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Population Aging, Unemployment, and Social Security

by

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Chapter 1 Introduction

The population aging due to the declining fertility and increasing life expectancy has been the common concern faced by many industrial countries in the world. The two main trends, as well as the retirement of generation of baby-boomers, born in 1950s, lead to the increases in the old-age dependency ratio. When pensioners become more, and labor forces become less, it put increasing pressure on the widely used Pay-As-You-Go (PAYG) pension, one of the main pillar of the contemporary social security system (SSS) of the world. Therefore, the sustainability of social security and the problem of pension reform has been the crucial issues of the current debate. On the other hand, in labor market, unemployment, as a macroeconomic factor, has existed for a long time. The relationship between population aging and social security should not only be considered in competitive economy, but also be analyzed in imperfect labor market.

In this dissertation "Population aging, Unemployment, and Social Security", I revisit the relationship between population aging and social security system, especially PAYG pension in labor market with unemployment due to different causes and dwell on the mechanism of how unemployment can affect fertility via social security in the channel of capital accumulation and interest rate changes in general equilibrium analysis of overlapping generations model. The first study is due to the constant minimum wage (Fanti and Gori, 2007), the second results from the union wage setting (Ono, 2007; 2010), and the last arises from efficiency wages (Shapiro and Stiglitz, 1984).

The rest of the dissertation is organized as follows. Chapter 2 presents the literature review on “Population aging, unemployment, and social security”. The review is divided into several parts. The first part is “Background and motivation” of this dissertation. It gives the “Definition of population aging”, and presents two important literature strands that motivated the studies of this dissertation: “Effects of demographic trends on social insurance” and “Population aging and the labor market”. The second is the literature on the “Fertility and Social Security system”, presenting several research strands on the reasons for fertility decline and what an extra child will bring to social security system. The third part is “Unemployment research on population aging and social security system”, summarizing previous studies on the mutual relationship between unemployment and population aging through social security system.

Chapter 3 “Fertility and Unemployment in a Social Security System” analyses the effect of a social security system composed of PAYG pension and child allowances on endogenous fertility in a two-period general-equilibrium overlapping generations (OLG) model with unemployment considered. The unemployment is due to constant minimum wage. It extends the study of Fanti and Gori (2007) and wonder whether it affects the mechanism of child allowances on fertility when pension is introduced to the economy. Contrary to the common sense that the development of social security system is always accompanied with decrease in fertility, the analysis reveals that the effect on fertility depends on the level of pensions. Furthermore, even with child allowances, the effect is not always positive.

Chapter 4 “Fertility, Union Wage Setting and Social Security System” analyzes how a social security system composed of a public pension, child allowances, and unemployment insurance affects endogenous fertility and unemployment when the wage level is endogenously set by monopolistic trade unions in an OLG model. The analysis reveals, first, that increased pension tax rates lead to a higher fertility rate when wages are higher but a lower rate when wages are lower. Second, an increased child allowances tax rates lead to an increased fertility rate when wages are lower but a decreased rate when wages are higher. Moreover, to improve social welfare, it is preferable to increase public pensions or the child allowances tax rate and reduce the unemployment insurance tax rate when wages are lower, while it is preferable to reduce the child allowances or the unemployment insurance level when wages are higher. Therefore, both social security and wage bargaining should be considered in order to improve fertility and reduce unemployment.

Chapter 5 “Fertility, Efficiency Wages and Social Security in an Overlapping Generations Model” analyses how fertility is affected by social security system when efficiency wages are considered in an overlapping generations model of a small open economy. Unemployment of this economy is due to the efficiency wages set by firms. It reveals that the effects of social security system on fertility depend on the level of childrearing cost: when the costs are at higher level, both pensions and unemployment insurances will decrease the fertility; while when the costs are at lower level the effects are opposite. On the other hand, the higher effort or labor efficiency level leads to higher ability to raise children, improving fertility.

Chapter 6 gives the closing remarks.

Chapter 2 Literature Review on "Population Aging, Unemployment, and Social Security"

2.1 Background and motivation

2.1.1 Definition of population aging

Population aging in many advanced countries has caused widespread concern among both researchers and policy makers. However, demographic trends can be seen as an amalgam of good news and bad news. The good news is that with the development of the social security system, people's health and well-being have substantially improved and mortality risks have reduced significantly. Thus, the average life expectancy and health status have increased constantly and significantly. On the other hand, with the liberation of the traditional view on female labor force participation and the demand for high-level production, female participation has become increasingly important in the labor market and occupied gradually increasing proportions of the labor force. Decreasing childcare time and increasing opportunity cost of staying home have pushed mothers to balance child procreation and market work, thus leading to fertility decline worldwide.

The two trends mentioned above can be seen as the main causes for population aging, today faced by many industrial countries. Furthermore, the old-age dependency ratio measured by the ratio of the elderly population (over 65) over the working population (20–64) has increased significantly. Therefore, the bad news is that this puts severe strain on the social security system, especially the widely used public pension system on a pay-as-you-go (PAYG) basis, which is financed by taxing the working population's income for the old-aged. Consequently, concerns about the sustainability and viability of this system have been in the forefront of the current debate in economic, political, and public areas. Moreover, population aging is affected by the social security system. Therefore, if we want to reform the social security system to adapt to the population aging trends, not only the effect of population aging on social security system, but also the feedback effect should be considered and explored (Boldrin et al., 2005). This dissertation focuses on the latter strand of research—to dwell on how the social security system affects population aging, especially fertility. In subsection 2.1.2, the former strand—the effect of population aging on social security—will be presented to review how this line of research has progressed. Moreover, in subsection 2.1.3, it is explored why unemployment should be considered in the analysis of the social security system's effect on fertility, and the effect of population aging on the labor market will be reviewed.

2.1.2 Effects of demographic trends on social insurance

Demographic trends have caused dramatic changes in the size and composition of the world population. The reductions in fertility and increases in life expectancy lead to an increase in the old-age dependency ratio and present a great concern about the sustainability of the current social security system, faced by not only developed countries, but also developing ones, whose populations are projected to decrease.[1,2] In this subsection, the studies relating to the impact of demographic trends on social security will be summarized. This topic is widely and popularly analyzed from both closed and open economy contexts using the most popular and useful tool, the OLG model (Attanasio et al., 2016).

In the closed economy analysis, in the context of demographic trends, both the institutional arrangement (such as social security system and annuity market) and savings decision of the individual lifecycle will affect the demand and supply of assets, and thus the equilibrium factor prices. The concern on the sustainability of the PAYG pension system can be resolved by the privatization of the social security system (Feldstein, 1998; Geanakoplos et al., 1998), and the estimate of welfare changes from the PAYG pension system to a privatized one is also addressed by many studies (Nishimura and Smetters, 2007; Huggett and Parra, 2010).[3] As the baby boom

[1] The dependency ratio is projected to rise from 22% to 38% in 2050 (Bell and Miller, 2005).

[2] Take China as an example; the one-child policy has pushed China into a short-cut trajectory of population aging.

[3] Nishimura and Smetters (2007) analyze one specific reform of the US social security system in an OLG model with heterogeneous agents of different elastic labor supply in respect to idiosyncratic earnings shocks and longevity risk. They conclude that privatization can produce efficiency gains by improving labor supply incentive, and that privatization performs better in a closed economy, where interest rates decline with capital accumulation, than in an open economy, and also performs better when an actuarially fair private annuity

generation retires, another concern is that the asset market will melt down. Therefore, the price of assets will also experience a rise because of the increasing demand for capital accumulation of this generation, and also a fall accompanied by retirement and consumption of wealth (Abel, 2001; 2003). However, Poterba (2001, 2004) argues that the results from OLG models are not realistic because it takes time for asset decumulation and that the demographic effects on asset prices are minor. On the other hand, population aging creates an increasing need for health care insurances(Attanasio et al., 2011; De Nardi et al., 2016), and its expenses play an important role in the saving behavior of the elderly (De Nardi et al., 2010) and the fiscal imbalances due to population aging (Braun and Joines, 2015; Kitao, 2015). It is projected that the social expenditures would range from an average of under 19% of GDP in 2000 to almost 26% of GDP by 2050 with old-age pension payments and expenditure on health care and long-term care (Dang, Antolin, & Oxley, 2001).[4] Kitao (2014) provides four options to make the US social security sustainable under the coming demographic shift.[5]

In the context of globalization, the regional differences in technology and factor endowments cannot be ignored when analyzing the impact of demographic trends on

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market does not exist.

