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Springback of Coiled Springs after Cold Working*

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Abstract

Simple formulae are presented for the springback ratio of coiled springs for various types of combined load of practical interests. For the analysis of deformation in the cold working process, the rigid, perfectly plastic theory under the combined loading is employed, and further, "proportional deformation" in the coiling process is assumed. The deformation in the springback process is, then, analysed under the assumption of the quasi-elastic behavior.

For the case of the coiling through pure bending, several experiments are performed. The experimental data are in good agreement with those predicted by the present theory, equally well as with laborious Gardiner's elasto-plastic solution.

I. Introduction

When the coiled spring is made by the cold working a considerable amount of springback occurs, which varies with the process of coiling, material used and spring index.

Gardiner and Carlson gave the theoretical and empirical formulae for this problem. They assumed this problem as the pure bending of the bar, and their analysis is appoximately right in the case of "hand coiling."

Recently automatic coiling machines are available and more exact analysis has been required. We made an analysis on the assumption that the wire is in the complete plastic condition during the cold working. We also used the new concept "proportional deformation," which is the notion similar to the "proportional loading" of the limit analysis. This approach avoids the necessity of having to analyze the bar in elasto-plastic portions, consequently achieving economy of thought.

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II. Nomenclature

α ······ pitch angle.
$1/\kappa$ ··· torsion ratio.
$1/\rho$ ··· curvature.
ν ······ Poisson's ratio.
σ_e ··· yield stress of wire.
σ_B ··· ultimate tensile strength of wire.
d ······ wire diameter.
D ······ mean diameter of spring, free position.
E ······ Young's modulus in tension and compression.
F ······ axial force in cold working.
G ······ modulus of rigidity.
I ······ geometrical moment of inertia of the cross section.
I_p ······ polar moment of inertia of the cross section.
M ··· bending moment in cold working.
p ······ pitch of coiled spring.
T ······ twisting moment in cold working.
K ······ dimensionless factor ; $\sigma_e D'/2Ed$

Subscripts

「 ' 」··· during winding operation. e.g. α', D'.
「 * 」··· yield point load of rigid-perfectly plastic thin rod under combined loading. e.g. F^*, M^*, T^*.
「 $_0$ 」··· yield point load of perfectly plastic thin rod under simple loading.

III. Analysis

III·1 Basic formulae

III·1·1 Characteristics of Circular Helix : Curvature and torsion ratio of a circular helix, which is formed by the centerline of the spring wire, are given as follows ;

curvature, $$1/\rho=(1+\cos 2\alpha)/D. \tag{1}$$

torsion ratio, $$1/\kappa=\sin 2\alpha/D. \tag{2}$$

from Eq. (1) & (2), $$\rho/\kappa=\tan\alpha=p/\pi D. \tag{3}$$

III·1·2 Yield point load of a rigid-perfectly plastic thin rod under combined loads : According to the mathematical theory of plasticity, the lower bound of the yield point load of a rigid-perfectly plastic thin rod submitted to the combination of bending moment, twisting moment and axial force is given as follows.[1)]

$$\left(\frac{F^*}{F_0}\right)^2+\left(\frac{T^*}{T_0}\right)^2+\frac{M^*}{M_0}\left\{1-\left(\frac{T^*}{T_0}\right)^2\right\}^{1/2}=1 \tag{4}$$

This is the equation of a surface, namely "interaction surface." (Fig. 1)

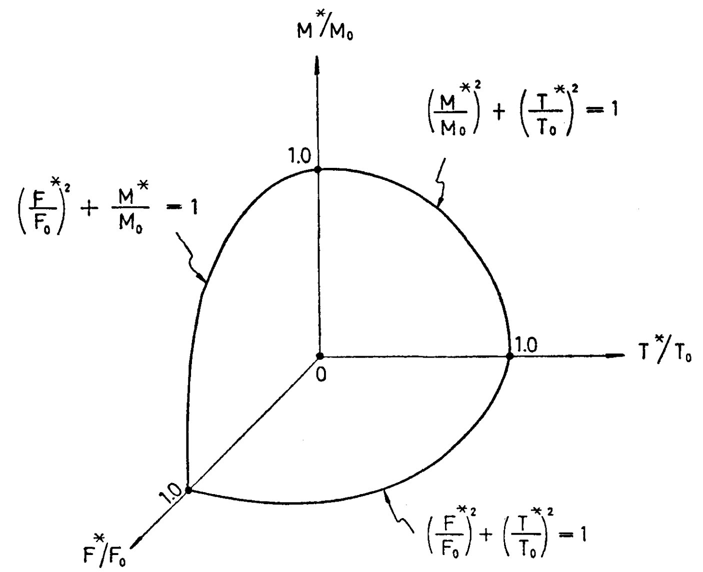

Fig. 1.

