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Three Essays on Economic Models Featuring Expectations

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Abstract

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Recent developments in economics have often emphasized the importance of considering economic agents' expectations on future economic activity. This dissertation consists of three essays on economic models featuring expectations.

The first essay (Chapter 1) theoretically studies the multiple equilibria that arise in standard New Keynesian models when the nominal interest rate is set according to the Taylor rule and constrained by a zero lower bound. One of these equilibria is deflationary and referred to as an expectations-driven liquidity trap (ELT) as it arises because of the de-anchoring of inflation expectations. This chapter demonstrates that a simple tax rule responding to inflation can prevent a liquidity trap from arising without increasing government spending or debt. We analytically investigate the necessary and sufficient conditions to prevent an ELT and show that both the frequency and persistence of ELT episodes affect the extent to which the tax rule must respond to inflation. In brief, the higher the frequency or the longer the persistence of the ELT, the greater the response of the tax rate must be. This chapter has been accepted for publication in *Macroeconomic Dynamics*.

The second essay (Chapter 2) proposes a novel methodology to derive nonlinear solutions of an indeterminate DSGE model in which the decision rules are affected by sunspot shocks. The proposed method converts an indeterminate system into a determinate system by introducing an auxiliary variable and an auxiliary equation as proposed by Bianchi and Nicolò, forthcoming and solves the model numerically by the projection method. We apply the method to the ELT and find that the model dynamics exhibit significant nonlinearity. Such nonlinearity arises because the zero lower bound ceases to bind once the inflation rate rises because of a temporary increase in inflation expectations.

The third essay (Chapter 3, coauthored with Takuji Fueki, Jouchi Nakajima, and Shinsuke Ohyama) empirically investigates the role of expectations focusing on the crude oil market. In this chapter, we propose a simple but comprehensive structural vector autoregressive model to examine the underlying factors of oil price dynamics. The distinguishing feature is to explicitly assess the role of expectations on future aggregate demand and oil supply in addition to the traditional realized aggregate demand and supply factors. The empirical analysis shows that identified future demand and supply shocks are as important as the traditional realized demand and supply shocks in explaining historical oil price fluctuations. The empirical result indicates that the influence of oil price changes on global output varies according to the nature of each shock. This chapter has been published in *International Finance*.

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Chapter 1

Tax Rules to Prevent Expectations-Driven Liquidity Traps

1.1 Introduction

In the aftermath of the global financial crisis, more than a decade has now passed since central banks found themselves constrained by a zero lower bound (ZLB) on their policy rates. However, despite the subsequent global economic recovery, inflation has remained stubbornly low in most countries and many central banks have largely kept their policy rates virtually at zero. We often refer to situations like these where the policy rate remains stuck at the lower bound and interest rates cannot fall further as a liquidity trap (LT). Because these LTs have become a global phenomenon, investigating how the existence of the ZLB affects the economy has become a central topic in modern macroeconomics.[1]

Among several issues arising from the existence of the ZLB, the seminal paper by Benhabib, Schmitt-Grohé, and Uribe, 2001 revealed that multiple equilibria emerge when the central bank targets a positive inflation rate and the nominal interest rate is constrained by a lower bound. Their study further showed that one of these equilibria is deflationary and the economy may become trapped in an LT without any changes in the fundamentals. In contrast to the conventional LTs triggered by large shocks to the fundamentals (fundamentals-driven LT, FLT hereafter), subsequent literature has often referred to this deflationary equilibrium as the expectations-driven

[1]For recent developments in the literature exploring how the ZLB affects the economy, see Debortoli, Galí, and Gambetti, 2019, Liu et al., 2019, and Ikeda et al., 2020 to mention a few.

LT (ELT) given it emerges solely by the de-anchoring of agents' inflation expectations.

With the prolonging of the shared experience of LTs and low inflation in many countries, the multiplicity of equilibria and the prevention of the formation of LTs have attracted a wide range of interest among academics and policymakers. Existing studies have revealed that we can prevent ELTs by the effective use of fiscal policy. For instance, Benhabib, Schmitt-Grohé, and Uribe, 2002 propose a fiscal policy rule that responds to inflation and leads to a violation of the transversality condition if the economy were to converge to the deflationary steady state. Similarly, Schmidt, 2016 demonstrates that a fiscal rule designed to keep the real marginal cost higher than a certain threshold can avoid any type of LT from arising.

However, large-scale fiscal stimulus to support the economy and the subsequent surge in government debt calls into question whether policy measures proposed in these extant studies are even plausible. This is because a policy that commits to increase government spending or debt as long as inflation remains low seems infeasible given the current situation where governments already face a huge amount of outstanding debt. More generally, policies designed to increase useless government spending only to create demand are considered unsustainable in the long run.

This chapter fills this gap and contributes to the literature by demonstrating that a simple tax rule responding to inflation can prevent an ELT from arising without any increase in government spending or debt. Rather than creating demand through government expenditure, the proposed tax rule lowers the labor income tax rate and encourages households to provide more labor once the pessimistic expectations of agents prevail. Given that a simultaneous decline in both inflation and output is the key element for the ELT to arise, the inflationary pressure caused by the increase in labor supply eliminates agents' beliefs that lead to ELTs.

The analysis builds on a standard linear New Keynesian dynamic stochastic general equilibrium (DSGE) model featuring different types of shocks. To model the self-fulfilling deflation, we introduce nonfundamental "regime shocks" that force the economy to move between the targeted regime, where the inflation rate is close to its target rate, and the unintended regime, where the inflation rate is negative. In addition, we introduce shocks to the real interest rate into the model to assess how the proposed tax rule operates with conventional FLTs.

The use of a log-linearized model enables us to investigate analytically the necessary and sufficient conditions that the fiscal authority must satisfy to prevent the ELT equilibrium. We show that the frequency and persistence of the ELT episodes both affect the extent to which the tax rule must respond to inflation, such that the higher the frequency or the longer the persistence of the ELT, the greater the response of the tax rate must be. Nonetheless, while the recurrence of ELTs—switching between the targeted regime and the unintended regime—has important implications for policy design, extant studies have paid relatively less attention to the issue.[2] Therefore, this study contributes to the literature by showing that both the frequency and persistence of ELT episodes are crucial for policy design.

This study demonstrates that the magnitude of the changes in the tax rate lies well within a realistic range under standard calibration. We show that to prevent the ELT equilibrium, the proposed tax rule requires the fiscal authority to cut the labor income tax rate from 20 to 15 percent in response to a two-percentage-point decline in annual inflation. However, we also show that if the response of the tax rate is not sufficient, the fiscal authority not only fails to prevent the ELT but also aggravates the declines in inflation and output in the ELT, relative to the case of no tax rate changes. We also consider the case where the real interest rate declines exogenously, and show that while this mitigates the decline in inflation, output is further depressed if the fiscal authority adjusts only the labor income tax rate to prevent the ELT equilibrium.

This study draws on the large literature focusing on policies to confront different types of LTs. Seminal work by Eggertsson and Woodford, 2003 shows that forward guidance can mitigate the declines in inflation and output in the FLT. Subsequent studies such as Sugo and Ueda, 2008 and Christiano and Takahashi, 2018 also argue that monetary policy can play a central role in avoiding the ELT, while Schmitt-Grohé and Uribe, 2017 find that raising the nominal interest rate to its intended target for an extended period can boost inflationary expectations and allow an escape from the ELT. These studies mainly focus on the use of monetary policy, yet

[2] Coyle and Nakata, 2019 show that the optimal inflation rate changes significantly when we assume ELT episodes to be recurrent.

several recent analyses have emphasized the importance of fiscal policy as a means to confront LTs.[3]

Although this chapter examines the role of fiscal *rules* to prevent LTs, several existing studies have already explored the effectiveness of *exogenous* policies in an LT. Correia et al., 2013 focus on the FLT and show that appropriate tax policy can deliver stimulus without the use of government spending. Mertens and Ravn, 2014 analytically consider fiscal policies in the ELT, concluding that supply-side policies, such as tax cuts, are more effective than conventional demand-side policies. Boneva, Braun, and Waki, 2016 also show that tax cuts are effective in increasing employment in the ELT. This study shares the finding that tax cuts are effective under the ELT with these extant studies.

By comparing the economic outcomes between FLTs and ELTs under the proposed tax rule, this study contributes to recent studies exploring effective policies under different LTs. Bilbiie, 2018 compares the effects of different monetary and fiscal policies between the two LTs and shows that neo-Fisherian policy is effective in the ELT. Nakata and Schmidt, 2019 provide a detailed analysis of both types of LTs, although their focus is on policymakers optimizing an assigned objective function and Cuba-Borda and Singh, 2019 compare the ELT and the secular stagnation equilibrium and obtain contrasting implications for different monetary and fiscal policies.

While the focus of this chapter is to investigate theoretically the properties of ELTs, some studies empirically explore their implications. Aruoba, Cuba-Borda, and Schorfheide, 2018 investigate whether the US and Japan have transitioned to a deflationary regime using a nonlinear DSGE model and suggest that Japan is likely to have moved to this regime in the late 1990s, whereas this is more unlikely for the US. Hirose, 2020 estimates a medium-scale DSGE model around the deflationary steady state, using Japanese data, and explores the model dynamics.

Finally, we can link this study to the literature on optimal taxation because it proposes the use of distortionary taxes to prevent LTs. It is known that tax smoothing is optimal if distortionary taxes are the only options (e.g., Barro, 1979 and Lucas and

[3] Investigating the effectiveness of fiscal policy at the ZLB is an active research area. For recent developments, see Bilbiie, Monacelli, and Perotti, 2019, Ercolani and Azevedo, 2019, and Ngo, 2019, among many others.

Stokey, 1983). However, recent studies have found that allowing variation in tax rates can be welfare improving under certain conditions. Hagedorn, 2010 studies a large class of models that feature various types of frictions and finds the conditions under which tax cycles (time-varying tax rates) are welfare improving. Arseneau and Chugh, 2012 show that instead of tax smoothing, "wedge smoothing" is desirable in a model with labor market search frictions. This study connects with this literature by showing that the effective use of distortionary taxes can improve welfare when different types of shocks disturb the economy.

The remainder of this chapter is organized as follows. Section 1.2 details the model. Section 1.3 analyzes the design of tax rules to prevent the ELT equilibrium and Section 1.4 examines how the assumption of recurrent ELT episodes influences the results. Section 1.5 considers how the proposed tax rule operates in a model with fundamental shocks. Section 1.6 concludes.

1.2 The model

This study builds on a canonical New Keynesian DSGE model, which consists of three equilibrium equations: the downward-sloping demand equation derived from the representative household's optimization problem, the upward-sloping supply equation derived from the firm's optimization problem, and the monetary policy rule.

With the absence of the ZLB on the nominal interest rate, there are no kinks in the first two equations and thus the equilibrium is determined uniquely. However, the existence of the ZLB generates a kink in the demand equation, which leads to multiple equilibria. Because only the most basic model is required to explore the key properties of the ELT, existing studies have largely built on this canonical three-equation model. In the following subsections, we provide the details of the model.

1.2.1 Household

A representative household gains utility from consumption and disutility from labor supply. The household maximizes its expected lifetime utility through the choice of consumption c_t, labor supply l_t, and bond holding b_t, given prices and subject to a budget constraint:

$$\max_{\{c_{t+s},l_{t+s},b_{t+s}\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\frac{c_{t+s}^{1-\sigma}-1}{1-\sigma} - \frac{l_{t+s}^{\eta+1}-1}{\eta+1} \right], \tag{1.1}$$

$$\text{s.t.} \quad c_t + \frac{b_t}{R_t} = (1-\tau_{w,t}) w_t l_t + \frac{b_{t-1}}{\Pi_t} + d_t, \tag{1.2}$$

where R_t and Π_t are the gross nominal interest rate and gross inflation rate, respectively, w_t is the real wage, d_t is the dividend from intermediate goods firms, and $\tau_{w,t}$ is labor income tax, which is allowed to vary over time.

From the first-order conditions, we derive the Euler equation (EE) and the wage equation as

$$c_t^{-\sigma} = \beta R_t \mathbb{E}_t \left[c_{t+1}^{-\sigma} \frac{1}{\Pi_{t+1}} \right], \tag{1.3}$$

$$\frac{c_t^{-\sigma}}{l_t^{\eta}} = \frac{1}{1-\tau_{w,t}} \frac{1}{w_t}. \tag{1.4}$$

The second equation shows that the labor income tax creates a wedge and affects labor supply. Simply assuming that the consumption (c_t) and the wage (w_t) are fixed, a lower income tax rate ($\downarrow \tau_{w,t}$) induces the household to increase labor supply ($\uparrow l_t$). This is the core mechanism through which the fiscal authority prevents the ELT equilibrium in what follows.

1.2.2 Firms

There are two types of firms in the economy: a continuum of intermediate goods producers and a final goods producer. The final goods producer uses intermediate goods as the only input and has constant elasticity of substitution production technology. The final goods producer is perfectly competitive and takes both output and input prices as given. The static profit maximization problem is as follows:

$$\max_{\{y_t,y_{i,t}\}} P_t y_t - \int_0^1 P_{i,t} y_{i,t} di, \tag{1.5}$$

$$\text{s.t.} \quad y_t = \left(\int_0^1 y_{i,t}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}. \tag{1.6}$$

Perfect competition drives final goods producers' profits to zero. From the first-order conditions, we derive the demand for intermediate goods and the associated price index:

$$y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\theta} y_t, \tag{1.7}$$

$$P_t = \left(\int_0^1 P_{i,t}^{1-\theta} di\right)^{\frac{1}{1-\theta}}. \tag{1.8}$$

There is a continuum of intermediate goods producers indexed by i. These producers are monopolistically competitive and incur quadratic price adjustment costs as in Rotemberg, 1982. Each producer uses labor as the input in production. Firm i chooses optimal price $P_{i,t}$ and labor input $l_{i,t}$ given the current aggregate output y_t and aggregate price level P_t. It then maximizes the present value of discounted dividends after tax $d_{i,t}$ as follows:

$$\max_{\{y_{i,t+s}, P_{i,t+s}, l_{i,t+s}\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} Q_{c,t+s} d_{i,t+s}, \tag{1.9}$$

$$\text{s.t.} \quad d_{i,t+s} = \frac{P_{i,t+s}}{P_{t+s}} y_{i,t+s} - w_{t+s} l_{i,t+s} - \frac{\psi}{2}\left(\frac{P_{i,t+s}}{P_{i,t+s-1}} - 1\right)^2 y_{t+s}, \tag{1.10}$$

$$y_{i,t+s} = l_{i,t+s}, \tag{1.11}$$

$$y_{i,t+s} = \left(\frac{P_{i,t+s}}{P_{t+s}}\right)^{-\theta} y_{t+s}, \tag{1.12}$$

where the real stochastic discount factor is defined as

$$Q_{c,t+s} \equiv \beta^s c_{t+s}^{-\sigma}. \tag{1.13}$$

Combining the first-order conditions and imposing symmetry across firms, we derive the Philips Curve (PC) as follows:

$$\psi(\Pi_t - 1)\Pi_t - \theta w_t + \theta - 1 = \beta \mathbb{E}_t \left[\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{y_{t+1}}{y_t} \psi(\Pi_{t+1} - 1)\Pi_{t+1}\right]. \tag{1.14}$$

The aggregate production function and dividend payouts are

$$y_t = l_t, \tag{1.15}$$

$$d_t = y_t - w_t l_t - \frac{\psi}{2}(\Pi_t - 1)^2 y_t. \tag{1.16}$$

1.2.3 Central bank and fiscal authority

The central bank sets the interest rate following the standard Taylor rule with the net nominal interest rate bounded from below by zero:

$$R_t = \max\left[1, \frac{1}{\beta}\Pi_t^{\phi_\pi}\right]. \tag{1.17}$$

For simplicity, we assume that the interest rate responds only to inflation and not to output.

In this study, we assume that only distortionary taxes are available to finance government spending. Moreover, we allow the tax rate on labor income to vary over time. The government's budget constraint is then

$$\frac{b_t}{R_t} + \tau_{w,t} w_t l_t = \frac{b_{t-1}}{\Pi_t} + g_t. \tag{1.18}$$

We further assume that government spending is determined endogenously. That is, the total amount of tax revenue constrains the amount of goods that the fiscal authority purchases:

$$\tau_{w,t} w_t l_t = g_t. \tag{1.19}$$

Therefore, the fiscal authority does not issue bonds in equilibrium ($b_t = 0$). Hence, a decrease in tax revenue results in a decrease in government spending, and this affects household income.

Most existing studies allow lump-sum transfers to isolate the marginal effect of government spending on household income. A model with a lump-sum transfer and government-spending rule nests in the results of this chapter, as shown in Appendix C. Therefore, the main results of this study carry over to models that retain exogenous government spending.[4]

Existing studies such as Benhabib, Schmitt-Grohé, and Uribe, 2002 and Schmidt, 2016 show that fiscal rules responding to inflation can avoid the ELT. Following these findings, we assume that the labor income tax rate is a function of inflation in the following form:

$$\tau_{w,t} = \tau_w \Pi_t^{\lambda_w}. \tag{1.20}$$

The fiscal authority adjusts the tax rate depending on the current inflation rate. The parameter λ_w governs how aggressively the fiscal authority adjusts the tax rate in response to changes in inflation and is referred to as the "tax response parameter." If the tax response parameter is positive, the fiscal authority lowers the tax rate in response to a decline in inflation.

There have been observations in the past where tax rates have changed depending on economic conditions. For example, the Jobs and Growth Tax Relief Reconciliation Act of 2003 in the US included cutting the tax rates on labor income and dividends, while the UK reduced its value-added tax rates throughout 2009. Although adjusting tax rates flexibly may face legislative challenges, we can view the assumption that tax rates vary depending on inflation as an extension to past examples where the adjustment in tax rates was to stimulate the economy.

Another rationale to link the tax rate with inflation lies in the fact that the current tax system is distortionary, and inflation has unequal effects on resource allocation. For example, Feldstein, 1999 and Ueda, 2001 estimate the welfare loss of distortionary taxation in the US and Japan, respectively, and discuss the benefits of low inflation. While the legislative feasibility remains a challenge, Feldstein, 1999 claims that indexing tax rates to inflation resolves such distortions and improves welfare.

[4] Endogenous government spending is motivated by the fact that a lump-sum transfer is not available in reality. When the fiscal authority increases (or at least maintains) its expenditure while its tax revenue decreases due to depressed economic activity, the fiscal authority must increase bond issuance. This decrease in tax revenue and an increase in expenditure have led to the current elevated government debt. By assuming endogenous government spending, the fiscal authority allows variation in the tax revenue and avoids the increase in government debt. However, the mechanism through which the fiscal authority prevents the ELT does not rely on this endogenous government-spending assumption, as shown in Appendix C.

In this study, the fiscal authority selects labor income tax as the policy instrument to influence the household's labor supply directly. As we show in the following analysis, the fiscal authority can prevent an ELT from arising by committing to lowering the labor income tax rate if the agent's pessimistic belief were to materialize and inflation to decline. Appendix B shows that the fiscal authority can prevent the ELT equilibrium by adjusting either the dividend tax or the consumption tax instead of the labor tax, while tax rates must be raised in response to a decline in inflation.

1.2.4 Equilibrium conditions

The resource constraint of the entire economy is derived by combining equations (1.2), (1.16), and (1.18) as

$$c_t + g_t + \frac{\psi}{2}(\Pi_t - 1)^2 y_t = y_t. \tag{1.21}$$

Equations (1.3), (1.4), (1.14), (1.15), (1.17), (1.20), and (1.21) comprise the equilibrium conditions. We summarize the nonlinear equilibrium conditions other than the tax rule as the following four equations:

$$c_t^{-\sigma} = \beta R_t \mathbb{E}_t \left[c_{t+1}^{-\sigma} \frac{1}{\Pi_{t+1}} \right], \tag{1.22}$$

$$\psi(\Pi_t - 1)\Pi_t - \frac{\theta c_t^{\sigma} y_t^{\eta}}{1 - \tau_{w,t}} + \theta - 1 = \beta \mathbb{E}_t \left[\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{y_{t+1}}{y_t} \psi(\Pi_{t+1} - 1)\Pi_{t+1} \right], \tag{1.23}$$

$$R_t = \max \left[1, \frac{1}{\beta} \Pi_t^{\phi_\pi} \right], \tag{1.24}$$

$$y_t \left[1 - \frac{\psi}{2}(\Pi_t - 1)^2 \right] = c_t + \frac{\tau_{w,t}}{1 - \tau_{w,t}} c_t^{\sigma} y_t^{\eta+1}. \tag{1.25}$$

Equilibrium conditions (1.22)–(1.25) may exhibit two different steady states. Let us call the deterministic steady state with positive inflation the "targeted steady state" and denote it by the subscript TSS. The steady-state values in the TSS are

$$R_{TSS} = \frac{1}{\beta}, \tag{1.26}$$

$$y_{TSS} = \left[\frac{\theta - 1}{\theta}(1 - \tau_w) \right]^{\frac{1}{\eta+\sigma}} \left[1 - \tau_w \frac{\theta - 1}{\theta} \right]^{-\frac{\sigma}{\eta+\sigma}}, \tag{1.27}$$

$$c_{TSS} = \left[\frac{\theta - 1}{\theta}(1 - \tau_w) \right]^{\frac{1}{\eta+\sigma}} \left[1 - \tau_w \frac{\theta - 1}{\theta} \right]^{\frac{\eta}{\eta+\sigma}}. \tag{1.28}$$

The following equilibrium conditions are derived by log-linearizing the tax rule (1.20) and the equilibrium conditions (1.22)–(1.25) around the TSS:

$$\hat{c}_t = \mathbb{E}_t\hat{c}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1}), \tag{1.29}$$

$$\hat{\pi}_t = \beta\mathbb{E}_t\hat{\pi}_{t+1} + \sigma\frac{\theta-1}{\psi}\hat{c}_t + \eta\frac{\theta-1}{\psi}\hat{y}_t - \frac{\theta-1}{\psi}\frac{\tau_w}{1-\tau_w}\hat{\tau}_{w,t}, \tag{1.30}$$

$$\hat{i}_t = \max[\log\beta, \phi_\pi\hat{\pi}_t], \tag{1.31}$$

$$\hat{\tau}_{w,t} = \lambda_w\hat{\pi}_t, \tag{1.32}$$

$$\gamma_y\hat{y}_t = \gamma_c\hat{c}_t + \gamma_{\tau,w}\hat{\tau}_{w,t}, \tag{1.33}$$

$$\text{where} \quad \gamma_y \equiv 1-(\eta+1)\frac{\theta-1}{\theta}\tau_w, \ \ \gamma_c \equiv \frac{c_{TSS}}{y_{TSS}} + \sigma\frac{\theta-1}{\theta}\tau_w, \ \ \gamma_{\tau,w} \equiv \frac{\theta-1}{\theta}\frac{\tau_w}{1-\tau_w}.$$

In equation (1.31), the ZLB is imposed on the nominal interest rate after log-linearization. The variables with hats are the log deviations from the TSS values, i.e., $\hat{x}_t \equiv \log x_t - \log x_{TSS}$.

After substitution, the equilibrium conditions simplify to the following EE and PC with two variables $\hat{\pi}_t$ and $\hat{y}_t$ as

$$\hat{y}_t = \xi\hat{\pi}_t + \mathbb{E}_t\hat{y}_{t+1} - \frac{1}{\sigma}(\max[\log\beta, \phi_\pi\hat{\pi}_t] - \mathbb{E}_t\hat{\pi}_{t+1}) - \xi\mathbb{E}_t\hat{\pi}_{t+1}, \tag{1.34}$$

$$(1+\zeta)\hat{\pi}_t = \kappa\hat{y}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1}, \tag{1.35}$$

$$\text{where} \quad \kappa \equiv \frac{\theta-1}{\psi}\left(\eta + \sigma\frac{\gamma_y}{\gamma_c}\right), \ \ \xi \equiv \frac{\gamma_{\tau,w}}{\gamma_y}\lambda_w, \ \ \zeta \equiv \frac{\theta-1}{\psi}\left(\sigma\frac{\gamma_{\tau,w}}{\gamma_c} + \frac{\tau_w}{1-\tau_w}\right)\lambda_w.$$

The log-linearized model allows us to derive closed-form solutions. Therefore, we explore effective tax rules using this model in the remainder of this chapter.

Because we focus on a large fall in the real interest rate in later analysis, it is useful to define the real interest rate as follows:

$$\hat{y}_t = \xi\hat{\pi}_t + \mathbb{E}_t\hat{y}_{t+1} - \frac{1}{\sigma}(\max[\log\beta, \phi_\pi\hat{\pi}_t] - \mathbb{E}_t\hat{\pi}_{t+1} - r_t^n) - \xi\mathbb{E}_t\hat{\pi}_{t+1}, \tag{1.36}$$

$$(1+\zeta)\hat{\pi}_t = \kappa\hat{y}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1}. \tag{1.37}$$

The above representation is a reduced form to capture changes in the real interest rate triggered by exogenous shocks, such as changes in household preferences. The real interest rate, r_t^n, is an exogenous process and zero at the normal state and negative at the crisis state.

1.2.5 Ensuring local determinacy around the targeted steady state

Local indeterminacy may arise under certain parameterizations given that our proposed tax rule depends on the current inflation rate. When the ZLB does not bind, we express the equilibrium conditions (1.34) and (1.35) as the following state space representation:

$$\begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t, \end{bmatrix} = A \begin{bmatrix} \mathbb{E}_t \hat{y}_{t+1} \\ \mathbb{E}_t \hat{\pi}_{t+1}. \end{bmatrix}, \tag{1.38}$$

$$\text{where } A \equiv \frac{\sigma^{-1}}{1+\zeta+\kappa\left(\frac{\phi_\pi}{\sigma}-\xi\right)} \begin{bmatrix} (1+\zeta)\sigma & (1+\zeta)(1-\xi\sigma)-\beta(\phi_\pi-\xi\sigma) \\ \kappa\sigma & \beta\sigma+\kappa(1-\xi\sigma). \end{bmatrix}.$$

Applying the Blanchard–Kahn conditions, we derive the following condition on the tax response parameters.

Proposition 1. *The log-linearized rational expectations equilibrium is locally determinate if the fiscal authority sets Λ lower than the threshold level Ψ^D as*

$$\Lambda < 1-\beta+\phi_\pi\frac{\kappa}{\sigma} \equiv \Psi^D, \tag{1.39}$$

$$\text{where } \Lambda \equiv \xi\kappa-\zeta = \frac{\theta-1}{\psi}\left(\eta\frac{\gamma_{\tau,w}}{\gamma_y}-\frac{\tau_w}{1-\tau_w}\right)\lambda_w.$$

Proof. See Appendix A.1. □

The proposition is a generalized statement of the Taylor principle, which requires the central bank to raise the nominal interest rate more than the increase in the inflation rate. Given that the tax response parameter λ_w as well as the Taylor coefficient ϕ_π affect the equilibrium inflation rate, both parameters must be chosen appropriately to assure local determinacy. Further, because a policy that generates indeterminacy locally around the TSS is not an appropriate option, we focus on tax rules satisfying the local determinacy condition (1.39) in the remaining analysis.

1.2.6 Calibration

In the model, we assume that a period corresponds to a quarter. The discount factor is set to $\beta = 0.996$, which yields an annual real interest rate of 1.6 percent. This relatively low level of real interest rate is common in recent studies on LTs. The elasticity of intertemporal substitution is chosen to be $\sigma = 1$, which is a standard calibration in the literature. The Frisch elasticity of labor substitution is set to $\eta = 0.4$, a value in line with the estimates by existing studies such as Boneva, Braun, and Waki, 2016 and Smets and Wouters, 2007, with parameter elasticities of 0.37 and 0.55, respectively. The elasticity of substitution between intermediate goods is set to $\theta = 6$, which yields a markup of 20 percent. This value is consistent with the estimates of existing studies such as Broda and Weinstein, 2006, which report that the median value of θ ranges from 3 to 4.3, while Denes, Eggertsson, and Gilbukh, 2013 estimate θ to be approximately 13. The price adjustment cost is set to $\psi = 400$, a calibration that lies between the estimates of Ireland, 2003 and Boneva, Braun, and Waki, 2016 of 162 and 495 for the parameter, respectively.

The target net inflation rate is set equal to zero (a stable price level) and the Taylor coefficient is set to $\phi_\pi = 1.5$, which is a standard value in New Keynesian models. The long-run labor income tax rate is set to $\tau_w = 0.20$ following existing studies such as Mertens and Ravn, 2014 and Boneva, Braun, and Waki, 2016. These calibrations in total yield a consumption-to-output ratio of $c_{TSS}/y_{TSS} = 0.83$ in the TSS.

1.3 Preventing ELT with nonfundamental shocks

In this section, we introduce nonfundamental shocks that bring the economy into an LT without any changes in the fundamentals and characterize analytically the conditions under which the fiscal authority can prevent the ELT from arising.

1.3.1 Nonfundamental regime shocks

Given that our primary interest is to investigate how we can design a fiscal policy to prevent an ELT, we abstract from fundamental shocks and assume that there are only two regimes in the economy: one is the “targeted regime” where inflation is near the central bank's target, while the other is the "unintended regime" where the central bank misses its inflation target and the interest rate is stuck at zero.

There is a nonfundamental shock s_t or "regime shock" that follows a two-state Markov process. The economy is in the "targeted regime" if $s_t = T$ and in the "unintended regime" if $s_t = U$. Regime shock s_t is revealed at the beginning of the period, which is observed by the household and firms. Private agents coordinate their decisions and therefore their information sets when forming expectations include the current realization of s_t. The transition probability is as follows:

$$\text{Prob}(s_t = T | s_{t-1} = T) = p_T, \tag{1.40}$$

$$\text{Prob}(s_t = U | s_{t-1} = U) = p_U. \tag{1.41}$$

Equilibrium inflation and output are denoted by T in the targeted regime and U in the unintended regime.

In general, the Taylor principle is not satisfied in the ELT because the nominal interest rate is constant at zero. Therefore, local indeterminacy may arise and an infinite number of equilibria can exist. One approach to handle this local indeterminacy is to introduce additional sunspot shocks as in Hirose, 2020. However, because the goal of this chapter is to prevent the ELT from arising and indeterminacy will not occur if it is achieved, we rule out such equilibria from our analysis.

1.3.2 Equilibrium inflation and output

We assume that the targeted regime is absorbing and impose the restriction $p_T = 1$ on the transition probability. The advantage of an absorbing steady-state assumption is that it allows a graphical representation of the relation between the PC and the EE. This assumption is relaxed in the following section.

Whether an equilibrium exists in the unintended regime depends on the calibration. Let us first assume that we do not exclude the unintended regime from the equilibrium. In the targeted regime, the inflation rate is close to the target and the ZLB on the interest rate does not bind, while inflation is low and the interest rate remains stuck at zero in the unintended regime. We state these assumptions more formally as

$$\hat{\pi}_T \geq \frac{\log \beta}{\phi_\pi} \text{ and } \hat{i}_T = \phi_\pi \hat{\pi}_T, \tag{1.42}$$

$$\hat{\pi}_U < \frac{\log \beta}{\phi_\pi} \text{ and } \hat{i}_U = \log \beta. \tag{1.43}$$

When the targeted regime is absorbing, equilibrium inflation and output in the targeted regime are equivalent to those in the deterministic TSS, i.e., $\hat{y}_T = \hat{y}_{TSS} = 0$ and $\hat{\pi}_T = \hat{\pi}_{TSS} = 0$. This indicates that the first assumption (1.42) is satisfied in the targeted regime.

