of $O_i - I_i < H$.

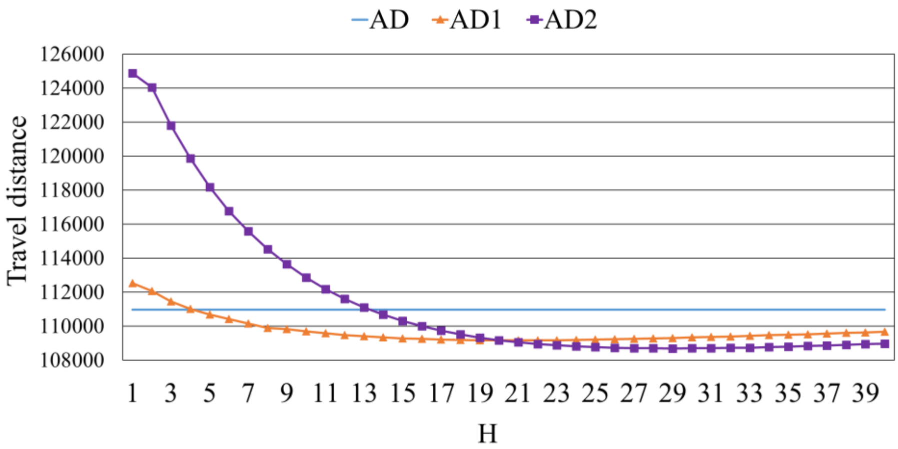
From **Figs. 4-17**, **4-18**, **4-19** and **4-20**, it is observed that when the *H* is small enough, the travel distance of the AD1 policy is shorter than that of the AD2 policy. Then as *H* increases, the AD2 gradually becomes to perform better than the AD and AD1 policies. In addition, the curves of AD2 are partially convex downward with respect to *H*. The optimal value H^*, which minimizes the travel distance, increases as the average storage duration of items increases. In some cases, there are multiple optimal values which all minimize the travel distance.

Similarly, the travel distance of the improved AD policies was investigated by changing the number of items arriving per day. The average storage duration is fixed at 20 days. In **Figs**. **4-17** b), **4-18** b), **4-19** b) and **4-20** b), the daily arrival number of items is set to 10, 25, 50 and 100. All the shapes of the curves representing AD, AD1 and AD2 have similar characteristics. It may be observed that H^* increased as the daily arrival number of items increased.

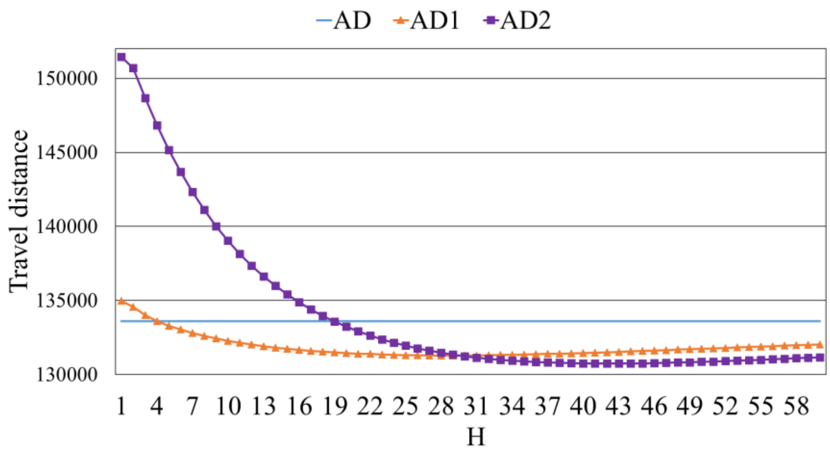
As a result, if *H* is set properly, the AD2 policy may perform superior to the AD and AD1 policies regarding the travel distance. At this point, a method or a function of specifying the value of H^* has not been clarified. However, in the next section, ways for determining the value of H^* are discussed.



a) The average storage duration is 10 days

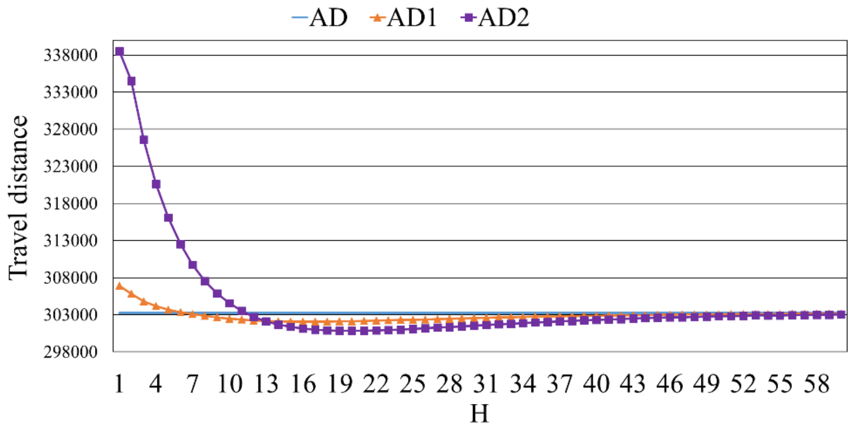


b) The average storage duration is 20 days

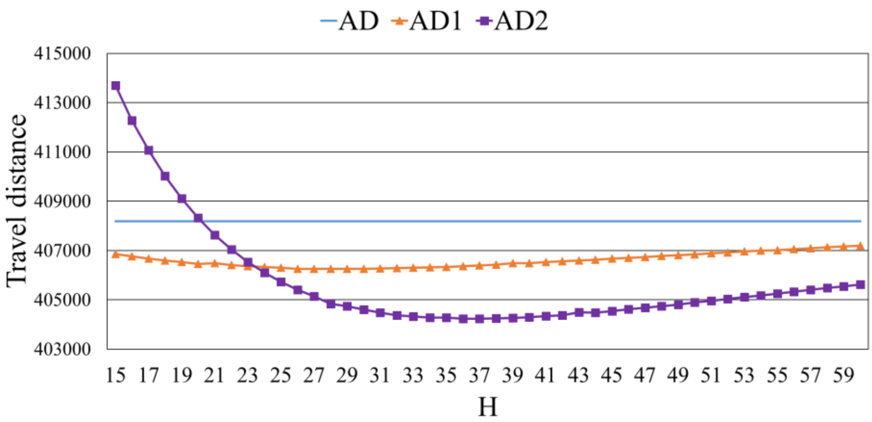


c) The average storage duration is 30 days

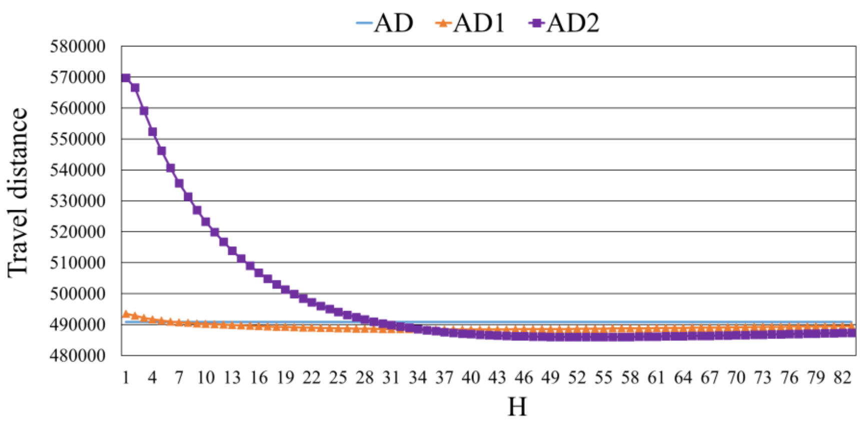
Fig. 4-17 Travel distances obtained with AD, AD1 and AD2 policies (10 items arrive each day.)



