

Title	On rate of reflection of sound-wave of long wave-length
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Abstract	Assuming the wave-length of sound wave (emitted from a point source) to be very long, the rate of reflection of the wave impinging on solid body (ellipsoid, circular cylinder etc.) has been calculated approximately. It was based on the theory of impulsive generation of fluid motion, as treated in Hydrodynamics.
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ワイバー一枚のみに頼っているので巻回間の絶縁強度は導體の四隅で弱點を形成する。川相中相山相のそれぞれのオッシロを比較するに矢印の隆起は巻回間矩絡に近い異常を明示する弱點であり、中相のものは隆起の程度は少いが波形最終部分の上昇が他と比較して小さいから主絶縁の劣化が他よりも進行しているものと想定された。猶この發電機は溝の開口は甚だ狭く線輪を溝に挿入する際これを軸方向に押し込む作業となるから溝部の線輪部分は機械的損傷を受け易く、且チェーン巻線のため線輪の端絶縁は現場作業に限定され、従つてその處理も亦不十分なことは免れ得ないことである。亦この機械のサーヂインピーダンスは他と比較して特に大きかつた。

GE 社方法に依る結果を述べると波頭がオッシロ挿入點に達するまでの波形の時間的經過はその變化不明瞭である。それ故波頭到着後の變化をしらべることにした。波形の窪みに相當する接地事故が検出されたが、これは前述の衝撃波法と比較して感度は低下し、亦故障位置を見出すことは不可能である。亦試験回数も原理的に前者よりも多いことは不便である。

高周波分の検出結果は相別に於て、コロナ發生電壓は山相が最も低く發生電壓と消失電壓とは略々一致する。一般に中性點側は端子側に比較して常時加壓されている電壓値が低いから、従つて劣化進捗の度合も亦相違し一般に中性點側はコロナ發生の度合は少い。本研究實施中特に現場試験に際し、その機會をあたえて下さつた日本發送電株式會社電力技術研究所の方々亦是現場係員の御厚意を感謝する次第である。亦本研究は昭和 24 年度文部省試験研究費及び 23 年度科學研究費の援助を得て行つた。

## On Rate of Reflection of Sound-Wave of Long Wave-Length.

Received Jan. 20, 1949

Fumiki Kitō\*

**Fumiko Kitō: On Rate of Reflection of Sound-Wave of Long Wave-Length.** Assuming the wave-length of sound wave (emitted from a point source) to be very long, the rate of reflection of the wave impinging on solid body (ellipsoid, circular cylinder etc.) has been calculated approximately. It was based on the theory of impulsive generation of fluid motion, as treated in Hydrodynamics.

(1) **A Sound Source is assumed to exist at a point  $P$  in space.** In front of that a rigid body is placed. Then the sound-wave emitted from  $P$  is reflected by the rigid body. To estimate the magnitude of reflected wave is one of the interesting problem in theoretical acoustics. When the wave-length is comparable with radius of the body, the estimation becomes much complicated. When the wave-length is very large, this problem of reflection may approximately be replaced by the following problem with regard to an incompressible fluid: In an incompressible

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fluid a rigid body is placed, and an impulsive pressure is applied at a point  $P$  external to the body, and we inquire the state of flow thus produced impulsively in this fluid region.

Let the pressure of the fluid be  $p$ . When it has very strong magnitude during a very short interval of time  $\tau$ , the amount of impulse  $\omega$  is defined by;

$$\omega = \int_0^\tau p dt$$

According to Hydrodynamics, the subsequent state of flow thus impulsively generated from the state of rest has velocity components  $u, v, w$  given by,

$$u = \frac{1}{\rho} \frac{\partial \omega}{\partial x}, \quad v = \frac{1}{\rho} \frac{\partial \omega}{\partial y}, \quad w = \frac{1}{\rho} \frac{\partial \omega}{\partial z}.$$

The impulse  $\omega$  must satisfy the Laplace's equation  $\nabla^2 \omega = 0$  throughout the region of the fluid. On the free surface we must have  $\omega = 0$ , while on the boundary surface made by rigid body immersed in fluid, we must have  $\partial \omega / \partial n$  (normal derivative)  $= 0$ . When a point source of impulse is applied in a fluid extending to infinity in all directions, we may represent it by the expression  $\omega_1 = m/R$ ; when  $R$  is the distance between the central point  $P$  of the applied impulse and any point  $(x, y, z)$  in fluid.