We obtain equilibrium inflation and output in the unintended regime by solving for the intersections of the EE and PC:

$$\hat{y}_U = \xi \hat{\pi}_U - \frac{1}{\sigma} \max[\log \beta, \phi_\pi \hat{\pi}_U] + p_U \left(\hat{y}_U + \frac{1}{\sigma} \hat{\pi}_U - \xi \hat{\pi}_U \right), \tag{1.44}$$

$$\hat{\pi}_U = \kappa \hat{y}_U - \zeta \hat{\pi}_U + \beta p_U \hat{\pi}_U. \tag{1.45}$$

The EE divides into two sections. When the ZLB does not bind, the Taylor rule is active with the EE expressed as

$$\hat{y}_U = \left[\frac{1}{\sigma} \frac{p_U - \phi_\pi}{(1 - p_U)} + \xi \right] \hat{\pi}_U. \tag{1.46}$$

Conversely, when the ZLB binds, the Taylor rule is inactive with the EE expressed as

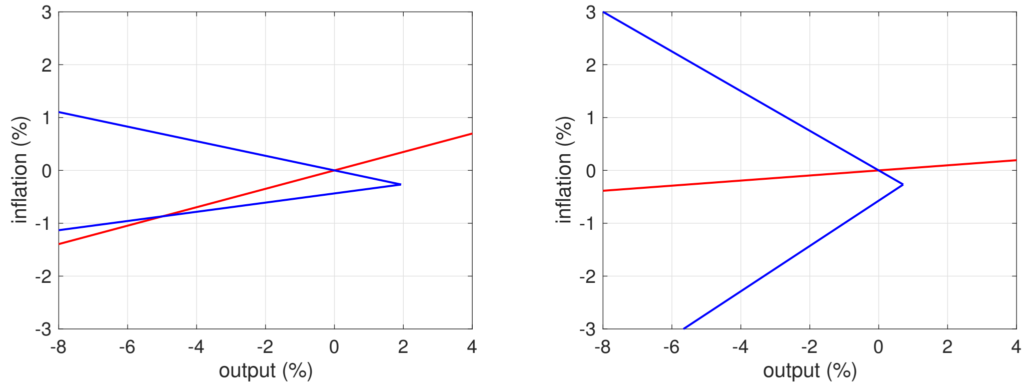
$$\hat{y}_U = -\frac{1}{\sigma} \frac{1}{(1 - p_U)} \log \beta + \left[\frac{1}{\sigma} \frac{p_U}{(1 - p_U)} + \xi \right] \hat{\pi}_U. \tag{1.47}$$

Assuming that condition (1.43) is satisfied, inflation and output in the unintended regime can be obtained by solving equations (1.45) and (1.47) as

$$\hat{\pi}_U = \frac{\frac{\log \beta}{1 - p_U} \frac{\kappa}{\sigma}}{\Lambda - (1 - \beta p_U) + \frac{p_U}{1 - p_U} \frac{\kappa}{\sigma}}, \tag{1.48}$$

$$\hat{y}_U = \frac{1 - p_U \beta + \zeta}{\kappa} \hat{\pi}_U. \tag{1.49}$$

Let us consider the case where the tax rate does not respond to inflation at all ($\lambda_w = 0$). Figure 1.1 illustrates the PC and the kinked EE in the unintended regime



(A) EE and PC in the unintended regime ($p_U = 0.92$ and $\lambda_w = 0$).

(B) An example where the ELT does not exist ($p_U = 0.70$ and $\lambda_w = 0$).

FIGURE 1.1: Euler equation and Philips curve with no policy intervention.

with $p_U = 0.92$. As shown, the EE is downward sloping in the region where the ZLB on the interest rate does not bind. In this case, the Taylor principle is satisfied and the central bank can lower the real interest rate in response to a decline in inflation. In contrast, the EE is upward sloping in the region where the ZLB binds. In this case, the central bank cannot lower its policy rate even if the inflation rate declines, which leads to an increase in the real interest rate.

The PC is upward sloping regardless of the inflation rate, which captures the standard relation that an increase in output creates upward pressure on inflation. The intersection of the PC and the upward-sloping part of the EE is the equilibrium in the unintended regime, which we refer to as the ELT equilibrium in the remainder of the chapter.

Even if the tax rate does not respond to inflation at all, the ELT equilibrium may not exist under certain parameterizations. Figure 1.1 depicts an example in which the ELT equilibrium does not exist. We derive the condition more formally as follows:

Proposition 2. *Assume that the tax rate does not respond to inflation ($\lambda_w = 0$). The ELT equilibrium exists if and only if $\underline{p} < p_U < 1$ is satisfied,*

$$\text{where } \underline{p} = \frac{1}{2}\left(1 + \frac{1}{\beta} + \frac{\kappa}{\sigma\beta}\right) - \sqrt{\frac{1}{4}\left(1 + \frac{1}{\beta} + \frac{\kappa}{\sigma\beta}\right)^2 - \frac{1}{\beta}}.$$

Proof. See Appendix A.2. □

Proposition 2 demonstrates that the existence of the ELT equilibrium is affected by the parameterization of the transition probability p_U. As we can observe from the right-hand side of Figure 1.1, a simultaneous decline in both output and inflation is the key element of the ELT equilibrium. When the probability of returning to the targeted regime is high ($p_U < \underline{p}$) and agents expect to escape from the ELT in a relatively short period, the firm's forward-looking price-setting behavior implies a higher current inflation rate in the ELT equilibrium. Such a higher current inflation rate is not consistent with the depressed output and labor input, which contradicts the existence of the ELT equilibrium.

The prolonged experience of a zero interest rate in many advanced economies suggests that the probability of remaining in the ELT is likely to be sufficiently high. For example, Boneva, Braun, and Waki, 2016 assume $p_U = 0.92$ for their baseline calibration, while Aruoba, Cuba-Borda, and Schorfheide, 2018 select $p_U = 0.95$. Therefore, it is natural to assume that the relevant case in our study is the situation shown on the left-hand side of Figure 1.1, where the probability of remaining in the unintended regime is high and policy intervention is necessary to prevent the ELT equilibrium. In the remaining analysis, we set $p_U = 0.92$ as the benchmark, which is higher than the lower bound $\underline{p} \simeq 0.89$ under our calibration.

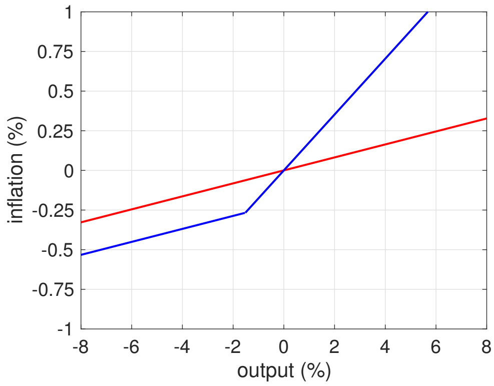
1.3.3 Preventing the ELT equilibrium by adjusting the labor income tax rate

Let us assume that the fiscal authority chooses a suitable tax response parameter λ_w to prevent the ELT equilibrium. Our previous observation that the existence of the ELT equilibrium depends on the model parameterization motivates us to explore whether fiscal policy can play an active role in preventing the ELT equilibrium.

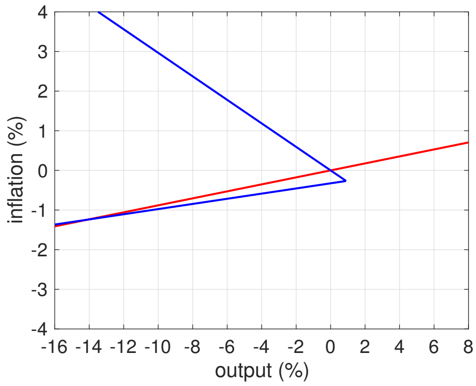
Proposition 3. *The ELT equilibrium does not exist if and only if the fiscal authority sets Λ equal to or lower than the threshold Ψ as*

$$\Lambda \leq 1 - \beta p_U - \frac{\kappa}{\sigma} \frac{p_U}{1 - p_U} \equiv \Psi. \tag{1.50}$$

Proof. See Appendix A.3. □



(A) Preventing the ELT through labor income tax rate adjustment ($\lambda_w = \Psi_w$).



(B) An example where the response of the tax rate is insufficient ($\lambda_w = 0.3\Psi_w$)

FIGURE 1.2: Euler equation and Philips curve with policy intervention.

Proposition 3 demonstrates that the fiscal authority can avoid the ELT equilibrium as long as the tax rate responds sufficiently in response to changes in inflation. Condition (1.50) can be modified as follows:

$$\lambda_w \geq -\frac{\psi}{(\theta-1)\eta}\left(\frac{1}{\eta}\frac{\tau_w}{1-\tau_w} - \frac{\gamma_{\tau,w}}{\gamma_y}\right)^{-1}\Psi \equiv \Psi_w > 0. \tag{1.51}$$

The above inequality shows that as long as the fiscal authority sets the tax response parameter λ_w equal to or higher than the threshold value Ψ_w, the ELT equilibrium does not exist.

The left-hand side of Figure 1.2 depicts the EE and the PC in the unintended regime when λ_w is set equal to Ψ_w. The slopes of the EE and the PC are parallel in the region where the ZLB binds, which indicates that the ELT equilibrium does not exist. The partial derivative $\partial\Psi_w/\partial p_U > 0$ shows that to avoid the ELT equilibrium, the fiscal authority must set the tax response parameter λ_w higher as the probability of remaining in the ELT becomes higher.

How does the labor income tax rate affect the equilibrium inflation and output? As we can observe from equation (1.4), changes in the labor income tax rate alter the effective real wage faced by the representative household and thus change its labor supply. Provided that λ_w is positive, an increase in the inflation rate raises the labor income tax rate, which discourages the household from supplying labor. This reduction in labor supply mitigates the inflationary pressure caused by an increase in the marginal cost and offsets the rise in inflation.

The tax rule also affects the EE because we have assumed that government spending is determined endogenously and changes in the labor income tax rate affect household income. An increase in the inflation rate leads to a rise in the labor income tax rate, and the resultant decline in the labor supply caused by the increase in the tax rate reduces household income. In the region where the ZLB does not bind, in addition to the intertemporal substitution effect caused by the increase in real interest rate, the decrease in income causes current consumption and output to decrease.

In the region where the ZLB binds, the Taylor rule is inactive and an increase in the inflation rate decreases the real interest rate, which induces the household to increase consumption through intertemporal substitution. However, the decrease in income caused by the rise in the labor income tax rate partly offsets this increase in consumption.

As shown in Appendix A.3, Ψ is always negative when $\underline{p} < p_U < 1$ is satisfied. The relation between the transition probability and the threshold level is as follows:

Proposition 4. *The threshold level Ψ is decreasing in transition probability p_U.*

Proof. See Appendix A.4. □

Thus, the greater the probability of remaining at the ELT, the more aggressively the fiscal authority must react to inflation. Namely, the fiscal authority must lower the labor income tax rate to a greater extent if the persistence of the ELT equilibrium becomes higher.

1.3.4 Practical relevance of the tax rule and some caveats

What is the magnitude of the variation in tax rate if the fiscal authority aims to prevent the ELT equilibrium? In the baseline calibration, the fiscal authority is required to set the tax response parameter to $\lambda_w \simeq 48$. Given that one period equals one quarter in the model, $\Delta\hat{\pi}_t = -0.5\%$ is equivalent to a two-percentage-point decline in the annual inflation rate. Because we calibrate the labor income tax rate τ_w to 20 percent, the magnitude of the variation in the tax rate is well within a realistic range:

that is, in response to a two-percentage-point decline in the annual inflation rate, the fiscal authority cuts the labor income tax rate from 20 to 15 percent.[5]

The caveat of the proposed tax rule is that if the response parameter λ_w is not sufficiently large, the fiscal authority not only fails to prevent the ELT equilibrium but also aggravates the declines in inflation and output in the ELT. The right-hand side of Figure 1.2 depicts the case where the policy parameter is set to $\lambda_w = 0.3\Psi_w$, which does not satisfy the condition stated in Proposition 3. Because the tax rate does not sufficiently respond to inflation, the fiscal authority fails to eliminate the private sector's pessimistic expectations and the economy becomes trapped in the ELT. The inflation rate and output are then even lower than in the case where the tax rate does not respond to inflation at all.

1.3.5 Connections with Schmidt, 2016

The key findings discussed so far are closely related to those of Schmidt, 2016. Both studies share the policy implication that a properly designed fiscal policy can prevent the ELT from arising by stopping real marginal cost from falling. However, some significant differences are worth noting.

First, Proposition 3 in this chapter entails both the necessary and sufficient conditions to prevent the ELT, while those presented in Schmidt, 2016 are sufficient conditions. More specifically, in the steady state, we express the nonlinear PC in equation (1.14) as follows:

$$\frac{c_{SS}^{\sigma} y_{SS}^{\eta}}{1-\tau_{w,SS}} - \frac{\theta-1}{\theta} = \frac{\psi}{\theta}(1-\beta)(\Pi_{SS}-1)\Pi_{SS}. \tag{1.52}$$

Schmidt, 2016 proposes a wide class of fiscal rules that prevent ELT by violating the above nonlinear equilibrium condition (1.52), while the present analysis chooses a specific tax rule and derives the condition to prevent ELT by directly working with the solutions of the log-linearized model. As such, this study formally derives the relation between the tax response parameters (Λ) and the persistence of the ELT

[5]The result is comparable to the magnitude of the tax variation proposed in Correia et al., 2013. In their benchmark case, consumption taxes increase from 5 to 14 percent and labor income taxes decrease from 28 to 21 percent to counteract the shock in the discount factor. Note that they also include an investment tax credit in their model, which jumps in the first period to 9 percent and then decreases gradually toward zero.

(p_U), and analytically shows the conditions under which the ELT equilibrium does not exist.

Second, the mechanism to prevent ELTs in Schmidt, 2016 is to create demand through government expenditure; the design of the expenditure rule is such that the marginal cost of labor never declines below a certain threshold, no matter the cause of the decline. However, if financed by the lump-sum tax, the proposed fiscal rule implicitly assumes the government raises its tax collection to match expenditure. In contrast to this demand-side policy, this chapter presents a supply-side policy that encourages the household to supply more labor if a self-fulfilling deflation were to emerge. We can then view the proposed rule as more favorable under the current situation where government debt has become large.

1.4 Preventing recurrent ELT episodes

In the previous section, we confirmed that the proposed tax rule prevents the ELT equilibrium under simplified assumptions. In this section, we relax the assumption that the unintended regime is absorbing and instead assume that the ELT is recurrent.

The recent study by Coyle and Nakata, 2019 considers a model that assumes recurrent ELT episodes and finds that even a small probability of switching back to the ELT can significantly affect the optimal inflation rate. We show that while the qualitative results in the absorbing case carry over to the recurrent case, the fiscal authority must adjust the tax rate more compared with the absorbing case.

1.4.1 Equilibrium inflation and output

We first do not exclude the unintended regime from the equilibrium. In the targeted regime, the inflation rate is close to the target and the ZLB on the interest rate does not bind, while inflation is low and the interest rate is stuck at zero in the unintended regime. We state these assumptions formally as follows:

$$\hat{\pi}_T \geq \frac{\log \beta}{\phi_\pi} \quad \text{and} \quad \hat{i}_T = \phi_\pi \hat{\pi}_T, \tag{1.53}$$

$$\hat{\pi}_U < \frac{\log \beta}{\phi_\pi} \quad \text{and} \quad \hat{i}_U = \log \beta. \tag{1.54}$$

When the ZLB does not bind as in inequality (1.53), the Taylor rule is active. Equilibrium conditions in the targeted regime are then

$$\hat{y}_T = \hat{y}_U - \frac{1}{\sigma}\frac{\phi_\pi - 1}{1 - p_T}\hat{\pi}_T + (\hat{\pi}_T - \hat{\pi}_U)\left(\xi - \frac{1}{\sigma}\right), \tag{1.55}$$

$$(1 - \beta p_T + \zeta)\hat{\pi}_T = \beta(1 - p_T)\hat{\pi}_U + \kappa \hat{y}_T. \tag{1.56}$$

Alternatively, when the ZLB binds as in inequality (1.54), the Taylor rule is inactive. Equilibrium conditions in the unintended regime are then

$$\hat{y}_U = \hat{y}_T - \frac{1}{\sigma}\frac{1}{1 - p_U}\log \beta + \frac{1}{\sigma}\frac{1}{1 - p_U}\hat{\pi}_U + (\hat{\pi}_U - \hat{\pi}_T)\left(\xi - \frac{1}{\sigma}\right), \tag{1.57}$$

$$(1 - \beta p_U + \zeta)\hat{\pi}_U = \beta(1 - p_U)\hat{\pi}_T + \kappa \hat{y}_U. \tag{1.58}$$

The four equations (1.55)–(1.58) comprise the equilibrium conditions for the two regimes.

Whether the above linear system has an equilibrium satisfying both assumptions (1.53) and (1.54) depends on the model parameters. Solving equations (1.55)–(1.58), we derive the following equilibrium inflation rates for each regime:

$$\hat{\pi}_U = \log \beta \frac{\Phi - \Omega\Lambda}{(1 - \Omega)\Lambda + \Upsilon}, \tag{1.59}$$

$$\hat{\pi}_T = \Omega(\hat{\pi}_U - \log \beta), \tag{1.60}$$

where $\Omega \equiv \frac{1}{\phi_\pi - 1}\frac{1 - p_T}{1 - p_U}$, $\Phi \equiv \beta(1 - p_T) + \frac{1 - \beta p_T}{\phi_\pi - 1}\frac{1 - p_T}{1 - p_U} + \frac{\kappa}{\sigma}\frac{1}{1 - p_U}\left(1 + \frac{1 - p_T}{\phi_\pi - 1}\right)$,
$\Upsilon \equiv \frac{\kappa}{\sigma}\frac{1}{1 - p_U}\left(p_U + \frac{1 - p_T}{\phi_\pi - 1}\right) - \beta(1 - p_T) + \frac{1 - \beta p_T}{\phi_\pi - 1}\frac{1 - p_T}{1 - p_U} - (1 - \beta p_U) + \frac{\beta(1 - p_T)}{\phi_\pi - 1}$.

Solutions (1.59) and (1.60) show that when both regimes are recurrent, the equilibrium inflation rate is affected by both transition probabilities p_T and p_U.

1.4.2 Conditions to prevent recurrent ELTs

Let us first assume that the tax rate does not respond to inflation ($\lambda_w = 0$). We can then derive the following condition under which the ELT equilibrium exists.

Proposition 5. *Assume that the tax rate does not respond to inflation ($\lambda_w = 0$). The ELT equilibrium exists if and only if $\Phi\phi_\pi > \Upsilon > 0$ is satisfied.*

Proof. See Appendix A.5. □

The above condition is analogous to that in Proposition 2, which claims that the ELT equilibrium exists only under a certain combination of parameters. In particular, the existence of the ELT equilibrium depends on both probabilities p_T and p_U.

We can derive the condition under which the fiscal authority prevents the ELT.

Proposition 6. *The ELT equilibrium does not exist if and only if the fiscal authority sets Λ equal to or lower than the threshold $\tilde{\Psi}$ as*

$$\Lambda \leq -\frac{\Upsilon}{1-\Omega} \equiv \tilde{\Psi}. \tag{1.61}$$

Proof. See Appendix A.6. □

The condition can be modified as

$$\lambda_w \geq -\frac{\psi}{(\theta-1)\eta}\left(\frac{1}{\eta}\frac{\tau_w}{1-\tau_w} - \frac{\gamma_{\tau,w}}{\gamma_y}\right)^{-1}\tilde{\Psi} \equiv \tilde{\Psi}_w > 0. \tag{1.62}$$

Taking the derivative of $\tilde{\Psi}$ with respect to transition probabilities p_U and p_T, we obtain the following two relations:

$$\frac{\partial \tilde{\Psi}}{\partial p_U} = -\beta - \frac{\kappa}{\sigma}\frac{1}{\left(p_U - 1 + \dfrac{1-p_T}{\phi_\pi - 1}\right)^2}, \tag{1.63}$$

$$\frac{\partial \tilde{\Psi}}{\partial p_T} = -\beta + \frac{\kappa}{\sigma}\frac{\phi_\pi - 1}{[1-p_T+(p_U-1)(\phi_\pi-1)]^2}. \tag{1.64}$$

The former derivative (1.63) is negative for any combination of p_U and p_T, which indicates that the fiscal authority must set λ_w larger as the persistence of the unintended regime becomes longer.

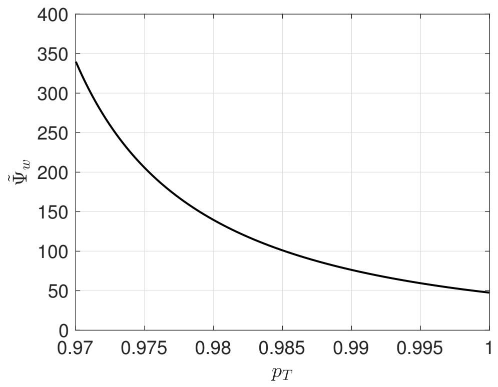


FIGURE 1.3: Threshold value $\tilde{\Psi}_w$ under different transition probabilities p_T.

The latter derivative (1.64) is positive when the following inequality is satisfied:

$$p_T > 1 + (p_U - 1)(\phi_\pi - 1) \equiv \underline{p}_T. \tag{1.65}$$

Under the benchmark calibration of $p_U = 0.92$, the cutoff value is $\underline{p}_T = 0.96$. The inequality shows that the fiscal policy needs to respond to inflation more as the probability of switching back to the unintended regime $(1 - p_T)$ becomes higher.

Figure 1.3 shows how the threshold $\tilde{\Psi}_w$ changes depending on the transition probability p_T. Given $\tilde{\Psi}_w \to \Psi_w$ holds as $p_T \to 1$, the qualitative results in the absorbing case carry over to the case of recurrent ELT episodes. Besides, because p_T is close to one, the size of the tax response parameter is little affected by the recurrence of the ELT equilibrium. For example, $p_T = 0.99$ is associated with the tax response parameter of $\tilde{\Psi}_w \simeq 76$. However, as the probability of switching back to the unintended regime becomes higher and approaches the bifurcation point $p_T \to \underline{p}_T$, $\tilde{\Psi}_w$ increases significantly.

Why do both the frequency and persistence of the unintended regime matter? Intuitively, both a higher frequency $(1 - p_T)$ and a longer persistence (p_U) increase the average duration of remaining in the unintended regime. As the duration becomes longer, agents come to believe that expected inflation remains low on average even in the intended regime. In such a case, a modest drop in current output and inflation suffices for the self-fulfilling deflation to materialize; the fiscal authority should not tolerate even a modest decline in inflation to avoid the ELT. To eliminate such deflationary expectations, the authority must commit to lowering the labor income tax rate more in response to a decline in inflation.

1.5 The role of tax rules with fundamental shocks

In this section, we examine how the inflation-sensitive tax rule performs in the stylized model with fundamental shocks. We show that the proposed tax rule can mitigate the declines in inflation in an LT triggered by a severe fall in the real interest rate, while output falls further if only the labor income tax rate adjusts.

1.5.1 Real interest rate shocks

Following the existing studies on FLTs such as Eggertsson and Woodford, 2003, we assume that the real interest rate r_t^n is stochastic.[6] Let us consider a two-state Markov model where the real interest rate takes $r_t^n = r_H^n$ in the "normal state" and $r_t^n = r_L^n$ in the "crisis state." The transition probability is

$$\text{Prob}(r_t^n = r_H^n | r_{t-1}^n = r_H^n) = p_H^*, \tag{1.66}$$

$$\text{Prob}(r_t^n = r_L^n | r_{t-1}^n = r_L^n) = p_L^*. \tag{1.67}$$

To facilitate comparability with existing work, let us assume that the normal state is absorbing and $p_H^* = 1$ in the remaining analysis.

1.5.2 Equilibrium inflation and output

When the normal state is absorbing, inflation and output in the normal state are equivalent to those in the targeted deterministic steady state, i.e., $\hat{y}_H = \hat{y}_{TSS} = 0$ and $\hat{\pi}_H = \hat{\pi}_{TSS} = 0$. The real interest rate in the normal state is $r_H^n = 0$.

Equilibrium inflation and output in the crisis state can be obtained by solving for the intersections of the EE and the PC with the real interest rate set to $r_t^n = r_L^n$ as

[6] In our model, r_t^n is expressed as the deviation from the steady-state gross real interest rate $R^n = 1/\beta$.

follows:

$$\hat{y}_L = -\frac{1}{\sigma}\max[\log\beta, \phi_\pi \hat{\pi}_L] + \frac{1}{\sigma} r_L^n + \xi \hat{\pi}_L + p_L^* \Big[\hat{y}_L + \frac{1}{\sigma}\hat{\pi}_L - \xi \hat{\pi}_L\Big], \tag{1.68}$$

$$\hat{\pi}_L = p_L^* \beta \hat{\pi}_L + \kappa \hat{y}_L - \zeta \hat{\pi}_L. \tag{1.69}$$

Let us assume that the fall in the real interest rate is sufficiently large and $r_L^n < \log\beta$.

Whether an equilibrium exists in the crisis state depends on the parameterization. Let us first assume that an equilibrium exists in the crisis state. Following similar steps to the case with regime shocks, inflation and output in the crisis state can be solved as

$$\hat{\pi}_L = \begin{cases} \dfrac{\dfrac{-r_L^n}{1-p_L^*}\dfrac{\kappa}{\sigma}}{\Lambda - (1-\beta p_L^*) - \dfrac{\phi_\pi - p_L^*}{1-p_L^*}\dfrac{\kappa}{\sigma}} & \text{if } \hat{\pi}_L \geq \dfrac{\log\beta}{\phi_\pi} \\ \dfrac{\dfrac{\log\beta - r_L^n}{1-p_L^*}\dfrac{\kappa}{\sigma}}{\Lambda - (1-\beta p_L^*) + \dfrac{p_L^*}{1-p_L^*}\dfrac{\kappa}{\sigma}} & \text{if } \hat{\pi}_L < \dfrac{\log\beta}{\phi_\pi} \end{cases}, \tag{1.70}$$

$$\hat{y}_L = \frac{1 - p_L^*\beta + \zeta}{\kappa}\hat{\pi}_L. \tag{1.71}$$

The above solution shows that whether the ZLB binds in the crisis state depends on the choice of the parameters.

Let us first consider the case where the tax rate does not respond to inflation ($\lambda_w = 0$). Similar to the case with regime shocks, we can establish the following proposition.

Proposition 7. *Assume that the tax rate does not respond to inflation ($\lambda_w = 0$). An equilibrium exists in the crisis state if and only if $0 < p_L^* < \underline{p}$. In particular, the equilibrium exhibits an LT if and only if $p_\dagger < p_L^* < \underline{p}$,*

where $p_\dagger = \frac{1}{2}\Big(1 + \frac{1}{\beta} + \frac{\kappa}{\sigma\beta}\Big) - \sqrt{\frac{1}{4}\Big(1 + \frac{1}{\beta} + \frac{\kappa}{\sigma\beta}\Big)^2 - \frac{1}{\beta} - \frac{\kappa}{\sigma\beta}\phi_\pi + \frac{\kappa}{\sigma\beta}\frac{\phi_\pi}{\log\beta} r_L^n}$.

Proof. See Appendix A.7. □

Proposition 7 asserts that p_L^* cannot be too high for the equilibrium to exist in the crisis state: this is a feature often pointed out in the literature. Note that for the equilibrium in both the crisis state and the unintended regime to exist, $p_L^* < \underline{p} < p_U$

must be satisfied. If the decline in the real interest rate is relatively short lived ($0 < p_L^* \leq p_\dagger$), the ZLB may not bind in the crisis state. Given that our primary interest is to explore policies in the LT, we rule out such occasions and restrict our focus to the case where ZLB binds in the crisis state ($p_\dagger < p_L^* < \underline{p}$). As a benchmark, we set the probability of remaining in the crisis state to $p_L^* = 0.85$, which is between the lower bound $p_\dagger \simeq 0.79$ and the upper bound $\underline{p} \simeq 0.89$ under our calibration.

Next, let us assume that the fiscal authority makes a suitable choice on the tax response parameter to avoid the ELT. The following proposition formally establishes the effect of the inflation-sensitive tax rule on equilibrium inflation in the crisis state.

Proposition 8. *If the fiscal authority targets in such a way as to avoid the ELT, the inflation rate in the crisis state is always higher than where the tax rate does not respond to inflation at all.*

Proof. See Appendix A.8. □

Figure 1.4 illustrates the EE and the PC in the crisis state with different tax response parameters. The dotted lines denote the case of $\lambda_w = 0$. In this case, the inflation rate is significantly low and output depressed in the crisis state. The solid lines show the EE and the PC with the response parameters set to $\lambda_w = \Psi_w$, where Ψ_w is chosen to avoid the ELT with the calibration $p_U = 0.92$. Indeed, inflation is higher than the case of no policy intervention in this case. However, the output is significantly depressed in the crisis state with an intervention because a cut in the labor income tax is associated with a decrease in government spending, which leads to a further decrease in aggregate demand.

The above observation shows that to mitigate the decline in both output and inflation in the crisis state, targeting a single tax rate is not sufficient. Similar shortcomings have been pointed out by recent studies that explore effective policies under different LTs. Bilbiie, 2018 summarizes the differences in the policy effects between the two LTs and shows that a temporary tax cut improves welfare in the ELT while its impact is negative in the FLT, which is close to our finding. Cuba-Borda and Singh, 2019 show that none of the major policy measures—government spending, supply shocks, and neo-Fisherian policies—are effective for both types of LTs, while a minimum wage policy can prevent both of the LTs.

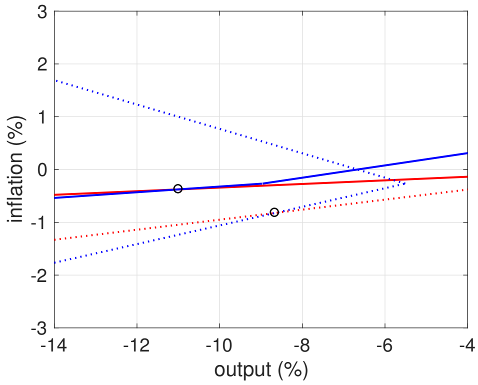


FIGURE 1.4: Euler equation and Philips curve in the crisis state

Note: Dotted lines represent the EE and the PC with $\lambda_w = 0$ and the solid lines the EE and the PC with $\lambda_w = \Psi_w$. Black circles indicate the intersections.

In Appendix B, we show that the fiscal authority can achieve both higher output and inflation in the crisis state if it combines different tax instruments appropriately. More concretely, fiscal policy can prevent the ELT equilibrium and improve welfare in the crisis state by affecting both household consumption demand and labor supply. However, the fiscal authority must lower the labor income tax rate on one hand and raise the dividend (or consumption) tax rate on the other hand to improve both inflation and output.

1.6 Conclusion

In this chapter, we showed that a simple tax rule that responds to inflation could prevent an economy from falling into an ELT, and investigated analytically the conditions under which the proposed tax rule can prevent the ELT. The study also investigated the effect on the fiscal rule when the ELT episodes are recurrent.

While this study shows that the proposed tax rule can improve allocations in ELTs, we have abstracted from other potentially relevant disturbances that may affect the welfare, such as cost-push and policy shocks. Given that this study relies on the use of distortionary taxes, desirable policies may differ when alternative shocks impact the economy.

The primary mechanism through which the fiscal authority prevents the ELT is by inducing the household to increase its labor supply when the inflation rate declines. Recent studies such as Schmitt-Grohé and Uribe, 2017 and Cuba-Borda and

Singh, 2019 assume inelastic labor supply. Under such cases, a minimum wage policy that installs a lower bound on deflation can be more effective than that affecting the marginal behavior of the household. Introducing other relevant frictions such as downward nominal wage rigidity may also affect our results. We defer such an in-depth investigation with different assumptions to future work.

Chapter 2

The Role of Nonlinearity in Indeterminate Models: An Application to Expectations-Driven Liquidity Traps

2.1 Introduction

Modern macroeconomic models represented by dynamic stochastic general equilibrium (DSGE) models are built on the premise that prices and allocations are uniquely determined by fundamental elements, such as technology, preferences, policy disturbances, etc. Such uniqueness of economic outcomes is referred to as model determinacy and plays a central role in macroeconomic modeling. At the same time, it is well known that particular constraints must be imposed on economic activity to ensure determinacy; otherwise, numerous prices and allocations emerge corresponding to a set of fundamental variables, which leads to a state of so-called indeterminacy.

When the economy faces indeterminacy, multiple equilibria arise and economic outcomes can be affected by nonfundamental elements. These nonfundamental elements are often expressed as "sunspots" or "animal spirits" and have been incorporated into economic models. How these nonfundamental elements affect economic activity has been an active research area since the seminal paper of Cass and Shell, 1983, in which the authors show that sunspots matter for real allocations under certain conditions.

One of the typical examples where a particular constraint must be imposed to ensure determinacy is the central bank's commitment to fight inflation, which is widely known as the Taylor principle in the standard New Keynesian framework.

The common wisdom that the nominal interest rate must respond more than one for one when the inflation rate deviates from the central bank's target is viewed as playing an essential role in stabilizing the economy. When the Taylor principle is violated, the economy suffers indeterminacy, and an infinite number of equilibrium paths converging to the steady state arise.

In recent years, most advanced economies have been confined to a situation where the Taylor principle is violated, namely the liquidity trap. As the interest rate is stuck at the ZLB, central banks have been unable to respond to changes in the inflation rate for a substantially extended period. Although many central banks have departed from conventional monetary policy that manipulates short-term interest rates and have adopted unconventional monetary policies, there is little room for such policies to react to inflation substantially. Such a state allows economic agents to form inflation expectations inconsistent with the central bank's long-run target and leads to indeterminacy.

The existence of the ZLB, at the same time, is known to generate nonlinearity in economic agents' behavior. As the central bank cannot lower the interest rate below the ZLB, a decrease in the inflation rate is associated with an increase in the real interest rate. As discussed by Fernández-Villaverde et al., 2015, a rise in the real interest rate aggravates the economic outcome by putting further downward pressure on output and inflation in a nonlinear manner at the ZLB; thus appropriately modeling the nonlinearity arising from the ZLB is considered to be important.