The lower bound of the yield point load for more simple types of loads can be derived from Eq. (4) as follows.

Combination of M^* & T^*; Putting $F^*=0$ in Eq. (4),

$$\left(\frac{M^*}{M_0}\right)^2+\left(\frac{T^*}{T_0}\right)^2=1. \tag{5}$$

Combination of M^* & F^*; Putting $T^*=0$ in Eq. (4),

$$\left(\frac{F^*}{F_0}\right)^2+\frac{M^*}{M_0}=1. \tag{6}$$

Curves of Eq. (5) and Eq. (6) are called "interaction curve."

III·1·3 Coiling Condition: We introduce a notion of "proportional deformation," in which we assume the ratio of curvature to torsional ratio is constant, or from Eq. (3), the pitch angle α' is constant throughout the deformation of rigid-perfectly plastic thin rod, or the plastic deformation of elasto-perfectly plastic one.

$$\tan \alpha' = \rho'/k' = (T^*/GI_p)/(M^*/EI)$$

Also, $I_p=2I$ for circular cross section, and $2G/E=1/(1+\nu)$.

Hence,
$$T^*=\frac{2GM^*\tan\alpha'}{E}=\frac{M^*\tan\alpha'}{1.3}. \tag{7}$$

Then, the ratio of twisting moment to bending moment is also kept constant.

III·1·4 Calculation of springback ratio: Let a straight spring wire be wound along a helix of α', D', $1/\rho'$, $1/\kappa'$, then unload it and get a helix of α, D, $1/\rho$, $1/\kappa$.

We may assume springback process to be quasi-elastic deformation as follows (Fig. 2).

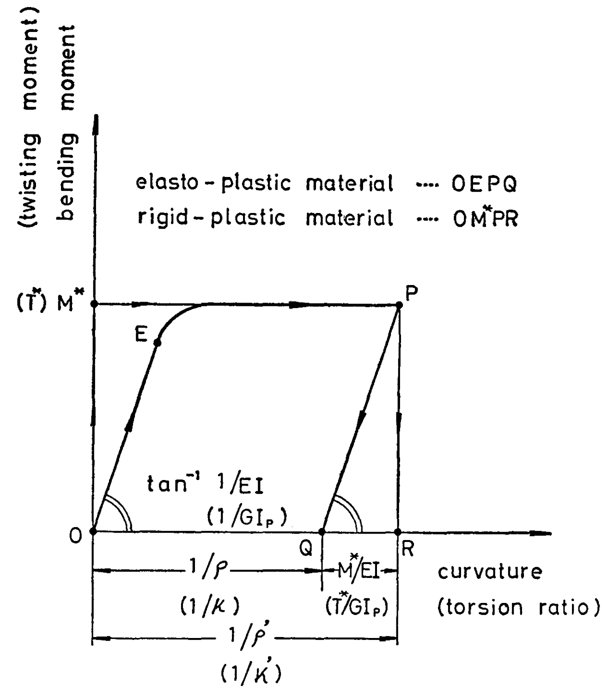

Fig. 2.

$$\frac{1}{\rho}=\frac{1}{\rho'}-\frac{M^*}{EI} \tag{8}$$

$$\frac{1}{\kappa}=\frac{1}{\kappa'}-\frac{T^*}{GI_p} \tag{9}$$

Substituting Eq. (2) & (7) into Eq. (9),

$$\frac{1}{\kappa}=\frac{\sin 2\alpha'}{D'}-\frac{2GM^*\tan\alpha'}{GI_pE}.$$

From Eq. (1), $$\frac{1}{D'}=\frac{1/\rho'}{1+\cos 2\alpha'}=\frac{1/\rho'}{2\cos^2\alpha'}.$$

Then, $$\frac{1}{\kappa}=\frac{2\sin\alpha'\cos\alpha'}{2\cos^2\alpha'}\cdot\frac{1}{\rho'}-\frac{M^*\tan\alpha'}{EI}$$

$$=\left(\frac{1}{\rho'}-\frac{M^*}{EI}\right)\tan\alpha'=\frac{1}{\rho}\tan\alpha'.$$

Hence, $$\frac{\rho}{\kappa}=\tan\alpha=\tan\alpha'.$$

We obtain $$\alpha=\alpha'. \tag{10}$$

Hence, the pitch angle is kept constant throughout the all deformation process, and the twisting moment is always proportional to the bending moment; namely "proportional deformation."