a) The average storage duration is 10 days



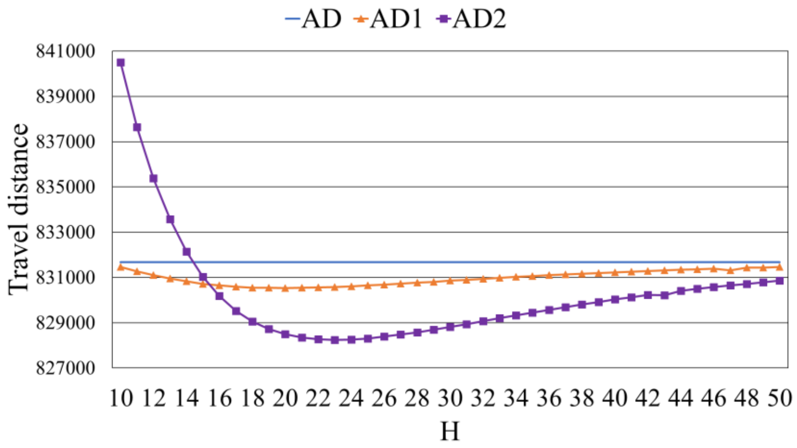
b) The average storage duration is 20 days



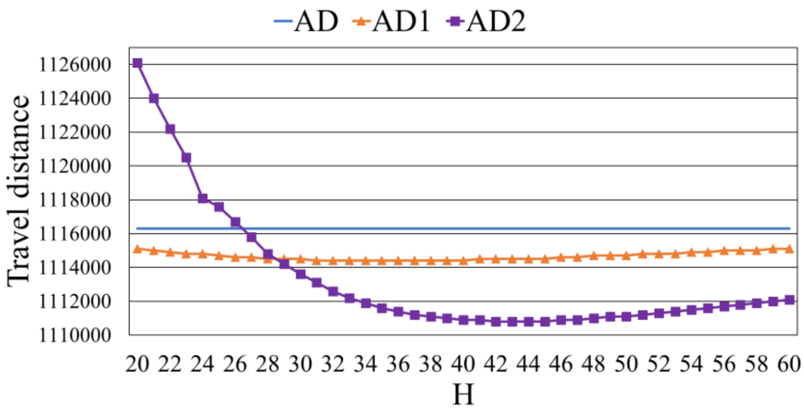
c) The average storage duration is 30 days

Fig. 4-18 Travel distances obtained with AD, AD1 and AD2 policies

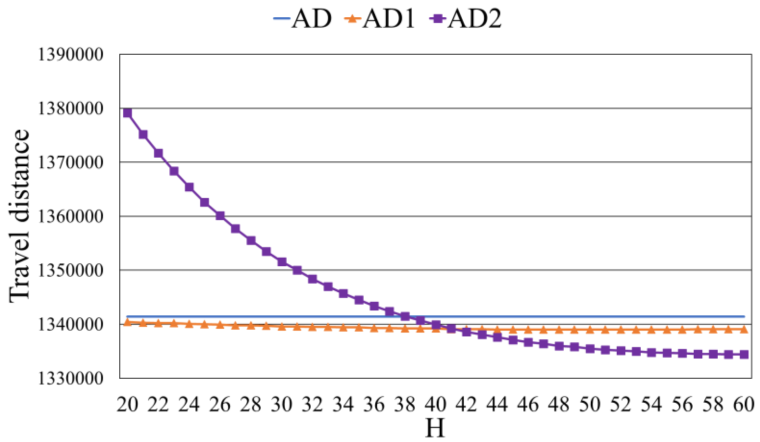
(25 items arrive each day.)



a) The average storage duration is 10 days

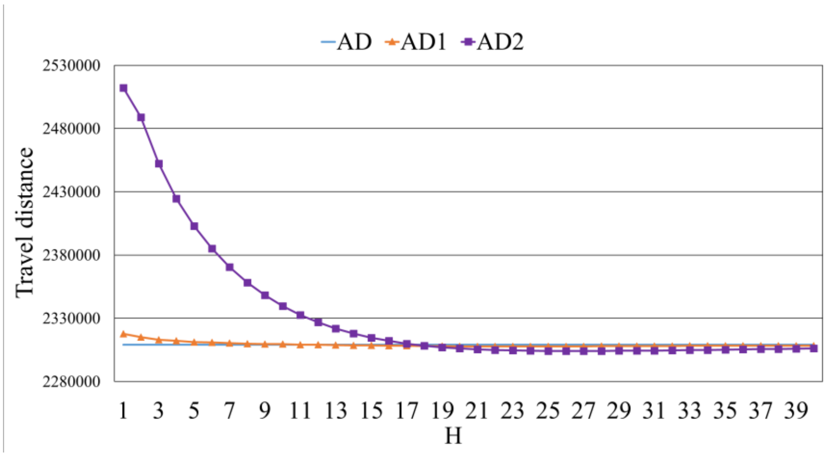


b) The average storage duration is 20 days

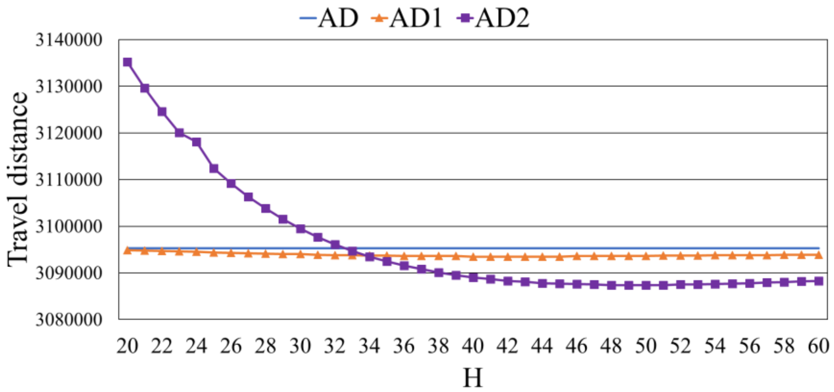


c) The average storage duration is 30 days

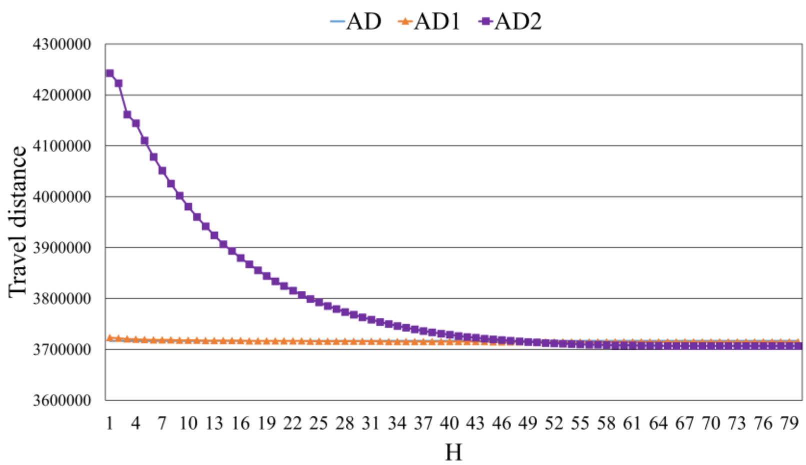
Fig. 4-19 Travel distances obtained with AD, AD1 and AD2 policies (50 items arrive each day.)



a) The average storage duration is 10 days



b) The average storage duration is 20 days



c) The average storage duration is 30 days

Fig. 4-20 Travel distances obtained with AD, AD1 and AD2 policies (100 items arrive each day.)

4.4.3 Optimal threshold of the storage duration

The threshold H of the storage duration of items works to compensate for the shortcomings of the AD policy, in which items with long storage durations occupy sections close to the PDP.

To investigate the property of this optimal threshold, α^* is introduced. It is the proportion of items whose storage duration is equal to or smaller than H^* among the total amount of items. **Table 4-2** and **Fig. 4-21** show the value of H^* and α^* of changing with the storage duration of items and the number of items arriving per day.

Focusing on the results with the same number of items that arrive, α^* hardly changes (i.e., decreases only slightly), even if the storage duration of items becomes long. Furthermore, α^* clearly increases as the number of items arriving per day increases. Therefore, the value of α^* is more strongly influenced by the number of items that arrive compared to the storage duration of them.