Although liquidity traps are the source of both indeterminacy and nonlinearity, there has been little investigation of the dynamics of a nonlinear indeterminate system. The aim of this chapter is to fill this gap regarding these two key aspects of liquidity traps. To this end, we present a novel methodology to derive a nonlinear solution of an indeterminate DSGE model in which the decision rules are affected by sunspot shocks. We consider sunspot shocks in a locally indeterminate nonlinear system and show that the nonlinear solutions can be derived by incorporating an auxiliary equation and an auxiliary variable proposed by the recent work of Bianchi and Nicolò, forthcoming into the projection method.

As an application of our newly developed solution method, we first consider a simple case in which the Taylor principle is not satisfied because of a passive monetary policy. We find that the intuition in the linear model carries over to the nonlinear model and the advantages of solving the model nonlinearly are limited. This is because it is extremely rare for the ZLB to be reached when monetary policy is passive, and such infrequent cases are not considered in deriving the nonlinear solutions. Therefore, the similarity of the two solutions can be attributed to the fact that the model is almost linear when the ZLB is not binding.

We then solve the model around the ELT—a liquidity trap that arises because of the ZLB constraint on the nominal interest rate and the de-anchoring of economic agents' expectations—and show that nonlinearity plays a significant role in the model dynamics. Nonlinearity emerges because the ZLB ceases to bind once the inflation rate increases because of a temporary rise in inflation expectations. These findings provide important insights into monetary policy conduct because inflation and consumption may temporarily increase to a level that lifts the interest rate above zero even if agents believe that the economy converges to a deflationary state in the long run.

Although extant studies have explored nonlinearity and indeterminacy arising in liquidity traps separately, none have succeeded in combining these two important elements. This study is the first to combine these two elements and derive a nonlinear solution that allows nonfundamental sunspot shocks to affect prices and allocations. Therefore, this study contributes significantly to the literature by linking these two key elements of liquidity traps; as such, it can be related to two different strands of the literature.

The first strand involves studies that focus on the effects of sunspot shocks on economic activity when the model exhibits indeterminacy. The seminal paper of Cass and Shell, 1983 studies how sunspots play a role in equilibrium allocation in both static and dynamic models and shows the conditions under which sunspots matter. Farmer and Guo, 1994 study a model with an aggregate technology that is subject to increasing returns and show that investors' "animal spirits" can generate business cycle fluctuations. The recent work by Farmer, 2019 provides a comprehensive survey of models featuring indeterminacy and sunspots.

As this study develops a method to derive nonlinear solutions of an indeterminate model, it is closely related to studies that explore methods to solve and estimate indeterminate models. The pioneering work of Lubik and Schorfheide, 2004 presents a methodology to solve and estimate an indeterminate model and applies it to US data. Farmer, Khramov, and Nicolò, 2015 propose a detailed methodology to solve linear indeterminate models and show how it could be applied to existing software packages. More recently, Bianchi and Nicolò, forthcoming propose a novel methodology to solve linear indeterminate models by introducing auxiliary equations and variables and apply their new methodology to a DSGE model with bubbles in the setup of Galí, forthcoming. They find that the US data support the presence of two degrees of indeterminacy, implying that the central bank was not reacting strongly enough to the bubble component.

The second strand comprises studies focusing on liquidity traps. Among liquidity traps arising from different causes, ELTs, which were first investigated in depth in the seminal paper of Benhabib, Schmitt-Grohé, and Uribe, 2001, have attracted attention from both empirical and theoretical perspectives. On the empirical side, Aruoba, Cuba-Borda, and Schorfheide, 2018 investigate whether the US and Japan have transitioned to a deflationary regime using a nonlinear DSGE model and suggest that Japan is likely to have moved to such a regime in the late 1990s, while it is unlikely for the US. As we discuss later, Aruoba, Cuba-Borda, and Schorfheide, 2018 select a particular solution and abstract from indeterminacy arising because of the ZLB. Hirose, 2020 adopts the method proposed by Bianchi and Nicolò, forthcoming and estimates a linear indeterminate DSGE model around an ELT using Japanese data.

On the theoretical side, recent studies have emphasized how fiscal policies can be implemented to deal with ELTs. Studies such as Benhabib, Schmitt-Grohé, and Uribe, 2002, Schmidt, 2016, and Tamanyu, 2019 focus on the use of fiscal policies to prevent ELTs. Other recent studies, such as Bilbiie, 2018 and Nakata and Schmidt, 2019 compare how monetary and fiscal policies can be implemented to confront ELTs.

As our findings highlight the importance of considering nonlinearity when the economy is in a liquidity trap, they can be related to the recent literature that investigates how model dynamics are affected by the existence of the ZLB. Fernández-Villaverde et al., 2015 argue for the importance of explicitly considering nonlinearities in a model that faces the ZLB and derive nonlinear decision rules using projection methods. Richter and Throckmorton, 2015 show that a tradeoff exists between the numerical convergence of a particular solution algorithm and the expected frequency and average duration of the ZLB events. Atkinson, Richter, and Throckmorton, 2020 compare the difference between a full nonlinear solution and a piecewise linear solution and find that there is a large practical advantage in using the latter.

The remainder of this chapter is organized as follows. In Section 2.2, the details of the model are provided. In Section 2.3, by considering passive monetary policy as an example, we present the new methodology to derive nonlinear solutions for indeterminate models. In Section 2.4, we apply our method to the ELT and explore the model dynamics. Section 2.5 concludes.

2.2 The model

As the most basic model suffices to explore the key aspects of the model dynamics under indeterminacy, this study builds on a canonical small-scale New Keynesian DSGE model. The model consists of three equilibrium equations: the downward-sloping demand equation derived from the representative household's optimization problem, the upward-sloping supply equation derived from the firm's optimization problem, and the monetary policy rule constrained by the ZLB.

To model price stickiness, we introduce price adjustment costs à la Rotemberg, 1982. As Rotemberg, 1982 pricing does not require an additional state variable, it is preferred in the studies concerned with nonlinear solution methods. In the following subsections, we provide the details of the model.

2.2.1 Household

There is a representative household that gains utility from consumption and disutility from labor supply. The household maximizes expected lifetime utility by choice of consumption c_t, labor supply l_t, and bond holdings b_t given prices and subject to a budget constraint as follows:

$$\max_{\{c_{t+s},l_{t+s},b_{t+s}\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \Big[\frac{c_{t+s}^{1-\sigma}-1}{1-\sigma} - \frac{l_{t+s}^{\eta+1}-1}{\eta+1} \Big], \tag{2.1}$$

$$\text{s.t.} \quad c_t + \frac{b_t}{R_t} = w_t l_t + \frac{b_{t-1}}{\Pi_t} + d_t. \tag{2.2}$$

R_t and Π_t are the gross nominal interest rate and the gross inflation rate respectively. w_t is the real wage and d_t is a dividend from intermediate goods firms. From the first-order conditions, we can derive the Euler equation and the wage equation as follows:

$$c_t^{-\sigma} = \beta R_t \mathbb{E}_t \Big[\frac{c_{t+1}^{-\sigma}}{\Pi_{t+1}} \Big], \tag{2.3}$$

$$\frac{c_t^{-\sigma}}{l_t^{\eta}} = \frac{1}{w_t}. \tag{2.4}$$

2.2.2 Firms

There are two types of firms in the economy: a continuum of intermediate goods producers and a final goods producer. The final goods producer uses intermediate goods as the only input and has CES production technology. The final goods producer is perfectly competitive and takes both output and input prices as given. The static profit maximization problem is given as follows:

$$\max_{\{y_t,y_{i,t}\}} P_t y_t - \int_0^1 P_{i,t} y_{i,t} di, \tag{2.5}$$

$$\text{s.t.} \quad y_t = \Big(\int_0^1 y_{i,t}^{\frac{\theta-1}{\theta}} di \Big)^{\frac{\theta}{\theta-1}}. \tag{2.6}$$

Perfect competition drives final good producers' profits to zero. From the first-order conditions, we can derive the demand for intermediate goods and the associated price index:

$$y_{i,t} = \Big(\frac{P_{i,t}}{P_{t+s}} \Big)^{-\theta} y_t, \tag{2.7}$$

$$P_t = \Big(\int_0^1 P_{i,t}^{1-\theta} di \Big)^{\frac{1}{1-\theta}}. \tag{2.8}$$

There is a continuum of intermediate goods producers indexed by i. They are monopolistically competitive and incur quadratic price adjustment costs as in Rotemberg, 1982. Each producer uses labor as an input in production. Firm i chooses optimal price $P_{i,t}$ and labor input $l_{i,t}$ given the current aggregate output y_t and aggregate price level P_t. It maximizes the present value of discounted dividends $d_{i,t}$ according to the following optimization problem:

$$\max_{\{y_{i,t+s}, P_{i,t+s}, l_{i,t+s}\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} Q_{t+s} d_{i,t+s}, \tag{2.9}$$

$$\text{s.t.} \quad d_{i,t+s} = \frac{P_{i,t+s}}{P_{t+s}} y_{i,t+s} - w_{t+s} l_{i,t+s} - \frac{\psi}{2}\Big(\frac{P_{i,t+s}}{P_{i,t+s-1}} - \Pi^*\Big)^2 y_{t+s}, \tag{2.10}$$

$$y_{i,t+s} = A_t l_{i,t+s}, \tag{2.11}$$

$$y_{i,t+s} = \Big(\frac{P_{i,t+s}}{P_{t+s}}\Big)^{-\theta} y_{t+s}, \tag{2.12}$$

where the real stochastic discount factor is defined as

$$Q_{t+s} \equiv \beta^s c_{t+s}^{-\sigma}. \tag{2.13}$$

Productivity is determined exogenously as

$$A_t = A_{t-1}^{\rho_a} \exp(\varepsilon_{a,t}), \quad i.i.d.\ \varepsilon_{a,t} \sim N(0, \sigma_a^2). \tag{2.14}$$

Combining the first-order conditions and imposing symmetry across firms, we derive the following Phillips curve:

$$\psi(\Pi_t - \Pi^*)\Pi_t - \theta\frac{w_t}{A_t} + \theta - 1 = \beta\mathbb{E}_t\Big[\Big(\frac{c_t}{c_{t+1}}\Big)^{\sigma}\Big(\frac{y_{t+1}}{y_t}\Big)\psi(\Pi_{t+1} - \Pi^*)\Pi_{t+1}\Big]. \tag{2.15}$$

The aggregate production function and dividend payouts are

$$y_t = A_t l_t, \tag{2.16}$$

$$d_t = y_t - w_t l_t - \frac{\psi}{2}(\Pi_t - \Pi^*)^2 y_t. \tag{2.17}$$

2.2.3 Central bank

The central bank sets the interest rate following the standard Taylor rule where the net nominal interest rate is bounded below by zero as follows:

$$R_t = \max\left[1, \frac{\Pi^*}{\beta}\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\exp(\varepsilon_{m,t})\right], \quad i.i.d.\ \varepsilon_{m,t} \sim N(0, \sigma_m^2). \tag{2.18}$$

We abstract from government spending for simplicity. Thus $b_t = 0$ holds for all t from Ricardian equivalence.

2.2.4 Equilibrium conditions

The resource constraint of the economy is derived by combining equations (2.2) and (2.17) as follows:

$$c_t + \frac{\psi}{2}(\Pi_t - \Pi^*)^2 y_t = y_t. \tag{2.19}$$

Equations (2.3), (2.4), (2.15), (2.16), (2.18), and the resource constraint (2.19) are the equilibrium conditions. The nonlinear equilibrium conditions can be summarized as the following two equations:

$$1 = \max\left[1, \frac{\Pi^*}{\beta}\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\exp(\varepsilon_{m,t})\right]\beta\mathbb{E}_t\left[\frac{1}{\Pi_{t+1}}\left(\frac{\{1-\psi(\Pi_{t+1}-\Pi^*)^2/2\}y_{t+1}}{\{1-\psi(\Pi_t-\Pi^*)^2/2\}y_t}\right)^{-\sigma}\right], \tag{2.20}$$

$$\left(\frac{y_t}{A_t}\right)^{\eta}\left[\left\{1-\frac{\psi}{2}(\Pi_t-\Pi^*)^2\right\}y_t\right]^{\sigma} - \frac{\theta-1}{\theta} = \frac{\psi}{\theta}(\Pi_t-\Pi^*)\Pi_t$$
$$-\frac{\psi}{\theta}\beta?\mathbb{E}_t\left[\left(\frac{\{1-\psi(\Pi_{t+1}-\Pi^*)^2/2\}y_{t+1}}{\{1-\psi(\Pi_t-\Pi^*)^2/2\}y_t}\right)^{-\sigma}(\Pi_{t+1}-\Pi^*)\Pi_{t+1}\frac{y_{t+1}}{y_t}\right]. \tag{2.21}$$

Let us denote the targeted steady state as TSS. The steady state values in the TSS can be derived as follows:

$$\Pi_{TSS} = \Pi^*, \tag{2.22}$$
$$R_{TSS} = \frac{\Pi^*}{\beta}, \tag{2.23}$$
$$y_{TSS} = c_{TSS} = \left(\frac{\theta-1}{\theta}\right)^{\frac{1}{\sigma+\eta}}. \tag{2.24}$$

When the Taylor principle is satisfied ($\phi_\pi > 1$), there is another steady state that exhibits deflation, which we call the unintended steady state (USS). The steady state values in the USS are as follows:

$$\Pi_{USS} = \beta, \tag{2.25}$$

$$R_{USS} = 1, \tag{2.26}$$

$$y_{USS} = \left[1 - \frac{\psi}{2}(\Pi^* - \beta)^2\right]^{-\frac{\sigma}{\sigma+\eta}} \left[\frac{\theta - 1}{\theta} - \frac{\psi\beta}{\theta}(1-\beta)(\Pi^* - \beta)\right]^{\frac{1}{\sigma+\eta}}, \tag{2.27}$$

$$c_{USS} = \left[1 - \frac{\psi}{2}(\Pi^* - \beta)^2\right]^{\frac{\eta}{\sigma+\eta}} \left[\frac{\theta - 1}{\theta} - \frac{\psi\beta}{\theta}(1-\beta)(\Pi^* - \beta)\right]^{\frac{1}{\sigma+\eta}}. \tag{2.28}$$

It is clear that the consumption level in the USS is lower than that in the TSS ($c_{USS} < c_{TSS}$) because there is a loss from the price adjustment cost in the USS.

2.2.5 Calibration

It is assumed that the model period corresponds to a quarter. The discount factor is set to $\beta = 0.99$, which yields an annual real interest rate of four percent. We set the elasticity of intertemporal substitution to $\sigma = 1$ and the Frisch elasticity of labor to $\eta = 1$, which yield log utility and linear disutility, respectively. The elasticity of substitution between intermediate goods is set to $\theta = 6$, which yields a markup of 20 percent. The price adjustment cost is set to $\psi = 58$, which is chosen to match the price stickiness of $\omega = 0.75$ under Calvo, 1983 price stickiness.[1] The parameters regarding the stochastic processes are set to $\rho_a = 0.9$ and $\sigma_a = 0.0025$ for productivity shocks, $\sigma_m = 0.001$ for monetary policy shocks, and $\sigma_v = 0.0025$ for sunspot shocks.

The target net inflation rate is set equal to zero, a stable price level. As for the Taylor coefficient, we set to $\phi_\pi = 1.5$ for the active case and $\phi_\pi = 0.5$ for the passive case.

2.3 Indeterminacy arising from passive monetary policy

This section presents the methodology to derive a nonlinear solution of an indeterminate model. As an application of the method, we first explore a case where the

[1] In a linearized model with zero steady state inflation, either assuming Rotemberg, 1982 or Calvo, 1983 price stickiness yields identical Phillips curves when the parameter is chosen to satisfy $\psi = \omega(\theta - 1)/[(1-\omega)(1-\beta\omega)]$.

Taylor coefficient of the interest rate rule is smaller than one ($\phi_\pi = 0.5$) and therefore exhibits indeterminacy.

We first investigate the properties of the solution of the linear indeterminate model using the stylized three-equation model. Then, following the intuition obtained in the linear case, we present how to derive nonlinear solutions of an indeterminate model and apply the method to the case of passive monetary policy.

2.3.1 Decision rules of linear indeterminate models: the case of the minimal state variable (MSV)

Let us begin our analysis by first investigating the dynamics of the linear model. By log-linearizing the equilibrium conditions (2.3), (2.15), and (2.18) around the TSS, we can obtain the stylized three-equation model as follows:

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \sigma^{-1}(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}), \tag{2.29}$$

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa \hat{y}_t, \tag{2.30}$$

$$\hat{R}_t = \phi_\pi \hat{\pi}_t + \varepsilon_{m,t}, \tag{2.31}$$

where $\kappa \equiv (\theta - 1)(\sigma + \eta)/\psi$. Variables with hats denote the log deviation from the TSS. Monetary policy shock ($\varepsilon_{m,t}$) is included as an exogenous disturbance.

When monetary policy is passive ($\phi_\pi < 1$) and the Taylor principle is not satisfied, the model exhibits indeterminacy. In this case, techniques proposed by Blanchard and Kahn, 1980 are not applicable to derive the decision rules because the number of stable roots does not equal the number of eigenvalues outside the unit circle.[2]

Even if the Blanchard and Kahn, 1980 conditions are not satisfied, we can obtain a particular solution based on the MSV criteria as discussed by McCallum, 1999. As the model is linearized around the steady state and the monetary policy shock is the only exogenous shock, we can conjecture that the MSV decision rules are linear

[2] Functions that solve a set of equilibrium conditions and map state variables onto control variables are often called policy functions. In this study, we call such functions decision rules of the economic agents following Fernández-Villaverde, Rubio-Ramírez, and Schorfheide, 2016.

functions of the exogenous process in the following form:

$$\hat{y}_t = A_0 + A_1 \varepsilon_{m,t}, \tag{2.32}$$

$$\hat{\pi}_t = B_0 + B_1 \varepsilon_{m,t}. \tag{2.33}$$

Substituting the above conjecture into the equilibrium conditions, we can derive the MSV decision rules as follows:

$$\hat{y}_t = -(\sigma + \kappa\phi_\pi)^{-1} \varepsilon_{m,t}, \tag{2.34}$$

$$\hat{\pi}_t = -\kappa(\sigma + \kappa\phi_\pi)^{-1} \varepsilon_{m,t}. \tag{2.35}$$

The decision rules expressed by equations (2.34) and (2.35) are one particular solution to the equilibrium conditions given by (2.29)–(2.31).

Figure 2.1 shows the impulse responses of output, inflation, and interest rate to a monetary policy shock in the case of the MSV solution. Both output and inflation rate respond negatively to a positive monetary policy shock because the real interest rate increases in response to monetary tightening and the household decreases its consumption. All the variables respond simultaneously to the shock and we do not observe any persistence in the dynamics.

As we have derived the decision rules by the so-called "guess and verify" method, the MSV decision rules solve the equilibrium conditions as if the system were determinate. The intuition of the MSV decision rules is that although nonfundamental sunspot shocks can potentially affect prices and allocations, agents coordinate to respond only against fundamental shocks. This MSV solution is often adopted in existing studies on ELTs because the researcher can work with fewer variables, which simplifies the analysis. As we will see in the next subsection, however, a larger set of solutions arises when the system is indeterminate.

2.3.2 Decision rules of linear indeterminate models: the case with sunspots

Let us derive the decision rules that allow sunspot shocks to affect prices and allocations. Existing studies such as Lubik and Schorfheide, 2003 and Farmer, Khramov,

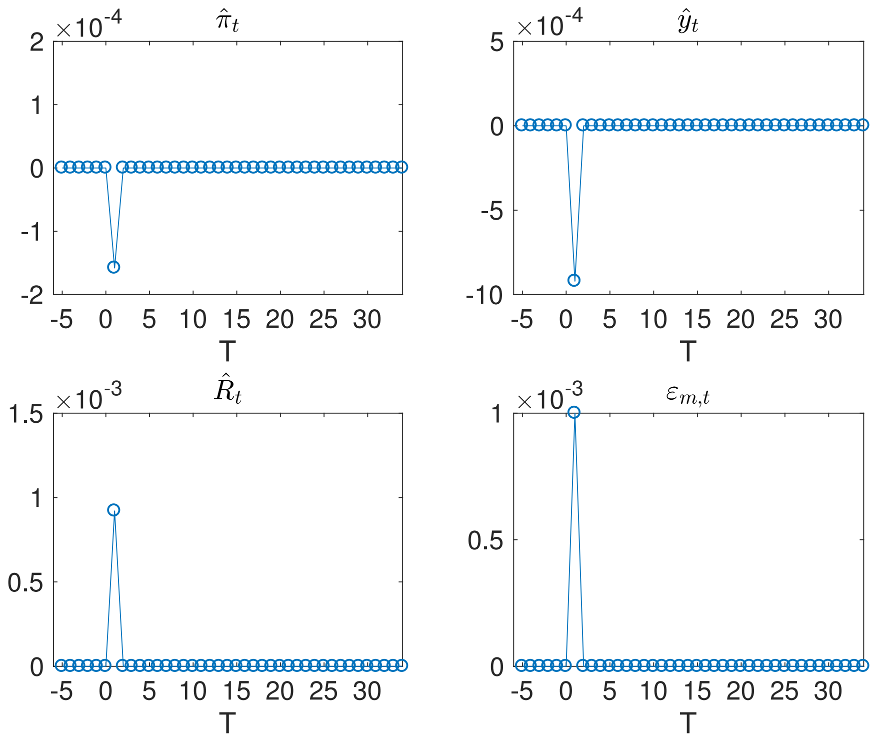


FIGURE 2.1: Impulse responses to a monetary policy shock (linear MSV case).

and Nicolò, 2015 propose methods to derive a complete set of solutions of linear indeterminate models. Along with the above studies, the recent work by Bianchi and Nicolò, forthcoming proposes a solution method that introduces an auxiliary variable $\hat{\omega}_t$ and converts an indeterminate system to a determinate system. For the case of the three-equation model, the auxiliary equation is introduced as follows:

$$\hat{\omega}_t = \frac{1}{\alpha}\hat{\omega}_{t-1} + \nu_t - \eta_t, \tag{2.36}$$

$$\text{where } \hat{\pi}_t = \mathbb{E}_{t-1}\hat{\pi}_t + \eta_t, \tag{2.37}$$

where ν_t is a sunspot shock and η_t is an expectational error. In this study, it is assumed that ν_t is white noise and is individually, identically, and normally distributed with mean zero and standard deviation of σ_ν.[3]

As proposed by Bianchi and Nicolò, forthcoming, the model can be converted to a determinate system when the parameter satisfies $0 < \alpha < 1$. In this case, $\hat{\omega}_t = 0$

[3]Sunspot shocks are often allowed to be correlated with other fundamental shocks. Empirical results in Hirose, 2020 show a significant positive correlation between sunspot shocks and investment adjustment costs and price markup shocks.

must hold for all t for a unique solution to exist, as $\hat{\omega}_t$ follows an explosive path.

Current inflation is affected by inflation expectations in the previous period $\mathbb{E}_{t-1}\hat{\pi}_t$ as well as the sunspot shock ν_t. The solution of the model can be derived as

$$\begin{pmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \mathbb{E}_t\hat{\pi}_{t+1} \end{pmatrix} = G\mathbb{E}_{t-1}\hat{\pi}_t + H \begin{bmatrix} \varepsilon_{m,t} \\ \nu_t \end{bmatrix}, \tag{2.38}$$

where G and H are matrices of parameters defined as

$$G = \begin{pmatrix} -\frac{a_2}{2\kappa} \\ 1 \\ \frac{a_1}{2\beta} \end{pmatrix}, \quad H = \begin{pmatrix} -\frac{2\beta\sigma^{-1}}{a_3} & \frac{2\kappa\sigma^{-1}(1-\beta\phi_\pi)-a_2}{a_3\kappa} \\ 0 & 1 \\ \frac{2\kappa\sigma^{-1}}{a_3} & -\frac{2(1+\kappa\phi_\pi\sigma^{-1})}{a_3} \end{pmatrix},$$

with $a_1 = (\beta - b_1 + \kappa\sigma^{-1} + 1)$, $a_2 = (a_1 - 2)$, $a_3 = (a_1 + 2b_1)$, $b_1 = [(1 + \beta + \kappa\sigma^{-1})^2 - 4\beta(1 + \kappa\phi_\pi\sigma^{-1})]^{-\frac{1}{2}}$, respectively.

The key feature of the decision rules provided by equation (2.38) is that when the original system described by equations (2.29)–(2.31) is indeterminate, an additional variable $\mathbb{E}_{t-1}\hat{\pi}_t$ enters as a state variable. The sunspot shock ν_t captures the temporary deviation in inflation expectations from the fundamentals, which leads to a multiplicity of equilibria.

Figure 2.2 displays the impulse responses of the variables to a positive monetary policy shock for the sunspot case.[4] Two major differences from the MSV case are worth noting. First, current inflation $\hat{\pi}_t$ does not respond to a monetary policy shock $\varepsilon_{m,t}$ on impact (T=1), which can be confirmed from the zero loading in the matrix H. As the current inflation rate $\hat{\pi}_t$ is predetermined by $\mathbb{E}_{t-1}\hat{\pi}_t$ in the previous period, the fundamental shock itself does not affect the inflation rate on impact.[5] Second, output decreases in response to a positive monetary policy shock, while inflation increases with a lag. This contrasts with the results in the MSV case, where the

[4] The impulse responses are computed using Dynare.

[5] Whether the current inflation rate responds to the fundamental shock depends on how the indeterminacy is modeled. We can model the indeterminacy by allowing current consumption to depend on the previous period's expectations as $\hat{c}_t = \mathbb{E}_{t-1}\hat{c}_t + \eta_t$ instead. Under certain conditions, we can show that both inflation and consumption indeterminacy yield identical results for the linear model. However, it is natural to assume that inflation expectations temporarily deviate from the central bank's target and fluctuate according to sunspot shocks.

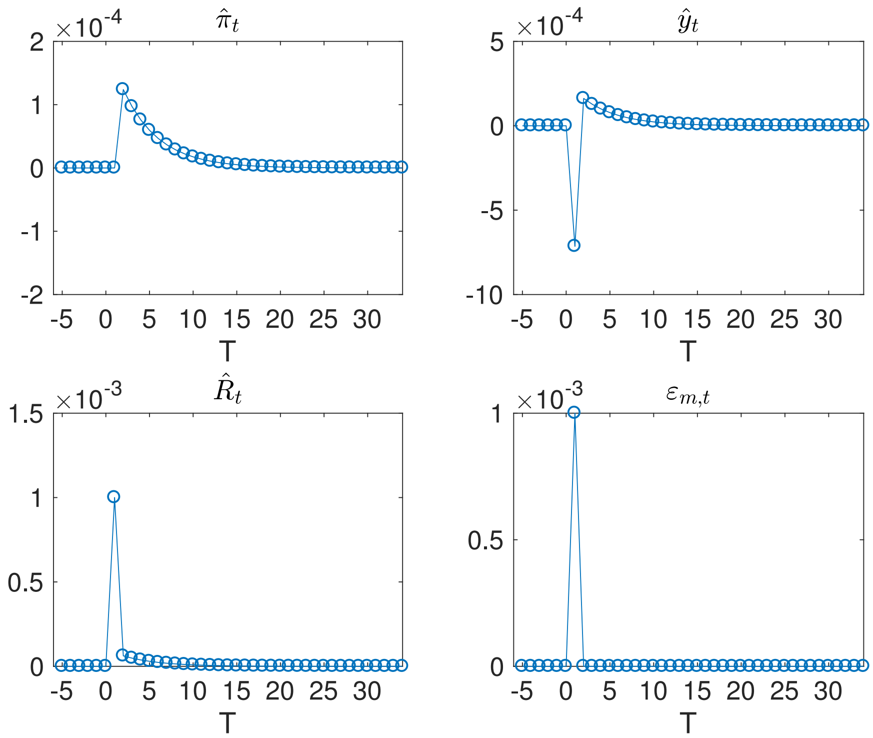


FIGURE 2.2: Impulse responses to a monetary policy shock (linear sunspot case).

inflation rate declines in response to a positive monetary policy shock.

To investigate the role of productivity shocks, we can derive the decision rules by replacing equation (2.30) by the following equation:

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa \hat{y}_t - \frac{(\theta - 1)(1+\eta)}{\psi} a_t. \tag{2.39}$$

The impulse responses of the variables to a positive productivity shock are shown in Figure 2.3 (A). Both inflation rate and output increase in response to a rise in productivity. This is in sharp contrast to the standard determinate case, in which the Taylor principle is satisfied and inflation declines in response to a positive productivity shock. In the indeterminate case, inflation increases because the monetary policy does not respond sufficiently to the inflation rate and the real interest rate decreases in response to an increase in productivity. This induces the household to further increase and overshoot consumption. In addition, inflation is predetermined and does not respond on impact but only with lags. Therefore, its response is hump-shaped and rises only gradually, reaching a peak after several periods.

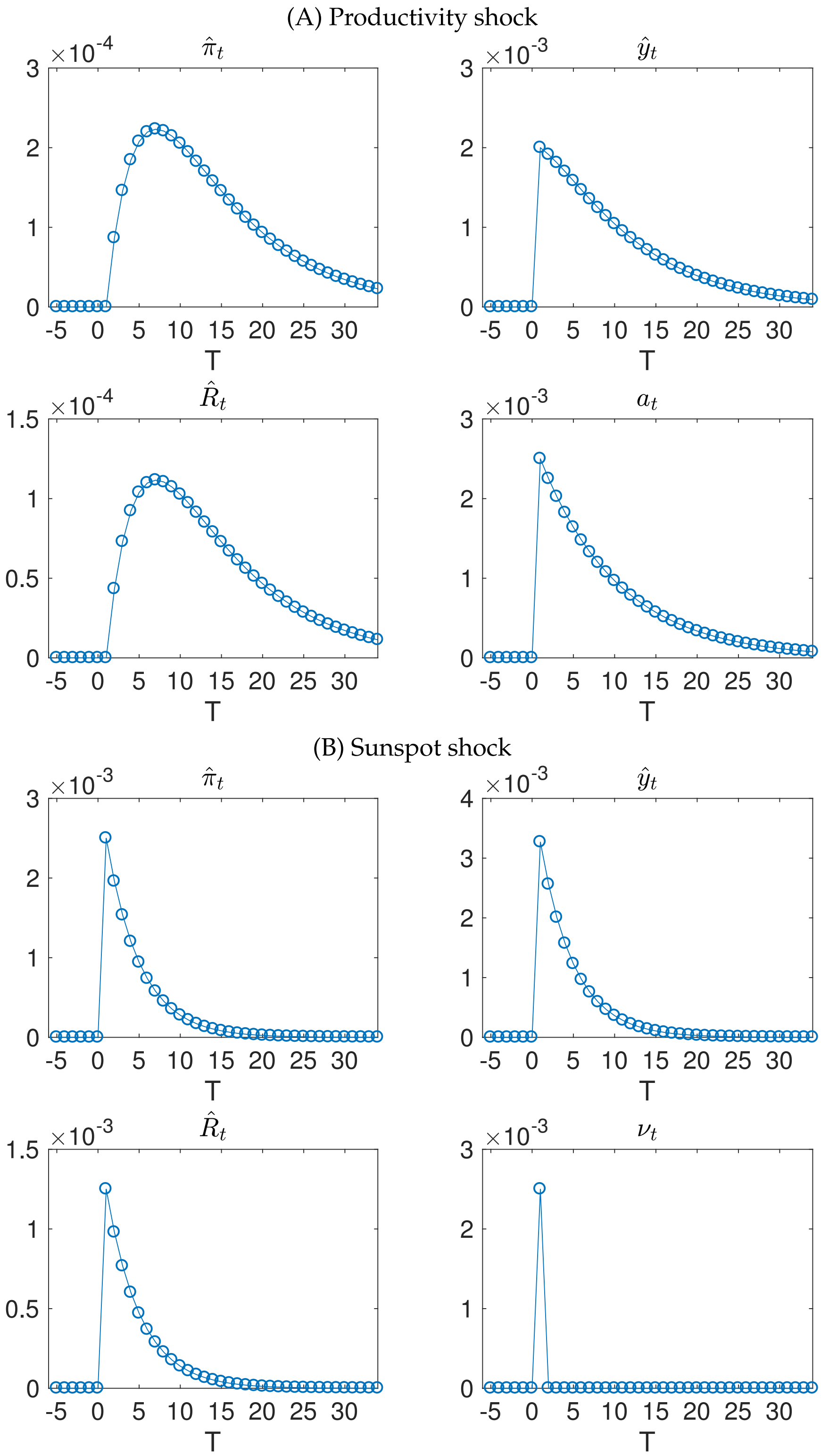


FIGURE 2.3: Impulse responses to different shocks (linear sunspot case).