We now consider the springback ratio $(D-D')/D$. Substituting Eq. (1) into Eq. (8),

$$\frac{1+\cos 2\alpha}{D}=\frac{1+\cos 2\alpha'}{D'}-\frac{M^*}{EI}.$$

Substituting Eq. (10) into the above relation, and rewriting it, we get

$$\frac{D-D'}{D}=1-\frac{D'}{D}=\frac{1}{2\cos^2\alpha'}\cdot\frac{M^*}{EI}\cdot D' \tag{11}$$

Bending moment M^* in Eq. (11) is found from Eq. (4), (5), (6) respectively.

III·2 Springback ratio for various types of combined loads of practical interest

III·2·1 Combined load of *M, T, F*: Substituting Eq. (7) into Eq. (4), and after some manipulation, we get

$$M^*=M_0\left[\frac{1.44\tan^2\alpha'\left\{1-\left(\frac{F^*}{F_0}\right)^2\right\}+1-\sqrt{2.88\tan^2\alpha'\left\{1-\left(\frac{F^*}{F_0}\right)^2\right\}\left(\frac{F^*}{F_0}\right)^2+1}}{1.44\tan^2\alpha'(1+0.72\tan^2\alpha')}\right]^{1/2} \tag{12}$$

where $\alpha'\neq 0$. s. Appendix [I]

Substituting this in Eq. (11) together with the relations $M_0=d^3\sigma_e/6$ and $I=\pi d^4/64$, we find

$$1-\frac{D'}{D}=\frac{1.7}{\cos^2\alpha'}\left[\frac{1.44\tan^2\alpha'\left\{1-\left(\frac{F^*}{F_0}\right)^2\right\}+1-\sqrt{2.88\tan^2\alpha'\left\{1-\left(\frac{F^*}{F_0}\right)^2\right\}\left(\frac{F^*}{F_0}\right)^2+1}}{1.44\tan^2\alpha'(1+0.72\tan^2\alpha')}\right]^{1/2}\cdot\frac{\sigma_e D'}{Ed}, \tag{13}$$

where $\alpha'\neq 0$.

Approximate Solution: Applying binomial theorem to the term under root in the braces of Eq. (13) and neglecting the terms of higher order, we get approximately

$$\sqrt{2.88\tan^2\alpha'\left\{1-\left(\frac{F^*}{F_0}\right)^2\right\}\left(\frac{F^*}{F_0}\right)^2+1}\doteqdot 1+\frac{1}{2}\times 2.88\tan^2\alpha'\left\{1-\left(\frac{F^*}{F_0}\right)^2\right\}\left(\frac{F^*}{F_0}\right)^2$$

s. Appendix [II]

Substituting this in Eq. (13), we get an approximate solution

$$1-\frac{D'}{D}\doteqdot 1.7\frac{1-\left(\frac{F^*}{F_0}\right)^2}{\cos^2\alpha'\sqrt{1+0.72\tan^2\alpha'}}\cdot\frac{\sigma_e D'}{Ed}. \tag{14}$$

III·2·2 Combined load of *M* and *T*: Substituting $F^*=0$ into Eq. (13), we get

$$1-\frac{D'}{D}=1.7\frac{1}{\cos^2\alpha'\sqrt{1+0.72\tan^2\alpha'}}\cdot\frac{\sigma_e D'}{Ed}. \tag{15}$$

III·2·3 Combined load of *M* and *F*: From Eq. (6),

$$M^*=M_0\left\{1-\left(\frac{F^*}{F_0}\right)^2\right\}. \tag{16}$$

also $T^*=0$ or from Eq. (7), $\alpha'=0$.