In general, as the number of items arriving per day increases, the number of storage sections used to store them increases on average. The penalty for using storage sections that are further away increases, since the distance of the j^{th} section from the PDP is determined by Eq. (15). Accordingly, as the number of items that arrive increases, the value of α^*, i.e., H^* becomes large to avoid the penalty, that curbs the use of distant

Table 4-2 Relationship between H^* and α^*

Items arrive per day	Storage duration of items		
	10 days	20 days	30 days
10	H^*=16 α^*=76.65%	H^*=29 α^*=74.87%	H^*=42, 43 α^*=74.62%
25	H^*=20 α^*=83.87%	H^*=36, 37 α^*=82.42%	H^*=53, 54 α^*=82.20%
50	H^*=23 α^*=87.65%	H^*=42-45 α^*=87.40%	H^*=62, 63 α^*=86.68%
100	H^*=27 α^*=91.41%	H^*=48-51 α^*=90.53%	H^*=69-75 α^*=90.20%

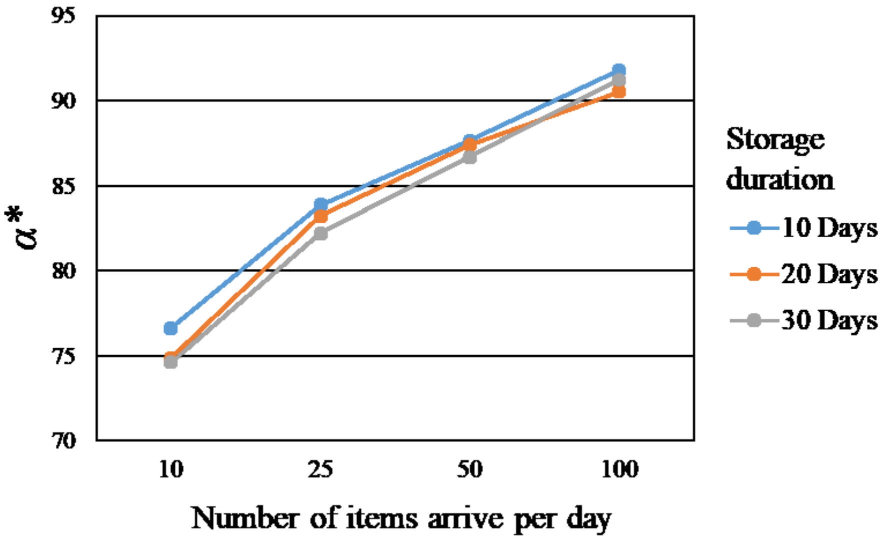


Fig. 4-21 Effect of the arriving number of items per day and storage duration on the value of α^*

storage sections. However, prolonging the storage duration does not necessarily mean an increase in the number of arriving items. Accordingly, H^* becomes large nominally to because the storage durations of items become longer.

In addition, how α^* was affected when the function that determines the distance of the j^{th} section was changed from Eq. (15) has been investigated. **Fig. 4-22** represents the comparison of α^* when storage duration is 30 days long, and the distance is set as $d_j = \sqrt{j}, j, j^2$. When $d_j = j^2$, the penalty for storing items in a distant section is high compared to $d_j = \sqrt{j}$ or $d_j = j$. Accordingly, in **Fig. 4-22**, as the number of items that arrive per day increases, the value of α^* is always highest in three cases of travel distances and increases gradually. When $d_j = \sqrt{j}$, the value of α^* is always lowest since the penalty is small. Regardless of which function is used to determine d_j, the difference in the value of α^* becomes smaller along with the increase of the number of items.

Although it is difficult to determine the exact value of H^*, the discussion here suggests that the introduction of α^* which can assist in estimating H^*. In the estimation, the number of items arriving per day strongly affects the estimation of α^*, but storage durations of items do not as much.

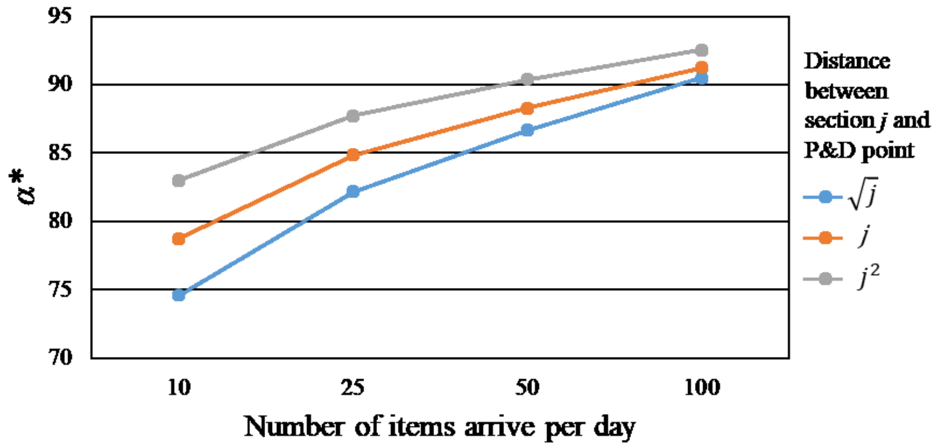


Fig. 4-22 Effect of the travel distance of each storage section from the PDP on the value of α^*

4.5 Conclusions of this chapter

In this chapter, the efficiency in distribution centers in terms of shortening the travel distance of storage items was investigated. For the problem in which all arrival and shipping dates of all items in the planning horizon are given, it was clarified that the SD policy was more effective than the AD and DOS policies. The properties of greedy policies were also clarified, and the optimal storage allocation of items were obtained. In addition, improved AD policies were proposed for the problem of incomplete arrival and shipping date information.

This chapter has contributed to utilize greedy policies to determine storage locations focusing on arrival and shipping dates. In the future, this study will be extended by considering the storage locations of items to shorten the travel distance for locating and collecting items. For instance, data on the extra travel distance resulting from congestion in aisles will be analyzed in different distribution centers, which has not been considered in this study. In the era of digital transformation, this would help to improve the efficiency of movement of automated guided vehicles used in distribution centers worldwide, which may contribute to reducing energy consumption.

Chapter 5. Configuration design of floor layout in inventory-type distribution centers

In inventory-type distribution centers, items are located beforehand and are picked by the order of customers. In this chapter, floor layout was changed, and class based storage was introduced to shorten the travel distance of items in distribution centers to raise the operational efficiency. Their performances have been evaluated with mathematical models.

Configuration designs of distribution centers have been discussed in a number of papers [37]. For example, directions of racks and aisles in Fishbone and Chevron layouts have been changed from Traditional layout [60] [61]. In previous works, the performances of proposed measures were compared to the original Traditional layout, and since the evaluation was done by numerical simulations, in which settings of the sizes or racks and aisles are needed, the capacity of each layout has not been the same. Therefore, comparing the measures was difficult. As a countermeasure, in this research, mathematical models with the mutual assumptions are set in order to make it possible to compare the performances. The aim of this chapter is not only to obtain results, but to make the characteristic of each measure clearer to help to give suggestions in determining which measure to implement to improve the efficiency in inventory-type distribution centers.

Firstly, a mathematical model of Traditional layout, which has racks and aisles placed vertically, has been explained in order to evaluate the performances of different measures later in the chapter.

Then, how inserting diagonal cross aisles or replacing the PDP contributes to shorten travel distance of items is investigated. Also, class based storage is implemented to Traditional layout because in practice, the organization would be simpler than the case in dedicated storage, and shared storage location system utilizes space more efficiently than random storage. The performances of different ways to divide floor into equal areas are evaluated in terms of the travel distance of items.

Lastly, measures of changing floor layout of distribution centers and storage location assignment according to the demand are integrated, and the performances are evaluated to see which measure should be prioritized to be introduced and how much synergy may be expected.

5.1 Model of Traditional layout

In Chapter 5, order picking in inventory-type distribution centers is considered, and single command operation (SCO), in which only one item per travel is carried, is assumed. Generally, SCO is conducted when picking items that are large and heavy, but in some cases, when several items are ordered, pickers pick under SCO, and items are sorted at the PDP. In addition, in some distribution centers, racks are carried to pickers by robot [88], and this movement of robots may also be considered as SCO.