Figure 2.3 (B) shows the impulse responses to a positive sunspot shock, which increases economic agents' inflation expectations exogenously. We can observe that all variables respond on impact, while the dynamics differ from the stylized determinate models: even though the shock itself is transitory, its impact is persistent. In addition, because the increase in the nominal interest rate is not sufficient to lower the real interest rate, both output and the inflation rate increase because of an increase in household consumption.

2.3.3 Decision rules of nonlinear indeterminate models

Let us now consider the nonlinear solutions. When the system is determinate, current prices and allocations can be pinned down uniquely by fundamental state variables including the exogenous processes as

$$y_t = f^y(X_t), \tag{2.40}$$

$$\Pi_t = f^\pi(X_t), \tag{2.41}$$

where X_t denotes the vector of fundamental state variables.

When the system is indeterminate, however, X_t is not sufficient to pin down current y_t and Π_t uniquely. For example, different prices and allocations may exist depending on nonfundamental variables in addition to the fundamental state variables:

$$y_t = f^y(Y_t, X_t), \tag{2.42}$$

$$\Pi_t = f^\pi(Y_t, X_t), \tag{2.43}$$

where Y_t is a vector of nonfundamental variables.[6]

To derive nonlinear solutions of the indeterminate model, in addition to the nonlinear equilibrium conditions (2.20) and (2.21), we introduce an auxiliary variable ω_t

[6] X_t and Y_t can include past realizations of each state variable.

and an auxiliary equation as follows:

$$\omega_t = \omega_{t-1}^{1/\alpha} \exp(\nu_t) \exp(-\eta_t), \tag{2.44}$$

$$\text{where } \Pi_t = \left(\mathbb{E}_{t-1}\Pi_t\right) \exp(\eta_t). \tag{2.45}$$

When the parameter is chosen to satisfy $0 < \alpha < 1$, the system has a unique solution if and only if $\omega_t = 1$ holds for all t, which corresponds to the case of $\hat{\omega}_t = 0$ in the linear case. Otherwise, equation (2.44) follows either an explosive path or converges to zero, which leads to a violation of the transversality condition. Substituting $\omega_t = 1$ for all periods, the auxiliary equation can be rearranged as

$$\Pi_t = \left(\mathbb{E}_{t-1}\Pi_t\right) \exp(\nu_t). \tag{2.46}$$

Let us consider two exogenous processes, productivity A_t and sunspot shock ν_t on the inflation expectations. Next-period inflation expectations can be considered to be an individual state variable, therefore we introduce a new auxiliary variable Φ_t to denote $\mathbb{E}_t\Pi_{t+1}$. Inflation expectations in the previous period are a predetermined variable. The auxiliary equation can be expressed as

$$\Pi_t = \Phi_{t-1} \exp(\nu_t). \tag{2.47}$$

When the model is indeterminate, the nonlinear decision rules that solve the equilibrium conditions (2.20), (2.21), and (2.47) can be expressed in a general form as follows:

$$\Pi_t = f^{\pi}(\Phi_{t-1}, \nu_t, A_t), \tag{2.48}$$

$$c_t = f^{c}(\Phi_{t-1}, \nu_t, A_t), \tag{2.49}$$

$$\Phi_t = f^{\Phi}(\Phi_{t-1}, \nu_t, A_t), \tag{2.50}$$

where Φ_{t-1} and ν_t are included in Y_t and A_t is included in X_t. The above decision rules are analogous to the linear rules summarized in equation (2.38). Note that the inflation expectations in the previous period Φ_{t-1} enter the decision rules as the nonfundamental predetermined variable.

When we approximate decision rules numerically, the choice of the variable is often crucial to obtain solutions efficiently. On applying the projection method, we consider an auxiliary variable $\mathcal{E}_t \equiv \beta \mathbb{E}_t \left[c_{t+1}^{-\sigma} / \Pi_{t+1} \right]$ instead of deriving the decision rule for consumption c_t. This is because consumption c_t is known to exhibit kinks when the ZLB binds, making it difficult to approximate the decision rules. $\mathcal{E}_t$, on the other hand, is smooth because the kink is smoothed out by the expectations operator. Therefore, the decision rules we approximate in this study are (2.48), (2.50), and

$$\mathcal{E}_t = f^{\mathcal{E}}(\Phi_{t-1}, \nu_t, A_t). \tag{2.51}$$

We derive the decision rules numerically by applying the projection method. In this study, we choose Chebychev polynomials as the basis function and use Smolyak sparse grids.[7] The details of the methodology to apply Smolyak sparse girds are provided in Judd et al., 2014.

When we use Chebychev polynomials as the basis function, we must choose the domain of the approximation because the variables must be standardized within the range of $[-1, 1]$. In this study, we choose the range to cover three standard deviations of the stationary distribution of each exogenous variable. In a standard model with active monetary policy, three standard deviations are large enough to include an occasion where the ZLB binds. However, because we assume the monetary policy to be passive, such an occasion does not occur within the range of three standard deviations; for example, when the Taylor coefficient is set to $\phi_\pi = 0.5$, the inflation rate must decline three times as much as in the case of $\phi_\pi = 1.5$ for the ZLB to bind. Therefore, although the solution is derived from the nonlinear equilibrium conditions with the ZLB, it actually never binds. As we will confirm later, this leads to the similar results between the linear and nonlinear solutions.

[7] In this study, we choose the degree of approximation using Smolyak sparse grids of $\mu = 2$.

Let us provide a brief sketch of the solution algorithm. The numerically approximated decision rules can be expressed as

$$\Pi_t = \hat{f}^{\pi}(\Phi_{t-1}, \nu_t, A_t|\theta^{\pi}), \tag{2.52}$$

$$\mathcal{E}_t = \hat{f}^{\mathcal{E}}(\Phi_{t-1}, \nu_t, A_t|\theta^{\mathcal{E}}), \tag{2.53}$$

$$\Phi_t = \hat{f}^{\Phi}(\Phi_{t-1}, \nu_t, A_t|\theta^{\Phi}), \tag{2.54}$$

where the hat shows that the functions are approximations. θ denotes the coefficients of the basis functions. Let us define the approximation residuals as

$$\mathcal{R}_t^{\mathcal{E}} \equiv \mathcal{E}_t - \beta \mathbb{E}_t\left[\frac{c_{t+1}^{-\sigma}}{\Pi_{t+1}}\right], \tag{2.55}$$

$$\mathcal{R}_t^{\pi} \equiv \left[\psi(\Pi_t - \Pi^*)\Pi_t - \theta w_t + \theta - 1\right] - \beta \mathbb{E}_t\left[\left(\frac{c_t}{c_{t+1}}\right)^{\sigma} \frac{y_{t+1}}{y_t}\psi(\Pi_{t+1} - \Pi^*)\Pi_{t+1}\right], \tag{2.56}$$

$$\mathcal{R}_t^{\Phi} \equiv \Phi_t - \mathbb{E}_t \Pi_{t+1}. \tag{2.57}$$

Using a Newton–Raphson-type of optimization algorithm, parameters $\hat{\theta}^{\pi}$, $\hat{\theta}^{\mathcal{E}}$, and $\hat{\theta}^{\Phi}$ solve an optimization problem that sets the residuals to $\mathcal{R}_t^{\mathcal{E}} = 0$, $\mathcal{R}_t^{\pi} = 0$, and $\mathcal{R}_t^{\Phi} = 0$. To calculate the expectations, exogenous shocks are discretized using Gauss–Hermite approximation. Further details of the solution algorithm can be found in Fernández-Villaverde, Rubio-Ramírez, and Schorfheide, 2016.

2.3.4 Comparison between linear and nonlinear decision rules

Once we obtain the decision rules numerically, we can investigate the dynamics of the model. Figure 2.4 shows the impulse response of the variables to productivity shock and sunspot shock, respectively. The impulse responses show similar dynamics to the linear case: positive productivity and sunspot shocks increase both inflation rate and output, while the shape of the responses differs between the two shocks.

It is known that there are several differences between linear and nonlinear decision rules. For example, the linear solution is derived around the deterministic steady state, thus it cannot capture the effects arising from uncertainty, while such

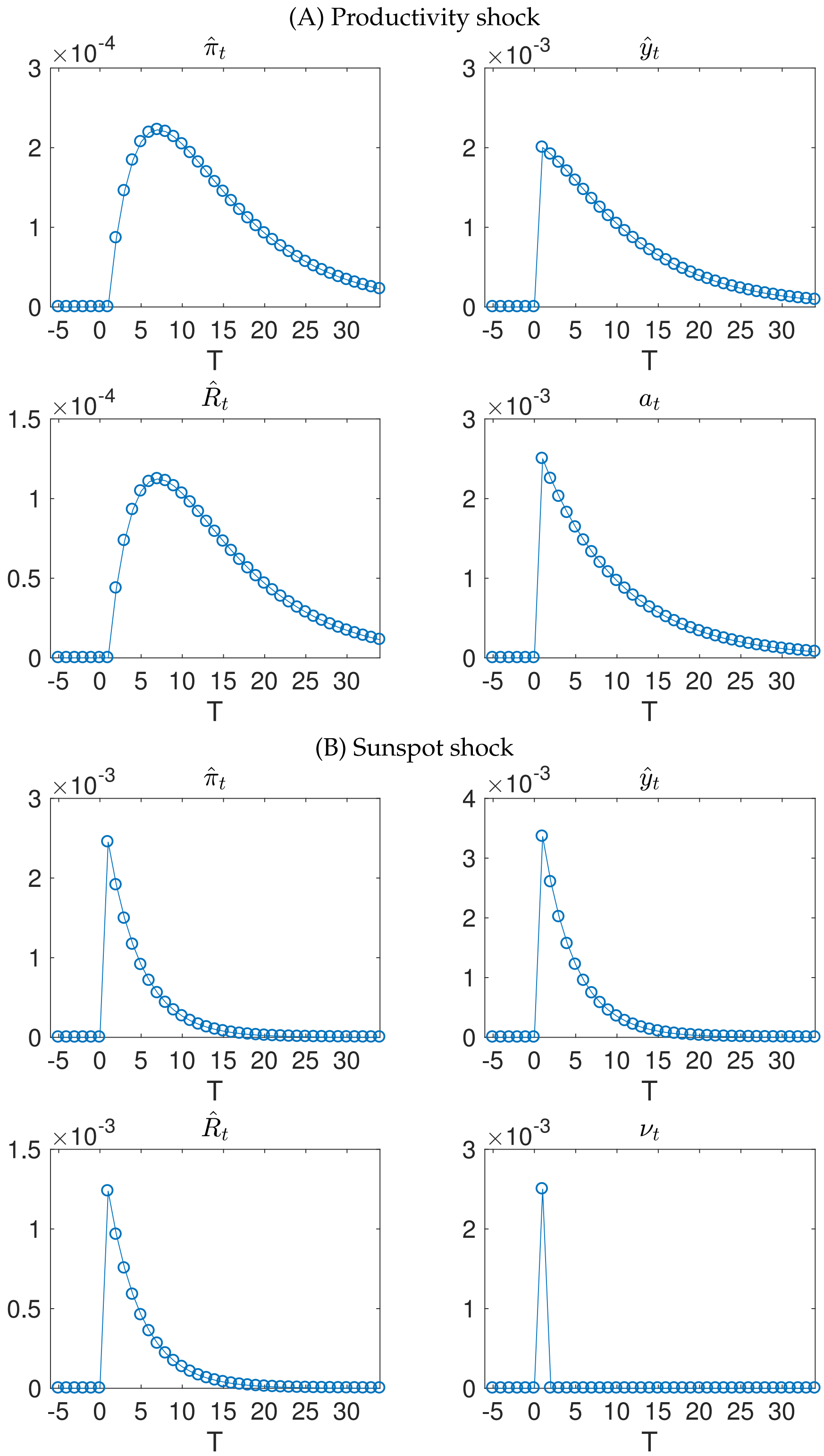


FIGURE 2.4: Impulse responses to different shocks (nonlinear sunspot case).

effects are included in nonlinear solutions. Another difference is that the quadratic price adjustment cost of Rotemberg, 1982 is always zero in the linear model, while it becomes positive in the nonlinear model.

These effects are generally relatively small in the neighborhood of the steady state, and linear approximation is known to perform effectively. These points can be confirmed by comparing Figure 2.3 and Figure 2.4: the dynamics of the impulse responses for linear and nonlinear models exhibit very similar results. The results are similar because the model is almost linear when the ZLB is not binding. Therefore, for the case of indeterminacy arising from passive monetary policy, the practical gain from applying nonlinear methods is limited.

Note that if we consider a shock large enough to force the central bank to lower its interest rate down to the ZLB, linear and nonlinear solutions can differ significantly. However, as we discussed earlier, such circumstances are extremely rare events when the Taylor principle is not satisfied. We do not consider such extreme cases in this study because liquidity traps are relatively infrequent events even in the case of active monetary policy.[8]

2.4 Indeterminacy arising in the expectations-driven liquidity trap

In the previous section, we investigated the characteristics of indeterminate models in which the Taylor coefficient is set lower than one. In this section, we assume that the Taylor principle is satisfied and explore the model dynamics when the economy is trapped in the ELT.

2.4.1 Indeterminacy described in Benhabib, Schmitt-Grohé, and Uribe, 2001

While the central focus of this study is on local indeterminacy, the existing literature has investigated two different types of indeterminacy: local and global indeterminacy. The term local indeterminacy is associated with the existence of multiple

[8] Fernández-Villaverde et al., 2015 find that the economy is at the ZLB during 5.53 percent of quarters with similar calibration to this study.

equilibrium paths from *different initial conditions* converging toward a single steady state or a stationary balanced growth path. The term global indeterminacy, however, concerns the existence of multiple equilibrium paths from a given initial condition converging toward *different steady states or convergence paths*.[9]

The main finding of Benhabib, Schmitt-Grohé, and Uribe, 2001 is that a wide class of models with nominal prices exhibits global indeterminacy when the nominal interest rate is determined by the Taylor rule and bounded below by the ZLB. They show that in addition to the TSS, the equilibrium path may converge either to the USS or to a limit cycle around the TSS. It is further discussed that whether the model exhibits a limit cycle depends on the parameterization, and their paper mainly focuses on the case of equilibrium paths converging to the USS. In this study, we choose the USS and solve the model nonlinearly, allowing sunspot disturbances to affect prices and allocations.

2.4.2 Nonlinear decision rules

As we discussed in Section 2.2, there are two steady states that solve the household's and firm's optimization problem: the TSS and the USS. To characterize the decision rules uniquely, one must choose which steady state is reached when all stochastic elements are shut down and set equal to zero.

We introduce an additional state variable s_t and define that if $s_t = T$, the economy is in the "targeted regime," in which the economy converges to the TSS, while if $s_t = U$, the economy is in the "unintended regime," in which the economy converges to the USS. In this study, we assume that s_t is fixed to either T or U for all periods and does not change over time.[10]

The choice of the regime is often attributed to agents' expectations on the state of the economy in the long run. If agents form an optimistic view on the future economy, inflation converges to the central bank's target. However, if agents form a pessimistic view, the central bank fails to achieve its goal, and the inflation rate converges to a negative value.

[9] For detailed discussions on local and global indeterminacy, see Brito and Venditti, 2010 and Antoci, Galeotti, and Russu, 2011, for example.

[10] Several recent studies that analyze ELTs assume Markov regime-switching between the two regimes. In many cases, the targeted regime is assumed to be absorbing to obtain closed-form solutions.

The model characterized by equilibrium conditions (2.20) and (2.21) is determinate around the TSS, thus the policy functions can be expressed as

$$y_t = f^y(X_t|s_t = T), \tag{2.58}$$

$$\Pi_t = f^\pi(X_t|s_t = T). \tag{2.59}$$

However, the model is indeterminate around the USS, therefore one natural candidate of the decision rules is that output and inflation rate are affected by inflation expectations and sunspot shocks as follows:

$$y_t = f^y(Y_t, X_t|s_t = U), \tag{2.60}$$

$$\Pi_t = f^\pi(Y_t, X_t|s_t = U). \tag{2.61}$$

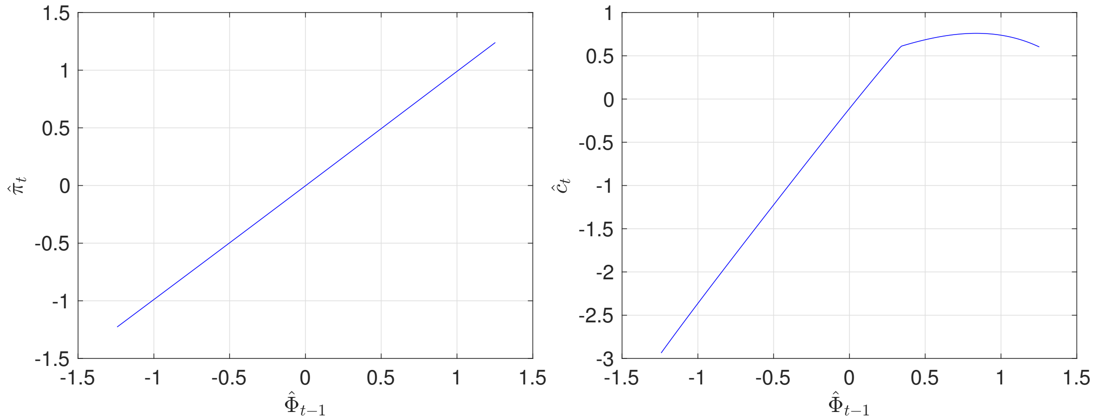
Note that the MSV decision rules can be derived for the nonlinear case as well. We assume that the economic agents respond only to fundamental elements and do not respond to nonfundamental elements. In this case, the decision rules can be expressed in the following form:

$$y_t = f^y(X_t|s_t = U), \tag{2.62}$$

$$\Pi_t = f^\pi(X_t|s_t = U). \tag{2.63}$$

For example, Aruoba, Cuba-Borda, and Schorfheide, 2018 select a particular solution that depends only on fundamental elements and derive decision rules in the ELT assuming the above functional form.

Note that when we derive decision rules of an indeterminate model, the solution may not be unique. That is, there can be multiple pairs of solutions that take the functional form of (2.60) and (2.61). Therefore, the solution presented in the following subsections should be viewed as a particular solution of the indeterminate model rather than a unique solution.



(A) Inflation (B) Consumption

FIGURE 2.5: Decision rules for inflation and consumption.

Note: Variables expressed as percentage deviations from the deterministic USS.

2.4.3 Dynamics of the stochastic model

Let us consider a stochastic model where the inflation expectations fluctuate according to sunspot shocks. We focus solely on the sunspot shocks and abstract from the rest of the fundamental shocks for computational simplicity. The decision rules are derived numerically by the projection method.

Decision rules for consumption and inflation are shown in Figure 2.5. Both rules are computed by taking different values for Φ_{t-1}, while keeping other variables fixed at their steady state values. We can confirm that higher inflation expectations generate higher realized inflation and consumption, which is similar to the results in the case of passive monetary policy. Consumption, however, starts to decline as inflation expectations exceed a certain threshold. This is because of the household's endogenous behavior; once the ZLB ceases to bind, increasing consumption and creating inflationary pressure induce the central bank to raise the interest rate, which leads to an increase in the real rate. Under such circumstances, it is suboptimal for the household to further increase consumption as there are no changes in the fundamentals such as productivity. Therefore, it becomes optimal for the household to refrain from increasing consumption once the monetary policy becomes active.

By combining the diagrams of Figure 2.5 and substituting out Φ_{t-1}, we can depict the convergence path for c_t and Π_t corresponding to different realizations of Φ_{t-1}. Figure 2.6 depicts the convergence path, which shows a strong nonlinearity in the

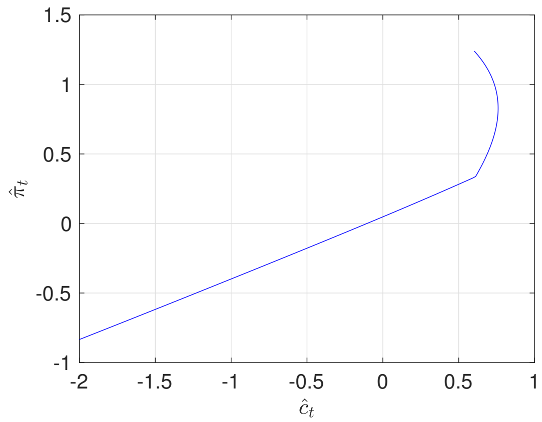


FIGURE 2.6: Convergence path corresponding to different Φ_{t-1}.

Note: Variables expressed as percentage deviations from the deterministic USS.

area where the ZLB does not bind.

Figure 2.7 shows the impulse responses of variables to a two-standard-deviation sunspot shock. All variables react positively on impact, and the nominal interest rate escapes from the ZLB as the inflation rate increases; the nominal interest rate is positive for three periods with the rise in the inflation rate. Even though the sunspot shock ν_t is white noise and transitory, the dynamics of the variables are persistent.

2.4.4 Dynamics of the deterministic model

To evaluate our results of the stochastic model and confirm that our results are not an artifact arising from computational methods, it is worth investigating the dynamics of the deterministic case in the ELT.

Figure 2.8 shows the equilibrium path converging to the USS in the deterministic setup. A small perturbation from the TSS, shown by "$\times$," leads to a de-anchoring of inflation expectations and converges to the USS, depicted by "$+$." This convergence path is similar to the path in the stochastic case shown in Figure 2.6. The area in grey shows the region where the ZLB binds, and we can observe that the equilibrium path starts to kink once it escapes from the area and the ZLB ceases to bind.[11] The inflation rate continues to increase, while consumption gradually starts to decline in this area. The mechanism by which such a curve emerges is similar to the stochastic case;

[11] As shown by Benhabib, Schmitt-Grohé, and Uribe, 2001, the deterministic model is indeed globally indeterminate; because there are only jump variables in the model, the economy can jump to the TSS or on the trajectory converging to the USS regardless of the past realization of the variables.

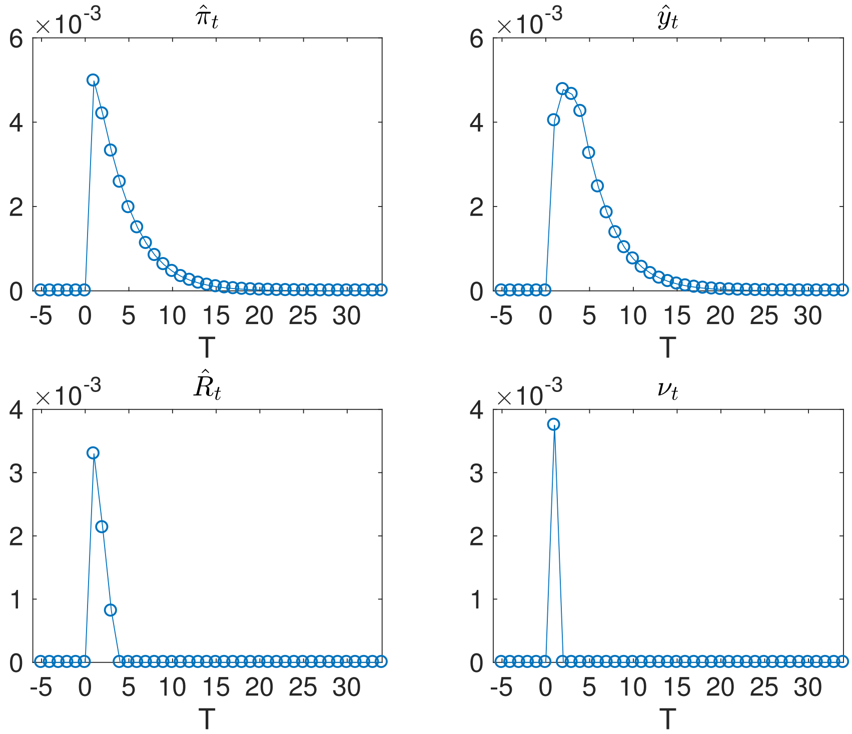


FIGURE 2.7: Impulse responses to a large sunspot shock.

Note: Variables expressed as log deviations from the stochastic USS.

because increasing consumption creates further inflationary pressure and induces the central bank to increase the interest rate, the household refrains endogenously from increasing consumption.

While the comparison between the deterministic and stochastic models shows that our methodology provides persuasive decision rules, some limitations are worth noting. As we fix our solution space to a certain domain when we approximate decision rules using Chebychev polynomials, solutions may not be accurate when the economy is far away from the USS. Especially in the ELT, there exist multiple prices and allocations corresponding to a certain inflation rate Π_t. For example, in Figure 2.8, there are more than two equilibrium prices and allocations that satisfy the equilibrium conditions with $\Pi_t = 1$. Therefore, not only the expectations on inflation but also expectations on consumption, for example, are further needed to pin down current consumption and inflation. As such, the decision rules derived by the projection method with a particular basis function should be regarded as a nonlinear approximation that holds in a relatively limited area around the steady state.

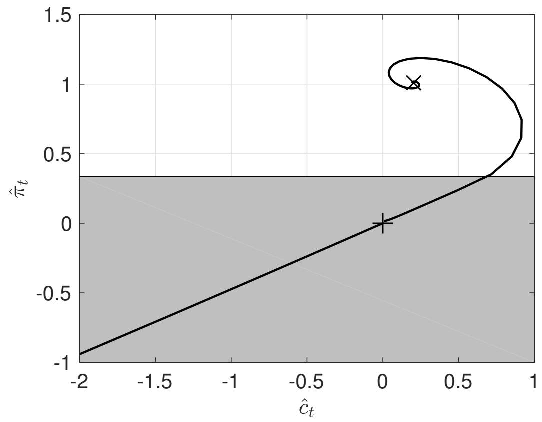


FIGURE 2.8: Equilibrium path converging to the deterministic USS.

Note: Variables with hats are measured as deviations from the deterministic USS. "×" and "+" denote the TSS and the USS, respectively.

Another limitation is that the solution we investigated in this study is one particular form of solution that incorporates sunspot shocks. As the model is nonlinear, the solution may not be unique and alternative solutions may exist. However, a nonlinear solution that allows inflation expectations to deviate from the fundamentals is intuitive; thus, it is regarded as a natural candidate of the solution of indeterminate models.

2.4.5 The role of nonlinearity in indeterminate models

We have confirmed that the nonlinear solution plays a key role in capturing the characteristic dynamics around the ELT. However, nonlinearity itself is often regarded as of second-order importance depending on the focus of the study.

Atkinson, Richter, and Throckmorton, 2020, for example, show that there is a large practical advantage in using a piecewise linear solution compared with a full nonlinear solution.[12] Such results reflect the fact that the major nonlinearity arises from a kink created by the occasionally binding nature of the ZLB.

However, the nonlinearity arising in the ELT is not a simple kink; the nonlinearity appears in a smooth and continuous manner, which can be seen from the curvature in Figure 2.6. In such a case, the piecewise linear solution cannot appropriately

[12] The authors apply the software package OccBin to implement the piecewise linear solution. Details of OccBin are provided by Guerrieri and Iacoviello, 2015.

approximate the decision rules. This fact strongly encourages the use of nonlinear methods to derive solutions of indeterminate models, especially in the ELTs.

2.5 Conclusion

In this study, we proposed a novel methodology to derive nonlinear solutions of an indeterminate model. We first applied the method to the case of passive monetary policy and found that linear and nonlinear decision rules exhibit similar dynamics, indicating that the practical gains from applying the nonlinear method is limited in the most basic setup. We then applied the method to the case of an ELT and found that nonlinearity plays a significant role in the model dynamics. These findings suggest the importance of considering both indeterminacy and nonlinearity when investigating the dynamics in a liquidity trap.

An important question that remains unanswered in this study is whether other solutions of the indeterminate model exist. As the solution presented in this study is one particular form that incorporates sunspot shocks with de-anchored inflation expectations, other forms of solutions may exist. Therefore, investigation of a more general set of solutions of nonlinear indeterminate models remains a challenging yet important direction for future work.

This chapter focused mainly on the technical aspect of the indeterminacy in DSGE models by presenting a solution in the ELT. However, whether the de-anchoring of inflation expectations is likely to be true in many advanced economies—and if so, how much it has affected the real economic outcomes—remains an important empirical question. Further investigation is left for future work to address such questions.

Chapter 3

Identifying Oil Price Shocks and Their Consequences: The Role of Expectations in the Crude Oil Market

3.1 Introduction

There is growing interest among academics, policymakers, and market practitioners in the causes and consequences of oil price fluctuations (e.g., World Bank, 2015). Various supply and demand factors are known to drive oil prices. Changes in oil prices can affect the economy in different ways, depending on the factors driving the change (e.g., Kilian, 2009, Ratti and Vespignani, 2013, and Basak and Pavlova, 2016). The ability to infer the respective drivers has important implications for the appropriate policy response to changing macro-financial conditions from both global and domestic perspectives (e.g., Gospodinov and Ng, 2013, Filardo and Lombardi, 2014, and Filardo et al., 2018).[1]

More recently, there is increasing recognition about the role of expectations about future developments of the oil markets which are not captured by traditionally-used, realized (or flow) demand and supply (e.g., Kilian and Murphy, 2014 and Kilian and Lee, 2014). However, little is known about the quantitative impacts of expectations about future oil supply and future oil demand on oil price fluctuations and business

[1] From the domestic perspectives, in particular, the following studies investigate oil price and its relationship with the macroeconomy and financial market for each country: Herrera and Rangaraju, 2020 for the United States, Park and Ratti, 2008 for the United States and European countries, Cunado and de Gracia, 2003 for European countries, Cunado and de Gracia, 2005 and Cunado, Jo, and de Gracia, 2015 for Asian countries.

cycles.[2] There are several key characteristics of the commodity market with which expectations can affect commodity prices. For instance, as oil is storable, not only realized demand and supply but also inventories affect investors' expectations about future oil supply and demand. Then, it is reasonable to consider that expectations and uncertainty about future demand and supply can play a significant role in determining the oil price. Furthermore, as is suggested in Kilian, 2009, the roles of future oil supply shocks can be potentially different from those of future oil demand shocks, which suggests the need to carefully examine each impact. These arguments make the standard decomposition of oil price fluctuations to demand and supply factors more challenging (e.g., Davig et al., 2015, Bernanke, 2016).

This study aims to fill this gap. We develop a simple but practically comprehensive structural vector autoregressive (SVAR) model which incorporates the role of expectations about future global aggregate demand and future oil supply, in addition to the traditionally used factors associated with realized aggregate demand and oil supply. We identify expected aggregate demand shocks and expected future oil supply shocks, exploiting revisions of global economic growth by professional forecasters and changes in oil inventory, respectively, to examine their impacts on the oil price in an endogenous manner. Using our proposed model, we identify four shocks driving oil price fluctuations: realized oil supply shocks, realized aggregate demand shocks, future aggregate demand shocks, and future oil supply shocks. We then disclose the mechanism of oil price developments as well as these shocks' influence on global industrial output, based on empirical evidence that expectations about future oil supply and demand have an important role in oil price fluctuations and the evolution of the economy. Our proposed model is closely connected to the literature on SVAR analysis for oil prices, which is addressed in the literature survey below.

Our main findings are twofold. First, our analysis sheds new light on the effects of expectations about the oil market: expected future oil supply shocks and expected aggregate demand shocks have a significant effect, compared with realized supply

[2] Davig et al., 2015 show that a large fraction of the recent oil price drop in 2014 is unexplained by those traditional supply and demand shocks, and discuss that the unexplained part reflects changes in expectations and uncertainty about future oil supply and demand.

and demand shocks. An estimated variance decomposition shows that future demand and supply shocks account for roughly 20% of oil price variance over twelve months, as much as realized demand and supply shocks explain.

Second, we show that the effects of oil price dynamics on the global economy depend on the factors behind them: for example, an unexpected increase in global oil supply will cause a small increase in global output as pointed out by Kilian, 2009. In our result, both realized and expected negative aggregate demand shocks bring global output down. More interestingly, both positive expected future oil supply shocks and negative oil price-specific shocks initially push global output down, probably reflecting the contractions in upstream investments of crude oil. Almost 1 year later, however, global output increases.

The main contribution of this study is that we find the importance of disentangling shocks to expectations about future global aggregate demand and future oil supply. Our results show that changes in the oil price due to shocks to expectations about future global aggregate demand have a remarkably different impact on the global economy than oil price fluctuations driven by shocks to expectations about future oil supply. Therefore, it is important for policymakers to learn more about those heterogeneities and to monitor sources of recent oil price fluctuations in real-time.

The remainder of this chapter is organized as follows. In section 3.2, we briefly review the existing literature. Section 3.3 describes the methodology and data to identify shocks as key determinants of real oil price dynamics. Section 3.4 provides empirical results and discussions, and Section 3.5 presents the robustness check. Section 3.6 concludes.