If a rigid body exists in the fluid, we take additional impulse  $\omega_2$ . If we choose  $\omega_2$  in such a manner that along the surface of the rigid body we have  $\partial(\omega_1 + \omega_2)/\partial n = 0$ , and at any point of fluid external to the body we have  $\omega_2$  regular and  $\nabla^2 \omega_2 = 0$ , then  $\omega = \omega_1 + \omega_2$  will satisfy all the requirements. When we regard  $\omega_1$  as arrival impulsive wave, then  $\omega_2$  may be looked upon as reflected impulsive wave.

(3) **Plane Boundary** It is already known that when the rigid boundary consists of an infinite plane wall, the solution may be obtained by so called method of images as in Electrostatics. For example, for a fluid region bounded by free surface

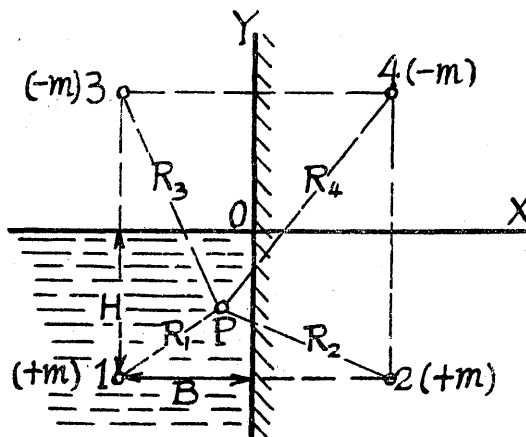


Fig. 1.

$y=0$  and rigid plane wall  $x=0$ , we arrange three images as shown in Fig. 1. In Fig. 2, we have shown distribution of impulse  $\omega$  along the wall at the plane  $z=0$ . If the rigid wall does not exist in this case (images "2", "4" being omitted), the value of  $\omega$  will be half that value. Thus the effect of rigid wall is to double the resultant value of impulse  $\omega$ , and this corresponds to the phenomenon of total reflection of sound wave

due to plane wall. If the rigid wall is made up of some closed curved surface, the rate of reflection may be lower than this case of plane wall. In what follows we shall make estimate of this rate of reflection for a few simple closed surfaces.

(3) **Ellipsoidal Rigid Body** A rigid body in form of an ellipsoid with radius  $a, b, c$  and with its center placed at origin of coordinates as shown in Fig. 3 is subjected to impulsive pressure which is generated by center of impulse at a point  $P(0, 0, h)$  in front of the ellipsoid. To fix the idea, we