[4] See also Grey (2005) for the review of population aging and health care expenditures.

[5] The four options are (1) increase payroll tax; (2) reduce replacement rate; (3) raise the retirement age; and (4) make the system means-tested and let the benefits decline one-to-one with income.

economic variables under institutional arrangements and fiscal policies. Both labor and capital mobility are important productive factors being considered in the literature.

Regarding labor mobility, some studies considered the immigration policy as a method to attenuate the pressure of population aging on fiscal crisis (Borjas, 1994; Storesletten, 2000; Fehr et al., 2004; Zimmermann, 2005) . This is because the inflow of working-age immigrants can broaden the tax base for government revenue, and skilled workers have significant and positive fiscal effect. Therefore, selective immigration can be used as an alternative to tax hikes and spending cuts for financing fiscal deficits (Storesletten, 2000).[6] However, due to the limitation of model construction—focusing only on developed countries and not considering developing countries—some studies question the implications of the above findings. Fehr et al. (2004) develop a multi-region (three developed regions: the US, Japan, and the EU) dynamic general equilibrium OLG model and confirm the former part of the conclusions, but argue that a significant expansion of immigration of any kinds of skill strategies does little to alter capital storage and tax hikes that characterize the demographic transition. Moreover, large-scale immigration poses political and social challenges to realize it for the government.

[6] Storesletten (2000) uses a calibrated OLG model to investigate whether immigration can mitigate the fiscal problems associated with population aging and notes that both age and skill of immigrants are important.

Besides labor migration, capital mobility is also a solution to the fiscal crisis associated with demographic trends. Attanasio, Kitao, and Violante (2006, 2007) develop a two-region general equilibrium OLG model calibrated to the North (more developed countries) and the South (less developed countries). In their 2006 paper, they evaluate quantitatively the impact of the observed demographic transition on aggregate variables (factor prices, saving rate, output growth), and on inter-generational welfare in developing economies. They find that the effects of the demographic trends for less developed regions depend on the degree of international capital mobility and the extent to which the large PAYG systems in place in the more developed world are reformed. In their 2007 paper, they investigate the sustainability of the current social security systems in the developed economies under the projected demographic trends, and compare two cases where capital cannot or can freely flow across regions (closed or open economy).

2.1.3 Population aging and the labor market

As population ages, how will the labor market be affected?[7] The conventional wisdom believes that population aging will make the labor supply scarcer and older workforces cannot produce as efficiently as younger workers, affecting the labor performances. As unemployment is not only an unfortunate period of work experience faced with exogenous shock during one's lifecycle, but also has become a long-run

[7] See also Dixon, 2003; D'Addio, 2010.

phenomenon from the view of the economy as a whole; thus, it cannot be ignored in the analysis on population aging. Many economists have investigated the effects of population aging on unemployment, and found that a shift in the age structure can modify the demand structure of goods, causing employment variations across sectors (Borsch-Supan, 2003; Fougere et al., 2007). Croix et al. (2013) examine the effects of population aging and pension reforms on the equilibrium unemployment, which is caused by labor market frictions, and imply that neglecting labor market frictions and employment rate dynamics may lead to underestimation of the effects of pension reforms. On the other hand, immigration policy is also considered to mitigate labor force crisis due to population aging because immigrant workers who are typically younger than the natives on average (Zimmermann, 2005), not only moderate the labor supply shortage, but also help relieve the tax burden of old-age expenditure (Borjas, 1994).

However, only few studies contribute insights on the effect of unemployment on population aging. Fanti and Gori (2007) argued that when unemployment is taken into consideration owing to constant minimum wages, the effect of child allowances on fertility is not positive, because the reduction of capital accumulation and increase in unemployment actually decrease fertility dramatically.[8] My research aims to demonstrate the effects of unemployment on population aging by analyzing how

[8] The first study of the dissertation has discussed whether the introduction of pension will change the effect of child allowances.

fertility is affected by social security systems in two-period overlapping generations environments with unemployment considered. This dissertation asks the question how the government improves fertility by modifying the social security taking into account the unemployment in the labor market.

2.2 Fertility and social security system

2.2.1 Reasons for fertility decline

A number of studies have focused on the relationship between fertility and the social security system with an overlapping generations model, the typical tool since Auerbach and Kotlikoff (1987). Many economists have viewed the problem of fertility decline as an exogenous shock, therefore taking fertility as an exogenous variable (Verbon, 1988; Breyer, 1989). However, in the contemporary societies, the fact that people can decide by themselves how many children they would like to procreate—the so-called endogeniety of fertility incentive—should be considered. Therefore, the features of offspring itself can help explain the downward tendency of fertility.

From the view of altruism and following Leibenstein (1957) and Becker (1960), offspring can be regarded as a consumption good: parents have children because they perceive children's lives as a continuation of their own and can derive satisfaction from rearing progeny. Therefore, the number of children is included in the utility function (Barro and Becker, 1989; Galor and Weil, 1996).[9] When the average wage increases, the opportunity cost to rear a child also increases, decreasing the demand for children.

From the view of egoism, children can be viewed as a private capital good (Cigno, 1992; Bental, 1989; Boldrin and Jones, 2002), serving as an insurance or investment against the risk of old-age dependency. When parents become old, children can provide old-age transfers. However, with the development of intergenerational transfer through pension projects and installation and extension of health care arrangements, this incentive has been depressed, leading to a dramatic decline in the fertility rate (Cigno and Rosati, 1996; Cigno et al., 2003).[10]

Another role of offspring is that of a public capital good (Cigno, 1993; Folbre, 1994).[11] When fertility increases, it contributes to broadening the tax base of

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[9] Parents could also derive direct utility from that of their offspring, which is considered as strong altruism (Razin and Ben-Zion, 1975; Zhang, 1995).

[10] Boldrin et al. (2005) examine the effect of social security on fertility choices in the two models, Barro and Becker (1989) and Boldrin and Jones (2002). They find that in the former model, the sign of effect depends on whether child-rearing costs are measured in goods or time, while the latter can account for most observations in reality, and predict that fertility is decreasing with the development of social security.

[11] Cigno (1993) constructed a three-period overlapping generations model in which the middle-aged population transfers to the elderly and children with the assumption of no altruism, in order to investigate the extent to which reproduction and intergenerational transfers can be explained by self-interest. He argued that within this family framework, such

intergenerational transfers from the young to the old in the PAYG pension system. However, the increased pension fund would not serve to pay the pension to the children's parents, but would be shared by all members eligible to the pension system. Therefore, instead of personal return, children have a public effect on society as a whole, which can be seen as a social externality. This externality induces parents to free ride on the public pension system by reducing the child-rearing cost or having fewer children (Sinn, 2001). It is proposed that making pensions contingent upon fertility (Bental, 1989; Kolmar, 1997; Abío et al., 2004; Sinn, 2005) or introducing child allowances (Groezen et al., 2003, 2008) are alternatives to correct this distortion.[12]

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transfers could be generated, and that this transfer system is vulnerable to developments in capital markets. Furthermore, the public pension system reduces the incentives to have children. If people raise more children, the future payment in the PAYG pension system will increase, but the benefit to a couple from having one child is too small to be taken into account in fertility choices. This externality results in an inefficient consumption allocation during the lifecycle.