3.2 A brief literature review on SVAR analysis for oil prices

Among the literature regarding the oil price shocks and their influence on economic activity, one of the most distinguished is Kilian, 2009. He proposes a novel SVAR model to identify three contributing factors in accounting for oil price fluctuations: flow demand shocks, flow supply shocks, and other factors involving oil-specific demand. The last component is designed to include any factors affecting swings in the real price of oil after controlling for oil supply and global demand shocks. He demonstrates the importance of separately identified shocks as they have considerably different effects on the oil price and economic activity (also see e.g., Barsky and Kilian, 2001, and Barsky and Kilian, 2004, Kilian, 2008).

Along with this seminal paper, a wide variety of extensions have been proposed. Among them, Ratti and Vespignani, 2013 extend the SVAR model by incorporating a monetary factor such as global real money stocks. They point out that global real money stocks have a statistically significant effect on oil prices, and that their historical impact is sizable in the phase of increasing oil prices from 2009 to 2011.

Kilian and Murphy, 2014 and Kilian and Lee, 2014 refine the original approach to allow for an explicit role of the speculative oil demand using oil inventories data. The key intuition of Kilian and Murphy, 2014 is that there exist some factors that are not captured by realized (or flow) demand and supply shocks, and that one of them can be "any expectations of a shortfall of future oil supply relative to future oil demand." They show in their empirical study that the "speculative oil demand shock" has a significant effect on the oil price by linking it to the oil inventories. Departing from their study, we explicitly identify the role of future demand and supply. From a methodological perspective, Kilian and Murphy, 2014 use the sign restriction technique to identify the structural shocks. In contrast, the current chapter proposes the simple, conventional triangular-type zero restriction to identify the shocks.[3]

More recently, there has been an increasing number of studies that focus on the causes and consequences of the large fall in oil prices from mid-2014 to 2016. World Bank, 2015 raises the following four causes of sharp oil price drop: a trend of greater-than-anticipated supply and less-than-anticipated demand, changes in OPEC objectives, fading geopolitical concerns about supply disruptions, and US dollar appreciation. While this World Bank's address is qualitative, several studies have examined quantitative assessments. On one hand, Baumeister and Kilian, 2015 show the evidence that more than half of the price decline from mid-2014 to 2016 was predictable as of June 2014, because it owes to the adverse shocks that hit the oil market before June 2014. On the other hand, Davig et al., 2015 decompose the oil price fluctuation

[3] Baumeister and Hamilton, 2019 propose a less restrictive approach to identify the conventional oil shocks than the triangular-type zero restrictions by utilizing a Bayesian prior-posterior analysis to include prior information about elasticity and equilibrium impacts of the shocks.

with the technique of Kilian, 2009 and find that oil-specific or precautionary demand shocks mostly drove the oil price decline.

The finding of Davig et al., 2015 reveals the limitation of the methodology developed in Kilian, 2009: it is not well defined enough to identify factors driving oil price-specific shocks, although we assume that it potentially reflects changes in expectations and uncertainty about future oil supply and future global real activity as well as financial shocks. Since "not all oil price shocks are alike," as is pointed out in Kilian, 2009, it would be difficult to examine the causes and consequences of the recent declines in oil prices without identifying factors that involve oil price-specific shocks. Our methodology aims to address this limitation by systematically employing the expectation-oriented factors that have been discussed in the existing literature into one simple but comprehensive model. This model enables us to decompose the contributions of oil price-specific shocks into the role of changes in expectations and uncertainty about future oil supply and future global real activity.

3.3 Methodology and data

3.3.1 Kilian's standard model

Kilian, 2009 proposes the three-variable SVAR model to identify underlying demand and supply shocks in the oil market. Specifically, the representation is expressed as follows:

$$A_0 z_t = \alpha + \sum_{i=1}^{k} A_i z_{t-i} + \varepsilon_t \tag{3.1}$$

where ε_t refers to the vector of serially and mutually uncorrelated innovations, and $z_t = (\Delta prod_t, rea_t, rpo_t)'$, where $\Delta prod_t$ represents the change in global crude oil production, rea_t the index of real economic activity, and rpo_t the real price of oil. Let e_t denote the reduced form VAR innovations such that $e_t = A_0^{-1}\varepsilon_t$.

The identification restrictions on A_0^{-1} are imposed by recursive exclusion as follows:

$$e_t \equiv \begin{pmatrix} e_t^{\Delta prod} \\ e_t^{rea} \\ e_t^{rpo} \end{pmatrix} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{pmatrix} \varepsilon_t^{oil\ supply} \\ \varepsilon_t^{aggregate\ demand} \\ \varepsilon_t^{oil-price\ specific} \end{pmatrix}.$$

The oil supply shocks are designed as unexpected innovations to global oil production. Innovations to global economic activity that cannot be explained by realized oil supply shocks refer to aggregate demand shocks. Finally, by construction, innovations to the real oil price could represent *any* factors having an impact on the real price of oil after controlling for those oil supply and aggregate demand shocks. As is discussed by Kilian, 2009, each shock could have a different dynamic impact on real oil prices and the real economy. To better quantify the causes and consequences of oil price fluctuations, it is required to disentangle each shock and properly separate the effects of each.

As discussed in the previous studies, the contribution of the oil-specific shock can be sometimes significantly large, which makes it difficult to interpret what causes the oil price changes. For example, it is well acknowledged that the main driver of declines in oil prices from mid-2014 to 2016 appears to be attributable to the oil-specific demand shocks (e.g., Davig et al., 2015). Our empirical analysis using the approach of Kilian, 2009, which is reported below, shows that oil supply and aggregate demand shocks explain only about 10% of the decline in oil prices and the remaining 90% are contributions of oil price-specific shocks. Kilian, 2009 and Davig et al., 2015 mention that the oil-specific shock may capture changes in the precautionary demand for oil. It potentially reflects some fluctuations in market expectations about the future supply or demand. This idea motivates us to identify the factors driving the oil price-specific shocks with the additional variables.

3.3.2 Our methodology

We extend the method by introducing two additional variables into the VAR model (3.1), which allows us to identify shocks on "expectations" about the future oil supply and aggregate demand. We use the oil inventory to address the future oil supply, and professional forecasts about global GDP growth to address the future aggregate demand. We use the terminology *realized* oil supply and demand shocks for the original variables in Kilian, 2009 and *future* oil supply and demand shocks for the newly proposed factors in this study.

This idea is implemented with a five-variable SVAR model with $z_t = (\Delta prod_t, rea_t, \Delta CF_t, \Delta Stock_t, rpo_t)'$ where $(\Delta prod_t, rea_t, rpo_t)$ are the same as above, ΔCF_t denotes the forecast revisions of the global GDP growth, and $\Delta Stock_t$ the change in the oil inventory. Based on Equation (3.1), we identify five shocks in the model as follows:

$$e_t \equiv \begin{pmatrix} e_t^{\Delta prod} \\ e_t^{rea} \\ e_t^{\Delta CF} \\ e_t^{\Delta Stock} \\ e_t^{rpo} \end{pmatrix} = \begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \begin{pmatrix} \varepsilon_t^{realized\ supply} \\ \varepsilon_t^{realized\ demand} \\ \varepsilon_t^{future\ demand} \\ \varepsilon_t^{future\ supply} \\ \varepsilon_t^{oil-price\ specific} \end{pmatrix}. \tag{3.2}$$

For simplicity of the terminology, we label original oil supply shocks and aggregate demand shocks as realized supply and demand shocks, respectively.

A rationale for the ordering of the variables is as follows. The realized supply shocks are defined as unexpected innovations to global oil production as in Kilian, 2009. Oil production is assumed not to respond to other shocks within the same month due to the adjusting cost of oil production and uncertainty about the future state of the oil market. The realized demand shocks correspond to shocks to global industrial production that cannot be explained by realized supply shocks. The realized demands for crude oil are assumed not to respond to shocks on the expected future demand and supply of crude oil and other below shocks within a month. We consider this assumption reasonable also due to the uncertainty.

The future demand shocks are innovations to professional forecasts about global

GDP growth which cannot be explained by realized aggregate demand and oil supply. This implies that when forecasters revise their expectations about future economic activity, they do not take into account the expectations about future oil supply. We consider that this assumption is not very restrictive because there are a variety of other factors that forecasters take into account when they forecast future economic activity. Expected future oil supply shocks are defined as innovations to the OECD oil inventory stocks which are attributable to neither innovations to the realized supply and demand nor those to expected future demand.[4] The future supply shocks are considered as shocks on the expectation of oil supply in the coming months or years. Lastly, the oil price-specific shocks are defined as innovations to the development of the real oil price after controlling for the effects of the above-mentioned factors.

3.3.3 Data

All data are monthly, and the sample period spans 30 years, from January 1990 to December 2019. While previous studies (e.g., Kilian and Murphy, 2014) use the post-1973 period data for the analysis of oil markets, our dataset starts in 1990 due to the availability of forecasts data obtained from Consensus Economics.[5] The data set is constructed as follows, and Figure 3.1 exhibits the time series of each variable.

Oil production ($\Delta prod$)

We use data on global oil production provided in the Monthly Energy Review of the Energy Information Administration (EIA). We take the log differences of seasonally adjusted, world oil production in millions of barrels pumped per day.

[4]In a simple identity, an increase in inventory implies that the current supply exceeds the current demand. The change in the inventory partly reflects unexpected changes in the realized supply and demand. Also, the inventory can change due to a revision of expected demand. These factors are captured by the parameters for contemporaneous relation among variables in Equation (3.2). The innovation due to the rest of the factors to drive the inventory is identified as the future supply shock. Our approach cannot precisely identify whether the change in inventory is intentional or not. Further, it can reflect several factors such as some lags between production and consumption due to shipping, the production cost per unit, and speculative behaviors. Because the elasticity of the inventory to these factors can vary over time, the identified expected supply shock can be partly associated with the current and expected demand factors that are not captured by the industrial production or the forecast revisions. Further analysis on this point is left for future work.

[5]As the studies of Hamilton, 1983 and Hamilton, 1985 discuss that in the pre-1973 period, the oil price was regulated by Texas Railroad Commission, it is well known in the literature that there was a major structural break in the time series behavior of oil price in 1973. Hence, earlier studies (e.g., Kilian, 2009, Kilian and Murphy, 2012 restrict their sample to post-1973 period.

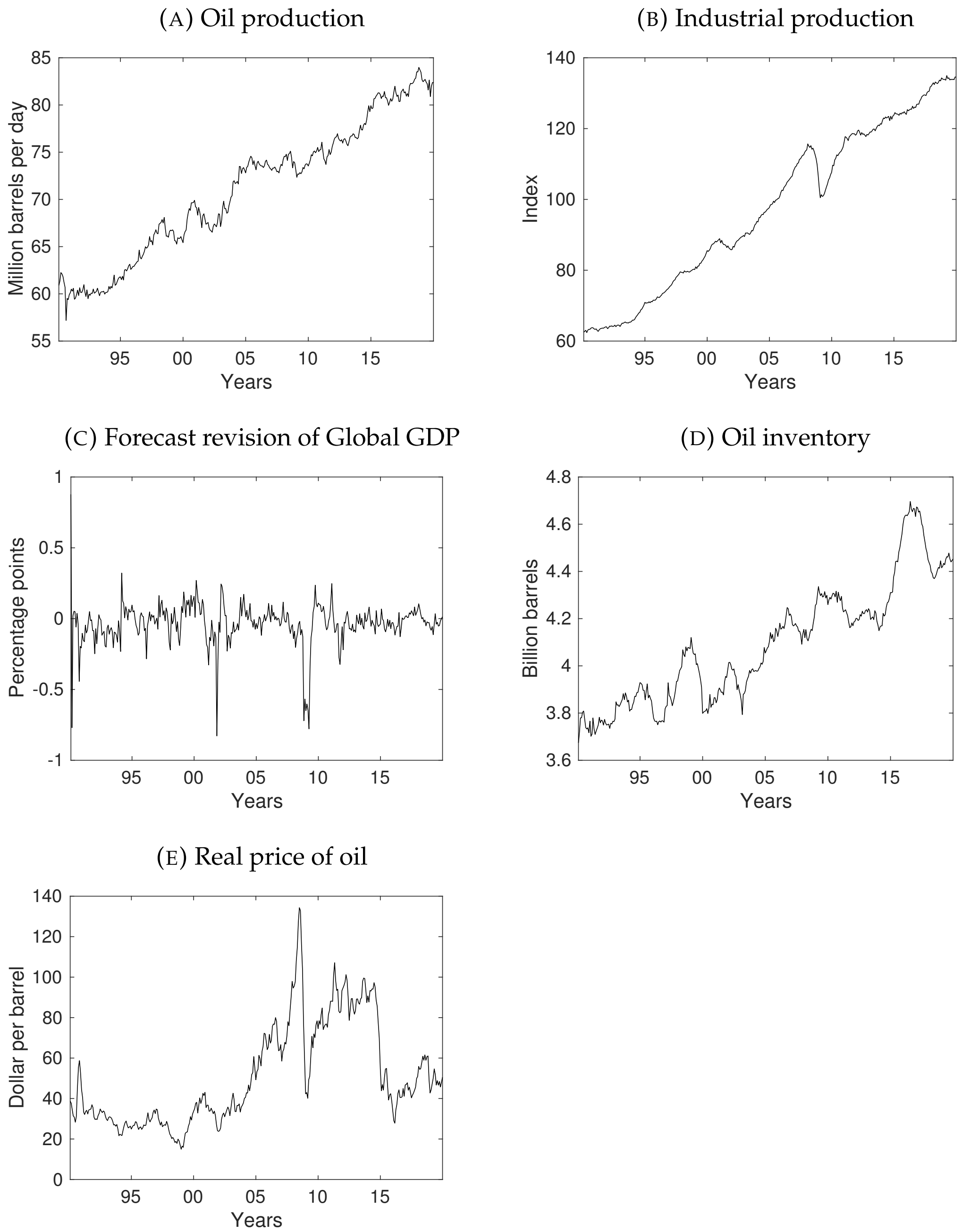


FIGURE 3.1: Time series of data.

Global real economic activity (rea)

Our measure of global real economic activity is the index of industrial production (IIP) of the OECD and 6 major countries (Brazil, China, India, Indonesia, Russia, and South Africa), which is computed and provided by Baumeister and Hamilton, 2019.[6] We apply the Hodrick-Prescott (HP) filter to calculate the trend and take the deviation from the trend to obtain a gap measure.

Revision of Consensus Forecast on global GDP of OECD countries (ΔCF)

Following Kilian and Hicks, 2013, we use the forecasts of annual real GDP growth for the next year and define the revisions of the forecast by taking the differences from the forecast delivered in the previous period. Specifically, let $CF_{i,j,t}$ denote the forecast of annual real domestic GDP growth for country i, for the next year at month j in year t. We use the series of the Consensus Forecast for OECD countries provided by Consensus Economics Inc. We focus on the 1-year forecast horizon because 1-year forecasts are more reliable and watched more closely by market participants than longer-horizon forecasts. The revisions of forecasts on real activity for country i are defined as follows:

$$\Delta CF_{i,j,t} = CF_{i,j,t} - CF_{i,j-1,t}.$$

Then, we take the weighted average for the aggregated revisions of the forecasts at month j in year t. That is, the aggregated revision $\Delta CF_{j,t}$ is defined as follows:

$$\Delta CF_{j,t} = \sum_i \omega_{i,t} \Delta CF_{i,j,t},$$

where $\omega_{i,t}$ denotes the PPP weights for country i in year t, which is aggregated within OECD countries. Note that, while countries included in $\omega_{i,t}$ changes as the

[6] We employ the IIP as a proxy for global aggregate demand, instead of using the BDI (Baltic Dry Index), a novel measure of global economic activity, proposed by Kilian, 2009. Although the BDI index usefully contains much information about global aggregate demand, it also includes some elements of expectations about future aggregate demand. The purpose of this chapter is to investigate the role of such expectations by explicitly incorporating the variable for future aggregate demand. Therefore, we use global IIP, which includes fewer expectation components than the BDI, for a better proxy for the realized aggregate demand. The correlation between global IIP and the future demand variable (revision of Consensus Forecast on global GDP) is lower than the one between the BDI and the future demand variable.

member of OECD countries have increased over the sample period, there is no severe break during the period as changes in the weight are smooth.

Oil inventory ($\Delta Stock$)

Following Kilian and Murphy, 2014, we treat OECD industry petroleum stocks as a proxy for global petroleum inventories. The series is provided by the EIA. We take the log differences of seasonally adjusted series.

Real price of oil (rpo)

Following Baumeister and Hamilton, 2019, we use the West Texas Intermediate (WTI) oil price as the nominal oil price. While there are several major references for oil prices, the WTI is one of the most popular references that practitioners address. The original series is deflated by the US CPI and the resulting real price of oil is expressed in log-levels.

3.3.4 Structural break test and lag length

As mentioned above, the oil market experienced many historical events during the 30 years of our sample period. These experiences can potentially lead to structural changes in the relationship among the oil prices and the other variables that we consider for the estimation. Hence, it is reasonable to take into account the possibility that the appropriate lag length and the parameter values of the VAR would change at some point during the 30 years (see, e.g., Hamilton, 1996, Herrera and Pesavento, 2009, Du, Yanan, and Wei, 2010).

To address this issue, we carefully examine the structural break of the VAR model by searching the break point that significantly divides the sample period into two subsample periods where an individual VAR model is estimated separately. We vary the break point through the sample periods and also alter the lag length from 1 to 12, and compute the Akaike Information Criterion (AIC) for each case. We find that the break point of September 2001 is significant for all the lag lengths considered. For the chosen lag length of 3, the VAR model with a structural break in September 2001 minimizes the AIC. The likelihood ratio test statistics of two VAR models for those two sub-sample periods (January 1990 to September 2001; and October 2001 to December 2019) over one VAR model for the full sample period is 634.4, with the 80 degrees of freedom. This indicates that the VAR model with structural break fits the data significantly better than the one VAR model without the structural break with a statistical significance level of 1%. Du, Yanan, and Wei, 2010 examine a standard Chow test to find that the structural break of the relationship among UK Brent crude oil price as a global oil price and macroeconomic variables is between December 2001 and January 2002, which is similar to our result.

In addition, we determine the number of lags in the VAR model for each subsample based on the AIC. For the baseline result reported in the next section, the number of lags for the three-variable VAR is 9 for the first subsample and 4 for the second subsample; for the five-variable VAR, 12 for the first subsample and 3 for the second subsample. Kilian, 2009 and several other studies point out that it is crucial to apply a sufficient lag length to account for a slow transmission of shocks associated with the oil supply and demand to the real price of oil. Also, the AIC is likely to suggest a short lag length in finite samples. To address these points, we report estimation results with different lag lengths of 12 and 24 in Section 3.5 to check the robustness of the results in the baseline setting of lag length.

There are several possible reasons why the optimal lag length has shortened from the first to the second subsample. One hypothesis is related to the structural changes in the entire economy due to significant technological progress: recent advances in supply chain management have enabled industrial producers to optimize their production in a substantially shorter period, which may have made the economy respond more quickly against shock realizations. Another possibility is the financialization of commodity products. Not only the market participants who utilize crude oil as inputs but also those who speculate on oil demand and supply have come to play a significant role in market activity. Compared to actual oil consumers, such speculative participants tend to react more on information on future activity, which is transmitted almost immediately throughout the globe these days. Another possibility is the fact that in recent years, shale oil producers, who can adjust their oil production much more flexibly than the traditional oil producers, have come to play a relevant role in the oil market. All of these elements can affect the economy to respond more swiftly against shocks and depend less on the longer lags. More specifically, as market participants including industrial producers and oil suppliers have become more forward-looking, the economy has changed so that it depends less on past information. This is the key intuition why it is more important to include information on expectations, rather than including longer lag lengths.

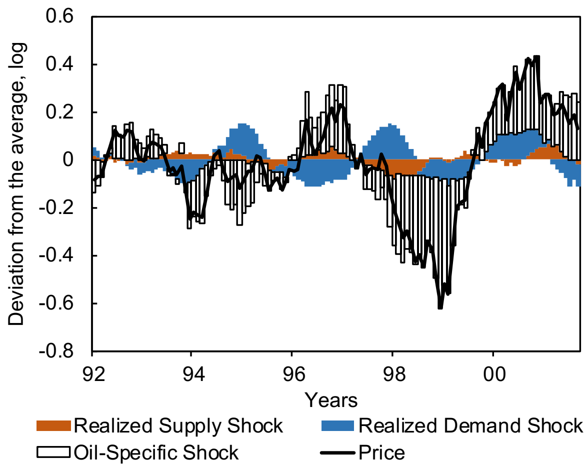
3.4 Empirical results

3.4.1 Identified shocks and the role of expectations

Figure 3.2 shows a historical decomposition of the real oil price, which presents the respective cumulative contributions of each shock identified by the three-variable VAR. The contribution of realized supply shocks is quite small, although the realized demand shock explains downward and upward streams of the real oil price around 2008-2009 and 2009-2010, respectively. In particular, what matters here is that most of the oil price fluctuations are left unexplained, as shown by the contributions of oil price-specific shocks. The variance decomposition estimates reported in Table 3.1 show that about 90% of oil price fluctuations are unexplained by either realized supply or demand shocks over 12-month horizons. Also, it is also notable that the large decline in the oil price from mid-2014 to 2016 is mostly left unexplained, as shown in Figure 3.2.

Figure 3.3 plots the historical decomposition based on the shocks identified from the five-variable VAR, additionally including future supply and demand shocks to the three-variable VAR. Focusing on the second subsample period, the contributions of oil price-specific shocks are smaller, compared with Figure 3.2. Most of them are accounted for by the contributions of future supply and demand shocks, which indicates that these expectations factors play important roles in explaining the oil price fluctuations. The contributions of the future supply and demand shocks are sizable. The variance decomposition estimate reported in Table 3.2 (B) indicates that roughly 20% of the variance in the oil price is explained by the future supply and demand shocks. It is also remarkable that in the five-variable model, the patterns of the contributions of realized demand and supply shocks for the second subsample period remain almost unchanged even after adding two variables to the three-variable VAR,

(A) For the first subsample period (from January 1990 to September 2001)



(B) For the second subsample period (from October 2001 to December 2019)

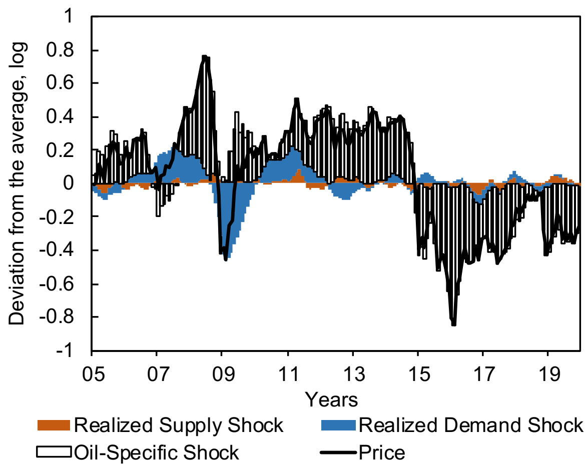


FIGURE 3.2: Historical decomposition of the real oil price with three variables in the baseline result.

(A) For the first subsample period (from January 1990 to September 2001)

Horizons (in months)	Realized Supply Shock	Realized Demand Shock	Oil-Price Specific Shock
1	0.1	1.5	98.4
12	0.9	5.4	93.7
24	2.9	7.8	89.3
48	5.1	9.8	85.1

(B) For the second subsample period (from October 2001 to December 2019)

Horizons (in months)	Realized Supply Shock	Realized Demand Shock	Oil-Price Specific Shock
1	1.6	2.0	96.4
12	0.8	8.1	91.2
24	0.8	9.0	90.2
48	0.9	8.8	90.3

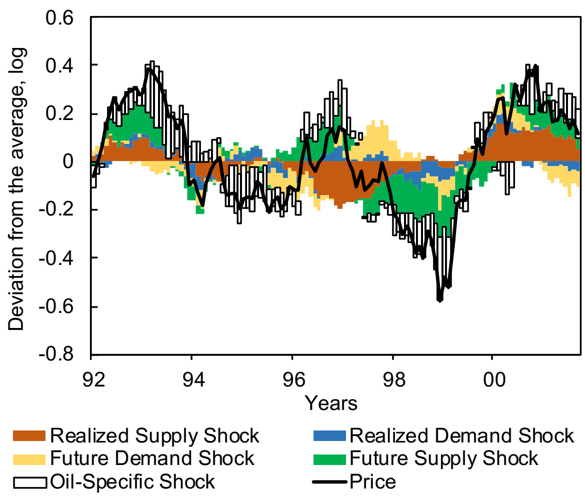
TABLE 3.1: Variance decomposition of the real oil price with three variables in the baseline result (in percent).

and additionally provides the contributions of future demand and supply shocks on them. This means that the proposed shocks improve the model as they explain roughly 20% of oil-price variance for the above-mentioned unexplained 90% component in the original three-variable VAR.

From 2007 to mid-2008, the West Texas Intermediate (hereafter WTI) hiked from 60 US dollars per barrel to 140 US dollars per barrel. In this period, realized demand shocks pushed the oil price up, indicating the demand-pull stemming from the unexpected, rapid growth of the emerging economies, in particular China and India. At the same time, shale-oil technology came into the oil industry and expectations of excess future supply were considered to put downward pressure on the oil price, which will be formally addressed later.

In the second half of 2008, the WTI fell dramatically from 140 US dollars per barrel to below 40 US dollars per barrel. Our historical decomposition shows that realized demand shocks mainly drove this decline, reflecting the recession just after the Global Financial Crisis (GFC). Expected aggregate demand shocks also contributed to the decline to some extent. From 2010 to early 2012, the WTI steadily increased from around 80 US dollars per barrel to over 100 US dollars per barrel. The main

(A) For the first subsample period (from January 1990 to September 2001)



(B) For the second subsample period (from October 2001 to December 2019)

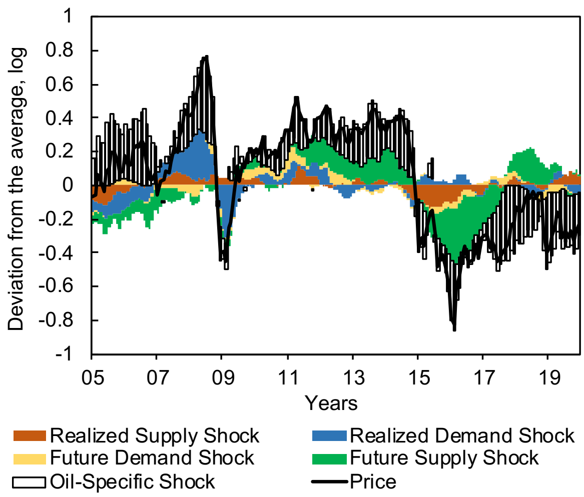


FIGURE 3.3: Historical decomposition of the real oil price with five variables in the baseline result.

(A) For the first subsample period (from January 1990 to September 2001)

Horizons (in months)	Realized Supply Shock	Realized Demand Shock	Future Demand Shock	Future Supply Shock	Oil-Price Specific Shock
1	2.4	1.9	0.1	0.1	95.5
12	17.2	2.8	2.9	19.8	57.3
24	19.6	2.2	3.2	20.8	54.2
48	17.7	2.9	4.3	20.2	54.9

(B) For the second subsample period (from October 2001 to December 2019)

Horizons (in months)	Realized Supply Shock	Realized Demand Shock	Future Demand Shock	Future Supply Shock	Oil-Price Specific Shock
1	2.0	2.3	1.3	3.8	90.7
12	2.9	10.4	1.0	16.2	69.6
24	3.1	8.8	1.4	18.2	68.4
48	3.1	8.8	1.4	18.7	68.0

TABLE 3.2: Variance decomposition of the real oil price with five variables in the baseline result (in percent).

contributors were realized demand shocks and future supply shocks. The former represented the steady growth of emerging economy and the United States after the GFC. The latter captured the uncertainty on oil supply stemming from the social instability in the Middle East and North Africa before and after the so-called Arab Spring.

From mid-2014, all shocks turned to decrease and push the oil price down, though the timings and magnitudes varied. This stream can be divided into two phases. The first period is from January 2014 to January 2015, when the real oil price plunged by about 50%. More than half of it is explained by the future oil supply shocks, which can be interpreted as influences arising from the expected increase in US shale oil, the recovery of Libyan oil production, and, most importantly, Saudi Arabia's public announcement that it would not act as the "swing producer." A decrease in the realized demand shocks had also contributed to the decline by about 10 percentage points. The second period is June 2015 to February 2016, the real oil price further decreased by about 30%. In this second period, both realized and future demand shocks played major roles in pulling the real oil price down by a sizable amount, which is a clear distinction from the first period. The realized demand shock can be linked to China's economic slowdown in manufacturing sectors, and the future demand shock to the downward revision of OECD economies' growth mainly in the IT and commodities sectors.

Figure 3.4 shows impulse responses of the real oil price to each of the five shocks in the five-variable VAR. Note that the size of each shock is set equal to one standard deviation. For the second subsample, an unexpected increase in global oil supply causes a certain decrease in the real oil price at the initial month and its impact on oil price turns out to be quite small afterward, which is consistent with the findings of Kilian, 2009.[7] Positive shocks in both realized and future demand lead to immediate, large increases in the real oil price. A positive future oil supply shock immediately causes a more persistent decrease in real oil prices than demand shocks. Shifts in expected supply schedules triggered by, for example, exogenous political events, are thought to create more persistent effects on oil price developments than realized demand shocks. An effect from oil price-specific shocks is also significant and persistent.

3.4.2 Influence of oil price shocks on global output

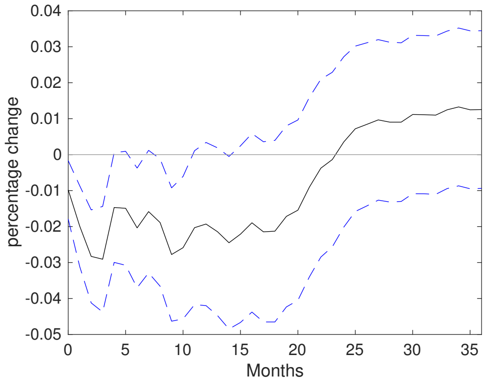
We examine the consequences of each shock identified in our model on global industrial output. Figure 3.5 shows the impulse responses of global output to shocks based on the five-variable VAR. First, as for the realized supply shock, an unexpected increase in global oil supply causes a small increase in global output. Second, realized and expected positive demand shocks bring global output up. This means that if a negative demand shock hits the economy, not only the real oil price but also global output decrease simultaneously for certain periods.

Third, positive future oil supply shocks initially push global output down, probably reflecting contractions mainly in the upstream investments of crude oil. Almost one year after these shocks, however, global output increases. This response of global output is considered as a positive impact on the global economy through the increase in real income or the decrease of production costs in oil-importing economies.

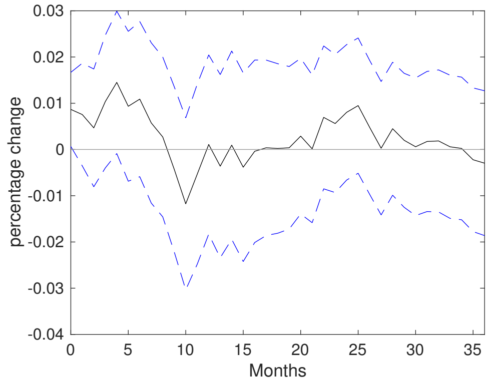
[7] One hypothesis to explain this result is that an unexpected increase in global oil supply causes an increase in oil inventory, leading to the expectation of a decrease in future oil supply.

(A) For the first subsample period (from January 1990 to September 2001)

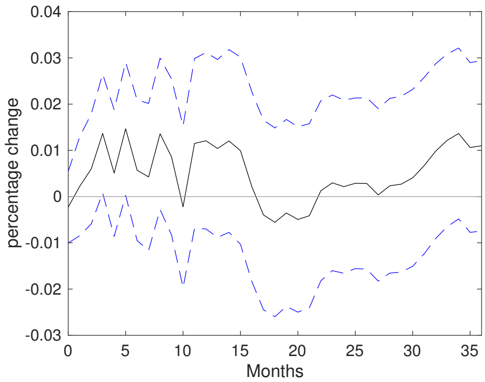
(A) Realized Supply Shock



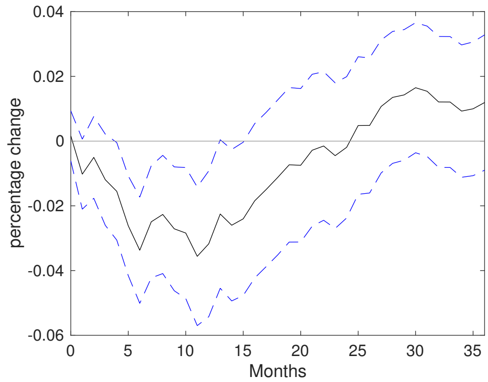
(B) Realized Demand Shock



(C) Future Demand Shock



(D) Future Supply Shock



(E) Price-specific Shock

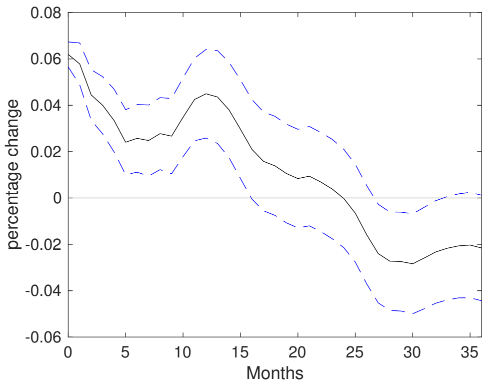
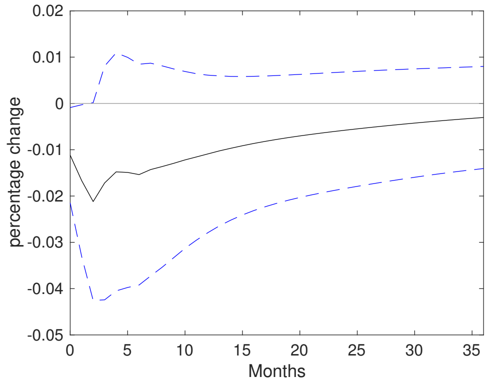


FIGURE 3.4: Impulse responses of the real oil price in the five variable VAR in the baseline result.