Under SCO, when considering the travel distance of items, adding another horizontal aisle in addition to the one at the bottom on which there is the PDP, as in **Fig. 2-1** does not reduce the travel distance. It rather pushes the racks further away, and the capacity of storage diminishes. Therefore, Traditional layout as in **Fig. 5-1**, in which vertical lines and a horizontal line at the bottom represent aisles, is assumed in this chapter.

Followings are prerequisites under which the travel distance in Traditional layout is considered.

1) The floor of distribution center is on x-y plane, and the area is 1 where the width is *a* and the length is *1/a.*
2) As an initial setting, the PDP is put on the origin of x-y plane, (0, 0).
3) The size of racks is small enough to be placed anywhere in the distribution center.
4) The width of aisles is small enough, and the horizontal aisle is placed at the bottom of the distribution center on which there is the PDP, and vertical aisles may be placed arbitrarily in the distribution center.
5) Pickers walk on the aisles given by 4).

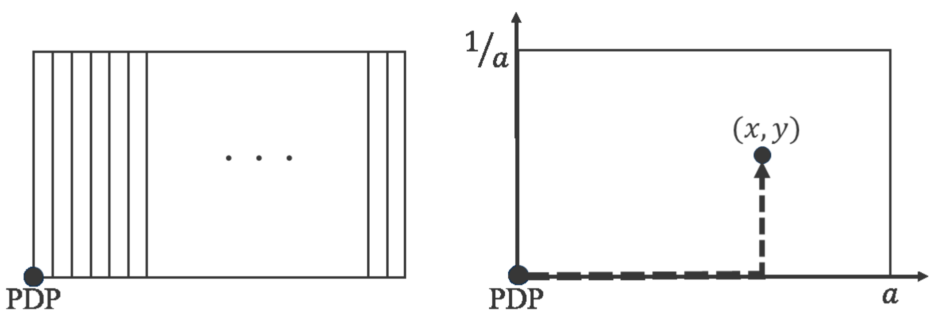


Fig. 5-1 Traditional layout discussed in this chapter

The prerequisites of 3) and 4) above are different from previous researches, in which the size of racks and the width of aisles are given, and the performance is evaluated under discrete simulation. However, results under discrete simulation would depend on the given size of racks or width of aisles. Also, when the layout of the floor is changed, the capacity of storage changes as well, which would be a problem when evaluating the performance of distribution centers. For these reasons, in Chapter 5, 3) and 4) are assumed, and mathematical analysis is done in order to evaluate the performance of distribution centers.

Under these prerequisites, as in **Fig. 5-1**, where the item is stored and is to be picked is on (x,y), the travel distance from the PDP to (x,y) is $D(x,y)$, and the probability of the storage location of each item follows a probability density function, $f(x,y)$. One-way travel distance of an item can be represented as Eq. (18).

$$TD = \int_0^a \int_0^{\frac{1}{a}} D(x,y) \cdot f(x,y) dydx \tag{18}$$

In Traditional layout as in **Fig. 5-1**, the travel distance is given by $D(x,y) = x + y$. When items are completely placed randomly, in other words, are determined following uniform function, then $f(x,y) = 1$, and the average travel distance TD_1 is the following Eq. (19).

$$TD_1 = \int_0^a \int_0^{\frac{1}{a}} (x+y) dydx = \frac{a^2+1}{2a} \tag{19}$$

This induces Eq. (20) below and second derivative function of TD_1 is positive in the range of $a > 0$. TD_1 is a convex function as for $a(>0)$.

$$\frac{d^2TD_1}{da^2} = \frac{1}{a^3} \tag{20}$$

As in Eq. (21), when $a = 1$, which means the distribution center is square shaped, $TD_1^* = 1$, and the travel distance of items is minimized.

$$\frac{dTD_1}{da} = \frac{a^2 - 1}{2a^2} \tag{21}$$

In actual distribution centers, it is common items are categorized and the floor is zoned according to that. It may be said that the above is an intuitive consequence, but when zoning the floor of distribution centers, it suggests that square shaped zones help to reduce the travel distance of items.

In Traditional layout, when the floor of the distribution center is a square, the travel distance of items is minimized. Hereafter, it is assumed $a = 1$, which means the floor is a square which has edges of 1.

5.2 Introduction of methods of order picking

The aim of this chapter is to increase the efficiency of order picking by changing floor layout and the storage location of items. Before this final goal, as changing floor layout, 1) inserting a diagonal cross aisle and 2) changing the placement of the PDP, and as changing the storage location of items, implementation of class based storage are investigated.

These may have already been carried out in practice by experiences, but the results have rarely been investigated in academic papers. These effects are investigated in this chapter using mathematical model discussed in Section 5.2.

5.2.1 Changing floor layout

(1) Utilizing diagonal cross aisles

In Traditional layout, since vertical and horizontal aisles cross at right angles, trajectories of items follow that as a consequence. By inserting diagonal cross aisles, pickers may be able to reach more directly from the PDP to racks. As in **Fig. 5-2**, one diagonal cross aisle from the PDP is inserted into Traditional layout. By this, the average travel distance of items is shortened.

When the diagonal cross aisle is represented as $y = tx$, when reaching a rack in the area above that, (R_1 in **Fig. 5-2**), the travel distance is shortest to walk on the diagonal aisle, then walk up on one of vertical aisles. When reaching a rack that is below the diagonal aisle, the region may be divided into R_2 and R_3. In order to pick an item in R_2, it is closest to walk on the diagonal aisle and then walk down on one of the vertical aisles.

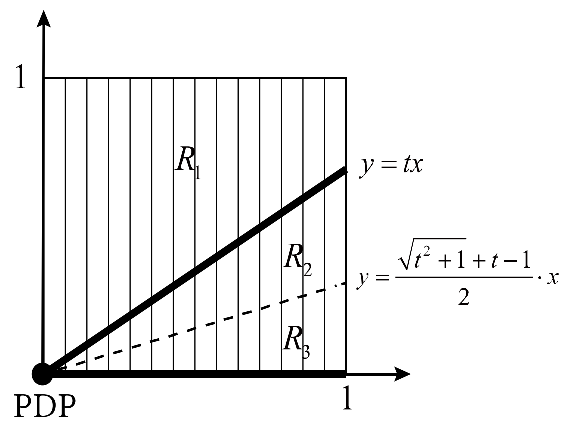


Fig. 5-2 Inserting a diagonal cross aisle

When reaching a rack in R_3, the travel distance is shorter by walking on the bottom horizontal aisle, and then walking up on the vertical aisles. The equation that divides the area below the diagonal aisle into R_2 and R_3 is given as Eq. (22).

From $x+y=\sqrt{x^2+(tx)^2}+tx-y$,

$$y=\frac{\sqrt{t^2+1}+t-1}{2}\cdot x \tag{22}$$

From this result, the average travel distance of items in Traditional layout with a diagonal cross aisle TD_2 is given as Eq. (23) below.

$$\begin{aligned} TD_2 &= \int\int_{R_1}\left(\sqrt{x^2+t^2x^2}+y-tx\right)dydx \\ &\quad + \int\int_{R_2}\left(\sqrt{x^2+t^2x^2}+tx-y\right)dydx + \int\int_{R_3}(x+y)dydx \\ &= \frac{t^2-2t-(t-4)\sqrt{t^2+1}+2}{6} \end{aligned} \tag{23}$$

Fig. 5-3 is the graph which shows the relationship between the average travel distance of items TD_2 and the slope of the diagonal cross aisle t. When $t=0.674$, which is the slope of 34°, the average distance $TD_2^*=0.853$ is the minimum value. This

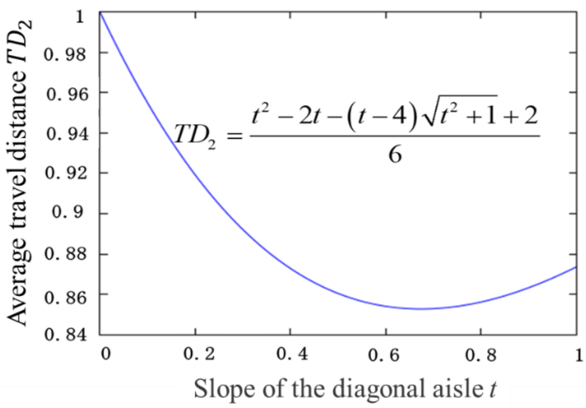


Fig. 5-3 Relationship between the average travel distance of items TD_2 and the slope of the diagonal cross aisle t

means that by adding this diagonal cross aisle, the average travel distance is shortened by 15% at the maximum compared to that in the original Traditional layout which has the average travel distance of $TD_1^* = 1$. The reason why the minimum value is not when the slope of the diagonal cross aisle is 45° is because in order to pick an item in R_2, the picker walks up on the diagonal aisle but then walks “back down” on one of the vertical aisles. In order to shorten the travel distance of walking “back down”, the area of R_2 needs to be smaller, and the travel distance on the vertical aisles needs to be shorter.