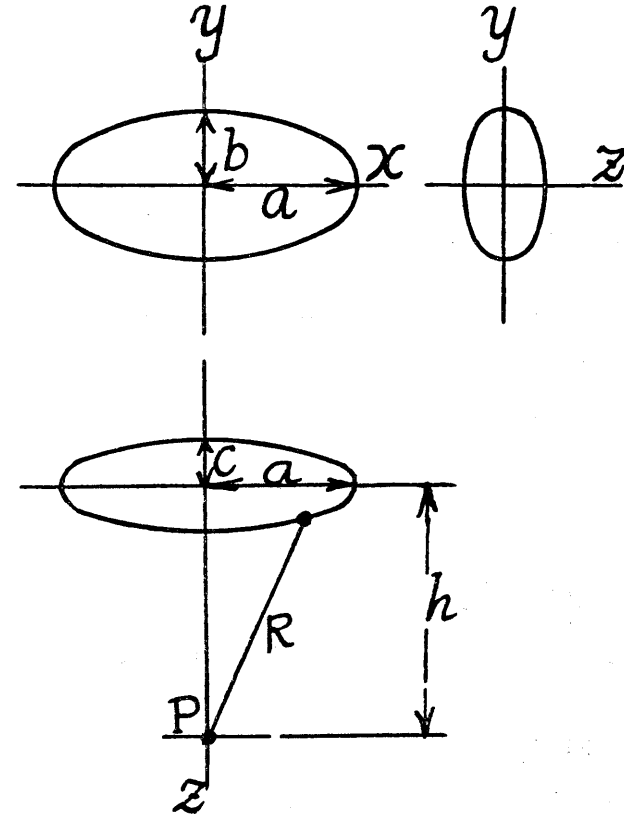


Fig. 3.

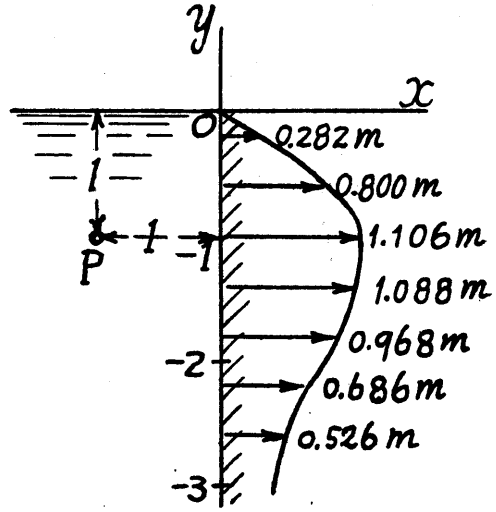


Fig. 2.

assume that  $a > b > c$ , and also assume that  $c^2/h^2$  may be neglected in comparison with unity. Use a system of confocal ellipsoidal coordinates  $(\lambda, \mu, \nu)$  instead of

rectangular coordinates.

Then we have

$$x^2 = \frac{(a^2 + \lambda)(a^2 + \mu)(a^2 + \nu)}{(a^2 - b^2)(a^2 - c^2)}$$

$$y^2 = \frac{(b^2 + \lambda)(b^2 + \mu)(b^2 + \nu)}{(a^2 - b^2)(b^2 - c^2)}$$

$$z^2 = \frac{(c^2 + \lambda)(c^2 + \mu)(c^2 + \nu)}{(a^2 - c^2)(b^2 - c^2)}$$

The coordinates  $(\lambda, \mu, \nu)$  are related to quadric surface

$$\frac{x^2}{a^2 + \theta} + \frac{y^2}{b^2 + \theta} + \frac{z^2}{c^2 + \theta} = 1$$

and the surface of given ellipsoid is represented by  $\lambda = 0$ . As we have

$$\omega_1 = m/R,$$

$$R^2 = (h - z)^2 + x^2 + y^2$$

the boundary condition at  $\lambda = 0$  becomes

$$\frac{\partial \omega_2}{\partial \lambda} = \frac{m}{2R^3} \left[ 1 - \frac{hz}{c^2} \right]$$

Further, taking the approximate value and writing

$$\frac{1}{R^3} = \frac{1}{h^3} \left[ 1 + \frac{3z}{h} \right]$$

the above condition reduces to

$$\frac{\partial \omega_2}{\partial \lambda} = \frac{m}{2h^3} \left[ 1 + \frac{z}{h} \left( 3 - \frac{h^2}{c^2} \right) \right]$$

The problem of finding  $\omega_2$  may be solved by following the method used in problem of translatory motion of an ellipsoid through fluid (ref. Lamb, Hydrodynamics; Chap. V, § 112). Thus, using two harmonics

$$\phi_a = \int_{\lambda}^{\infty} \frac{d\lambda}{\Delta}, \quad \phi_b = zF(\lambda)$$

where

$$F(\lambda) = \int_{\lambda}^{\infty} \frac{d\lambda}{\Delta(c^2 + \lambda)}$$

$$\Delta^2 = (\alpha^2 + \lambda)(b^2 + \lambda)(c^2 + \lambda)$$

$$\omega_2 = A\phi_a + B\phi_b.$$

$A$  and  $B$  being arbitrary constants, we determine them so as to satisfy the boundary condition. Then we obtain

$$\omega_2 = -\frac{mabc}{2h^3} \int_{\lambda}^{\infty} \frac{d\lambda}{\Delta} + \frac{mabc}{h^2} \left( 1 - 3 \frac{c^2}{h^2} \right) \frac{z}{2-K} \int_{\lambda}^{\infty} \frac{d\lambda}{(c^2 + \lambda)\Delta}$$

where we put for shortness,

$$K = abck = abc \int_{\lambda}^{\infty} \frac{d\lambda}{(c^2 + \lambda)\Delta}$$