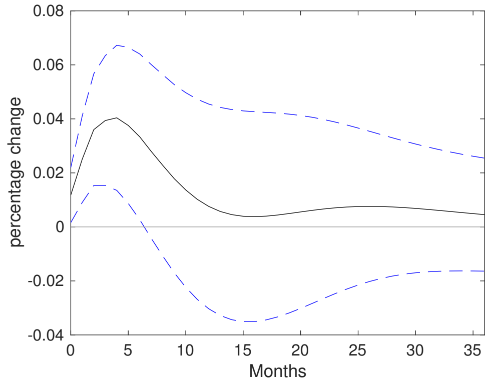
Note: The dashed lines refer to 95 percent confidence intervals. The horizontal axis refers to months from the shock.

(B) For the second subsample period (from October 2001 to December 2019)

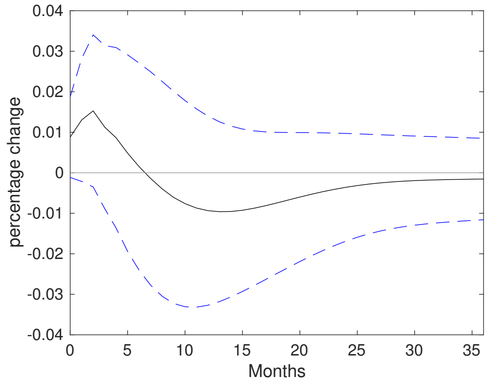
(A) Realized Supply Shock



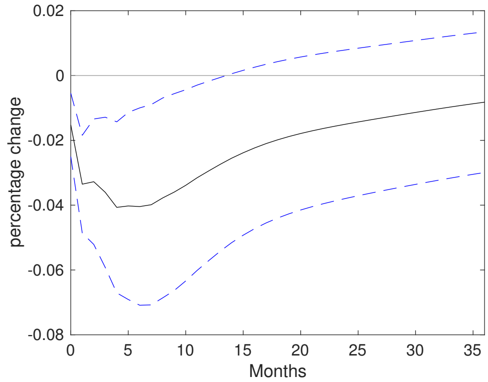
(B) Realized Demand Shock



(C) Future Demand Shock



(D) Future Supply Shock



(E) Price-specific Shock

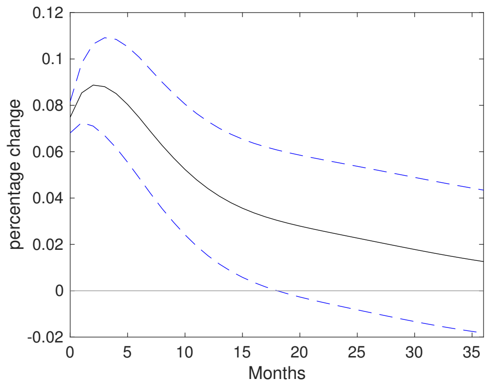


FIGURE 3.4: Continued.

(A) For the first subsample period (from January 1990 to September 2001)

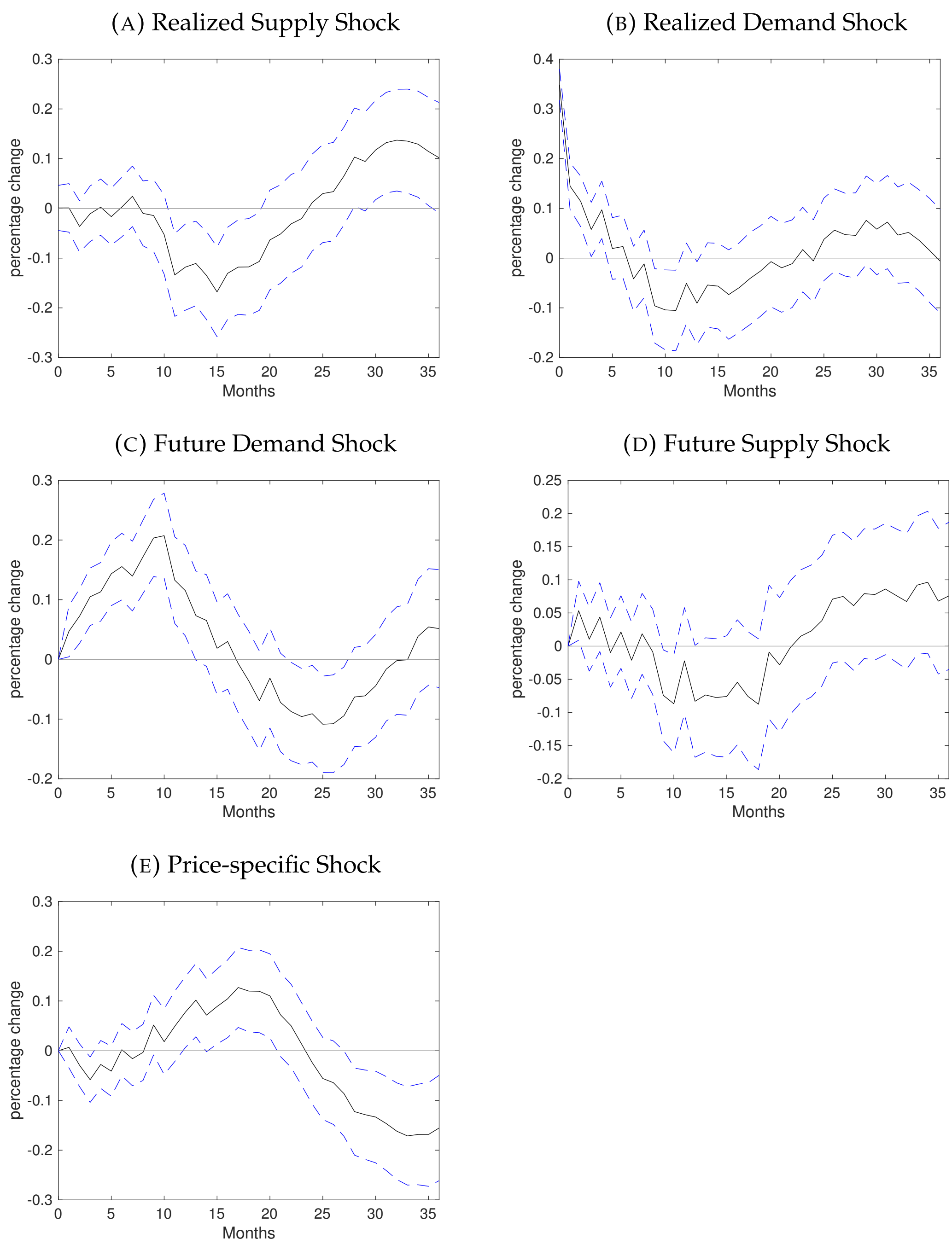


FIGURE 3.5: Impulse responses of the global output in the five variable VAR in the baseline result.

Note: The dashed lines refer to 95 percent confidence intervals. The horizontal axis refers to months from the shock.

(B) For the second subsample period (from October 2001 to December 2019)

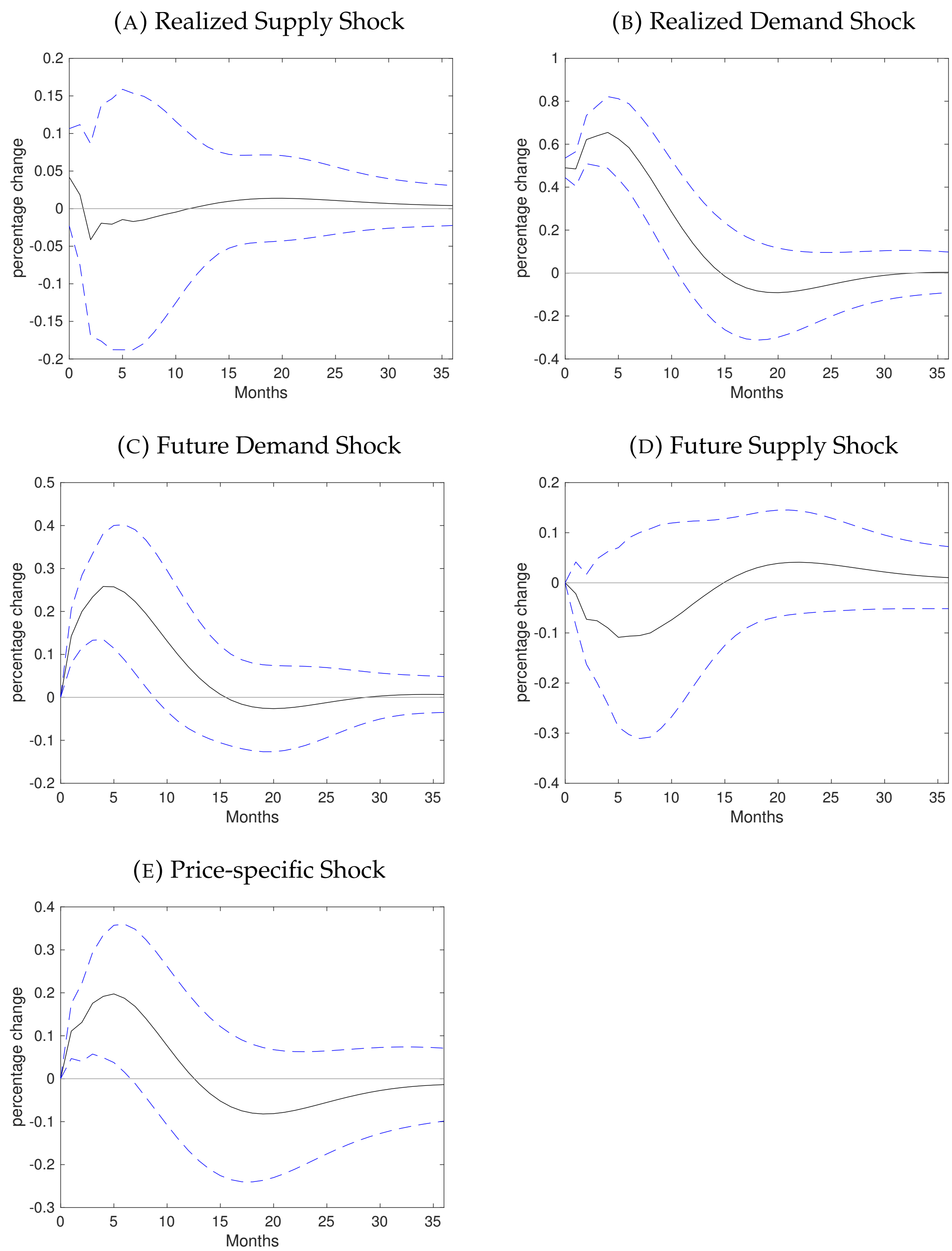


FIGURE 3.5: Continued.

All of these features clearly show that "not all oil price shocks are the same" in accounting for the development of real oil prices and global output. Bearing these findings in mind, one has to identify the shocks behind an oil price decline when evaluating its consequences on global output.

3.5 Robustness check

This section examines the robustness of our main results. Based on the five-variable VAR, we conduct two types of robustness checks: lag lengths for the VAR, and the filtering method to detrend the variable of global economic activity.

3.5.1 Lag length

The statistical test shows the optimal lag length of 3 for our baseline model. However, Kilian, 2009 proposes a lag length of 24. To study how our results depend on the choice of the lag length, we examine lag lengths of 12 and 24.

Table 3.3 reports the variance decomposition of the real oil price estimated by the VAR with the lag length of 12 and 24, respectively, for the second subsample period. Compared with our benchmark results in Table 3.2 (B), with the results for the lag length of 12, we find that the contribution of the future demand shock is slightly larger than the baseline result, while that of the future supply shock is smaller. The contribution of realized supply shock is larger compared with the baseline result. In contrast, concerning the lag length of 24, the contribution of the future demand shock is more pronounced than the baseline and that of the future supply shock remains at almost the same level as the baseline. While the results depend on the choice of the lag length to some extent, the main implication remains valid.

Also, Figure 3.6 exhibits the impulse responses of the real oil price for the second subsample period with the lag length of 12 and 24, respectively. Compared with the baseline result in Figure 3.4 (B), the impulse responses appear to be bumpy due to the long lag length, while the overall direction of the impulse responses remains similar to that in the baseline result.

(A) Lag length of 12

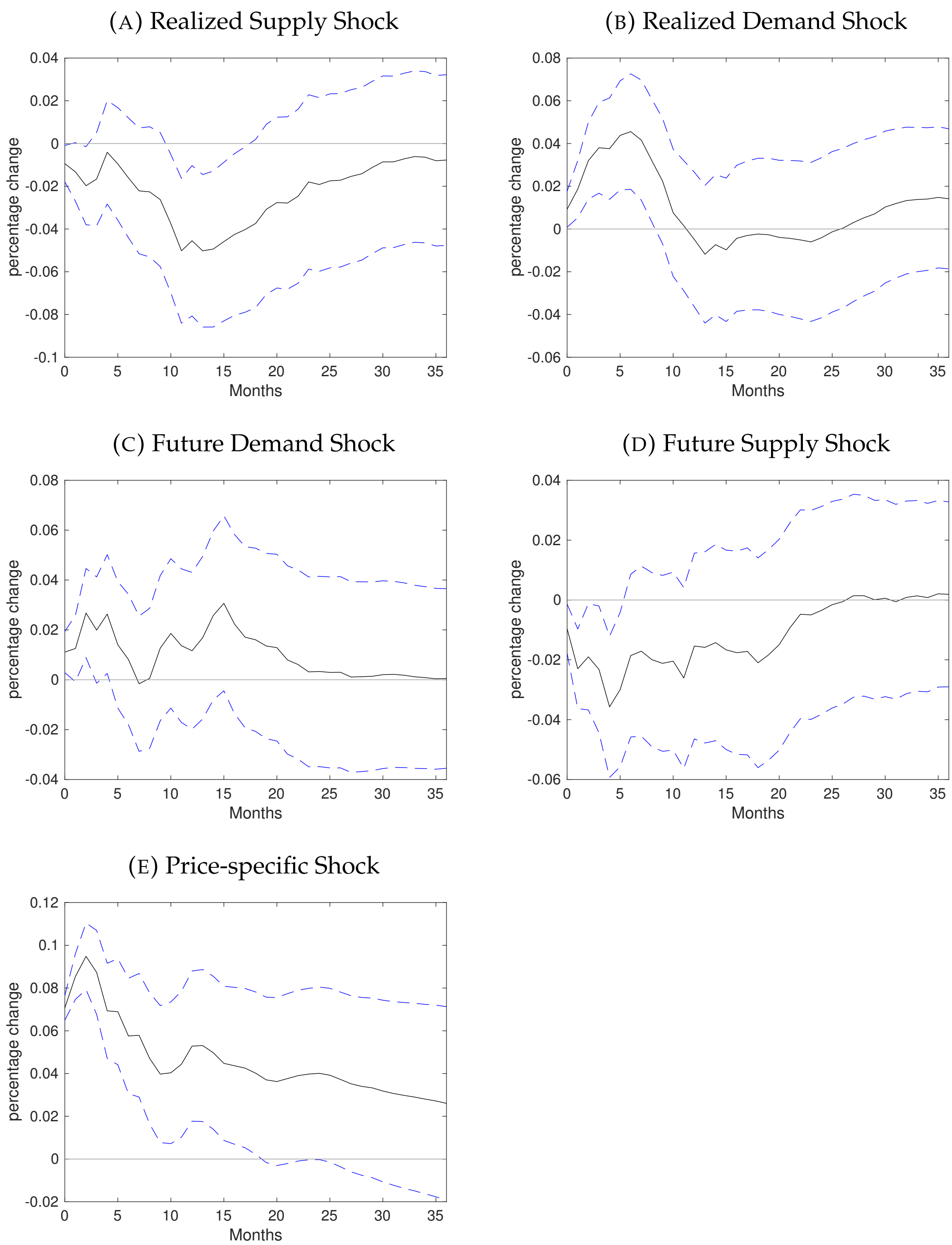
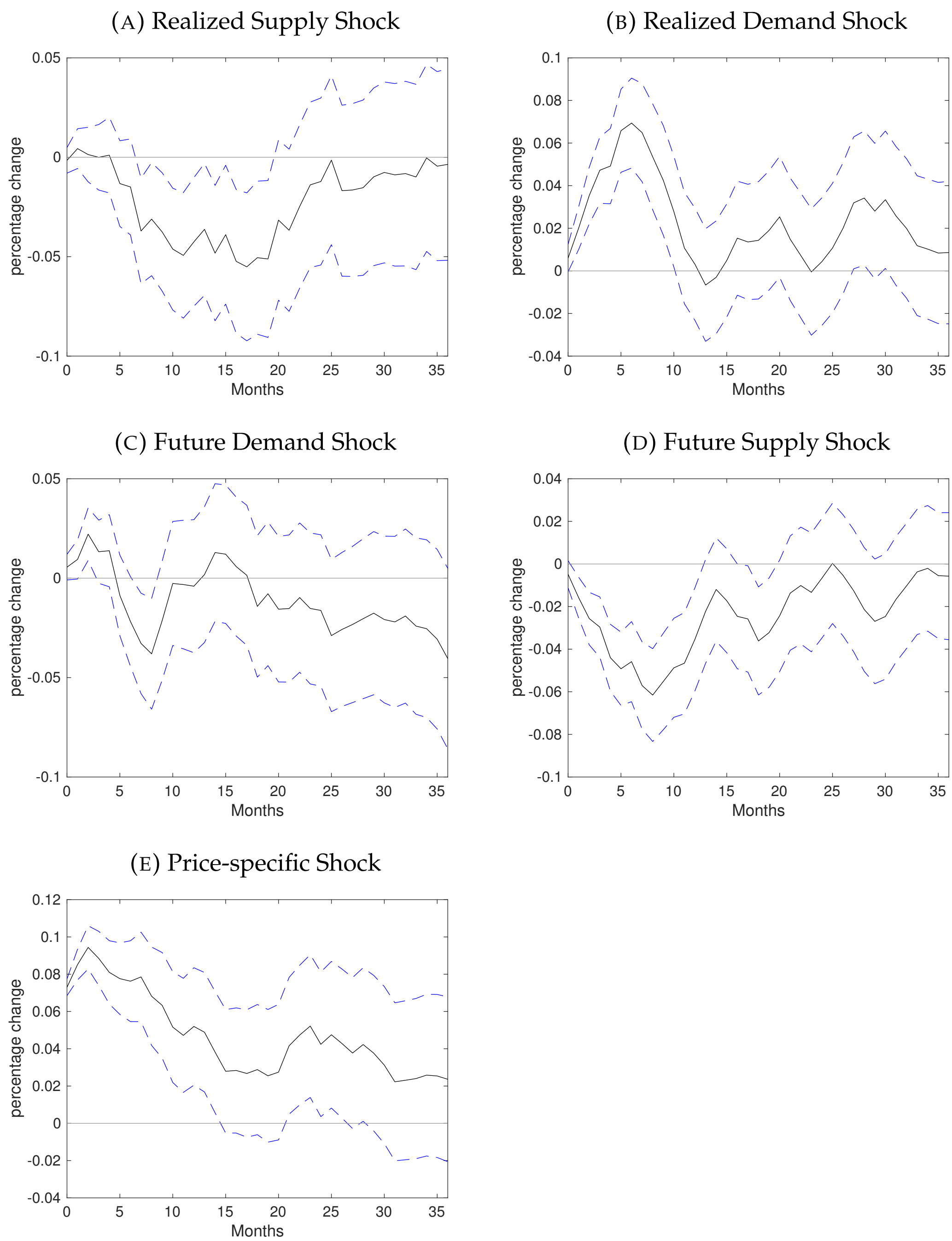


FIGURE 3.6: Impulse responses of the real oil price in the five variable VAR in the robustness check with different lag lengths for the second subsample (from October 2001 to December 2019).

Note: The dashed lines refer to 95 percent confidence intervals. The horizontal axis refers to months from the shock.

(B) Lag length of 24



(A) Realized Supply Shock

(B) Realized Demand Shock

(C) Future Demand Shock

(D) Future Supply Shock

(E) Price-specific Shock

FIGURE 3.6: Continued.

(A) Lag length of 12

Horizons (in months)	Realized Supply Shock	Realized Demand Shock	Future Demand Shock	Future Supply Shock	Oil-Price Specific Shock
1	1.6	1.6	2.3	1.7	92.8
12	8.6	14.4	3.8	7.8	65.4
24	19.1	9.5	5.2	7.1	59.1
48	18.1	9.7	4.5	6.0	61.7

(B) Lag length of 24

Horizons (in months)	Realized Supply Shock	Realized Demand Shock	Future Demand Shock	Future Supply Shock	Oil-Price Specific Shock
1	0.0	0.7	0.6	0.4	98.3
12	6.8	19.6	3.5	17.8	52.3
24	16.8	15.3	3.3	16.7	47.9
48	14.3	15.6	10.8	14.3	45.0

TABLE 3.3: Variance decomposition of the real oil price with five variables in the robustness check with different lag lengths for the second subsample period (in percent).

3.5.2 Detrending with the filter proposed by Hamilton

The HP filter is applied to detrend global industrial production in the baseline result. However, Hamilton, 2018 points out the potential drawbacks of using the HP filter. That is, the HP filter may cause spurious dynamic relations that have no basis in the underlying data-generating process. Therefore, to confirm the robustness of our results, we also examine how our baseline results may be altered when global industrial production is detrended by the Hamilton filter (Hamilton, 2018).[8] We re-estimate our five-variable VAR, focusing on the second subsample period.

Table 3.4 shows the variance decomposition of the real oil price. We can confirm that the overall contribution of both realized and expected supply and demand shocks remain unchanged, while the contribution of oil price specific shock is lower compared to the baseline results. Figure 3.7 presents the impulse responses of the real oil price. Again, the results are similar to the baseline case shown in Figure 3.4 (B).

[8] Parameters for the Hamilton filter is set to $h = 24$ and $p = 12$. The lag length is set to 6, which minimizes the AIC using the global industrial production detrended by the Hamilton filter.

(A) Lag length of 12

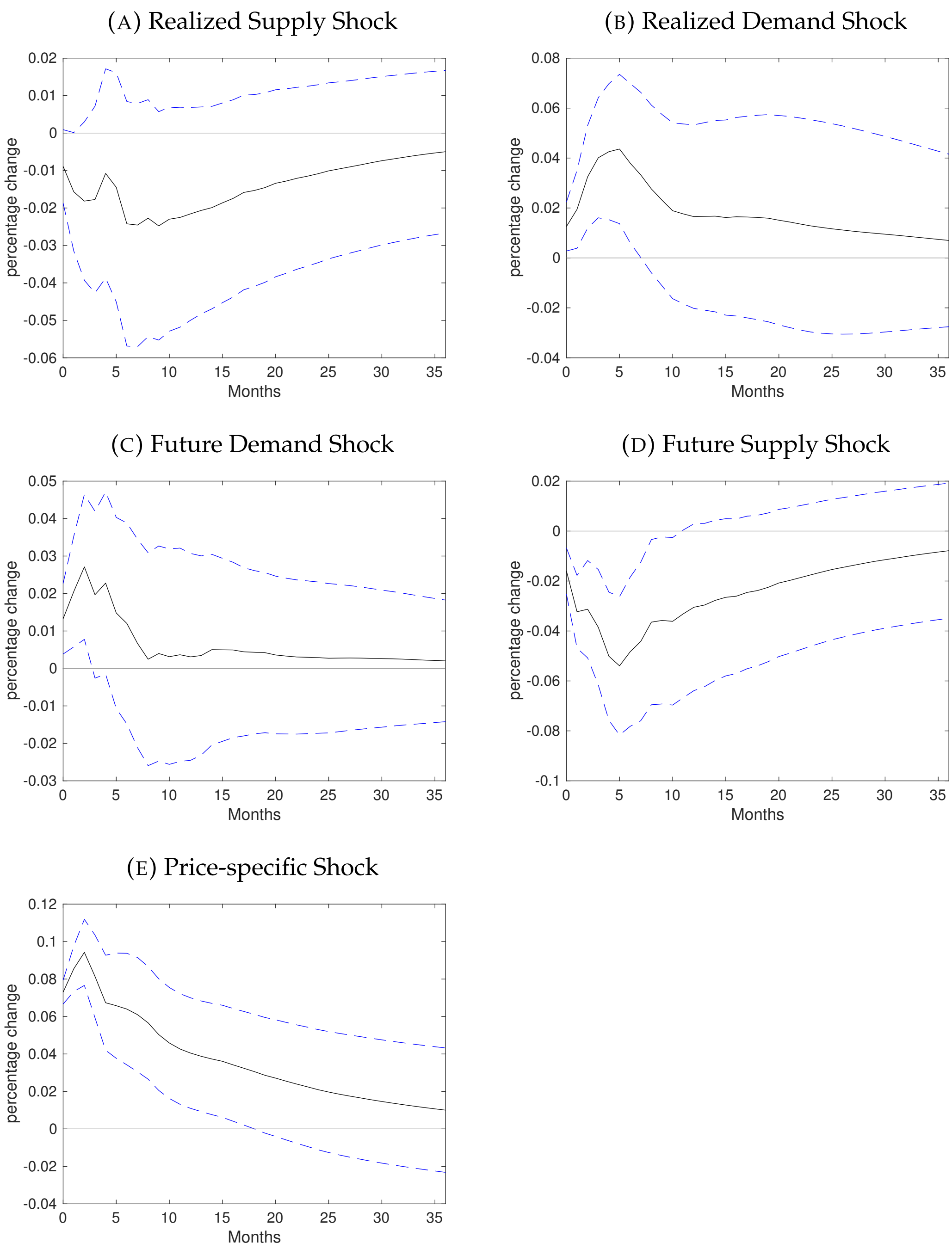


FIGURE 3.7: Impulse responses of the real oil price in the five variable VAR in the robustness check applying the Hamilton filter for the second subsample (from October 2001 to December 2019).

Note: The dashed lines refer to 95 percent confidence intervals. The horizontal axis refers to months from the shock.

Horizons (in months)	Realized Supply Shock	Realized Demand Shock	Future Demand Shock	Future Supply Shock	Oil-Price Specific Shock
1	1.3	2.6	2.9	4.2	88.9
12	5.1	12.5	2.9	20.1	59.4
24	6.7	12.2	2.5	21.8	56.8
48	7.0	12.6	2.4	22.1	55.9

TABLE 3.4: Variance decomposition of the real oil price with five variables in the robustness check applying the Hamilton filter for the second subsample period (in percent).

3.6 Conclusion

This chapter proposes a novel SVAR model of the real oil price to shed light on the role of expectations in the crude oil market. Our model enables one to quantitatively examine the respective importance of shocks on expectations about future aggregate demand and oil supply in addition to traditionally-used realized aggregate demand and oil supply factors. We find that future demand and supply shocks explain roughly 20% of oil price variance and that, among those shocks, expected future supply shocks have the largest influence on the oil price. The cumulative contribution of oil price shocks based on historical decomposition reveals the mechanism behind major episodes of oil price fluctuations. We also find that the influence of oil price changes on global output varies according to the nature of each shock. Our result suggests that it is important to understand the causes of oil price developments in evaluating their macroeconomic influence.

One area of future research is on the mechanism through which the shocks identified in this study affect the real economy. A richer structural VAR model is required to examine this link, which is left for future work. From another perspective, it is of interest to develop the time-varying structure of the proposed VAR model (e.g., Primiceri, 2005, Byrne, Lorusso, and Xu, 2017).

Appendix A

Proofs

This Appendix provides the details of the proofs for the propositions in the main article.

A.1 Proof of proposition 1

A system is locally determinate if the eigenvalues of the matrix A lie within the unit circle. Let us denote the eigenvalues of the matrix A as λ_1 and λ_2. Following Bullard and Mitra, 2002, the conditions can be expressed as

$$|\lambda_1 \lambda_2| = |\det(A)| < 1, \tag{A.1}$$

$$|\lambda_1 + \lambda_2| = |\text{trace}(A)| < 1 + \det(A). \tag{A.2}$$

The first inequality (A.1) can be modified as

$$\begin{aligned}
&|\det(A)| < 1 \\
\Leftrightarrow &\left| \frac{1+\zeta}{1+\zeta+\left(\frac{\phi_\pi}{\sigma}-\xi\right)\kappa} \times \frac{\beta + \frac{\kappa}{\sigma}(1-\xi\sigma)}{1+\zeta+\left(\frac{\phi_\pi}{\sigma}-\xi\right)\kappa} - \right. \\
&\left. \frac{\kappa}{1+\zeta+\left(\frac{\phi_\pi}{\sigma}-\xi\right)\kappa} \times \frac{\frac{1}{\sigma}(1+\zeta)(1-\xi\sigma) - \frac{\beta}{\sigma}(\phi_\pi - \xi\sigma)}{1+\zeta+\left(\frac{\phi_\pi}{\sigma}-\xi\right)\kappa} \right| < 1 \\
\Leftrightarrow &\left| \frac{\beta}{1+\zeta+\left(\frac{\phi_\pi}{\sigma}-\xi\right)\kappa} \right| < 1 \\
\Leftrightarrow &\left| \frac{\beta}{1+\frac{\phi_\pi}{\sigma}\kappa - \Lambda} \right| < 1.
\end{aligned} \tag{A.3}$$

Therefore, Λ must satisfy the following conditions:

$$\Lambda < -\beta + 1 + \frac{\phi_\pi}{\sigma}\kappa, \tag{A.4}$$

or

$$\Lambda > \beta + 1 + \frac{\phi_\pi}{\sigma}\kappa. \tag{A.5}$$

The second inequality (A.2) can be modified as

$$\begin{aligned}
&|\text{trace}(A)| < 1 + \det(A) \\
\Leftrightarrow & \left| \frac{1+\zeta}{1+\zeta+\left(\frac{\phi_\pi}{\sigma}-\xi\right)\kappa} + \frac{\beta+\frac{\kappa}{\sigma}(1-\xi\sigma)}{1+\zeta+\left(\frac{\phi_\pi}{\sigma}-\xi\right)\kappa} \right| \\
& < \frac{\beta}{1+\zeta+\left(\frac{\phi_\pi}{\sigma}-\xi\right)\kappa} + 1 \\
\Leftrightarrow & \left| \frac{1+\beta+\frac{\kappa}{\sigma}-\Lambda}{1+\frac{\phi_\pi}{\sigma}\kappa-\Lambda} \right| < \frac{\beta}{1+\frac{\phi_\pi}{\sigma}\kappa-\Lambda} + 1.
\end{aligned} \tag{A.6}$$

First, assuming $\Lambda < -\beta + 1 + \frac{\phi_\pi}{\sigma}\kappa$, we obtain the following relation:

$$\begin{aligned}
& \frac{1+\beta+\frac{\kappa}{\sigma}-\Lambda}{1+\frac{\phi_\pi}{\sigma}\kappa-\Lambda} < \frac{\beta}{1+\frac{\phi_\pi}{\sigma}\kappa-\Lambda} + 1 \\
\Leftrightarrow \ & \phi_\pi > 1.
\end{aligned} \tag{A.7}$$

This is satisfied from our assumption.