[12] Gorezen et al. (2003) showed that the introduction of child allowances to the PAYG pensions is an efficient way to achieve Pareto-improvement because child allowances internalize the externalities of children caused by the PAYG scheme. In particular, they find that the optimal subsidy to parents is equal to the present value of a child's contribution to the intergenerational redistribution scheme during his working life.

2.2.2 Effects of an extra birth on social security system

The effect of fertility on the social security system can be concluded as three effects: the intergenerational transfer effect, the capital dilution effect, and the child quality effect (Cipriani, 2014; Groezen et al., 2003; Groezen and Meijdam, 2008).[13] The first effect is the externality mentioned in the previous section, which can be called the *positive social externality* or *intergenerational effect* or *dependency-ratio effect* of fertility associated with the pension system (Cigno, 2006; Ehrlich and Lui, 1998; Nishimura and Zhang, 1992; Rosati, 1996; Wigger, 1999; Cremer et al., 2006; Alders and Broer, 2005). The second one states that an extra birth requires a higher capital stock in order to keep the per-capita production constant. In a small open economy, the pressure will be shared through the world labor market; while in a closed economy, the parents have to save more. Therefore, the increase in fertility dilutes the capital stock, and thus, this effect can be called the *capital-dilution effect* or *negative social externality* (Michel and Pestieau, 1993; Cigno, 1993).[14] Another effect is the *child quality effect* (Cipriani, 2014). This effect stems from the fact that parental care has invested more in the quality of their offspring than quantity; thus, the cumulated human capital will bring up the wage level and production level, and thus

[13] Cipriani (2014) showed the effect of fertility decline on the PAYG pension system in an OLG model allowing both endogenous and exogenous fertility and concluded that increased longevity implies a reduction in pension payouts.

[14] Whether the number of offspring has reached the optimum depends on the relative size of the two effects (Groezen and Meijedam, 2008).

the tax revenue for the pension system (Cremer et al., 2011; Cipriani and Makris, 2012).

2.3 Unemployment research on population aging and social security

2.3.1 Introduction of unemployment theories

Unemployment matters because it reduces output and aggregate income. It also increases the concern for living conditions personally and risk for security socially. Unemployment theories can be understood in both micro and macro perspectives. The success to seek new jobs depends on two circumstances: whether the characteristics of job seekers match those of vacant jobs, and whether the demand and supply are balanced in the labor market. Corresponding to the two circumstances, two categories of approaches are used to explain unemployment.

The approach for the first circumstance focuses on the heterogeneity of jobs and jobseekers. The different propensities between jobs and workers lead to the fact that job search is time consuming, and that it takes time to arrive at the final matches. Search models are a popular example taking this approach, requiring the specification of parameters of the job creation and destruction, separation of workers from the current jobs, and the flow into or out of unemployment pools.

The other approach emphasizes microeconomic imperfections that lead to an imbalance between labor demand and supply in the aggregate labor market with the assumption of homogeneity of labor. These imperfections are attributed to government interferences, such as minimum wages and unemployment benefits, or the deviation between firms and workers, by eliminating which ways, such as efficiency wages and union wage setting, are often taken into consideration. The case models are the efficiency-wage model, the contract model, and the insider-outsider model.

In this dissertation, the latter approach is taken. Therefore, unemployment occurs due to the imbalance between labor supply and demand, and the causes have been identified as minimum wages, union wage setting, and efficiency wages, respectively in different studies.

2.3.2 Unemployment, growth and social security system

The association between unemployment and growth attracts much attention of politicians, researchers, and laymen because of the well-known fact that many developed countries are plagued with slow growth and high unemployment.

On one hand, many researchers are interested to explore not only the effect of growth on unemployment but also the feedback effect of unemployment on growth. For the former, a typical mechanism is that growth reduces unemployment because an increase in growth raises the expected returns to open new job vacancies, stimulating the flow out of the unemployment pool (Pissarides, 2000). Moreover, a representative alternative presents that higher growth associated with innovation and creative destruction results in higher rate of labor allocation, more keen competition, and thus higher unemployment rate (Aghion and Howitt, 1994). For the latter, a rise in equilibrium unemployment lowers the income of the young, reducing the savings and hence decreasing the equilibrium growth rate (Bean and Pissarides, 1993; Daveri and Tabellini, 2000). However, empirical studies cannot show a definite or robust relationship between the two factors. Therefore, the sign of the correlation between growth and unemployment can be positive or negative.[15] On the other hand, this association can also be understood from the different causes of unemployment. Since the causes of unemployment involve union wage bargaining, minimum wages, and search frictions among others, the relationship between unemployment and growth also changes consequently. Brauninger (2000) and Lingens (2003) explore the correlations of growth and unemployment caused by wage bargaining, and conclude negative relations; Aghion and Howitt (1994) and Pissarides (2000) focus on the relation between growth and unemployment caused by search frictions, and insist positive relations. Cahuc and Michel (1996) examined the relation between growth and unemployment caused by minimum wages, and found that the minimum wage "can have positive effects on growth by inducing more human capital accumulation," thereby increasing the demand for skilled workers.[16]

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[15] Aghion and Howitt (1994) insisted a positive relationship, while Daveri and Tabellini (2000), Brauninger (2000), and Brauninger and Pannenberg (2000) concluded a negative relationship.

[16] Many studies in the empirical literature show that an increase in minimum wages will aggravate the negative effect on employment (See Bazen and Martin, 1991).

On the other hand, since the relationship relates to the changes in income, factors affecting income also affect the sign of the correlation. The social security system (SSS) is a popular consideration associated with this topic (Nickell and Layard, 1999; Saint-Paul, 1992). Corneo and Marquardt (2000) was the first study to consider the relation among SSS, unemployment, and growth by developing an OLG model that studies the interaction between public pensions and unemployment insurance programs in the presence of unemployment caused by a union wage setting. They assumed the following: (1) labor efficiency is determined by capital per employed worker; and (2) a monopolistic trade union whose objective function was developed by Pencavel (1984). They concluded that first, unemployment is independent of contribution rates to the pension system; and second, there is no link between unemployment and capital accumulation. However, their conclusions are doubted by many other researchers: (1) the first conclusion contradicts the findings of empirical studies (e.g., Daveri and Tabellini, 2000); and (2) the second conclusion implies that the model ignores the endogenous response of capital to a union wage setting (Brauninger, 2000; Kaas and Thadden, 2003; 2004). Therefore, Brauninger (2005) extended Corneo and Marquardt (2000) by changing the assumptions as (1) labor efficiency depends on capital per population, and (2) monopolistic unions exist, each of which maximizes utility of representative members by bargaining wages with its correspondent firm. They argued that an increase in unemployment reduces growth; therefore, an increase in pension or replacement ratio can lead to lower growth by increasing unemployment. Moreover, pensions reduce the motivation to save, and thus negatively affect the accumulation of capital and human capital, declining growth. Ono (2007) focused on the effect of public pensions on unemployment in an OLG model with a wage setting by a trade union, extending Corneo and Marquardt (2000) by (1) assuming that labor efficiency depends on the capital per population (Brauninger, 2005) and (2) generalizing the objective function of the trade union to a CES function, including the Cobb-Douglas function in Corneo and Marquardt (2000) as a particular case. It is demonstrated that a higher contribution rate to the pension system is favorable to employment and stability of the economy. Ono (2010) extended Ono (2007) by comparing the effects of two types of pensions—the lump-sum pension and the proportionate pension—on the trade-off between unemployment and growth, motivated by the conjecture that the different savings motives between the two systems may affect unemployment and growth differently.[17] His study assumed that (1) productive externality is taken as an engine of endogenous growth, and that (2) monopolistic trade unions aim to maximize the expected utility of union members, and argued that a reduction in the firms' contribution to pension leads to a trade-off between growth and unemployment in the proportionate pension system, while no trade-off in the lump-sum one (Cigno, 2008). The results implied that the choice of a pension system can affect the relations between growth and unemployment.