(2) Changing the placement of the PDP

The PDP is where picked items are accumulated, inspected and packed. It may be possible to change the placement of the PDP by changing where items are inspected and packed. Also, the placement of the PDP may be restricted by the placements of vertical conveyors or platforms of tracks for transportation. However, in distribution centers, in many cases, there are several placements of vertical conveyors or platforms of tracks for transportation, so it is often possible to change the placement of the PDP.

In this section, taking this situation in consideration, the effect of changing the placement of the PDP from the initial setting, in which it is placed on the origin of x-y plane, (0, 0), is investigated. In Traditional layout, the PDP is placed on the bottom aisle in the distribution center because that is where items that are picked from racks are accumulated, and items are shipped from the distribution center. Taking these into consideration, the PDP is placed arbitrarily on the bottom aisle of distribution centers. As in **Fig. 5-4**, when the travel distance between the origin and the PDP is set as $b(0 \le b \le 1)$, the average travel distance of items TD_3 is given as Eq. (24).

$$TD_3 = \int_0^b \int_0^1 (b - x + y)dydx + \int_b^1 \int_0^1 (x - b + y)dydx$$
$$= (b - \frac{1}{2})^2 + 0.75 \qquad (24)$$

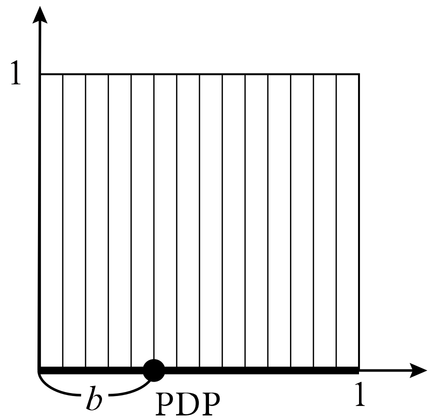


Fig. 5-4 Changing the placement of the PDP

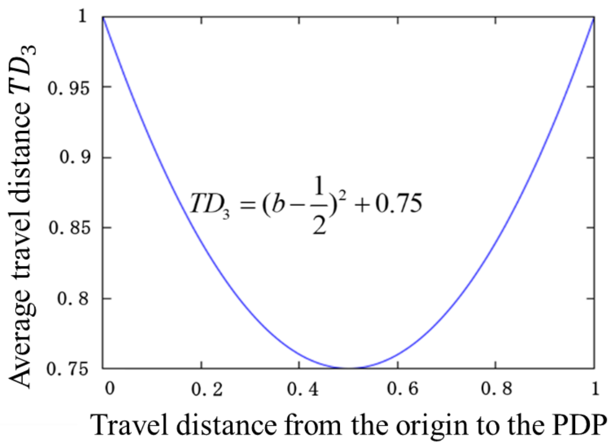


Fig. 5-5 Relationship between the average travel distance TD_3 of items and the travel distance from the origin to the PDP

Fig. 5-5 is the graph that shows the relationship between the average travel distance of items TD_3 and *b,* which is the travel distance from the origin to the PDP. When $b = 0.5$, TD_3 is the minimum value, which is $TD_3^* = 0.75$. Due to the curve of the graph of TD_3, when *b* increases from 0 to 0.5, the average travel distance of items becomes shorter. From this, in distribution centers, the further the PDP is placed from the middle of the bottom aisle, which means the nearer it is placed to the origin or (1, 0), more effects of shortening the travel distance there would be by replacing it closer to the middle. In terms of shortening the travel distance of items, changing the placement of the PDP has an effect of reduction by 25% at the maximum, whereas inserting a diagonal cross aisle has that by 15%. When it is possible to place the PDP near the middle of the bottom aisle of the distribution center, it is more effective compared to inserting a cross diagonal aisle.

5.2.2 Implementing class based storage

It has been claimed that class based storage is to divide items into classes according to characteristics of items by [79]. In practice, there are cases that items are classified according to their demand in order to manage items efficiently in distribution centers. In this chapter, the storage location of items under class based storage is investigated.

Firstly, the case Traditional layout is vertically divided into three equal sized zones to manage class based storage as in **Fig. 5-6** is considered.

In zone A in **Fig. 5-6**, items with the highest demand are stored, those with the lowest demand are stored in zone C, and those which are neither stored in zone A nor zone C are stored in zone B. As a result, the storage location of items is changed so that pickers visit zone A with the probability of p, zone B with $p(1-p)$ and zone C with $(1-p)^2$. This means that zone A has items with the demand of p, and zone B has that of $p(1-p)$, and zone C has that of $(1-p)^2$. As a consequence, the sum of probabilities of visiting each zone is 1, and with the range of $0.5 < p < 1$. Eq. (25) below is established.

$$p > p(1-p) > (1-p)^2 \tag{25}$$

Assuming that in each zone, items are stored randomly. In other words, items may be stored in any place within the determined zone following a uniform distribution, since each zone has an area of 1/3 because the area of distribution center is 1, the probability density function of zones A, B and C are $3p$, $3p(1-p)$ and $3(1-p)^2$. Therefore, when items are managed with class based storage location in floor layout as in **Fig. 5-6**, the average travel distance of items TD_4 is given as Eq. (26).

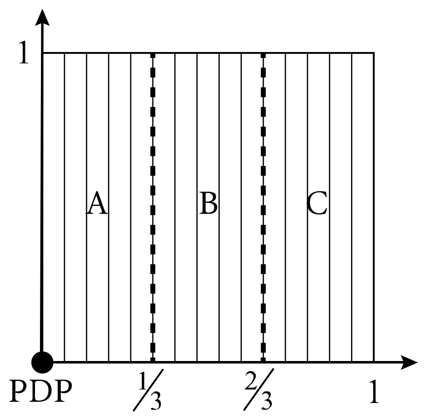


Fig. 5-6 Class based storage location in vertically divided floor layout

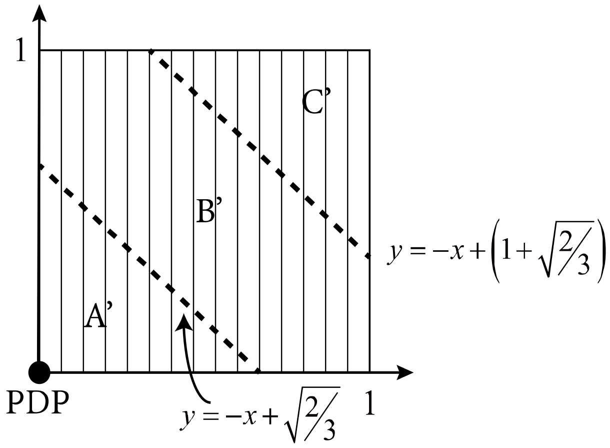


Fig. 5-7 Class based storage location in diagonally divided floor layout

$$TD_4 = \int\int_A \{(x+y)\cdot 3p\}\,dydx + \int\int_B \{(x+y)\cdot 3p(1-p)\}dydx$$
$$+\int\int_C \{(x+y)\cdot 3(1-p)^2\}dydx$$
$$=\frac{(p-{}^{3}/_{2})^2}{3}+\frac{7}{12} \tag{26}$$