Especially, on the surface  $\lambda=0$  of the rigid ellipsoid,

$$\omega_2 = -\frac{mabc}{2h^3} \int_0^{\infty} \frac{d\lambda}{\Delta} + \frac{2m}{h^2} \left( 1 - 3 \frac{c^2}{h^2} \right) \frac{K}{2-K}$$

$$\omega_1 = \frac{m}{R} = \frac{m}{h} \left( 1 + \frac{z}{h} \right)$$

We may also make more accurate evaluation, taking into account terms containing  $z^2, \dots$

(4) **Ellipsoid of Revolution** In the above solution, let  $b=c$ . Then

$$K = \frac{ab}{a^2 - b^2} \left[ \frac{a}{b} - \frac{h}{2a} M \right]$$

$$M = \frac{a}{\sqrt{a^2 - b^2}} \log \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}}$$

and values of  $\omega_1$  and  $\omega_2$  along the rigid wall  $\lambda=0$  becomes;

$$\omega_1 = \frac{m}{h} \left( 1 + \frac{z}{h} \right)$$

$$\omega_2 = -\frac{b^2}{2h^2} M + \frac{z}{h} \left( 1 - 3 \frac{a^2}{h^2} \right) \frac{K}{2-K}$$

Taking for example the case of  $a=5b$ , we have  $M=4.70$ ,  $K=0.944$ , so that on the surface of the body;

$$\omega = \omega_1 + \omega_2 = \frac{m}{h} \left( 0.906 + 1.79 \frac{z}{h} \right)$$

But, if this body did not exist, there would arise an impulse of amount  $\omega_0 = m/h$  at the center (the origin). Taking the ratio  $\omega/\omega_0$  we have

$$\frac{\omega}{\omega_0} = 0.906 + 1.79 \frac{z}{h}$$

It is seen that the impulsive pressure  $\omega$  over the surface of the ellipsoid of revolution is eccentrically distributed in such a way that it is the maximum at the front ( $z=c$ ), and minimum at the back ( $z=-c$ ).

(5) **Spherical Wall** This case can be solved rigorously by aid of spherical harmonics. Corresponding to impressed impulse of

$$\omega_1 = \frac{m}{R} = \frac{m}{h} \sum_{n=0}^{\infty} (-)^n \left(\frac{r}{h}\right)^n P_n(\cos \theta)$$

where  $r^2 = x^2 + y^2 + z^2$  and  $\cos \theta = x/r$ , we have

$$\omega_2 = \frac{m}{h} \sum_{n=0}^{\infty} \frac{A_n}{r^{n+1}} P_n(\cos \theta)$$

$$A_n = (-)^n \frac{n}{n+1} \frac{a^{2n+1}}{h^n}$$

So that we have on the surface of the spherical wall  $r=a$ ;

$$\begin{aligned} \omega &= \omega_1 + \omega_2 \\ &= \frac{m}{h} \sum_{n=0}^{\infty} (-)^n \frac{2n+1}{n+1} \left(\frac{a}{h}\right)^n P_n(\cos \theta) \end{aligned}$$