Consequently, this current dissertation also considers the effects of social

[17] In the lump-sum pension system, everyone can receive pensions, while in the proportionate system, only the contributors can receive pensions.

security systems—public pensions, child allowances, and unemployment insurance—on unemployment, by analyzing the alteration in capital accumulation in OLG models.

2.3.3 Unemployment and fertility

Since fertility always concerns the child-care time and costs of parents, especially those of mothers, the relations between fertility and (un)employment are a popular topic in economic research on population aging (Blau and Robins, 1989; Ahn and Mira, 2001).[18] On one hand, mountainous studies shed light on the correlation between total fertility rate (TFR) and female participation rate (FPR). Butz and Ward (1979) investigated the cyclical behavior of fertility by using US aggregate time series data, and predicted that the fertility responses to business cycle would change from procyclical to countercyclical due to the increases in female participation rate.[19] However, their prediction contradicts with the well-known fact that the correlation had been negative during 1970s and up to early 1980s, and became positive by the

[18] By analyzing the cases in Spain, Ahn and Mira (2001) looked for the evidence of a link between "unemployment increases" and "fertility decreases," and concluded that the uncertainty brought by high youth unemployment with a rising proportion of temporary contracts inhibit marriage and childbearing, which can help explain the decreasing fertility.

[19] Butz and Ward (1979) concluded that the fertility will decrease as female participation rate increases because the opportunity cost of childrearing becomes higher for working mothers.

late 1980s in OECD countries (Ahn and Mira, 2002).[20] This intriguing phenomenon has aroused the interests of many researchers for figuring out the explanations for why the correlation reversed its sign. Some researchers argued that since child-care costs affect the family labor supply decision (Blau and Robins, 1989),[21] the emergence of the purchased child care and the changes in its prices can affect female participation in labor markets (Ermish, 1989; Ahn and Mira, 2002; Martinetz and Iza, 2004).[22]

Another explanation is based on the comparative analysis of the incidences of wage increases between the two different groups—working women and new entrants. When the wage increases, it brings only the income effect on the former group, increasing the fertility rate of the group, while it brings both stronger substitution

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[20] Ahn and Mira (2002) also noted that the reversal in the correlation of TFR and FPR is paralleled with the emergence of high and persistent unemployment rates.

[21] Blau and Robins (1989) set and estimated a model of family labor supply incorporating both market and nonmarket child care, by using data of the Employment Opportunity Pilot Project, and suggested that both the decision of employment and market child care purchase are affected by the child-care costs.

[22] Martinez and Iza (2004) stressed that in the first phase, female labor supply and fertility have a negative relationship due to the relative expensiveness of market caretaker services, while in the second phase, it is the positive effect of female mean wages increase on fertility rates, and the negative effect of skill premium caused by the skill-biased technological changes on the relative cost of unskilled child care services that generates the positive relationship between fertility rates and female employment rate.

effect and income effect to the latter, decreasing fertility dramatically. If the countries develop from a low FPR and wage, the latter will bring more negative effect to fertility; furthermore, as fewer and fewer women are not working, the effect will become smaller (Ahn and Mira, 2002).[23] It is also believed that unemployment can also affect fertility (Adsera, 2011).[24] Unemployment brings (negative) income effect to male employees, and both (positive) substitution effect and (negative) income effect to female workers on fertility.[25] In low FPR countries, the income effects should be more significant, and thus, fertility rates will decline dramatically—the positive correlation between FPR and TFR (Ahn and Mira, 2001).[26] Therefore, the

[23] This implication is based on the assumption of the fixed working hour restrictions. Batz and Ward (1979) assumed continuous working hours, which means people can choose their working hours every day. They concluded that when more women become employed, fertility will decrease due to the increasing substitution effect, that is, fertility is becoming countercyclical.

[24] Adsera (2011)found that high and persistent unemployment in a country is associated with delays in childbearing and second births; and that women with temporary contracts, mostly in Southern Europe, are the least likely to give birth to a second child.

[25] Income effect implies that more income will improve fertility, and substitution effect implies that women can trade off childcare time in favor of work, which will decrease fertility.

[26] According to Ahn and Mira (2001), faced with unemployment, more women may choose to participate in the labor market for income concern or insurance strategy against the possible unemployment of their husbands, and thus, they would prefer to have no children or more children.

pessimistic prospect concerning income and career brings down the overall fertility.[27] Some researchers also emphasized that the employment uncertainty and labor market arrangements mold childrearing and participation decisions greatly (Adsera, 2004; Adsera, 2011; Bono et al., 2015). Adsera (2004) found that countries with higher certainty of reentrance into the labor market (e.g.,the US) or stable job security (e.g., Northern Europe), which accommodates the entry-exit of the labor market, will have lower unemployment and higher fertility. However, the areas that are lack of employment stability or temporary contract will experience sharp reduction of fertility (e.g., Southern Europe) (Bono et al., 2015).[28] Gender gap in unemployment also affects fertility, and the larger the gap, the lower the fertility (Galor and Weil, 1996; Adsera, 2005). Moreover, business cycles may affect household income via unemployment, which makes fertility more procyclical (Ahn and Mira, 2001).

[27] Adsera (2004) noted three factors of income loss due to unemployment: forgone earnings during childbearing, a lower wage growth due to career breaks, and a potential increase in unemployment risk.

[28] Bono et al. (2015) demonstrated that labor market institutions associated with higher uncertainty about employment and lower career and promotion opportunities such as temporary, part-time, or zero-hours contracts can significantly reduce women's demand for children.

Chapter 3 Fertility and unemployment in a social security system

3.1 Introduction

The problems of fertility reduction and unemployment increase have puzzled many governments over the past decades. Can the widely used social security system help solve these problems?

Analyses concerning fertility consider children as a consumption good or an investment good. The former, the altruism hypothesis, argues that parents procreate because they derive satisfaction from raising children (Barro and Becker, 1989 ; Galor and Weil, 1996), while the latter, the egoism hypothesis, insists that parents raise children because of old-age security considerations (Cigno, 1993; Bental, 1989; Boldrin and Jones, 2002). Concerning the effects of pension on fertility, many empirical studies believe that egoism prevails over altruism (e.g., Cigno and Rosati, 1992; Hohm, 1975). However, few studies consider that people have incentives to raise more children when their future lives are guaranteed by pensions.

The study analyses the effect of a social security system on endogenous fertility in a two-period OLG model with unemployment considered. The unemployment is due to constant minimum wage. The social security system is composed of defined-benefit PAYG pension and child allowances.

The motivation of this study arises from what if the pension is introduced to

Fanti and Gori (2007) because their conclusion is intriguing and argue that for any constant minimum wage, the child allowance will affect fertility negatively, which contrasts with the common sense that child allowances will help decreasing childrearing cost, thus increasing fertility. Moreover, the mutual effect between pension and fertility is a popular topic. So the research question can also be expressed as "how the pension and child allowances affect fertility allowing for unemployment due to constant minimum wage". The present study extends the OLG model by incorporating not only child allowances but also a PAYG public pension into the economy and examines the effects of the two subsidies on endogenous fertility and unemployment using comparative statics.

The analysis reveals that the effect depends on the level of pensions. For pension's effect on fertility, when pension is at higher level, the effect on fertility will be positive; when pension is at lower level, the effect will be negative. The effect of child allowances also depends on the level of pensions. If pension's level is lower, even with child allowances, the final effect will be negative. But if the pension's level is higher, even though the child allowances decreases, the final effect on fertility will also increase.