In Traditional layout, the travel distances from the PDP to racks on the slope of -1 with the same intercept are constant. Using this property, as in **Fig. 5-7**, dividing x-y squared plane into three equal zones with slopes of -1 is considered. When the probabilities of visiting sections A', B', C' are p, $p(1-p)$ and $(1-p)^2$, the average travel distance of items TD_5 can be obtained by changing Eq. (26) of sections A, B, C to A', B', C', which is Eq. (27) below.

$$TD_5 = \int\int_{A'} \{(x+y)\cdot 3p\}\,dydx$$
$$+\int\int_{B'} \{(x+y)\cdot 3p(1-p)\}dydx$$
$$+\int\int_{C'} \{(x+y)\cdot 3(1-p)^2\}dydx$$
$$=\frac{1}{9}\left(9-2\sqrt{6}\right)\left(\left(p-\frac{3}{2}\right)^2+\frac{1}{76}\left(13+24\sqrt{6}\right)\right) \tag{27}$$

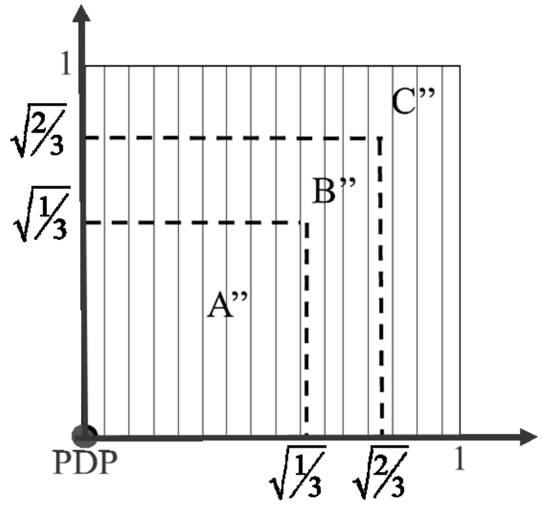


Fig. 5-8 Floor layout of class based storage location divided by square-shaped boundaries

Another intuitive way to introduce class based storage location into Traditional layout is to divide floor into squared shapes as in **Fig. 5-8**. This may be easier to implement than class based stage location dividing with diagonal boundaries, but it may not shorten as much travel distance. When the probabilities of visiting sections A'', B'', C'' are p, $p(1-p)$ and $(1-p)^2$, the average travel distance of items TD_6 can be obtained by changing Eq. (26) of sections A, B, C to A'', B'', C'', which is Eq. (28) below.

$$TD_6 = \int\int_{A''} \{(x+y)\cdot 3p\}\,dydx + \int\int_{B''} \{(x+y)\cdot 3p(1-p)\}dydx + \int\int_{C''} \{(x+y)\cdot 3(1-p)^2\}dydx$$

$$= \frac{p^2}{\sqrt{3}} - 4\sqrt{\frac{2}{3}}p^2 + 3p^2 + 2\sqrt{6}p - 6p - 2\sqrt{\frac{2}{3}} + 3 \qquad (28)$$

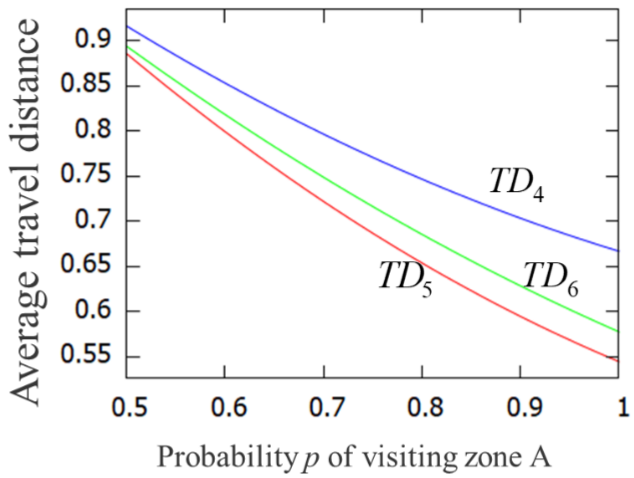


Fig. 5-9 Average travel distances under class based storage location

Fig. 5-9 is the graph that shows the relationship between probability of visiting zone A, $p(0.5 < p < 1)$ on x-axis and travel distance of items, TD_4 (vertical boundaries), TD_5 (diagonal boundaries) and TD_6 (square-shaped boundaries) on y-axis. When $p = 0.5$, the average travel distance of TD_5 is 3.5% shorter than that of TD_4, and as p increases, the difference becomes more significant. The average travel distance is longer in class based storage with square-shaped boundaries compared to that with diagonal boundaries divided by slopes of -1 even though the management of items would be easier considering that racks are placed vertically**,** but shorter compared to that with vertical boundaries, in which the management would be more difficult.

To compare the performance of class based storage system with other measures, class based storage with square-shaped boundaries is chosen because the performance and the difficulty of management of items is between that of class based storage with vertical boundaries and class based storage with diagonal boundaries divided by slopes of -1.

5.2.3 Comparison of measures

Fig. 5-10 shows the comparison of average travel distances of items when a diagonal cross aisle is inserted, the placement of the PDP is changed and class based storage with square-shaped boundaries is implemented. The performance is not as improved when a diagonal aisle is inserted even though the cost to introduce a diagonal cross aisle may be high. Since it still shortens the travel distance by 15% at the maximum compared to that in Traditional layout, it may be considered as a substitution when other measures are not available. When probability p of visiting zone A is relatively small, changing the placement of the PDP in which the travel distance is shortened by 25% at the maximum compared to that in Traditional layout is recommended, but when p is relatively large, implementing class based storage is preferred. Also, when demand of items is uncertain, the performance of changing the placement of the PDP would be stable while implementing class based storage may cause the travel distance to be longer because the performance depends on p.

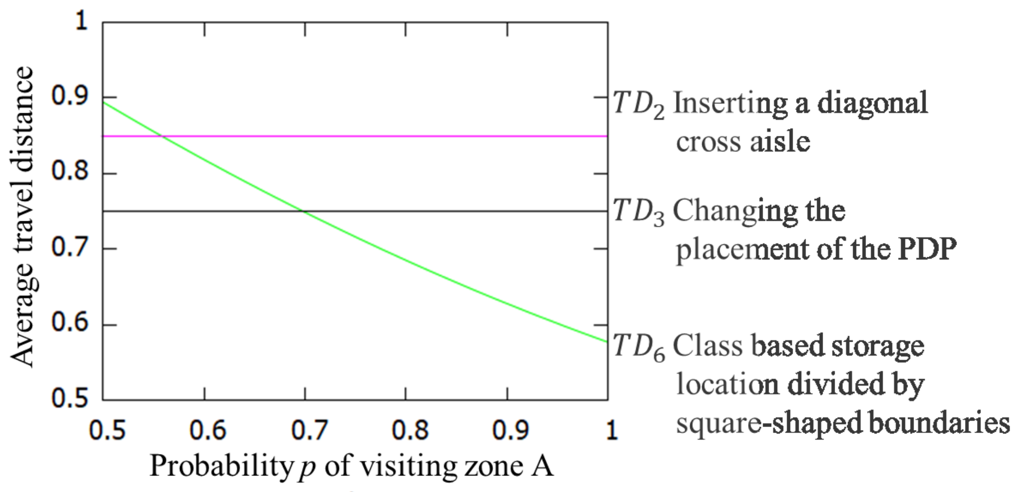


Fig. 5-10 Average travel distances under different measures

5.3 Integration measures to improve efficiency in inventory-type distribution centers

In this section, measures for improving the efficiency in inventory-type distribution centers considered in Section 5.3 are implemented simultaneously in order to shorten the average travel distance of items. In Subsection 5.4.1, how two of three measures are combined is explained, and preparation of mathematical analysis is discussed, then in Subsection 5.4.2, the performances of proposed integration measures for improving the efficiency are investigated. In Subsection 5.4.3, all of three measures that have been explained in Section 5.3 are integrated, and the performance is evaluated.

5.3.1 The explanation of integrating two measures and mathematical analysis preparation

In this section, as a measure to shorten the travel distance of items, two of three measures explained in Section 5.3 are combined to propose three different integration measures of changing floor layout and storage location assignment as in **Fig. 5-11**.