Substituting these into Eq. (11)

$$1-\frac{D'}{D}=1.7\left\{1-\left(\frac{F^*}{F_0}\right)^2\right\}\frac{\sigma_e D'}{Ed}. \tag{17}$$

III·2·4 Simple load of *M* or pure bending: Substituting $F^*=0$ into Eq. (17), we obtain

$$1-\frac{D'}{D}=1.7\frac{\sigma_e D'}{Ed}. \tag{18}$$

VI. Experimental results and its comparison to the theoretical values

In order to prove the usefulness of the theoretical analysis, and to know the relation between ultimate tensile strength σ_B and σ_e in Eq. (18), some test springs were coiled from music wire of

$$\sigma_B=166\sim220\ \text{kg/mm}^2,$$

$$E=21000\ \text{kg/mm}^2,$$

$$d=1.0\sim5.0\ \text{mm},$$

and arbor diameters; $D'-d=2.0\sim30.0$ mm.

Test results are shown in Fig. 3. By taking $\sigma_e=\sigma_B$, Eq. (18) is in good agreement with the test data. Consequently, we rewrite Eq. (18) and get a design formula ;

$$1-\frac{D'}{D}=1.7\frac{\sigma_B D'}{Ed}. \tag{19}$$

It is now interesting to compare our result Eq. (19), based on rigid-perfectly plastic analysis, with those obtained by Gardiner and Carlson.

Gardiner has developed a theorerical springback formula, based on elasto-perfectly plastic analysis for the case of the coiling through pure bending.[2] His formula in our notation may be written as follows:

$$1-\frac{D'}{D}=\frac{2}{\pi}\left\{\sin^{-1}2K+\frac{2}{3}K\sqrt{1-4K^2}(5-8K^2)\right\}, \tag{20}$$

where $$K=\frac{1}{2}\cdot\frac{\sigma_e}{E}\cdot\frac{D'}{d}.$$

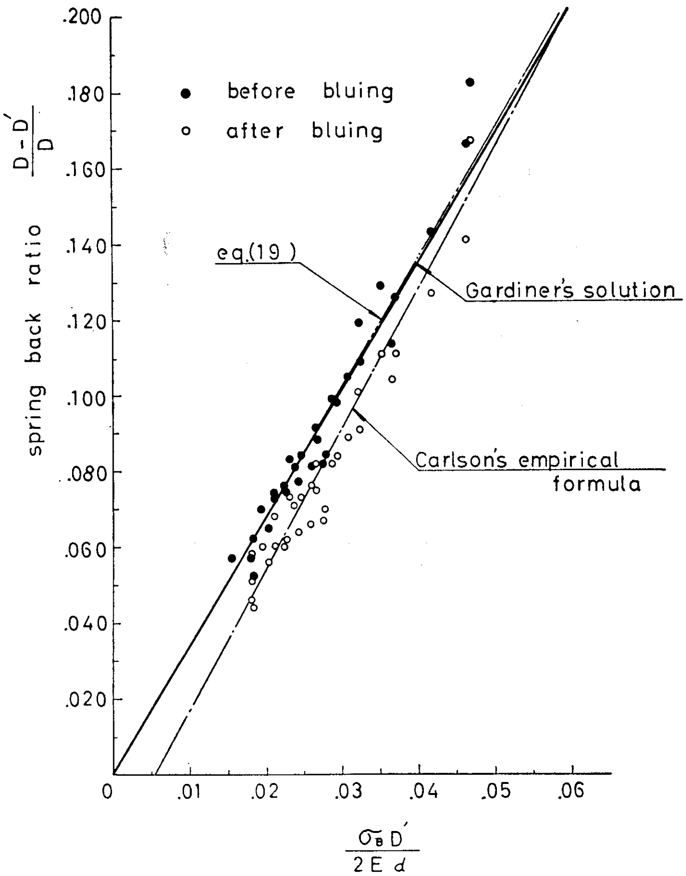

Fig. 3.

Carlson and Gardiner also have shown that best correlation between experimental data and Eq. (20) is found when the ultimate tensile strength is used for the term σ_e.