This study provides the intuition that if people expect to obtain sufficient benefits in old age, they do not have to worry about their future life, so they prefer to raise more children; if the pension is not sufficiently high, people save for their future needs rather than procreate even though they can obtain child allowances.

The remainder of the chapter is organized as follows: Section 2 introduces the model; Section 3 discusses the equilibria of the two endogenous variables; Section 4 reviews the comparative statics; and Section 5 concludes.

3.2 Model

In this section, we will discuss the basic model by analyzing consumers, production, the government, and capital market equilibrium in a closed economy. A general equilibrium OLG model is considered to be used, and the individual's life is divided into youth and old.

3.2.1 The government

Taxes from workers' income are used to finance the government's public pension and child allowances:[29]

$$\pi_t \omega L_t = \theta N_{t-1} + \varphi n_t N_t \,, \tag{1}$$

where π_t is the income tax rate of social security involving the pension and the child allowances; ω is the constant minimum wage, which is set to exceed the competitive wage;[30] N_t is the population of generation t; L_t is the labor force. And $L_t = N_t(1 - u_t)$, where u_t is the unemployment rate. The relationship between

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[29] When the model was first set, the unemployment benefits were also considered. However, these had no effect on the final result, and they were thus removed from the model for simplicity.

[30] See Fanti and Gori (2007, 2010).

populations of adjacent generations is linked by the endogenous fertility as $N_{t+1} = N_t n_t$.

3.2.2 Consumption

Consider a two-period general equilibrium OLG model in a closed economy. Individuals gain utility from youth consumption, old-age consumption, and child rearing:

$$U\left(c_{1,t}, c_{2,t+1}, n_t\right) = \alpha\, ln\left(c_{1,t}\right) + \beta\, ln\left(c_{2,t+1}\right) + \gamma\, ln(n_t), \tag{2}$$

where $\alpha, \beta, \gamma \in (0,1)$ are the weights of utility components, and $\alpha + \beta + \gamma = 1$. n_t is the number of children. The budget constraint of the young in generation t is

$$c_{1,t} + s_t = \omega(1 - u_t)(1 - \pi_t) - (m - \varphi)n_t. \tag{3}$$

where m and φ are the average cost of raising children and the child allowances level $(0 < \varphi < m < \omega)$. In the youth period, the wages of workers support consumption $(c_{1,t})$, savings (s_t), social security payments and the cost of raising children. In the retirement period, old-age consumption $\left(c_{2,t+1}\right)$ comes from savings $(s_t R_{t+1})$ and the PAYG pension$(\theta,\ 0 < \theta < \omega)$:

$$c_{2,t+1} = s_t R_{t+1} + \theta, \tag{4}$$

where $R_{t+1} = 1 + r_{t+1}$, and r_{t+1} is the interest rate in period $t + 1$. According to the utility maximization,

$$s_t = \beta\omega(1 - u_t) - \beta\varphi n_t - \frac{\beta\theta}{n_{t-1}} - \frac{(1 - \beta)\theta}{R_{t+1}}, \tag{5}$$

$$n_t = \frac{\gamma}{m - \varphi + \gamma\varphi}\left[\omega(1 - u_t) - \frac{\theta}{n_{t-1}} + \frac{\theta}{R_{t+1}}\right]. \tag{6}$$

3.2.3 Production

It is assumed that innumerable identical firms act competitively. The Cobb–Douglas production function is $Y_t = K_t{}^{\delta} L_t{}^{1-\delta}$, where K_t and L_t denote the capital and labor input, and $\delta \in (0,1)$ represents the weight of the capital input. To maximize profits,

$$\omega_t = (1-\delta)\left(\frac{K_t}{L_t}\right)^{\delta}, \tag{7}$$

$$r_t = \delta\left(\frac{K_t}{L_t}\right)^{\delta-1} - 1 \ . \tag{8}$$

Based on the relationship between labor and total population, the wage rate and the interest rate can be derived as

$$\omega = (1-\delta)\left(\frac{K_t}{L_t}\right)^{\delta}, \tag{7}$$

$$r_t = \delta\left(\frac{K_t}{L_t}\right)^{\delta-1} - 1. \tag{8}$$

Therefore, the capital–labor ratio and the interest factor are constant.

$$R_{t+1} = 1 + r_{t+1} = \delta\left(\frac{K_{t+1}}{L_{t+1}}\right)^{\delta-1} = \frac{\delta}{\varepsilon} = R. \tag{12}$$

As $L_t = N_t(1-u_t)$,

$$\frac{K_t}{L_t} = \frac{k_t}{1-u_t},$$

where $k_t = K_t/N_t$. Thus,

$$u_t = 1 - k_t\left(\frac{1-\delta}{\omega}\right)^{\frac{1}{\delta}}. \tag{9}$$

3.2.4 Capital market equilibrium

Consider the equilibrium in capital markets. At the beginning of the next period, the savings of previous period t are seen as a resource to invest the capital of this period $t+1$. We can write it as $S_t = K_{t+1}$. Therefore, the relationship of savings and capital per-capita can be written as follows:

$$\frac{s_t}{n_t} = k_{t+1} \,. \tag{18}$$

Since the interest rate of the economy is constant, how can the equilibrium be reached? If the capital of period $t + 1$ is less than savings of period t ($K_{t+1} < S_t$) , the unemployment rate of period t+1 (u_{t+1}) is big as the wage rate is constant (equation (15)). At this time, the labor supply in period $t + 1$ (L_{t+1}) is inadequate, and thus the production level (Y_{t+1}) is low. Accordingly, the total income of the society is small, and then the savings level of the period t+1 (S_{t+1}) becomes lower. From this analysis, even though the savings level in the period t (S_t) is higher, in the next period (S_{t+1}), it will lower down, in which way, capital and savings will get close to balance ($S_{t+1} = K_{t+2}$). Similarly, If the capital of period $t + 1$ is bigger than the savings of period t ($K_{t+1} > S_t$), capital and savings will get close to balance ($S_{t+1} = K_{t+2}$).

Substituting Equations (5) and (6) into Equation (10), the per-capita capital is

$$k_{t+1} = \frac{\beta(m-\varphi)}{\gamma} - \frac{\theta\varepsilon}{\delta n_t} \,. \tag{11}$$

This equation means that when workers are levied the pension tax, part of the resources have to pay for the tax, which affects the capital accumulation of the society. On the other hand, if there is no pension existing, just as Fanti and Gori (2007) did, the capital per-capita will stay unchanged as the constant minimum wage.

This equation also tells us the positive relationship between the capital per-capita of current period and the fertility rate of the previous period. When the fertility of previous period becomes larger, which means the number of people who bear the burden of pension tax increases, the capital accumulation of the following period becomes bigger consequently. This inter-generational transfer connects the generations of the same period, and tells us that population changes can affect the capital accumulation per-capita positively.

This implies that capital accumulation is obstructed because of pension enforcement.[31] When the fertility rate increases, the population bearing the pension tax burden increases. When the tax burden per capita decreases, the savings level increases, and the next-period capital accumulation per capita is fostered, indicating that the population alteration positively affects the per-capita capital accumulation.

3.3 Equilibrium

In this part, dynamic systems of the three endogenous variables: the fertility rate and the unemployment rate will be discussed, and a thorough and comprehensive understanding about how the equilibrium in the economy is reached and how the three endogenous variables are mutually affected can be clearly obtained.[32]

Substituting Equation (11) into (9), and then into Equation (6), the dynamic fertility equilibrium is

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[31] If there is no pension, as shown by Fanti and Gori (2007), the capital per capita remains unchanged.

[32] In section 3, this chapter focuses on the analysis in the first quadrant of the equilibria of three variables.

$$n_t = \frac{\gamma}{m - \varphi + \gamma\varphi}\left[\frac{\beta(m-\varphi)(1-\delta)}{\gamma\varepsilon} + \frac{\theta\varepsilon}{\delta} - \frac{\theta}{\delta n_{t-1}}\right], \tag{12}$$

where $\varepsilon := \omega^{\frac{1-\delta}{\delta}}(1-\delta)^{\frac{\delta-1}{\delta}}$.