In Integration measure 1, the floor layout is changed by placing the PDP on the middle of the bottom aisle, (1/2, 0), and from that, two diagonal cross aisles are inserted toward the upper corners of the distribution center, (0, 1) and (1, 1). In Integration measure 2, a diagonal aisle is inserted from (0, 0), where the PDP is, to (1, 1), and class based storage is implemented. In Integration measure 3, the PDP is replaced to the middle of the bottom aisle, (1/2, 0), and class based storage is implemented.

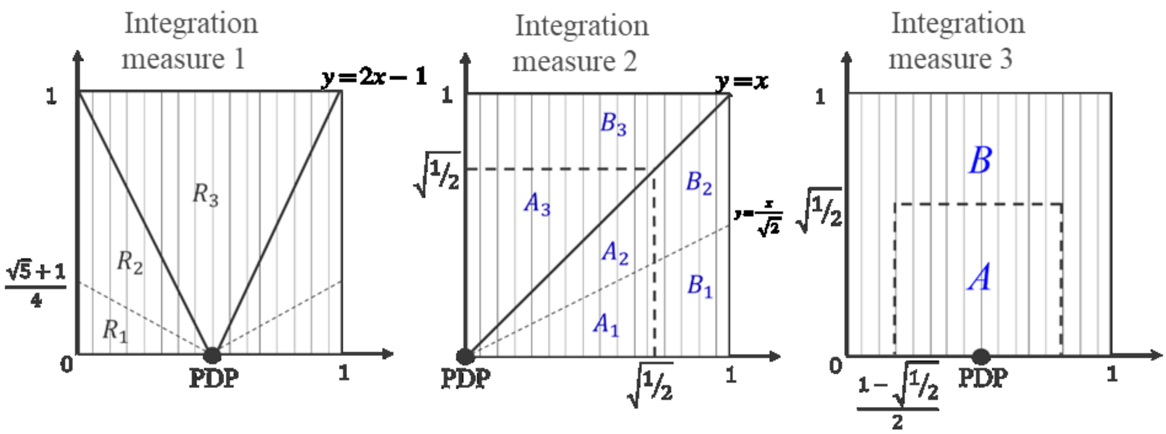


Fig. 5-11 Integration measures

<Integration measure 1>

In Integration measure 1, which is the left figure of **Fig. 5-11**, two diagonal cross aisles with slopes of ± 2 are inserted from the middle of the bottom aisle (1/2, 0) where the PDP is located, to the left and right corners of the distribution center, (0, 1) and (1, 1). In this case, aisles and racks are placed vertically and directions of racks have not been changed as in Fishbone or Chevron layouts. In this Integration measure 1, the mathematical formula of the diagonal cross aisles is given as Eq. (29).

$$y = \pm 2x \mp 1 \tag{29}$$

Similarly to Eq. (22), the line that divides the zone below the diagonal aisle into zone R_1, which is reached by walking on the bottom aisle, and zone R_2, which is reached by walking on the diagonal cross aisle, is solved as Eq. (30) shown below.

From $\left(\frac{1}{2} - x\right) + y = \sqrt{\left(\frac{1}{2} - x\right)^2 + (1 - 2x)^2} + (1 - 2x) - y$,

$$y = -\frac{\sqrt{5} + 1}{2} \cdot x + \frac{\sqrt{5} + 1}{4} \tag{30}$$

The average travel distance in Integration measure 1 is solved as Eq. (31).

$$\begin{aligned} TD_7 = & \int\int_{R_1} \left\{\left(\frac{1}{2} - x + y\right) \cdot 4\right\} dydx \\ & + \int\int_{R_2} \left\{\left(\sqrt{\left(\frac{1}{2} - x\right)^2 + (1 - 2x)^2} + (1 - 2x) - y\right) \cdot 4\right\} dydx \\ & + \int\int_{R_3} \left\{\left(\sqrt{\left(\frac{1}{2} - x\right)^2 + (1 - 2x)^2} + y - (1 - 2x)\right) \cdot 4\right\} dydx \\ & = \frac{5\sqrt{5} + 5}{24} \end{aligned} \tag{31}$$

<Integration measure 2>

In Integration measure 2, which is the middle figure of **Fig. 5-11**, a diagonal aisle with a slope of 1 is inserted from (0, 0), where the PDP is, to (1, 1), and class based storage with squared-shaped boundaries, which divide the floor into two equal areas, is implemented.

When probabilities of visiting zones A and B are *p* and *(1-p)* (where $p \geq 0.5$), the average travel distance of items TD_8 is given as Eq. (32).

$$
\begin{aligned}
TD_8 = &\int\int_{A_1} \{(x+y)\cdot 2p\}dydx \\
&+\int\int_{A_2} \left\{\left(\sqrt{2x^2}-y+x\right)\cdot 2p\right\}dydx \\
&+\int\int_{A_3} \left\{\left(\sqrt{2x^2}+y-x\right)\cdot 2p\right\}dydx \\
&+\int\int_{B_1} \{(x+y)\cdot 2(1-p)\}dydx \\
&+\int\int_{B_2} \left\{\left(\sqrt{2x^2}-y+x\right)\cdot 2(1-p)\right\}dydx \\
&+\int\int_{B_3} \left\{\left(\sqrt{2x^2}+y-x\right)\cdot 2(1-p)\right\}dydx \\
&=\frac{28}{25}-\frac{1}{2}\cdot p
\end{aligned} \tag{32}
$$

<Integration measure 3>

In Integration measure 3, which is the right figure of **Fig. 5-11**, the PDP is replaced to the middle of the bottom aisle (1/2, 0), and class based storage with square-shaped boundaries that divide the floor into two equal areas is implemented.

When probabilities of visiting zones A and B, are *p* and *(1-p)*, (where $p \geq 0.5$), the average travel distance of items TD_9 is given as Eq. (33).

$$
\begin{aligned}
TD_9 = &\int\int_A \left\{\left(\tfrac{1}{2}-x+y\right)\cdot 4p\right\}dydx + \int\int_B \left\{\left(\tfrac{1}{2}-x+y\right)\cdot 4(1-p)\right\}dydx \\
&=\frac{49}{50}-\frac{11}{25}\cdot p
\end{aligned} \tag{33}
$$

5.3.2 Performance evaluation when two measures are introduced at the same time

Fig. 5-12 shows the performance evaluation when two of three measures are introduced at the same time. Among the integration measures, the average travel distance of items in Integration measure 2, in which a diagonal cross aisle and class based storage with square-shaped boundaries are implemented at the same time was the longest. When the probability *p* of visiting zone A is relatively large, Integration measure 3, in which the placement of the PDP is changed and class based storage are introduced, has shorter travel distance. On the other hand, when the probability *p* of visiting zone A is relatively small, Integration measure 1, in which the placement of the PDP is changed and a diagonal cross aisle is inserted at the same time, shows better performance in terms of travel distance of items. When the demand of items is uncertain, the performance of Integration measure 1 may constantly shorten the travel distance by 35% at the maximum. Since the performance of Integration measure 3 depends on *p,* it may not show as much performance when the demand is uncertain, but it may shorten the travel distance by 45% at the maximum. It may be concluded that since Integration measures 1 and 3 involve the replacement of the PDP while Integration measure 2 does not, prioritizing the replacement of the PDP as one of measures is recommended in order to improve efficiency in inventory-type distribution centers.