Refering to their experiments, Carlson established an empirical formula of practical interests.[3] That is, in our notation;

$$1-\frac{D'}{D}=1.85\frac{\sigma_B}{E}\cdot\frac{D'}{d}-0.02\,. \tag{21}$$

Curves of Eq. (19), (20), (21) are given in Fig. 3 for small springback zone and Fig. 4 for larger one. The experimental data are in good agreement with those curves of Eq. (19) and of more laborious Gardiner's solution Eq. (20). On the other hand the curve given by Carlson's formula dosen't agree with our experimental data, and it gives minus value of springback ratio for small value of $\sigma_B D'/2Ed$. Theoretically it should converge to zero as Eq. (19) or Eq (20). Since most spring material (except stainless steel) shows decrease of diameter of springs after bluing, we distinguish them by mark "o" in Fig. 3. It can be seen that Carlson's formula rather agrees with the test data of after bluing. In larger springback zone our assumption dosen't hold good any more, and both Eq. (19) and Eq. (21) show a considerable deviation from Gardiner's solution Eq. (20).

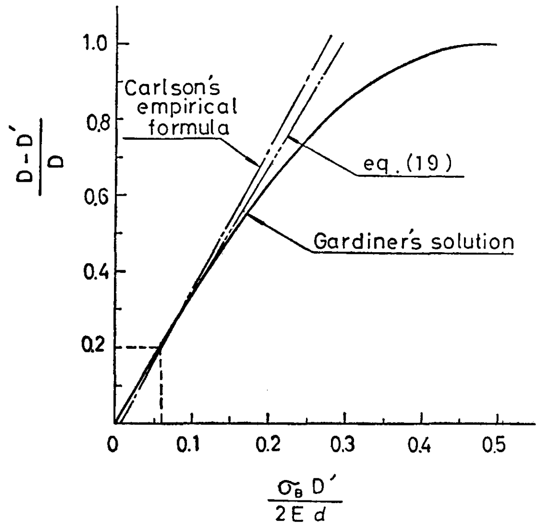

Fig. 4.

Conclusion

A simplified mathematical analysis is shown for the springback ratio of coiled springs for various types of combined loads. It was proved that the analitical results are right for the case of coiling through almost pure bending. For the case of combined loads, experiments are under consideration and the results will be reported in the next paper.

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1) R. Hill and M. P. L. Siebel, J. Mech. Phys. Solids, Vol. 1~3, p. 207, 1953.
2) Frank J. Gardiner, The Spring-Back of Metals, Trans. ASME, Vol. 79~1, p. 1, 1957.
3) Frank J. Gardiner and H. C. R. Carlson, The Spring-Back of Coil springs Mech. Eng., Vol. 80, p. 74~76, 1958.

Appendix

(I) Derivation of Eq. (12) ;
Substituting Eq. (7) into Eq. (4), we have

$$\left(\frac{F^*}{F_0}\right)^2+\left(\frac{\tan\alpha'}{1.3}\right)^2\cdot\left(\frac{M^*}{T_0}\right)^2+\frac{M^*}{M_0}\left\{1-\left(\frac{\tan\alpha'}{1.3}\right)^2\cdot\left(\frac{M^*}{T_0}\right)^2\right\}^{1/2}=1,$$

$$\left(\frac{F^*}{F_0}\right)^2+\left(\frac{\tan\alpha'}{1.3}\right)^2\cdot\left(\frac{M_0}{T_0}\right)^2\cdot\left(\frac{M^*}{M_0}\right)^2+\frac{M^*}{M_0}\left\{1-\left(\frac{\tan\alpha'}{1.3}\right)^2\cdot\left(\frac{M_0}{T_0}\right)^2\cdot\left(\frac{M^*}{M_0}\right)^2\right\}^{1/2}=1.$$

Put $F^*/F_0\equiv f$, and $M^*/M_0\equiv m$ into the final equation, noting $M_0=d^3\sigma_e/6$, $T_0=\pi d^3\tau_e/12$, $\sigma_e=\sqrt{3}\,\tau_e$ (von Mises-Hencky yield condition), we have

$$f^2+Am^2+m(1-Am^2)^{1/2}=1,$$

where $$A\equiv\left(\frac{\tan\alpha'}{1.3}\right)^2\cdot\left(\frac{M_0}{T_0}\right)^2=0.72\tan^2\alpha'.$$