In this model, corresponding to the upper and lower bound of capital, there exists an upper and a lower limit on the fertility level. The upper limit, which ensures the existence of unemployment, can be expressed as $\bar{n} = \theta\gamma\varepsilon(1-\delta)[\beta\delta(m-\varphi)(1-\delta) - \delta\omega\gamma\varepsilon]^{-1}$. The lower limit of fertility is set to ensure the non-negative capital per-capita, and it can be expressed as $\underline{n} = \theta\gamma\varepsilon[\beta\delta(m-\varphi)]^{-1}$. Therefore, to make the system meaningful, we restrict fertility rate on this scope: $\underline{n} < n_{t-1} < \bar{n}$. From the conditions above, we can depict the dynamics of fertility in Figure 1. N_2 and O are locally stable, whereas N_1 is unstable.

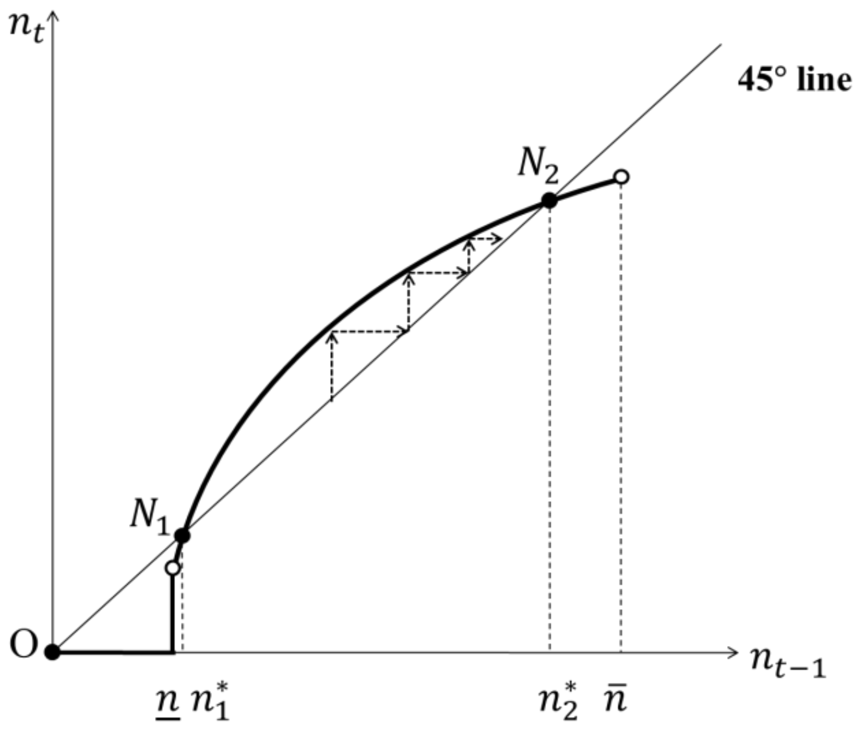

Figure 3.1. Fertility equilibrium

Substituting Equation (11) into (9), and replacing n_t and n_{t-1} with u_{t+1} and u_t, and then into Equation (12), the unemployment equilibrium dynamics are

$$u_{t+1} = \frac{\theta\sigma^2\varepsilon^2(m-\varphi+\gamma\varphi)}{\sigma[\theta\gamma\varepsilon^2-\beta(m-\varphi)\delta^2]+\gamma\delta(1-u_t)} + \frac{\gamma-\beta(m-\varphi)\sigma}{\gamma}, \tag{13}$$

where $\sigma = (1-\delta)^{\frac{1}{\delta}}\omega^{-\frac{1}{\delta}}$. Equilibria of the two endogenous variables are determined by Equations (12) and (13).[33] The equilibria are depicted in Figure 2, and U_2 and P are locally stable, whereas U_1 is unstable.

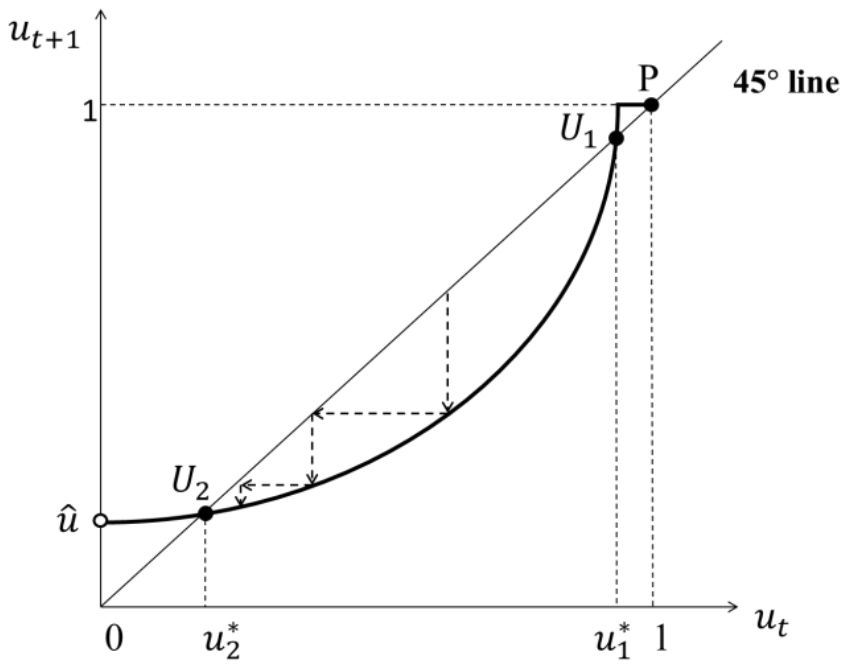

Figure 3.2. Unemployment equilibrium

The trajectories of the equilibrium of the endogenous variables is like this: when the fertility rate (n_{t-1}) increases, the per-capita capital accumulation (k_t) increases. As the capital-labor ratio is constant, the labor demand ($L_{t,d}$) increases. Then, the unemployment rate (u_t) decreases, which means that the labor supply ($L_{t,s}$) is increasing. Consequently, production (Y_t) and the fertility rate (n_t) increase. Thus, the per-capita capital (k_{t+1}) increases, and the unemployment rate (u_{t+1}) decreases.

[33] There are three equilibria in each dynamics, but this study focuses on the neighborhoods of the stable and economically meaningful steady states.

Proposition 1. *Multiple equilibria exist in the PAYG economy with unemployment.*

Proposition 1 shows the different results from Fanti and Gori (2007), which showed there is only one equilibrium in the steady state. It is obvious that the difference comes from the enforcement of public pensions. The reason why the multiple equilibria appeared is that: first of all, public pension is used as an income tax in this model, which calls for the capital accumulation stage in the very beginning; secondly, public pension acts as an inter-generational transfer to connect generations, which extends the previous model set up by Fanti and Gori (2007) which contains only intra-generational transfer-child subsidy. The second reason connects these endogenous variables of adjacent periods into one mechanism, and the dynamic equilibria are derived in this way.

3.4 Comparative statics

In order to determine the impact of social security system on fertility and the unemployment rate, we will analyze the effects of social security system consisting of pension and child subsidy on the two variables respectively in steady state.

3.4.1 Public pension effect

Proposition 2.

For any given minimum wage value, when the pension level is higher than $\frac{\delta}{\gamma\varepsilon^2}\left(2A - B + 2\sqrt{A(A-B)}\right)$*, it improves the fertility rate and decreases the unemployment rate; when the pension level is lower than* $\frac{\delta}{\gamma\varepsilon^2}\left(2A - B - 2\sqrt{A(A-B)}\right)$ *or* $\frac{\delta}{\gamma\varepsilon^2}\cdot\frac{2A(A-B)}{A+B}$*, it depresses the fertility rate and increases the unemployment rate. Here,* $A = m - \varphi + \gamma\varphi$ *and* $B = \beta(m-\varphi)(1-\delta)$.