Next, assuming $\Lambda > \beta + 1 + \frac{\kappa}{\sigma}\phi_\pi$, we obtain the following relation:

$$\begin{aligned}
& \frac{1+\beta+\frac{\kappa}{\sigma}-\Lambda}{1+\frac{\phi_\pi}{\sigma}\kappa-\Lambda} < \frac{\beta}{1+\frac{\phi_\pi}{\sigma}\kappa-\Lambda} + 1 \\
\Leftrightarrow \ & \phi_\pi < 1.
\end{aligned} \tag{A.8}$$

This contradicts our assumption. Therefore, the condition to ensure local determinacy around the targeted steady state is

$$\Lambda < 1 - \beta + \phi_\pi \frac{\kappa}{\sigma} \equiv \Psi^D. \tag{A.9}$$

(End of Proof)

A.2 Proof of proposition 2

The ELT equilibrium exists if and only if the following inequality is satisfied:

$$\begin{aligned} \hat{\pi}_U = \frac{\frac{\log \beta}{1-p_U}\frac{\kappa}{\sigma}}{-(1-\beta p_U) + \frac{p_U}{1-p_U}\frac{\kappa}{\sigma}} &< \frac{\log \beta}{\phi_\pi} \\ \Leftrightarrow \frac{\frac{\kappa}{\sigma}}{\frac{\kappa}{\sigma}p_U - (1-\beta p_U)(1-p_U)} &> \frac{1}{\phi_\pi}. \end{aligned} \tag{A.10}$$

Since the right-hand side of the inequality is positive, the denominator in the left-hand side must be also positive to satisfy the inequality. Hence the following two inequalities are the necessary and sufficient conditions for the ELT equilibrium to exist:

$$\frac{\kappa}{\sigma}p_U - (1-\beta p_U)(1-p_U) > 0, \tag{A.11}$$

and

$$\frac{\kappa \phi_\pi}{\sigma} > \frac{\kappa}{\sigma}p_U - (1-\beta p_U)(1-p_U). \tag{A.12}$$

Inequality (A.12) can be modified as

$$\left[p_U - \frac{1}{2}\left(1 + \frac{1}{\beta} + \frac{\kappa}{\sigma\beta}\right)\right]^2 + \frac{\kappa\phi_\pi}{\beta\sigma} + \frac{1}{\beta} - \frac{1}{4}\left(1 + \frac{1}{\beta} + \frac{\kappa}{\sigma\beta}\right)^2 > 0. \tag{A.13}$$

Above inequality is satisfied under standard calibration. The solution of (A.11) is

$$\underbrace{\frac{1}{2}\left(1+\frac{1}{\beta}+\frac{\kappa}{\sigma\beta}\right)-\sqrt{\frac{1}{4}\left(1+\frac{1}{\beta}+\frac{\kappa}{\sigma\beta}\right)^2-\frac{1}{\beta}}}_{\equiv \underline{p}} < p_U$$
$$< \underbrace{\frac{1}{2}\left(1+\frac{1}{\beta}+\frac{\kappa}{\sigma\beta}\right)+\sqrt{\frac{1}{4}\left(1+\frac{1}{\beta}+\frac{\kappa}{\sigma\beta}\right)^2-\frac{1}{\beta}}}_{\equiv \bar{p}}. \tag{A.14}$$

Since $\bar{p} > 1$ holds, the necessary and sufficient condition is

$$\underline{p} < p_U < 1. \tag{A.15}$$

(End of Proof)

A.3 Proof of proposition 3

The same steps are taken as in proposition 2. The ELT equilibrium exists if and only if the following inequality is satisfied:

$$\hat{\pi}_U = \frac{\frac{\log\beta}{1-p_U}\frac{\kappa}{\sigma}}{\Lambda-(1-\beta p_U)+\frac{p_U}{1-p_U}\frac{\kappa}{\sigma}} < \frac{\log\beta}{\phi_\pi}$$
$$\Leftrightarrow \frac{\frac{1}{1-p_U}\frac{\kappa}{\sigma}}{\Lambda-(1-\beta p_U)+\frac{p_U}{1-p_U}\frac{\kappa}{\sigma}} > \frac{1}{\phi_\pi}. \tag{A.16}$$

The numerator of the left-hand side is positive. Therefore, inequality (A.16) holds for the following Λ:

$$\underbrace{1-\beta p_U-\frac{p_U}{1-p_U}\frac{\kappa}{\sigma}}_{\equiv \Psi} < \Lambda < \underbrace{1-\beta p_U+\frac{\phi_\pi-p_U}{1-p_U}\frac{\kappa}{\sigma}}_{\equiv \Psi^N}. \tag{A.17}$$

Taking the contraposition, the ELT does not exist if and only if

$$\Psi^N \leq \Lambda, \tag{A.18}$$

or

$$\Lambda \leq \Psi. \tag{A.19}$$

Since $\Psi < \Psi^D < \Psi^N$, the second inequality (A.19) satisfies the determinacy condition (A.9) while the first inequality (A.18) does not.

Note that if we assume the ELT equilibrium exists without any policy intervention, (A.11) indicates:

$$\Psi \equiv (1 - \beta p_U) - \frac{p_U}{1 - p_U}\frac{\kappa}{\sigma} < 0. \tag{A.20}$$

Therefore, the threshold Ψ is negative. (End of Proof)

A.4 Proof of proposition 4

Taking the derivative of Ψ with respect to p_U shows

$$\frac{\partial \Psi}{\partial p_U} = -\beta - \frac{\kappa}{\sigma}\frac{1}{(p_U - 1)^2} < 0.$$

Therefore, the threshold level Ψ is decreasing in transition probability p_U. (End of Proof)

A.5 Proof of proposition 5

Λ is equal to 0 when $\lambda_w = 0$. The ELT equilibrium exists if and only if

$$\hat{\pi}_U = \log \beta \frac{\Phi}{\Upsilon} < \frac{\log \beta}{\phi_\pi} \tag{A.21}$$

$$\Leftrightarrow \frac{\Phi}{\Upsilon} > \frac{1}{\phi_\pi}. \tag{A.22}$$

Note that Φ is positive from our assumption. Since the right-hand side of the inequality (A.22) is positive, Υ must be also positive ($\Upsilon > 0$). Then, the condition can be arranged as

$$\Phi\phi_\pi - \Upsilon > 0. \tag{A.23}$$

(End of Proof)

A.6 Proof of proposition 6

The ELT equilibrium does not exist if and only if the following inequality holds:

$$\hat{\pi}_U \geq \frac{\log\beta}{\phi_\pi}. \tag{A.24}$$

Equation $\hat{\pi}_U = \log\beta(\Phi - \Omega\Lambda)[(1-\Omega)\Lambda + \Upsilon]^{-1}$ can be regarded as a hyperbola taking $\hat{\pi}_U$ in the vertical axis and Λ in the horizontal axis. The equation can be arranged as

$$\hat{\pi}_U + \frac{\Omega}{1-\Omega}\log\beta = \frac{\Omega}{1-\Omega}\log\beta\frac{\Upsilon + \frac{1-\Omega}{\Omega}\Phi}{(1-\Omega)\Lambda + \Upsilon}. \tag{A.25}$$

Following inequality shows that the horizontal asymptote of the hyperbola is higher than the threshold level:

$$\frac{\log\beta}{\phi_\pi} < -\frac{\Omega}{1-\Omega}\log\beta. \tag{A.26}$$

There are two regions of Λ that satisfies (A.24). The first region is

$$\Lambda \geq \frac{\Phi\phi_\pi - \Upsilon}{\Omega(\phi_\pi - 1) + 1}. \tag{A.27}$$

However, above inequality contradicts the determinacy condition given in (A.9). The second region is

$$\Lambda \leq -\frac{\Upsilon}{1-\Omega} \equiv \tilde{\Psi}. \tag{A.28}$$

Above region satisfies the determinacy condition. (End of Proof)

A.7 Proof of proposition 7

An equilibrium exists in the crisis state if and only if the following inequalities are satisfied in each case:

$$\hat{\pi}_L = \frac{\frac{-r^n_L}{1-p^*_L}\frac{\kappa}{\sigma}}{-(1-\beta p^*_L) - \frac{\phi_\pi - p^*_L}{1-p^*_L}\frac{\kappa}{\sigma}} \geq \frac{\log \beta}{\phi_\pi}, \tag{A.29}$$

$$\hat{\pi}_L = \frac{\frac{\log \beta - r^n_L}{1-p^*_L}\frac{\kappa}{\sigma}}{-(1-\beta p^*_L) + \frac{p^*_L}{1-p^*_L}\frac{\kappa}{\sigma}} < \frac{\log \beta}{\phi_\pi}. \tag{A.30}$$

(i) When the ZLB does not bind

The first inequality can be modified as

$$\frac{\frac{\kappa}{\sigma}}{\frac{\kappa}{\sigma}(p^*_L - \phi_\pi) - (1-\beta p^*_L)(1-p^*_L)} \geq -\frac{\log \beta}{\phi_\pi r^n_L}. \tag{A.31}$$

The denominator of the left-hand side in inequality (A.31) is negative under standard calibration:

$$-\left[p^*_L - \frac{1}{2}\left(1 + \frac{1}{\beta} + \frac{\kappa}{\sigma\beta}\right)\right]^2 - \frac{\kappa\phi_\pi}{\beta\sigma} - \frac{1}{\beta} + \frac{1}{4}\left(1 + \frac{1}{\beta} + \frac{\kappa}{\sigma\beta}\right)^2 < 0. \tag{A.32}$$

Therefore inequality (A.31) can be modified as

$$\frac{\kappa}{\sigma}(p^*_L - \phi_\pi) - (1-\beta p^*_L)(1-p^*_L) \leq \frac{\kappa}{\sigma}\frac{\phi_\pi}{\log \beta}(-r^n_L). \tag{A.33}$$

The solution to the inequality is

$$p_L^* \leq \underbrace{\frac{1}{2}\left(1+\frac{1}{\beta}+\frac{\kappa}{\sigma\beta}\right) - \sqrt{\frac{1}{4}\left(1+\frac{1}{\beta}+\frac{\kappa}{\sigma\beta}\right)^2 - \frac{1}{\beta} - \frac{\kappa}{\sigma\beta}\phi_\pi + \frac{\kappa}{\sigma\beta}\frac{\phi_\pi}{\log\beta}r_L^n}}_{\equiv p_\dagger}, \tag{A.34}$$

or

$$\underbrace{\frac{1}{2}\left(1+\frac{1}{\beta}+\frac{\kappa}{\sigma\beta}\right) + \sqrt{\frac{1}{4}\left(1+\frac{1}{\beta}+\frac{\kappa}{\sigma\beta}\right)^2 - \frac{1}{\beta} - \frac{\kappa}{\sigma\beta}\phi_\pi + \frac{\kappa}{\sigma\beta}\frac{\phi_\pi}{\log\beta}r_L^n}}_{\equiv p^\dagger} \leq p_L^*. \tag{A.35}$$

Since $0 < p_\dagger$ and $1 < p^\dagger$, the condition is

$$0 < p_L^* \leq p_\dagger. \tag{A.36}$$

(ii) When the ZLB binds

The second inequality can be modified as

$$\frac{\frac{\kappa}{\sigma}}{\frac{\kappa}{\sigma}p_L^* - (1-\beta p_L^*)(1-p_L^*)} < \frac{\log\beta}{\phi_\pi(\log\beta - r_L^n)}. \tag{A.37}$$

Inequality (A.37) holds if and only if the following two inequalities are satisfied:

$$\underbrace{-(1-\beta p_L^*) + \frac{p_L^*}{1-p_L^*}\frac{\kappa}{\sigma}}_{\equiv -\Psi^F} < 0, \tag{A.38}$$

and

$$\frac{\kappa}{\sigma}p_L^* - (1-\beta p_L^*)(1-p_L^*) > \frac{\kappa}{\sigma}\frac{\phi_\pi}{\log\beta}(-r_L^n) + \frac{\kappa}{\sigma}\phi_\pi. \tag{A.39}$$

Inequalities (A.38) and (A.39) can be solved as

$$p_L^* < \underline{p} \quad \text{or} \quad \bar{p} < p_L^*, \tag{A.40}$$

and

$$p_\dagger < p_L^* < p^\dagger. \tag{A.41}$$

Since $p_{\dagger} < \underline{p} < 1 < p^{\dagger}$, the conditions can be summarized to

$$p_{\dagger} < p_L^* < \underline{p}. \tag{A.42}$$

Combining the two conditions (A.36) and (A.42), we obtain the condition as follows:

$$0 < p_L^* < \underline{p}. \tag{A.43}$$

Note that $p_L^* < \underline{p} < p_U$ implies $\Psi < 0 < \Psi^F$. (End of Proof)

A.8 Proof of proposition 8

Let us denote the equilibrium inflation and output as $\hat{\pi}_L^{NI}$ and $\hat{y}_L^{NI}$ (NI stands for "No Intervention") in the case where the tax rate does not respond to inflation ($\Lambda = 0$). Inflation rate in the crisis state is higher compared to the case without the tax rule if the following inequality holds:

$$\hat{\pi}_L > \hat{\pi}_L^{NI}. \tag{A.44}$$

Since we have restricted our focus to the case where the ZLB binds in the crisis state, we can modify the condition as

$$\frac{\frac{\log\beta - r_L^n}{1-p_L^*}\frac{\kappa}{\sigma}}{\Lambda - (1-\beta p_L^*) + \frac{p_L^*}{1-p_L^*}\frac{\kappa}{\sigma}} > \frac{\frac{\log\beta - r_L^n}{1-p_L^*}\frac{\kappa}{\sigma}}{-(1-\beta p_L^*) + \frac{p_L^*}{1-p_L^*}\frac{\kappa}{\sigma}}. \tag{A.45}$$

The numerator in both sides are positive while the denominator in the right-hand side ($-\Psi^F$) is negative under the assumption $p_{\dagger} \le p_L^* < \underline{p}$ from (A.38). Therefore, above inequality can be solved as

$$\Lambda < 0 \text{ or } (1-\beta p_L^*) - \frac{p_L^*}{1-p_L^*}\frac{\kappa}{\sigma} < \Lambda. \tag{A.46}$$

The first condition $\Lambda < 0$ is satisfied when the fiscal authority sets $\Lambda \le \Psi$ to avoid the ELT equilibrium since $\Psi < 0$. Therefore, as long as the fiscal authority targets to prevent the ELT equilibrium, $\hat{\pi}_L > \hat{\pi}_L^{NI}$ holds. (End of Proof)

Appendix B

Models with Different Tax Instruments

This Appendix provides a full description of the optimization problem and the conditions to prevent the ELT equilibrium with different tax instruments. For completeness, dividend tax and consumption tax are also included as the fiscal authority's target in addition to the labor income tax. In the following analysis, steady state tax rates are calibrated to $\tau_w = 0.2$, $\tau_c = 0.2$, and $\tau_d = 0.2$ respectively.

B.1 Optimization problem

B.1.1 Household

A representative household maximizes its lifetime utility subject to the budget constraint:

$$U = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \Big[\frac{c_{t+s}^{1-\sigma} - 1}{1-\sigma} - \frac{l_{t+s}^{\eta+1} - 1}{\eta + 1} \Big],$$

$$(1+\tau_{c,t})c_t + \frac{b_t}{R_t} = (1-\tau_{w,t})w_t l_t + \frac{b_{t-1}}{\Pi_t} + (1-\tau_{d,t})d_t. \tag{B.1}$$

The Lagrangian can be set up as follows:

$$\mathcal{L} = \mathbb{E}_t \sum_{s=0}^{\infty} \Big\{ \beta^s \Big[\frac{c_{t+s}^{1-\sigma} - 1}{1-\sigma} - \frac{l_{t+s}^{\eta+1} - 1}{\eta + 1} \Big] - \mu_{t+s} \Big[(1+\tau_{c,t+s})c_{t+s} + \frac{b_{t+s}}{R_{t+s}} - (1-\tau_{w,t+s})w_{t+s}l_{t+s} - \frac{b_{t+s-1}}{\Pi_{t+s}} - (1-\tau_{d,t+s})f_{t+s} - \tau_{t+s} \Big] \Big\}. \tag{B.2}$$

Household takes prices $\{w_t, P_t, R_t\}_{t=0}^{\infty}$ as given. The first order conditions can be derived as

$$\text{w/r } c_{t+s} \ : \beta^s c_{t+s}^{-\sigma} - \mu_{t+s}(1+\tau_{c,t+s}) = 0, \tag{B.3}$$

$$\text{w/r } l_{t+s} \ : -\beta^s l_{t+s}^{\eta} + \mu_{t+s}(1-\tau_{w,t+s}) w_{t+s} = 0, \tag{B.4}$$

$$\text{w/r } b_{t+s} \ : -\frac{\mu_{t+s}}{R_{t+s}} + \mathbb{E}_{t+s}\frac{\mu_{t+s+1}}{\Pi_{t+s+1}} = 0. \tag{B.5}$$

Equilibrium conditions are

$$\frac{c_t^{-\sigma}}{1+\tau_{c,t}} = \beta R_t \mathbb{E}_t\Big[\frac{c_{t+1}^{-\sigma}}{1+\tau_{c,t+1}}\frac{1}{\Pi_{t+1}}\Big], \tag{B.6}$$

$$\frac{c_t^{-\sigma}}{l_t^{\eta}} = \frac{1+\tau_{c,t}}{1-\tau_{w,t}}\frac{1}{w_t}. \tag{B.7}$$

B.1.2 Firms

The optimization problem for the final goods producer is

$$\max_{\{y_t, y_{i,t}\}} P_t y_t - \int_0^1 P_{i,t} y_{i,t} di - \lambda\Big[y_t - \Big(\int_0^1 y_{i,t}^{\frac{\theta-1}{\theta}} di\Big)^{\frac{\theta}{\theta-1}}\Big]. \tag{B.8}$$

First order conditions are

$$\text{w/r } y_t \ : \ P_t = \lambda, \tag{B.9}$$

$$\text{w/r } y_{i,t} \ : \ P_{i,t} = \lambda\Big[\int_0^1 y_{i,t}^{\frac{\theta-1}{\theta}} di\Big]^{\frac{1}{\theta-1}} y_{i,t}^{-\frac{1}{\theta}}. \tag{B.10}$$

Substituting out the Lagrange multiplier, we obtain

$$y_{i,t} = \Big(\frac{P_{i,t}}{P_{t+s}}\Big)^{-\theta} y_t, \tag{B.11}$$

$$P_t = \Big(\int_0^1 P_{i,t}^{1-\theta} di\Big)^{\frac{1}{1-\theta}}. \tag{B.12}$$

The optimization problem for the intermediate goods producers are

$$\max_{\{y_{i,t+s}, P_{i,t+s}, l_{i,t+s}\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} Q_{c,t+s}(1-\tau_{d,t+s}) d_{i,t+s}, \tag{B.13}$$

$$\text{s.t.} \quad d_{i,t+s} = \frac{P_{i,t+s}}{P_{t+s}} y_{i,t+s} - w_{t+s} l_{i,t+s} - \frac{\psi}{2}\Big(\frac{P_{i,t+s}}{P_{i,t+s-1}} - 1\Big)^2 y_{t+s}, \tag{B.14}$$

$$y_{i,t+s} = l_{i,t+s}, \tag{B.15}$$

$$y_{i,t+s} = \Big(\frac{P_{i,t+s}}{P_{t+s}}\Big)^{-\theta} y_{t+s}. \tag{B.16}$$

where the stochastic discount factor is defined as

$$Q_{c,t+s} \equiv \beta^s \frac{c_{t+s}^{-\sigma}}{1+\tau_{c,t+s}}. \tag{B.17}$$

Note that the consumption tax rate is included in the stochastic discount factor. We can set up the Lagrangian as

$$\mathcal{L} = E_t \sum_{s=0}^{\infty} \Big\{ Q_{c,t+s}(1-\tau_{d,t+s}) \Big[\frac{P_{i,t+s}}{P_{t+s}} y_{i,t+s} - w_{t+s} l_{i,t+s} - \frac{\psi}{2}\Big(\frac{P_{i,t+s}}{P_{i,t+s-1}} - 1\Big)^2 y_{t+s} \Big]$$
$$- \mu_{t+s}(y_{i,t+s} - l_{i,t+s}) - \phi_{t+s}\Big(y_{i,t+s} - \Big(\frac{P_{i,t+s}}{P_{t+s}}\Big)^{-\theta} y_{t+s} \Big) \Big\}. \tag{B.18}$$

First order conditions of the optimization problem for the firms are

$$\text{w/r } l_{i,t} \ : \mu_t - Q_{c,t}(1-\tau_{d,t}) w_t = 0, \tag{B.19}$$

$$\text{w/r } y_{i,t} \ : Q_{c,t}(1-\tau_{d,t}) \frac{P_{i,t}}{P_t} - \mu_t - \phi_t = 0, \tag{B.20}$$

$$\text{w/r } P_{i,t} \ : Q_{c,t}(1-\tau_{d,t}) \Big[\frac{y_{i,t}}{P_t} - \psi\Big(\frac{P_{i,t}}{P_{i,t-1}} - 1\Big) \frac{y_t}{P_{i,t-1}} \Big] - \theta \phi_t \Big(\frac{P_{i,t}}{P_t}\Big)^{\theta} \frac{y_t}{P_{i,t}}$$
$$= \mathbb{E}_t \Big[Q_{c,t+1}(1-\tau_{d,t+1}) \psi\Big(\frac{P_{i,t+1}}{P_{i,t}} - 1\Big)\Big(- \frac{P_{i,t+1}}{P_{i,t}^2}\Big) y_{t+1} \Big]. \tag{B.21}$$

Substituting out the Lagrange multipliers and imposing symmetry across firms, we can derive the Philips curve as

$$\psi(\Pi_t - 1)\Pi_t - \theta w_t + \theta - 1 = \beta \mathbb{E}_t \Big[\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{y_{t+1}}{y_t} \frac{1-\tau_{d,t+1}}{1-\tau_{d,t}} \psi(\Pi_{t+1} - 1)\Pi_{t+1} \Big]. \tag{B.22}$$

We can observe that both consumption tax and dividend tax are included in the PC.

B.1.3 Central bank and fiscal authority

The central bank sets the interest rate following the standard Taylor rule. The net nominal interest rate is bounded below by zero:

$$R_t = \max\left[1, \frac{1}{\beta}\Pi_t^{\phi_\pi}\right]. \tag{B.23}$$

The government's budget constraint with consumption tax, dividend tax, and labor income tax is

$$\frac{b_t}{R_t} + \tau_{c,t}c_t + \tau_{w,t}w_t l_t + \tau_{d,t}d_t = \frac{b_{t-1}}{\Pi_t} + g_t. \tag{B.24}$$

It is further assumed that the government spending is determined endogenously. Namely, the total amount of tax revenue constrains the amount of goods that the fiscal authority purchases:

$$\tau_{c,t}c_t + \tau_{w,t}w_t l_t + \tau_{d,t}d_t = g_t. \tag{B.25}$$

Therefore, the fiscal authority does not issue bonds in the equilibrium ($b_t = 0$).

B.2 Equilibrium conditions

The complete set of equilibrium conditions with eleven variables $\{c_t, y_t, l_t, d_t, g_t, w_t, \Pi_t, R_t, \tau_{w,t}, \tau_{c,t}, \tau_{d,t}\}$ are as follows:

$$\frac{c_t^{-\sigma}}{1+\tau_{c,t}} = \beta R_t \mathbb{E}_t\Big[\frac{c_{t+1}^{-\sigma}}{1+\tau_{c,t+1}}\frac{1}{\Pi_{t+1}}\Big], \tag{B.26}$$

$$\frac{c_t^{-\sigma}}{l_t^{\eta}} = \frac{1+\tau_{c,t}}{1-\tau_{w,t}}\frac{1}{w_t}, \tag{B.27}$$

$$\frac{c_t^{-\sigma}}{1+\tau_{c,t}}(1-\tau_{d,t})\Big[\psi(\Pi_t-1)\Pi_t - \theta w_t + \theta - 1\Big] = \beta\mathbb{E}_t\Big[\frac{c_{t+1}^{-\sigma}}{1+\tau_{c,t+1}}(1-\tau_{d,t+1})\psi(\Pi_{t+1}-1)\Pi_{t+1}\frac{y_{t+1}}{y_t}\Big], \tag{B.28}$$

$$y_t = l_t, \tag{B.29}$$

$$d_t = y_t - w_t l_t - \frac{\psi}{2}(\Pi_t - 1)^2 y_t, \tag{B.30}$$

$$R_t = \max\Big[1, \frac{1}{\beta}\Pi_t^{\phi_\pi}\Big], \tag{B.31}$$

$$y_t = c_t + g_t + \frac{\psi}{2}(\Pi_t-1)^2 y_t, \tag{B.32}$$

$$g_t = \tau_{c,t}c_t + \tau_{w,t}w_t l_t + \tau_{d,t}d_t, \tag{B.33}$$

$$\tau_{c,t} = \tau_c\Pi_t^{\lambda_c}, \quad \tau_{w,t} = \tau_w\Pi_t^{\lambda_w}, \quad \tau_{d,t} = \tau_d\Pi_t^{\lambda_d}. \tag{B.34}$$

Equilibrium conditions other than the tax rules can be summarized to the following four equations with four variables:

$$\frac{c_t^{-\sigma}}{1+\tau_{c,t}} = \beta R_t \mathbb{E}_t\Big[\frac{c_{t+1}^{-\sigma}}{1+\tau_{c,t+1}}\frac{1}{\Pi_{t+1}}\Big], \tag{B.35}$$

$$\frac{1-\tau_{d,t}}{1+\tau_{c,t}}\Big[\psi(\Pi_t-1)\Pi_t - \theta\frac{1+\tau_{c,t}}{1-\tau_{w,t}}c_t^{\sigma}y_t^{\eta} + \theta - 1\Big] = \beta\mathbb{E}_t\Big[\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}}\frac{1-\tau_{d,t+1}}{1+\tau_{c,t+1}}\frac{y_{t+1}}{y_t}\psi(\Pi_{t+1}-1)\Pi_{t+1}\Big], \tag{B.36}$$

$$R_t = \max\Big[1, \frac{1}{\beta}\Pi_t^{\phi_\pi}\Big], \tag{B.37}$$

$$(1-\tau_{d,t})y_t\Big[1 - \frac{\psi}{2}(\Pi_t-1)^2\Big] = (1+\tau_{c,t})c_t + (\tau_{w,t}-\tau_{d,t})\frac{1+\tau_{c,t}}{1-\tau_{w,t}}c_t^{\sigma}y_t^{\eta+1}. \tag{B.38}$$

Steady state values are

$$R_{TSS} = \frac{1}{\beta}, \tag{B.39}$$

$$y_{TSS} = (1+\tau_c)^{\frac{\sigma}{\eta+\sigma}} \left[1 - \tau_d - (\tau_w - \tau_d)\frac{\theta-1}{\theta}\right]^{-\frac{\sigma}{\eta+\sigma}} \left(\frac{\theta-1}{\theta}\frac{1-\tau_w}{1+\tau_c}\right)^{\frac{1}{\eta+\sigma}}, \tag{B.40}$$

$$c_{TSS} = (1+\tau_c)^{-\frac{\eta}{\eta+\sigma}} \left[1 - \tau_d - (\tau_w - \tau_d)\frac{\theta-1}{\theta}\right]^{\frac{\eta}{\eta+\sigma}} \left(\frac{\theta-1}{\theta}\frac{1-\tau_w}{1+\tau_c}\right)^{\frac{1}{\eta+\sigma}}. \tag{B.41}$$

Log-linearized equilibrium conditions are

$$\hat{c}_t = \mathbb{E}_t\hat{c}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1}) - \frac{1}{\sigma}\frac{\tau_c}{1+\tau_c}(\hat{\tau}_{c,t} - \mathbb{E}_t\hat{\tau}_{c,t+1}), \tag{B.42}$$

$$\hat{\pi}_t = \beta\mathbb{E}_t\hat{\pi}_{t+1} + \sigma\frac{\theta-1}{\psi}\hat{c}_t + \eta\frac{\theta-1}{\psi}\hat{y}_t + \frac{\theta-1}{\psi}\frac{\tau_c}{1+\tau_c}\hat{\tau}_{c,t} - \frac{\theta-1}{\psi}\frac{\tau_w}{1-\tau_w}\hat{\tau}_{w,t}, \tag{B.43}$$

$$\hat{i}_t = \max[\log\beta, \phi_\pi\hat{\pi}_t], \tag{B.44}$$

$$\gamma_y\hat{y}_t = \gamma_c\hat{c}_t + \gamma_{\tau,c}\hat{\tau}_{c,t} + \gamma_{\tau,w}\hat{\tau}_{w,t} + \gamma_{\tau,d}\hat{\tau}_{d,t}, \tag{B.45}$$

$$\hat{\tau}_{c,t} = \lambda_c\hat{\pi}_t, \;\; \hat{\tau}_{w,t} = \lambda_w\hat{\pi}_t, \;\; \hat{\tau}_{d,t} = \lambda_d\hat{\pi}_t, \tag{B.46}$$

where

$$\gamma_y \equiv 1 - \tau_d - (\eta+1)(\tau_w - \tau_d)\frac{\theta-1}{\theta}, \quad \gamma_c \equiv (1+\tau_c)\frac{c_{TSS}}{y_{TSS}} + \sigma(\tau_w - \tau_d)\frac{\theta-1}{\theta},$$

$$\gamma_{\tau,c} \equiv \left[(1+\tau_c)\frac{c_{TSS}}{y_{TSS}} + (\tau_w - \tau_d)\frac{\theta-1}{\theta}\right]\frac{\tau_c}{1+\tau_c}, \quad \gamma_{\tau,w} \equiv \frac{\theta-1}{\theta}\frac{\tau_w}{1-\tau_w}, \quad \gamma_{\tau,d} \equiv \frac{1}{\theta}\tau_d.$$

Above equilibrium conditions can be further summarized to following equations:

$$\begin{aligned}\hat{y}_t &= \mathbb{E}_t\hat{y}_{t+1} - \frac{1}{\sigma}(\max[\log\beta, \phi_\pi\hat{\pi}_t] - \mathbb{E}_t\hat{\pi}_{t+1}) + \left(\frac{\gamma_{\tau,c}}{\gamma_y} - \frac{1}{\sigma}\frac{\tau_c}{1+\tau_c}\frac{\gamma_c}{\gamma_y}\right)\hat{\tau}_{c,t} + \frac{\gamma_{\tau,w}}{\gamma_y}\hat{\tau}_{w,t} \\ &+ \frac{\gamma_{\tau,d}}{\gamma_y}\hat{\tau}_{d,t} + \left(\frac{1}{\sigma}\frac{\tau_c}{1+\tau_c}\frac{\gamma_c}{\gamma_y} - \frac{\gamma_{\tau,c}}{\gamma_y}\right)\mathbb{E}_t\hat{\tau}_{c,t+1} - \frac{\gamma_{\tau,w}}{\gamma_y}\mathbb{E}_t\hat{\tau}_{w,t+1} - \frac{\gamma_{\tau,d}}{\gamma_y}\mathbb{E}_t\hat{\tau}_{d,t+1},\end{aligned} \tag{B.47}$$

$$\begin{aligned}\hat{\pi}_t &= \beta\mathbb{E}_t\hat{\pi}_{t+1} + \frac{\theta-1}{\psi}\left(\eta + \sigma\frac{\gamma_y}{\gamma_c}\right)\hat{y}_t + \frac{\theta-1}{\psi}\left(\frac{\tau_c}{1+\tau_c} - \sigma\frac{\gamma_{\tau,c}}{\gamma_c}\right)\hat{\tau}_{c,t} \\ &- \frac{\theta-1}{\psi}\left(\sigma\frac{\gamma_{\tau,w}}{\gamma_c} + \frac{\tau_w}{1-\tau_w}\right)\hat{\tau}_{w,t} - \sigma\frac{\theta-1}{\psi}\frac{\gamma_{\tau,d}}{\gamma_c}\hat{\tau}_{d,t},\end{aligned} \tag{B.48}$$

$$\hat{\tau}_{c,t} = \lambda_c\hat{\pi}_t, \;\; \hat{\tau}_{w,t} = \lambda_w\hat{\pi}_t, \;\; \hat{\tau}_{d,t} = \lambda_d\hat{\pi}_t. \tag{B.49}$$

After substitution, the equilibrium conditions simplify to the following EE and PC with two variables $\hat{\pi}_t$ and $\hat{y}_t$:

$$\hat{y}_t = \xi\hat{\pi}_t + \mathbb{E}_t\hat{y}_{t+1} - \frac{1}{\sigma}(\max[\log\beta, \phi_\pi\hat{\pi}_t] - \mathbb{E}_t\hat{\pi}_{t+1}) - \xi\mathbb{E}_t\hat{\pi}_{t+1}, \tag{B.50}$$

$$\hat{\pi}_t = \kappa\hat{y}_t - \zeta\hat{\pi}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1}, \tag{B.51}$$

where
$$\kappa \equiv \frac{\theta-1}{\psi}\left(\eta + \sigma\frac{\gamma_y}{\gamma_c}\right),$$
$$\xi \equiv \left(\frac{\gamma_{\tau,c}}{\gamma_y} - \frac{1}{\sigma}\frac{\tau_c}{1+\tau_c}\frac{\gamma_c}{\gamma_y}\right)\lambda_c + \frac{\gamma_{\tau,w}}{\gamma_y}\lambda_w + \frac{\gamma_{\tau,d}}{\gamma_y}\lambda_d,$$
$$\zeta \equiv \frac{\theta-1}{\psi}\left(\sigma\frac{\gamma_{\tau,c}}{\gamma_c} - \frac{\tau_c}{1+\tau_c}\right)\lambda_c + \frac{\theta-1}{\psi}\left(\sigma\frac{\gamma_{\tau,w}}{\gamma_c} + \frac{\tau_w}{1-\tau_w}\right)\lambda_w + \sigma\frac{\theta-1}{\psi}\frac{\gamma_{\tau,d}}{\gamma_c}\lambda_d.$$

Equations (B.50) and (B.51) are identical to equations (1.34) and (1.35) in the main article with different definitions for ξ and ζ. Therefore, all propositions hold for models in this Appendix by replacing Λ with

$$\Lambda \equiv \xi\kappa - \zeta = \frac{\theta-1}{\psi}\left[\eta\left(\frac{\gamma_{\tau,c}}{\gamma_y} - \frac{1}{\sigma}\frac{\tau_c}{1+\tau_c}\frac{\gamma_c}{\gamma_y}\right)\lambda_c + \left(\eta\frac{\gamma_{\tau,w}}{\gamma_y} - \frac{\tau_w}{1-\tau_w}\right)\lambda_w + \eta\frac{\gamma_{\tau,d}}{\gamma_y}\lambda_d\right].$$

Note that equations in the main article are particular cases with $\tau_c = 0$, $\gamma_{\tau,c} = 0$, $\tau_d = 0$, and $\gamma_{\tau,d} = 0$.