#### (6) Infinitely long Circular Cylinder

Let an infinitely long circular cylinder of radius  $a$  be placed as shown in Fig. 4, in front of which, at a distance of  $OP=D$  a point source of impulse is impressed. Using, for  $1/R$ , an expression devised by Dougall, this impressed impulse may be expressed in the following form;

$$\begin{aligned} \omega_1 &= \frac{m}{R} \\ &= \frac{4m}{\pi} \int_0^{\infty} \cos \lambda z \left[ \sum_{m=0}^{\infty} K_m(\lambda D) I_m(\lambda r) \cos m\theta \right] d\lambda \end{aligned}$$

where we use cylindrical coordinates  $(r, \theta, z)$ . This expression holds only for the case of  $D > r$ , and the sign of summation  $\sum$  must be understood to mean sum for  $m=1, 2, 3, \dots$ , together with the half value for  $m=0$ . The corresponding expression for  $\omega_2$  is assumed to be

$$\omega_2 = \frac{4m}{\pi} \int_0^{\infty} \cos \lambda z \left[ \sum_{m=0}^{\infty} K_m(\lambda D) C_m(\lambda) K_m(\lambda r) \cos m\theta \right] d\lambda$$

where  $C_m(\lambda)$  are unknown functions. This is justified by the fact that terms of the form

$$\cos \lambda z K_m(\lambda r) \cos m\theta$$

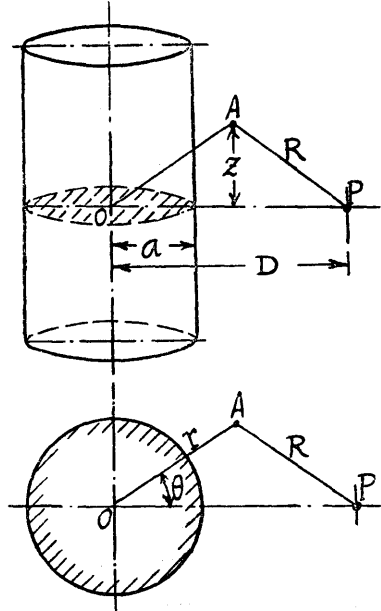


Fig. 4.

are harmonics. The boundary condition  $\partial\omega/\partial r=0$  for  $r=a$ ,  $\omega=\omega_1+\omega_2$  are satisfied by putting

$$C_m(\lambda) = -I_m'(\lambda a)/K_m(\lambda a)$$

Thus the value of  $\omega=\omega_1+\omega_2$  at any point has been obtained. Especially the value of  $\omega$ , around the meridian section  $z=0$  of cylindrical surface  $r=a$ , which we call  $\omega_m$ , becomes as follows:—

$$\omega_m = \frac{2m}{\pi a} [A_0 + A_1 \cos \theta + A_2 \cos 2\theta + \dots]$$

where  $\beta=D/a$  and

$$A_0 = \int_0^\infty \frac{K_0(\beta\xi)}{\xi K_1(\xi)} d\xi$$

$$A_m = 4 \int \frac{K_m(\beta\xi)}{K_{m+1}(\xi) + K_{m-1}(\xi)} \frac{d\xi}{\xi}$$

( $m=1, 2, \dots$ ). The Author, not knowing suitable method of calculating  $A_0, A_1$  etc., has estimated them by numerical integration. Values obtained are tabulated in Table 1.

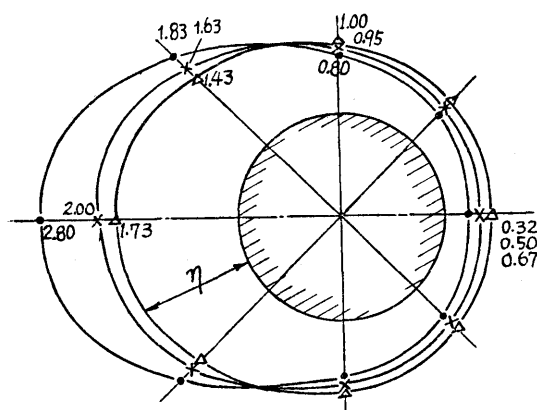


Fig. 5.

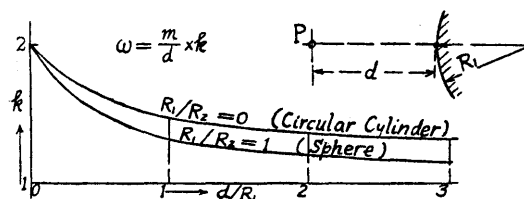


Fig. 6.