Proof. See Appendix A-1.

To illustrate the proposition, the parameters are set as follows: $m = 0.25, \omega = 1.00, \alpha = 0.45, \beta = 0.10, \gamma = 0.45, \delta = 0.33, \varepsilon = 2.25$ $(\theta < \omega, 0 < \varphi < m < \omega)$. Figure 3 shows that when the pension level is low ($\theta \leq \theta_2$), the pension reduces the fertility rate (if $\varphi = 0.1, \theta_2 = 1.92 * 10^{-5}$; if $\varphi = 0.14$, $\theta_2 = 1.16 * 10^{-5}$). Figure 4 indicates that when the pension level is high ($\theta \geq \theta_1$), the pension improves the fertility rate (if $\varphi = 0.1$, $\theta_1 = 0.11$; if $\varphi = 0.14$, $\theta_1 = 0.0977$).

Discussion.

Equation (12) indicates that the fertility rate in the steady state is

$$n^* = \frac{\beta(m-\varphi)(1-\delta)}{\varepsilon(m-\varphi+\gamma\varphi)} + \frac{\gamma}{m-\varphi+\gamma\varphi}\left[\frac{\theta}{R} - \frac{\theta}{\delta n^*}\right]. \tag{15}$$

In the right-hand side of the equation, when the first term in the bracket $\frac{\theta}{R}$ is much bigger than the second, $\frac{\theta}{\delta n^*}$ ($n^* \gg \frac{R}{\delta}$), meaning that the future income is much higher than the tax burden for the pension, the final effect on fertility will be positive;

otherwise, the effect will be negative.[34]

The effects of pension on fertility can be summarized as direct effect and indirect effect:

$$n_2^* = n^*\left\{\underbrace{\theta}_{+or-}, \underbrace{u^*[k^*(\theta)]}_{-}\right\}.$$

The direct effect, as the first argument shows, has two parts: the pension is collected from the income of workers as a social security tax; it then reduces the income and the fertility rate, and this is defined as a *tax effect*, which is negative. Further, it is an income because it will be derived in retirement period; this increases the fertility rate, and it is defined as an *income effect*, which is positive. Indirectly, as the second argument shows, the pension affects fertility via capital accumulation, which is hindered as the pension increases, increasing the unemployment rate. Thus, the fertility rate drops.

Therefore, the effects of pension on fertility involve 3 kinds of effects: one positive effect and two negative effects. When the positive effect surpasses the negative effects, the fertility will be increased. The positive effect is the income effect because pension can be seen as a future income, the current value of which relates to interest rate. The negative effects include a tax effect and an indirect effect. For the tax effect, pension is equal to a social security tax for working generation, thus decreasing income of workers. On the other hand, the pension will affect capital accumulation negatively, since the capital-labor ratio stays constant because of the

[34] Refer to the seminal paper of Aaron in 1960 where the *Aaron condition* tells the same story.

constant wage, the labor force will shrink, so the unemployment rate increases, decreasing fertility rate. This two effects can be seen as the tax burden, so in per capita, these effects are determined by the fertility of pervious period.

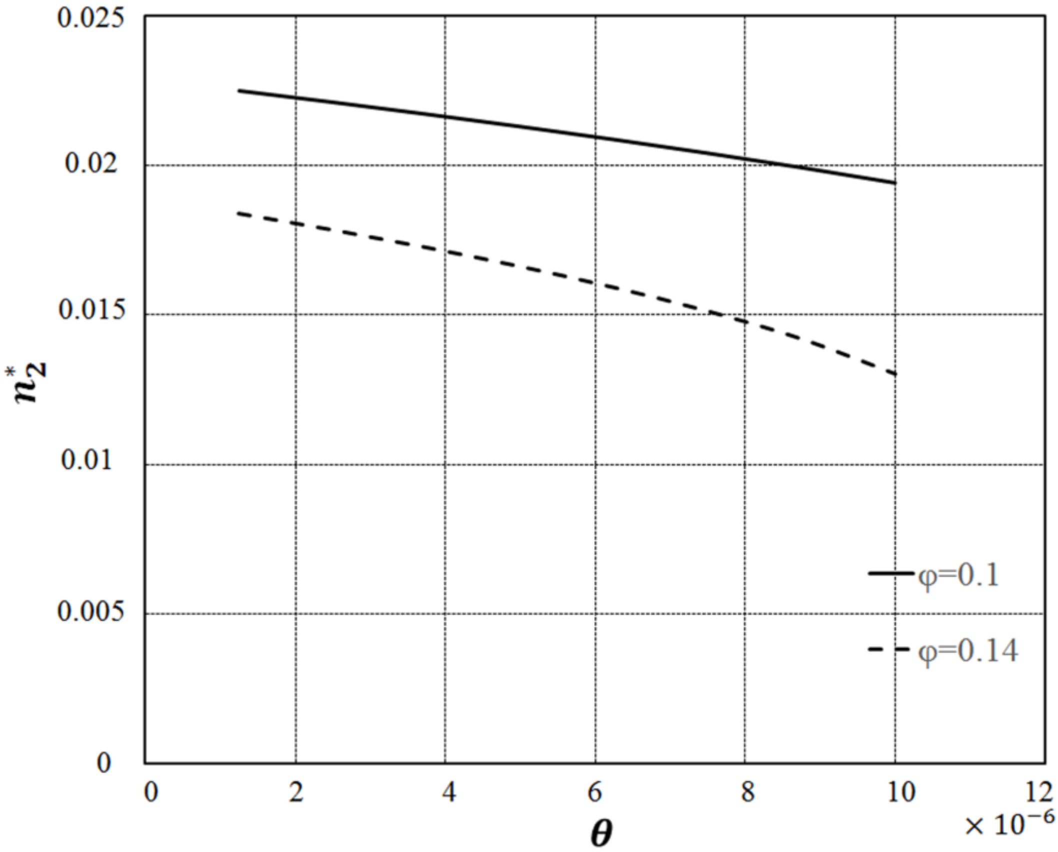

Figure 3.3. Effects of public pension on fertility ($\boldsymbol{\theta}$ is low)

(Note: $\theta \leq \theta_{2,\varphi=0.1} = 1.92 * 10^{-5}, \theta_{2,\varphi=0.14} = 1.16 * 10^{-5}$)

The intuition is that the higher pension implies a higher expected future income for old-age life. In this context, people do not worry about their old age and save less, the incentive to rear children becomes higher. So the tax burden becomes lighter, the negative effects become lower, and the synthesized effect is thus positive.