Putting $m^2\equiv q$ into the above equation, we have a quadratic in q,

$$A(A+1)\cdot q^2-\{2(1-f^2)A+1\}\cdot q+(1-f^2)^2=0.$$

Noting $\alpha'\neq 0$ and $|\alpha'|<\pi/2$, we have $A\neq 0$.
Then the quadratic has two roots,

$$q=\frac{2(1-f^2)A+1\pm\sqrt{\{2(1-f^2)A+1\}^2-4A(A+1)(1-f^2)^2}}{2A(A+1)}.$$

Noting $0<M^*/M_0\leqq 1$, we have the Eq. (12),

$$M^*=M_0\sqrt{q}=M_0\left[\frac{1.44\tan^2\alpha'\left\{1-\left(\frac{F^*}{F_0}\right)^2\right\}+1-\sqrt{2.88\tan^2\alpha'\left\{1-\left(\frac{F^*}{F_0}\right)^2\right\}\left(\frac{F^*}{F_0}\right)^2+1}}{1.44\tan^2\alpha'(1+0.72\tan^2\alpha')}\right]^{1/2}.$$

(II) Neglecting the Terms of Higher Order in the Expansion;
Applying binomial theorem to the term under root in the braces of Eq. (13), we have

$$\sqrt{2.88\tan^2\alpha'\left\{1-\left(\frac{F^*}{F_0}\right)^2\right\}\cdot\left(\frac{F^*}{F_0}\right)^2+1}$$

$$=1+\frac{1}{2}\times 2.88\tan^2\alpha'\left\{1-\left(\frac{F^*}{F_0}\right)^2\right\}\left(\frac{F^*}{F_0}\right)^2-\frac{1}{8}\times(2.88)^2\tan^4\alpha'\left\{1-\left(\frac{F^*}{F_0}\right)^2\right\}^2\left(\frac{F^*}{F_0}\right)^4+\cdots$$

Putting $2.88\tan^2\alpha'\left\{1-\left(\frac{F^*}{F_0}\right)^2\right\}\cdot\left(\frac{F^*}{F_0}\right)^2\equiv x$, and ε as the summation of the all terms of higher order, we have,

$$\sqrt{1+x}=1+\frac{x}{2}+\varepsilon,$$

hence, $$|\varepsilon|=\left|\frac{-\frac{x^2}{4}}{\sqrt{1+x}+\left(1+\frac{x}{2}\right)}\right|\leqq\frac{x^2}{4\sqrt{1+x}}.$$

Since most of coiled springs of practical interest have pitch angle $\alpha\,(=\alpha')$ of less than 30 degree, we get

$$\tan\alpha' \leqq \left(\frac{1}{\sqrt{3}}\right)^2 .$$

Noting $0 \leqq (F^*/F_0) < 1$, we have

$$\left\{1-\left(\frac{F^*}{F_0}\right)^2\right\}\left(\frac{F^*}{F_0}\right)^2 \leqq \frac{1}{4} .$$ s. Appendix [III].

From these, $x \leqq 2.88\left(\frac{1}{\sqrt{3}}\right)^2 \cdot \frac{1}{4} = 0.24 .$

Substituting this relation into the above inequality, we get

$$|\varepsilon| \leqq \frac{(0.24)^2}{4\sqrt{1+0.24}} = 0.013 \ll \left(1+\frac{0.24}{2}\right) .$$

Consequently, if the pitch angle is less than 30 degree, we can neglect the terms of higher order and put,

$$\sqrt{2.88\tan^2\alpha'\left\{1-\left(\frac{F^*}{F_0}\right)^2\right\}\left(\frac{F^*}{F_0}\right)^2+1} \doteqdot 1+1.44\tan^2\alpha'\left\{1-\left(\frac{F^*}{F_0}\right)^2\right\}\left(\frac{F^*}{F_0}\right)^2$$

with good accuracy.

(III) Derivation of inequality, $\left\{1-\left(\frac{F^*}{F_0}\right)^2\right\}\left(\frac{F^*}{F_0}\right)^2 \leqq \frac{1}{4}$;

Putting $\left\{1-\left(\frac{F^*}{F_0}\right)^2\right\}\left(\frac{F^*}{F_0}\right)^2 \equiv y$, and $\left(\frac{F^*}{F_0}\right)^2 \equiv t$,

$$y=(1-t)t, \quad \frac{dy}{dt}=1-2t, \quad \frac{d^2y}{dt^2}=-2<0,$$

hence, $y_{t=\frac{1}{2}} = y_{\max} = \frac{1}{4}$,

we obtain, $y \equiv \left\{1-\left(\frac{F^*}{F_0}\right)^2\right\}\left(\frac{F^*}{F_0}\right)^2 \leqq \frac{1}{4}$.