B.3 Preventing the ELT equilibrium

Proposition 3 in the main article claims that the fiscal authority prevents the ELT equilibrium if the tax response parameters satisfy the following condition:

$$\Lambda \leq 1 - \beta p_U - \frac{\kappa}{\sigma}\frac{p_U}{1-p_U} \equiv \Psi \tag{B.52}$$

In the following subsections, we discuss how the use of different tax instruments affects the equilibrium outcome.

B.3.1 Consumption tax rate adjustment

Changes in the consumption tax rate operate through the demand side, while it can prevent the ELT equilibrium as long as the utility function of the representative household is not a logarithmic utility. Let us set λ_w and λ_d to zero.

(i) When $\sigma = 1$

The utility function takes the form of log, therefore income effect and substitution effect perfectly offset each other. This is reflected in the coefficients

$$\frac{\gamma_{\tau,c}}{\gamma_y} - \frac{1}{\sigma}\frac{\tau_c}{1+\tau_c}\frac{\gamma_c}{\gamma_y} = 0$$

on λ_c. Therefore, altering λ_c cannot affect the equilibrium, and whether the ELT equilibrium exists or not does not depend on the choice of λ_c.

(ii) When $\sigma \neq 1$

Inequality (B.52) simplifies to

$$\lambda_c \leq -\frac{\psi}{(\theta-1)\eta}\left(\frac{1}{\sigma}\frac{\tau_c}{1+\tau_c}\frac{\gamma_c}{\gamma_y} - \frac{\gamma_{\tau,c}}{\gamma_y}\right)^{-1}\Psi \equiv \Psi_c < 0. \tag{B.53}$$

Since Ψ_c is negative, the fiscal authority raises the consumption tax rate in response to a decrease in the inflation rate.

B.3.2 Dividend tax rate adjustment

The dividend tax rate operates through the demand side and affects household income. Let us set λ_w and λ_c equal to zero. Then, the condition (B.52) simplifies to

$$\lambda_d \leq \frac{\psi}{(\theta-1)\eta}\frac{\gamma_y}{\gamma_{\tau,d}}\Psi \equiv \Psi_d < 0. \tag{B.54}$$

Since Ψ_d is negative, the fiscal authority raises the dividend tax rate in response to a decline in the inflation rate.

The mechanism through which the demand-side policy affects the equilibrium is as follows. The negative Ψ_d implies that an increase in inflation causes the dividend tax rate to decline and increases household income. When the ZLB does not bind, the increase in consumption induced by this increase in income partially offsets the decline in consumption caused by the increase in real interest rate through intertemporal substitution.

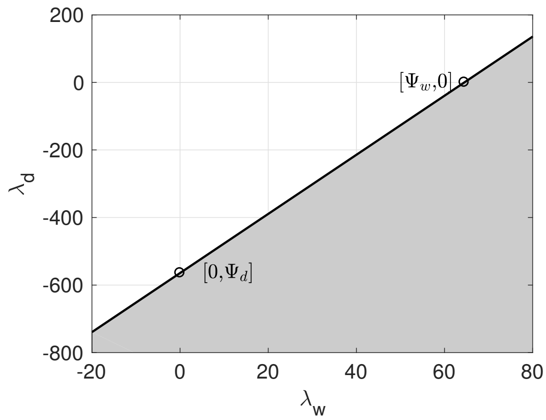


FIGURE B.1: Parameter space where the ELT equilibrium does not exist.

Alternatively, when the ZLB binds, the Taylor rule is inactive, and an increase in inflation decreases the real interest rate, which induces the household to increase current consumption. However, in addition to the increase in consumption according to the household's intertemporal substitution, the increase in income caused by the decline in the dividend tax rate also operates to increase consumption.

Changes in the dividend tax rate also affect the PC. When the inflation rate rises, output increases, driven by the rise in household income. This induces the household to increase its labor supply, which adds further inflationary pressure.

B.3.3 Combining different tax rates

We have confirmed that both supply-side and demand-side policies affect labor supply and consumption in different ways. Although we have examined each tax individually, we can combine different taxes to achieve our goal.

Let us focus on the labor income tax and the dividend tax. Any combination of λ_w and λ_d that satisfies inequality (B.52) can prevent the ELT equilibrium. The condition stated in proposition 3 of the main article can be rearranged as

$$\lambda_d \leq \frac{\gamma_y}{\gamma_{\tau,d}}\left(\frac{1}{\eta}\frac{\tau_w}{1-\tau_w} - \frac{\gamma_{\tau,w}}{\gamma_y}\right)\lambda_w + \frac{\gamma_y}{\gamma_{\tau,d}}\frac{\psi}{(\theta-1)\eta}\Psi. \tag{B.55}$$

Figure B.1 displays the area that satisfies inequality (B.55). Both the edge and the area in gray depict the parameter space where the ELT equilibrium does not exist.

Any linear combination $\mu\Psi_w + (1-\mu)\Psi_d$ lies on the edge and therefore satisfies (B.55). For simplicity, let us restrict our focus on the case with $0 \leq \mu \leq 1$.

The mechanism through which the inflation-sensitive tax rule prevents the ELT equilibrium can be summarized as follows. The existence of the ELT equilibrium requires both inflation and output to fall simultaneously. However, the proposed tax rule counteracts the decline in output when the inflation rate declines by inducing the household to increase its labor supply. If the tax rule drives the household to supply more labor, ceteris paribus, firms are operating at too low a marginal revenue product of their labor input, and the monopolistic competitor reacts by raising prices. Therefore, the private agents' belief that a decline in inflation and output occurs without any changes in the fundamentals becomes inconsistent under the fiscal authority's commitment.

B.4 Endogenous government spending

In the benchmark case, the labor income tax rate was the only tax instrument available for the fiscal authority. In such a case, whether government spending g_t increases or decreases according to changes in the inflation rate was determined by λ_w.

However, when the fiscal policy manipulates more than two different tax rates, whether g_t increases or not depends on the combination of the tax response parameters. This section investigates how the government spending g_t is affected by the choice of tax response parameters.

B.4.1 Increasing government spending when inflation becomes lower

Let us assume that the fiscal authority adjusts both λ_w and λ_d as its policy instrument and chooses μ that satisfies $\mu\Psi_w + (1-\mu)\Psi_d$. It is not obvious whether government spending increases or decreases in response to a decline in inflation since spending is determined endogenously.

On the one hand, the more the fiscal authority relies on the use of the labor income tax rate (higher μ) to prevent the ELT equilibrium, the more government spending is likely to decline due to the reduction in tax revenue. On the other hand, relying more on the dividend tax rate (lower μ) increases government spending as inflation declines. Therefore, the fiscal authority can control the variation in government spending by combining the labor income tax and the dividend tax.

Log-linearizing the government budget constraint (B.24) around the deterministic steady state and substituting out the rest of the endogenous variables, we obtain the following representation:

$$\hat{g}_t = \Gamma_\pi \hat{\pi}_t + \Gamma_y \hat{y}_t, \tag{B.56}$$

$$\text{where } \Gamma_\pi \equiv \frac{\left(\frac{\tau_w w_{TSS} l_{TSS}}{g_{TSS}} - \frac{1-2w_{TSS}}{1-w_{TSS}}\frac{\tau_w}{1-\tau_w}\right)\lambda_w + \frac{\tau_d d_{TSS}}{g_{TSS}}\lambda_d}{1 - \frac{1-2w_{TSS}}{1-w_{TSS}}\sigma\frac{g_{TSS}}{c_{TSS}}},$$

$$\Gamma_y \equiv \frac{2 + \frac{1-2w_{TSS}}{1-w_{TSS}}\left(\eta - \sigma\frac{y_{TSS}}{c_{TSS}}\right)}{1 - \frac{1-2w_{TSS}}{1-w_{TSS}}\sigma\frac{g_{TSS}}{c_{TSS}}}.$$

Γ_y is positive under standard calibration, which implies that controlling for $\hat{\pi}_t$, $\hat{g}_t$ increases as $\hat{y}_t$ increases. By imposing the restriction $\Gamma_\pi < 0$, the fiscal authority can ensure that $\hat{g}_t$ increases as $\hat{\pi}_t$ declines, controlling for $\hat{y}_t$.

Although one of the main findings of this study is that we can prevent ELT without increasing $\hat{g}_t$, the above restriction may be desirable when the fiscal authority prefers to avoid a decrease in government spending when the economy is experiencing deflation. For the government spending to be decreasing in inflation, fiscal authority is required to put a larger weight on the dividend tax than the labor income tax.

B.4.2 Increasing output in the crisis state

In the main article, we confirmed that fiscal authority cannot improve both output and inflation in the crisis state if it adjusts only one tax instrument. Here we derive the following condition under which both inflation and output improve in the crisis state by allowing the fiscal authority to adjust more than two tax instruments.

Proposition 9. *Output in the crisis state is higher than where the tax rates do not respond to inflation if the fiscal authority chooses λ_w and λ_d to satisfy the following condition:*

$$\lambda_d < -\frac{\Psi^F\left(\sigma\frac{\gamma_{\tau,w}}{\gamma_c} + \frac{\tau_w}{1-\tau_w}\right) + (1-p_L^*\beta)\left(\eta\frac{\gamma_{\tau,w}}{\gamma_y} - \frac{\tau_w}{1-\tau_w}\right)}{\Psi^F\sigma\frac{\gamma_{\tau,d}}{\gamma_c} + (1-p_L^*\beta)\eta\frac{\gamma_{\tau,d}}{\gamma_y}}\lambda_w, \tag{B.57}$$

$$\text{where } \Psi^F \equiv (1-\beta p_L^*) - \frac{p_L^*}{1-p_L^*}\frac{\kappa}{\sigma}. \tag{B.58}$$

Proof. Output in the crisis state is higher compared to the case where the tax rates do not respond to inflation ($\Lambda = 0$) if the following inequality holds:

$$\hat{y}_L > \hat{y}_L^{NI}. \tag{B.59}$$

Since we have restricted our focus on the case where the ZLB binds in the crisis state ($\Psi^F > 0$), $\Lambda < \Psi^F$ holds from $\Psi < 0 < \Psi^F$. We can modify the condition as

$$\begin{aligned}
\frac{1-p_L^*\beta+\zeta}{\kappa} \times \frac{\frac{\log\beta - r_L^n}{1-p_L^*}\frac{\kappa}{\sigma}}{\Lambda - (1-\beta p_L^*) + \frac{p_L^*}{1-p_L^*}\frac{\kappa}{\sigma}} &> \frac{\frac{\log\beta - r_L^n}{1-p_L^*}\frac{\kappa}{\sigma}}{-(1-\beta p_L^*) + \frac{p_L^*}{1-p_L^*}\frac{\kappa}{\sigma}} \times \frac{1-p_L^*\beta}{\kappa} \\
\Leftrightarrow \frac{1-p_L^*\beta+\zeta}{\Lambda - \Psi^F} &> -\frac{1-p_L^*\beta}{\Psi^F} \\
\Leftrightarrow \zeta\Psi^F &< -\Lambda(1-p_L^*\beta).
\end{aligned} \tag{B.60}$$

The inequality can be arranged as

$$\lambda_d < -\frac{\Psi^F\left(\sigma\frac{\gamma_{\tau,w}}{\gamma_c} + \frac{\tau_w}{1-\tau_w}\right) + (1-p_L^*\beta)\left(\eta\frac{\gamma_{\tau,w}}{\gamma_y} - \frac{\tau_w}{1-\tau_w}\right)}{\Psi^F\sigma\frac{\gamma_{\tau,d}}{\gamma_c} + (1-p_L^*\beta)\eta\frac{\gamma_{\tau,d}}{\gamma_y}}\lambda_w. \tag{B.61}$$

□

Figure B.2 depicts the region where the proposed tax rule mitigates the decline in output in the crisis state. The result shows that adjusting both the labor income tax and the dividend tax is desirable when the economy suffers from real interest rate shocks.

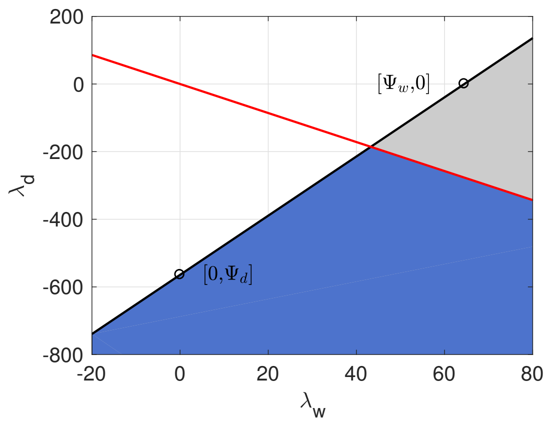


FIGURE B.2: Parameter space where the output is higher in the crisis state with policy intervention.

Note: The blue area shows the parameter space where output is higher in the crisis state compared to where the tax rates do not respond to inflation at all.

Appendix C

Models with Alternative Fiscal Policies

In the baseline model, we assumed that the government debt outstanding is always set equal to zero. In this Appendix, we investigate two alternative cases: the case with lump-sum transfer and the case with government debt targeting.

C.1 A model with lump-sum transfer

This section shows that introducing an inflation-sensitive tax rule prevents the ELT equilibrium when the lump-sum transfer is available. We assume that government spending is set proportional to the output.

Let us consider a case where the lump-sum transfer is used to balance the budget. Household's budget constraint is

$$(1+\tau_{c,t})c_t + \frac{b_t}{R_t} = (1-\tau_{w,t})w_t l_t + \frac{b_{t-1}}{\Pi_t} + (1-\tau_{d,t})d_t - \tau_t. \tag{C.1}$$

Assume that the government spending is set proportional to the output:

$$g_t = s_g y_t, \tag{C.2}$$

$$g_t + \frac{b_t}{R_t} = \frac{b_{t-1}}{\Pi_t} + \tau_t + \tau_{c,t}c_t + \tau_{w,t}w_t l_t + \tau_{d,t}d_t. \tag{C.3}$$

Although Ricardian equivalence did not hold in the baseline model of the main article, it holds with the lump-sum transfer and we can set $b_t = 0$ without loss of generality. The government budget simplifies to

$$s_g y_t = \tau_t + \tau_{c,t}c_t + \tau_{w,t}w_t l_t + \tau_{d,t}d_t. \tag{C.4}$$

In this case, the lump-sum transfer instead of the government spending is determined endogenously. Equilibrium conditions can be summarized as

$$\frac{c_t^{-\sigma}}{1+\tau_{c,t}} = \beta R_t \mathbb{E}_t\Big[\frac{c_{t+1}^{-\sigma}}{1+\tau_{c,t+1}}\frac{1}{\Pi_{t+1}}\Big], \tag{C.5}$$

$$\frac{1-\tau_{d,t}}{1+\tau_{c,t}}\Big[\psi(\Pi_t-1)\Pi_t - \theta\frac{1+\tau_{c,t}}{1-\tau_{w,t}}c_t^{\sigma}y_t^{\eta}+\theta-1\Big] = \beta\mathbb{E}_t\Big[\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}}\frac{1-\tau_{d,t+1}}{1+\tau_{c,t+1}}\frac{y_{t+1}}{y_t}\psi(\Pi_{t+1}-1)\Pi_{t+1}\Big], \tag{C.6}$$

$$c_t = \Big(1-s_g-\frac{\psi}{2}(\Pi_t-1)^2\Big)y_t, \tag{C.7}$$

$$R_t = \max\Big[1, \frac{1}{\beta}\Pi_t^{\phi_\pi}\Big], \tag{C.8}$$

$$\tau_{w,t} = \tau_w\Pi_t^{\lambda_w}, \quad \tau_{c,t} = \tau_c\Pi_t^{\lambda_c}, \quad \tau_{d,t} = \tau_d\Pi_t^{\lambda_d}. \tag{C.9}$$

By log-linearizing these equations around the TSS, we obtain the following equilibrium conditions:

$$\hat{c}_t = \mathbb{E}_t\hat{c}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1}) - \frac{1}{\sigma}\frac{\tau_c}{1+\tau_c}(\hat{\tau}_{c,t} - \mathbb{E}_t\hat{\tau}_{c,t+1}), \tag{C.10}$$

$$\hat{\pi}_t = \beta\mathbb{E}_t\hat{\pi}_{t+1} + \sigma\frac{\theta-1}{\psi}\hat{c}_t + \eta\frac{\theta-1}{\psi}\hat{y}_t + \frac{\theta-1}{\psi}\frac{\tau_c}{1+\tau_c}\hat{\tau}_{c,t} - \frac{\theta-1}{\psi}\frac{\tau_w}{1-\tau_w}\hat{\tau}_{w,t}, \tag{C.11}$$

$$\hat{i}_t = \max[\log\beta, \phi_\pi\hat{\pi}_t], \tag{C.12}$$

$$\hat{y}_t = \hat{c}_t, \tag{C.13}$$

$$\hat{\tau}_{w,t} = \lambda_w\hat{\pi}_t, \tag{C.14}$$

$$\hat{\tau}_{c,t} = \lambda_c\hat{\pi}_t. \tag{C.15}$$

Above equations are identical to (B.42) – (B.46) with parameters set to the following values:

$$\gamma_y = 1, \gamma_c = 1, \gamma_{\tau,w} = \gamma_{\tau,c} = \gamma_{\tau,d} = 0. \tag{C.16}$$

Therefore, all propositions established in the main article hold in the model discussed in this Appendix by replacing ξ, ζ and Λ to appropriate values. Note that $\gamma_{\tau,d} = 0$ indicates that the dividend tax does not affect the equilibrium outcome. This is because changes in the dividend tax do not affect the marginal behavior of the representative household as long as the aggregate demand is kept constant by

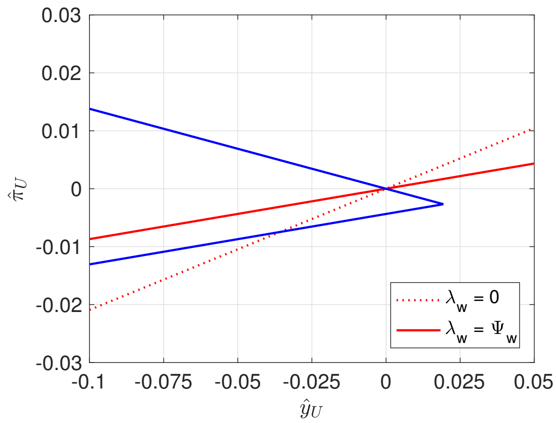


FIGURE C.1: Euler equation and Philips curve with lump-sum transfer.

the use of the lump-sum transfer.

Figure C.1 shows the case where the tax response parameter is set to $\lambda_w = \Psi_w$ and others to zero. We can observe that only the supply side is affected by the introduction of the tax rule since changes in the demand are isolated by the lump-sum transfer.

The case where the government spending is always zero ($g_t = 0$) and the variation in distortionary taxes is fully funded by the lump-sum transfer can be obtained by setting $s_g = 0$. Log-linearized equations (C.10) – (C.15) are not affected by the choice of s_g, therefore the results remain unchanged if we assume balanced government spending.

C.2 A model with endogenous government debt

We assumed that the fiscal authority runs a balanced budget in the baseline model and keeps government debt to zero at all periods. A natural question that arises here is whether the results would be affected if we relax the balanced budget assumption and allow the government debt to vary over time.

To address this question, we assume that the government spending is determined by the following government debt targeting rule:

$$g_t = s_g y_t \left(\frac{b_{t-1}}{b_{target}} \right)^{\phi_b}. \tag{C.17}$$

The parameter s_g determines the ratio of government spending to output. The fiscal authority sets the target level of government debt b_{target} equal to the steady state level of government debt b_{TSS}, which is determined by s_g and τ_w as well as other parameters.

Let us assume that the lump-sum transfer is not available. As shown in the seminal paper of Leeper, 1991, the parameter ϕ_b must be appropriately selected for a unique equilibrium to exist. More concretely, the government spending rule must be designed so that government debt does not follow an explosive path.

Let us further assume that $\phi_b < 0$ is satisfied. Then, for a fixed level of output y_t, the fiscal authority reduces government expenditure g_t when the government debt level b_{t-1} is higher than its target b_{target}.

In the rest of the analysis, the parameter is set to $\phi_b = -1$. For simplicity, we assume that the fiscal authority adjusts only the labor income tax rate and the steady state tax rate is set to $\tau_w = 0.2$, $\tau_d = 0$, and $\tau_c = 0$. In this case, equilibrium conditions consist of the following equations:

$$c_t^{-\sigma} = \beta R_t \mathbb{E}_t \left[c_{t+1}^{-\sigma} \frac{1}{\Pi_{t+1}} \right], \tag{C.18}$$

$$\psi(\Pi_t - 1)\Pi_t - \frac{\theta c_t^{\sigma} y_t^{\eta}}{1 - \tau_{w,t}} + \theta - 1 = \beta \mathbb{E}_t \left[\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{y_{t+1}}{y_t} \psi(\Pi_{t+1} - 1)\Pi_{t+1} \right], \tag{C.19}$$

$$y_t = c_t + s_g y_t \left(\frac{b_{t-1}}{b_{target}} \right)^{\phi_b} + \frac{\psi}{2}(\Pi_t - 1)^2 y_t, \tag{C.20}$$

$$\frac{b_t}{R_t} + \frac{\tau_{w,t}}{1 - \tau_{w,t}} c_t^{\sigma} y_t^{\eta+1} = \frac{b_{t-1}}{\Pi_t} + s_g y_t \left(\frac{b_{t-1}}{b_{target}} \right)^{\phi_b}, \tag{C.21}$$

$$R_t = \max \left[1, \frac{1}{\beta} \Pi_t^{\phi_\pi} \right]. \tag{C.22}$$

Equation (C.20) and (C.21) represent the aggregate resource constraint and the government budget constraint respectively. The steady state values can be calculated as

$$y_{TSS} = \left[\frac{\theta - 1}{\theta} (1 - \tau_w) \right]^{\frac{1}{\eta+\sigma}} (1 - s_g)^{-\frac{\sigma}{\eta+\sigma}}, \tag{C.23}$$

$$c_{TSS} = \left[\frac{\theta - 1}{\theta} (1 - \tau_w) \right]^{\frac{1}{\eta+\sigma}} (1 - s_g)^{\frac{\eta}{\eta+\sigma}}. \tag{C.24}$$

The steady state value of the government debt can be derived from the government budget constraint as

$$b_{TSS} = \frac{1}{1-\beta}\Big[\tau_w \frac{\theta-1}{\theta} - s_g\Big] y_{TSS}. \tag{C.25}$$

The steady state amount of government debt outstanding is positive only if the right-hand side of equation (C.25) is positive. In the remaining analysis, the spending ratio is calibrated to $s_g = 0.16$, which yields the debt-to-output ratio of $b_{TSS}/y_{TSS} = 1.67$.

The log-linearized equilibrium conditions consist of following equations:

$$\hat{c}_t = \mathbb{E}_t \hat{c}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}), \tag{C.26}$$

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \sigma \frac{\theta-1}{\psi}\hat{c}_t + \eta \frac{\theta-1}{\psi}\hat{y}_t - \frac{\theta-1}{\psi}\frac{\tau_w}{1-\tau_w}\hat{\tau}_{w,t}, \tag{C.27}$$

$$\hat{i}_t = \max[\log\beta, \phi_\pi \hat{\pi}_t], \tag{C.28}$$

$$\hat{\tau}_{w,t} = \lambda_w \hat{\pi}_t, \tag{C.29}$$

$$\hat{c}_t = \hat{y}_t - \frac{s_g}{1-s_g}\phi_b \hat{b}_{t-1}, \tag{C.30}$$

$$\beta\gamma_b \hat{b}_t = \gamma_{y,b}\hat{y}_t - \sigma\frac{\theta-1}{\theta}\tau_w \hat{c}_t - \gamma_b \hat{\pi}_t + (\gamma_b + s_g\phi_b)\hat{b}_{t-1} + \beta\gamma_b \hat{i}_t - \gamma_{\tau,w}\hat{\tau}_t, \tag{C.31}$$

where $\gamma_b \equiv \frac{b_{TSS}}{y_{TSS}}$, $\gamma_{y,b} \equiv s_g - (\eta+1)\frac{\theta-1}{\theta}\tau_w$.

After substitution, equilibrium conditions simplify to the following EE and PC with three variables $\hat{\pi}_t$, $\hat{y}_t$ and $\hat{b}_{t-1}$:

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \frac{1}{\sigma}(\max[\log\beta, \phi_\pi\hat{\pi}_t] - \mathbb{E}_t\hat{\pi}_{t+1}) + \frac{s_g}{1-s_g}\phi_b(\hat{b}_{t-1} - \hat{b}_t), \tag{C.32}$$

$$\Big(1 + \frac{\theta-1}{\psi}\frac{\tau_w}{1-\tau_w}\lambda_w\Big)\hat{\pi}_t = (\sigma+\eta)\frac{\theta-1}{\psi}\hat{y}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1} - \sigma\frac{\theta-1}{\psi}\frac{s_g}{1-s_g}\phi_b\hat{b}_{t-1}, \tag{C.33}$$

$$\begin{aligned}\beta\gamma_b\hat{b}_t = \Big(\gamma_{y,b} - \sigma\frac{\theta-1}{\theta}\tau_w\Big)\hat{y}_t + \beta\gamma_b\max[\log\beta, \phi_\pi\hat{\pi}_t] - (\gamma_b + \gamma_{\tau,w}\lambda_w)\hat{\pi}_t \\ + \Big[\sigma\frac{\theta-1}{\theta}\tau_w\frac{s_g}{1-s_g}\phi_b + (\gamma_b + s_g\phi_b)\Big]\hat{b}_{t-1}.\end{aligned} \tag{C.34}$$

Let us focus on the case where the economy fluctuates around the TSS. The ZLB does not bind and monetary policy is active around the TSS. In this case, the equilibrium conditions can be expressed in the following state space representation:

$$B\begin{bmatrix}\hat{b}_t\\ \mathbb{E}_t\hat{y}_{t+1}\\ \mathbb{E}_t\hat{\pi}_{t+1}\end{bmatrix} = C\begin{bmatrix}\hat{b}_{t-1}\\ \hat{y}_t\\ \hat{\pi}_t\end{bmatrix}, \tag{C.35}$$

where

$$B \equiv \begin{bmatrix}-\frac{s_g}{1-s_g}\phi_b & 1 & \frac{1}{\sigma}\\ 0 & 0 & \beta\\ \beta\gamma_b & 0 & 0\end{bmatrix},$$

$$C \equiv \begin{bmatrix}-\frac{s_g}{1-s_g}\phi_b & 1 & \frac{1}{\sigma}\phi_\pi\\ \sigma\frac{\theta-1}{\psi}\frac{s_g}{1-s_g}\phi_b & -(\sigma+\eta)\frac{\theta-1}{\psi} & 1+\frac{\theta-1}{\psi}\frac{\tau_w}{1-\tau_w}\lambda_w\\ \sigma\frac{\theta-1}{\theta}\tau_w\frac{s_g}{1-s_g}\phi_b+(\gamma_b+s_g\phi_b) & \gamma_{y,b}-\sigma\frac{\theta-1}{\theta}\tau_w & \beta\gamma_b\phi_\pi-(\gamma_b+\gamma_{\tau,w}\lambda_w)\end{bmatrix}.$$

Since there are two control variables ($\hat{y}_t$, $\hat{\pi}_t$) and one predetermined variable ($\hat{b}_{t-1}$), one of the eigenvalues of $C^{-1}B$ must lie outside the unit-circle and two of them within the unit-circle.

The solution of the linear system can be represented as

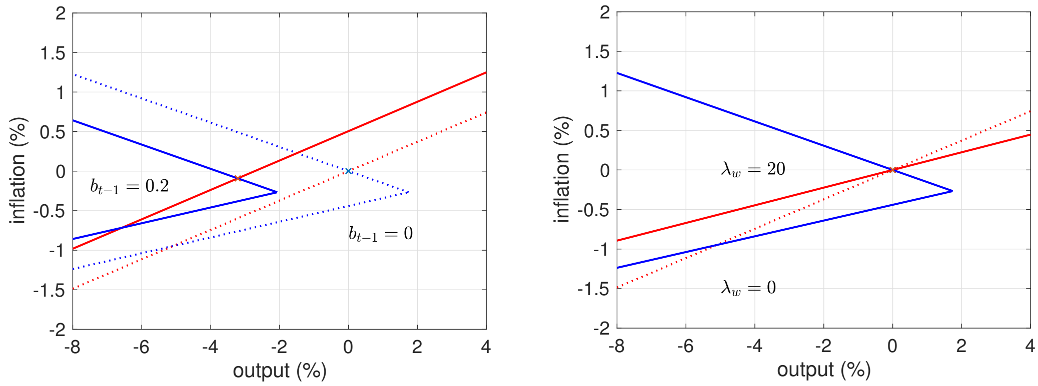
$$\hat{y}_t = a_1\hat{b}_{t-1}, \quad \hat{\pi}_t = a_2\hat{b}_{t-1}, \quad \hat{b}_t = a_3\hat{b}_{t-1}. \tag{C.36}$$

Assuming $\lambda_w = 0$, the solution of the linear system can be obtained numerically as follows:[1]

$$\begin{bmatrix}a_1\\ a_2\\ a_3\end{bmatrix} = \begin{bmatrix}-0.169\\ -0.005\\ 0.910\end{bmatrix}. \tag{C.37}$$

When there is no uncertainty, we can replace $\mathbb{E}_t\hat{y}_{t+1} = \hat{y}_{t+1} = a_1\hat{b}_t = a_1a_3\hat{b}_{t-1} = a_3\hat{y}_{t-1}$ and $\mathbb{E}_t\hat{\pi}_{t+1} = \hat{\pi}_{t+1} = a_2\hat{b}_t = a_2a_3\hat{b}_{t-1} = a_3\hat{\pi}_{t-1}$, which holds regardless of the value of b_{t-1}.

[1]The program `gensys.m` by Sims, 2002 is used to compute the solution.



(A) Cases with different level of government debt outstanding.

(B) Cases with $\hat{b}_{t-1} = 0$ and different values of λ_w.

FIGURE C.2: Euler equation and Philips curve with endogenous government debt around the TSS.

To investigate how demand and supply are affected by the amount of debt outstanding and the size of the tax response parameter, figure C.2 shows the EE and PC under different values of b_{t-1} and λ_w. As shown in the left-hand figure, fiscal authority reduces government spending as b_{t-1} becomes larger, which shifts the EE downwards and the PC upwards. While the equilibrium inflation is little affected, the output is depressed when there is a positive amount of government debt since the fiscal authority cuts government spending. The key feature here is that both EE and PC shift in a parallel manner, and the slope of these two equations are not affected by the level of $\hat{b}_{t-1}$.

The right-hand figure shows that changing λ_w only affects the supply side. This contrasts with the baseline model, where the demand curve was also affected by the level of λ_w. Since government spending is determined by equation (C.17), it is isolated from the tax revenue; a marginal change in the tax revenue does not affect government spending. Therefore, the demand curve remains unchanged. The key observation here is that when the government debt fluctuates over time, the fiscal authority can isolate the demand effects and the supply effects of the fiscal policy.

The above analysis has abstracted from the possibility of switching between the targeted regime and the unintended regime. Simple models that do not include predetermined variables (b_{t-1} in this case) are relatively straightforward to solve even with the regime-switching structure. However, once a predetermined variable enters the model, closed-form solutions are not available, and different algorithms are required to solve the model (see Farmer, Waggoner, and Zha, 2009). A comprehensive study on ELTs with government debt is left for future work.

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