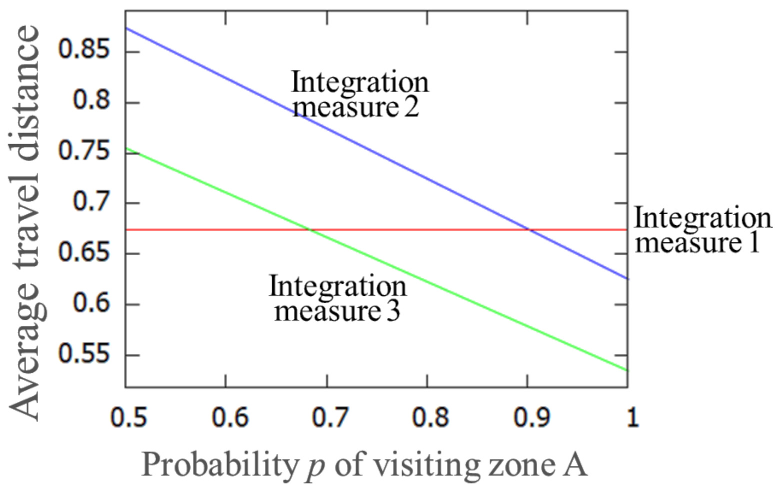


Fig. 5-12 Comparison of average travel distances when two measures are integrated

5.3.3 The examination of integrating three measures

In Subsection 5.4.1, two of three measures of inserting a diagonal cross aisle, replacement of the PDP and introducing class-based storage location have been implemented at the same, and the performances have been compared and evaluated in Subsection 5.4.2. Here, all three measures are conducted at the same time, and the performance is compared to that of Integration measures 1 and 3, in which the average travel distance of items is shorter than in Integration measure 2.

Integration measure 4 is shown in **Fig. 5-13**, in which the PDP is replaced to (1/2, 0), and two diagonal aisles are inserted toward (0, 1) and (1, 1). Class based storage location with square-shaped boundaries which divide the floor into half is conducted. The mathematical model of this is Eq. (34).

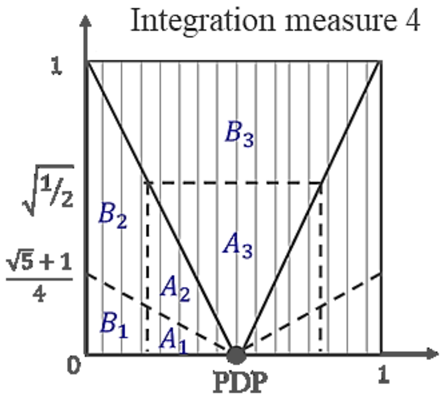


Fig. 5-13 Integration measure 4 in which all measures are introduced

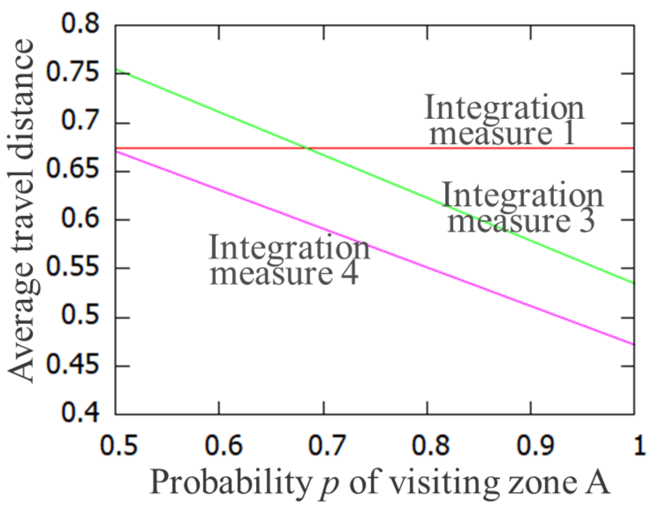


Fig. 5-14 Average travel distances of Integration measures 1,3, and 4

$$TD_{10} = \iint_{A_1} \left\{\left(\frac{1}{2} - x + y\right) \cdot 4p\right\} dydx$$

$$+ \iint_{A_2} \left\{\left(\sqrt{\left(\frac{1}{2} - x\right)^2 + (1-2x)^2} + (1-2x) - y\right) \cdot 4p\right\} dydx$$

$$+ \iint_{A_3} \left\{\left(\sqrt{\left(\frac{1}{2} - x\right)^2 + (1-2x)^2} + y - (1-2x)\right) \cdot 4p\right\} dydx$$

$$+ \iint_{B_1} \left\{\left(\frac{1}{2} - x + y\right) \cdot 4(1-p)\right\} dydx$$

$$+ \iint_{B_2} \left\{\left(\sqrt{\left(\frac{1}{2} - x\right)^2 + (1-2x)^2} + (1-2x) - y\right) \cdot 4(1-p)\right\} dydx$$

$$+ \iint_{B_3} \left\{\left(\sqrt{\left(\frac{1}{2} - x\right)^2 + (1-2x)^2} + y - (1-2x)\right) \cdot 4(1-p)\right\} dydx$$

$$= \frac{9}{10} + \frac{2}{5}p \qquad (34)$$

Fig. 5-14 is the graph of average travel distance of Integration measure 4 compared to that of Integration measures 1 and 3. Among these measures, in terms of travel distance of items, Integration measure 4 always has the best performance, and it reduces the travel distance by 50% at the maximum compared to that in Traditional layout. This result shows that if possible, introducing all measures of inserting diagonal aisles, changing the placement of the PDP and introducing class based storage at the same time is recommended.

5.4 Conclusions of this chapter

In this chapter, in order to improve the efficiency of order picking in inventory-type distribution centers, 1) changing floor layout and 2) changing the storage location of items were implemented. Mathematical models were utilized to evaluate the efficiency in distribution centers, and measures have been implemented to Traditional layout which has vertical and horizontal aisles, to shorten average travel distance. 1) above is done by introducing diagonal cross aisles and changing the location of the PDP, and management of the storage location of items has been introduced by 2). Integration measures in which two of three measures and all three measures are combined have been investigated and the performance was verified. In order to shorten the travel distance of items, it is preferred to prioritize replacement of the PDP to the middle of the bottom aisle. If it is possible to implement all measures, it is recommended to do so because it shows the best performance among measures investigated in this chapter, and there are no offset effects.

Chapter 6. Conclusions and future research

6.1 Conclusions

Supply chain management has become more and more globalized and complicated, and since the efficiency in distribution centers has become the source of competitive supply chain, it has become more important to increase the efficiency. There have been many previous researches which aimed to improve the performance in distribution centers.

In this paper, measures to make improvements in distribution centers have been discussed. Several methods have been proposed to shorten the travel distance of items in transfer-type and inventory-type distribution centers.

In Chapter 4, storage location assignment problem (SLAP) based on arrival and shipping dates of items in transfer-type distribution centers has been investigated. Firstly, greedy methods utilizing arrival and shipping dates of items to determine storage location to shorten the travel distance of items have been evaluated and compared with greedy methods utilizing duration of stay of items which have been discussed in previous researches. Their properties have been proved, and they were also utilized to obtain optimal solutions. Then under the situation with incomplete information of arrival and shipping dates of items, which may happen in practice, new improved greedy policies that utilize only arrival dates of items have been considered to find near-optimal solutions to shorten the travel distance of items.

In Chapter 5, configuration design of floor layout in inventory-type distribution centers has been discussed with mathematical models. Measures of inserting diagonal cross aisles, changing the placement of pick-and-deposit point (PDP) and implementation of class based storage have been discussed. Then, the performances of Integration measures have been verified. It has been concluded that implementing all measures at the same time would make the performance of inventory-type distribution center the most efficient among measures discussed, if possible.

6.2 Limitations of this research and future work

In this research, the improvements of efficiency in distribution centers have been investigated. However, the settings deviate from that in distribution centers in practice.

In transfer-type distribution centers, when the arrival and shipping dates of the items are uncertain, it has been assumed that information of items that arrive on the day of arrival and the following day are gradually revealed. In some cases, shipping dates of items are not known on the arrival dates. Some items may arrive and be shipped on different days from the initial expected days. Also, it may take time after items arrive until they are ready to be stored temporarily in transfer-type distribution centers. Some items may need to be replaced inside the distribution centers. There is even a probability that a worker puts items in unintended racks by an accident. Those uncertainties need to be taken into account in future research.

In inventory-type distribution centers, it has been assumed that items enter from the PDP and are shipped from the same place. However, items may arrive at the different place from where they are shipped, in practice. In this research, the scope of reducing travel distance of items has been limited under single command operation (SCO), but further research could be extended to the situation under multi command operation (MCO) because congestion may be seen when multiple pickers are working in distribution centers. Especially in inventory-type distribution centers, even if the travel distance of items is shortened, other processes after picking items, such as packing, also have to be considered in order to make further improvements in distribution centers. There are also probabilities that workers may deviate from the planned route.

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List of figures and tables

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