Table 1.

	$\beta=2$	$\beta=3$	$\beta=4$
$A_0$	0.93	0.58	0.43
$A_1$	0.84	0.36	0.20
$A_2$	0.30	0.08	0.04
$A_3$	0.13	0.03	0.01
$A_4$	0.04	0.01	0.002

If the cylinder does not exist, the value of  $\omega$  at the origin 0 would be  $\omega_0=m/D$ . The ratio  $\eta=\omega_m/\omega_0$  represents the degree of reflection of impulsive pressure by presence of cylindrical rigid wall. In Fig. 5, some distribution curves of the ratio  $\eta$  around the meridian section of the cylindrical wall surface are shown, which were calculated by means of the above formula.

#### (8) Summing up; the Rela-

**tion between the curvature of rigid wall and the Rate of Reflection** It was stated at the beginning that, when there exist a plane wall extending to infinity, the value of impulsive pressure at the wall is doubled. Considering a convex closed surface, it is shown in Differential Geometry that at any point on it, there exist two radii of curvature  $R_1$  and  $R_2$ . The two extreme cases  $R_1/R_2=1$  (Spherical

surface) and  $R_1/R_2=0$  (Cylindrical surface) has been treated in the above. Hence we could draw curves shown in Fig. 6. From the curves it is seen that the values  $k$  of the rate of reflection of impulsive pressure are considerably close to each other. The case of other rigid walls (for example,  $R_1/R_2=2$ ) will all lie between the two curves shown, and we may easily imagine to what extent the rate of reflection is affected by the curvature and distance of wall from the point source of impulse.

## On Vibration of Drum-Type Diaphragm in Water

Received Jan. 20, 1949

Fumiki Kitō\*

**Fumiki Kitō: On Vibration of Drum-type Diaphragm in Water** When a Diaphragm of drum-type vibrates in a fluid, it emits sound waves in all directions. In this paper, confining ourselves to a case in which the wave length is very long in comparison with the radius of the drum, the so-called virtual mass of the diaphragm has been estimated theoretically.

**Section 1. Introduction** The Author has shown in a recent paper (Several Examples of generation and prevention of vibration due to vortex-streets in Naval Engineering, Misc. Notes of Institution of Naval Engineers, Japan, No. 277, 1949) that it may be of some use to take up the problem of vibration of drum-type diaphragm as shown in Fig. 1, which vibrates in the water. In this report, some results of calculation made by the Author on this respect are given. Thus, the Author has shown a rough estimate on the effect of phase difference of vibration of two sides of drum upon wave propagation and virtual

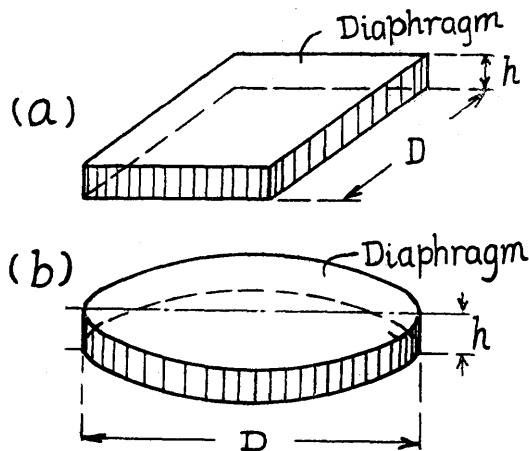


Fig. 1. Drum-type Vibrator

mass of surrounding water. In our case, the wave length of pressure wave being very large in comparison with the diameter of the drum, the equation of pressure  $p$  may be taken approximately to be  $\nabla^2 p = 0$ , that is, the Laplace's Equation.  $\phi$  being the amplitude of the pressure  $p$ , we have  $p = \phi \cos \omega t$ , and  $\phi$  must also be a solution of Laplace's equation, viz.,  $\nabla^2 \phi = 0$ . The instantaneous value of the kinetic

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