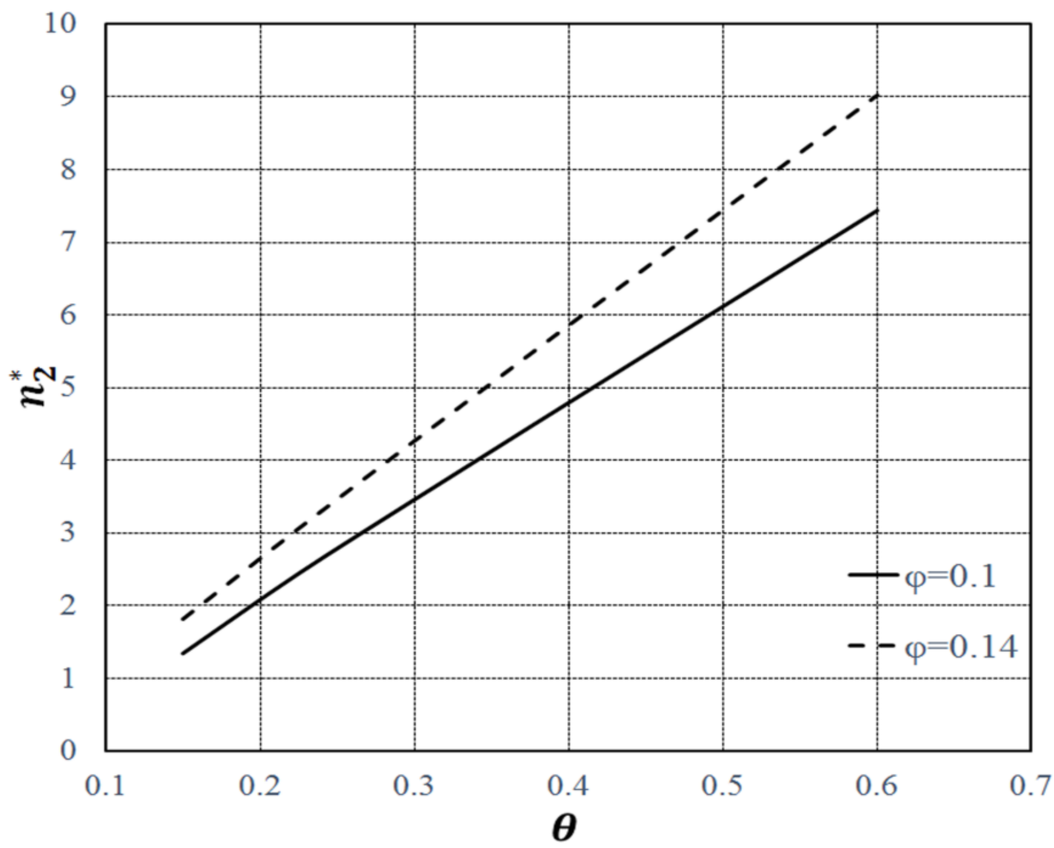

Figure 3.4. Effects of public pension on fertility (θ is higher)

(Note: $\theta > \theta_{1,\varphi=0.1} = 0.11, \theta_{1,\varphi=0.14} = 0.0977$)

When pension increases at a high level, the positive effect increases much more quickly than the negative ones do, which means when the old-aged services are guaranteed, people would like to raise more children; while pension increases at a low level, the negative effects increase much faster than the positive one does, which means when people have to worry about the old-aged life, they prefer to save money for themselves rather than have more children. In summary, the effects of pension on fertility depend on the dominance within the pension positive and negative effects.

3.4.2 Child subsidy effect

Proposition 3.

(1) *The effects of child allowances on fertility are uncertain because of the implementation of the PAYG pension policy.*

(2) i) *When the pension level is higher than* $\frac{\delta}{\gamma\varepsilon^2}\left(2A - B + 2\sqrt{A(A-B)}\right)$ *or* θ_4*, child allowances improve the fertility rate and decrease the unemployment rate;* **ii)** *When the pension level is lower than* $\frac{\delta}{\gamma\varepsilon^2}\left(2A - B - 2\sqrt{A(A-B)}\right)$ *or* θ_5*, child allowances decrease the fertility rate and increase the unemployment rate. Here,* θ_4 *and* θ_5 *are the two solutions of* $\left(\frac{\beta(m-\varphi)(1-\delta)}{\gamma\varepsilon} + \frac{\theta\varepsilon}{\delta} + \sqrt{\Delta}\right)\left(\frac{1-\gamma}{\gamma} - \frac{\beta(1-\delta)(m-\varphi+\gamma\varphi)}{\gamma^2\varepsilon\sqrt{\Delta}}\right) + \frac{2\theta(1-\gamma)(m-\varphi+\gamma\varphi)}{\gamma^2\delta\sqrt{\Delta}} = 0$ *and* $\theta_4 > \theta_5$.

Proof. Appendix A-2.

To illustrate the propostion, the paramters are set as same as Section 4.1. Figure 5 shows that when the pension level is low, child allowances decrease the fertility rate. Figure 6 indicates that when the pension level is high, child allowances increase the fertility rate.

Discussion.

From Equation (15), the effects of child allowances on fertility can be explained independently and dependently:

$$n_2^* = n^*\left\{\underbrace{\varphi}_{-}, \underbrace{\overbrace{\varphi}^{+} \cdot \overbrace{\theta}^{+or-}}_{+or-}\right\}.$$

Independently, as the first argument shows, the income effect increases the fertility rate, whereas the tax effect reduces it. Fanti and Gori (2007) imply that the tax effect is stronger:

$$\frac{\partial n^*}{\partial \varphi}|_{\theta=0} = \frac{-\gamma m \beta(1-\delta)}{\varepsilon(m-\varphi+\gamma\varphi)^2} < 0\,.$$

When the pension policy is not implemented, child allowances negatively affect fertility.

Dependently, as the second argument shows, the effects of child allowances involve those of the public pension. This is because in this two-period OLG model, child allowances relate to the intra-generational relationship whereas pension concerns inter-generational correlations. The intra-generational alteration will also lead to inter-generational changes, and the effects of child allowances thus depend on those of pension.

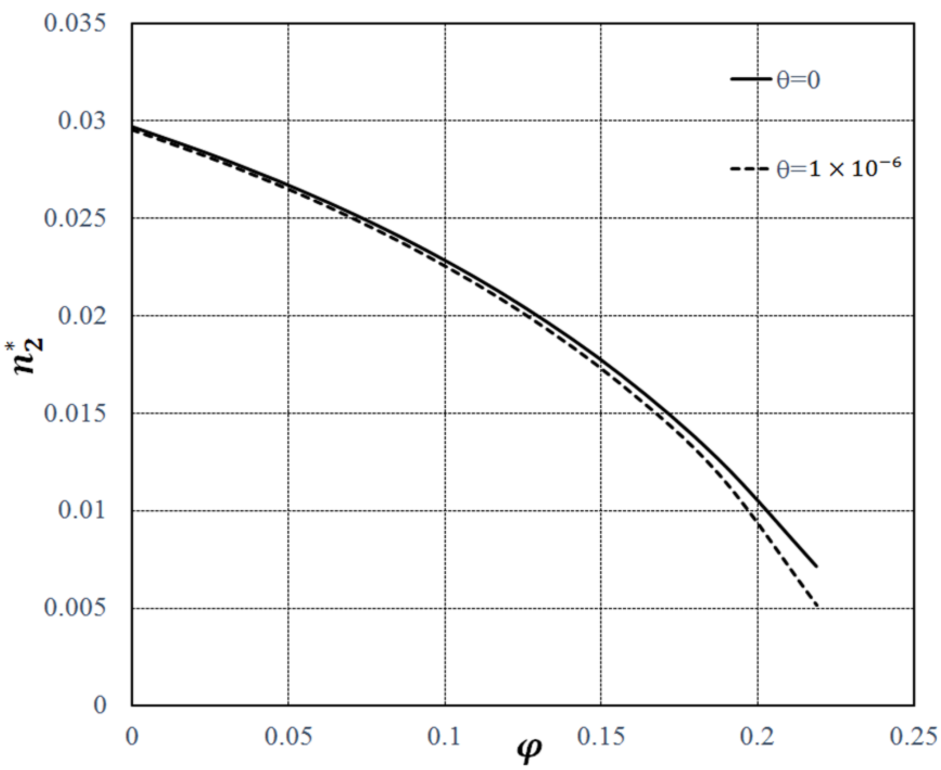

Figure 3.5. Effects of child allowances on fertility (θ is low)

(Note: $\theta = 0, 10^{-6}$)

The two propositions provide the intuition that if people expect to obtain sufficient benefits in old age, they prefer to raise more children; if the pension is not sufficiently high, people save for their future needs rather than procreate even though they can obtain child allowances (that are less than the average cost of raising children).

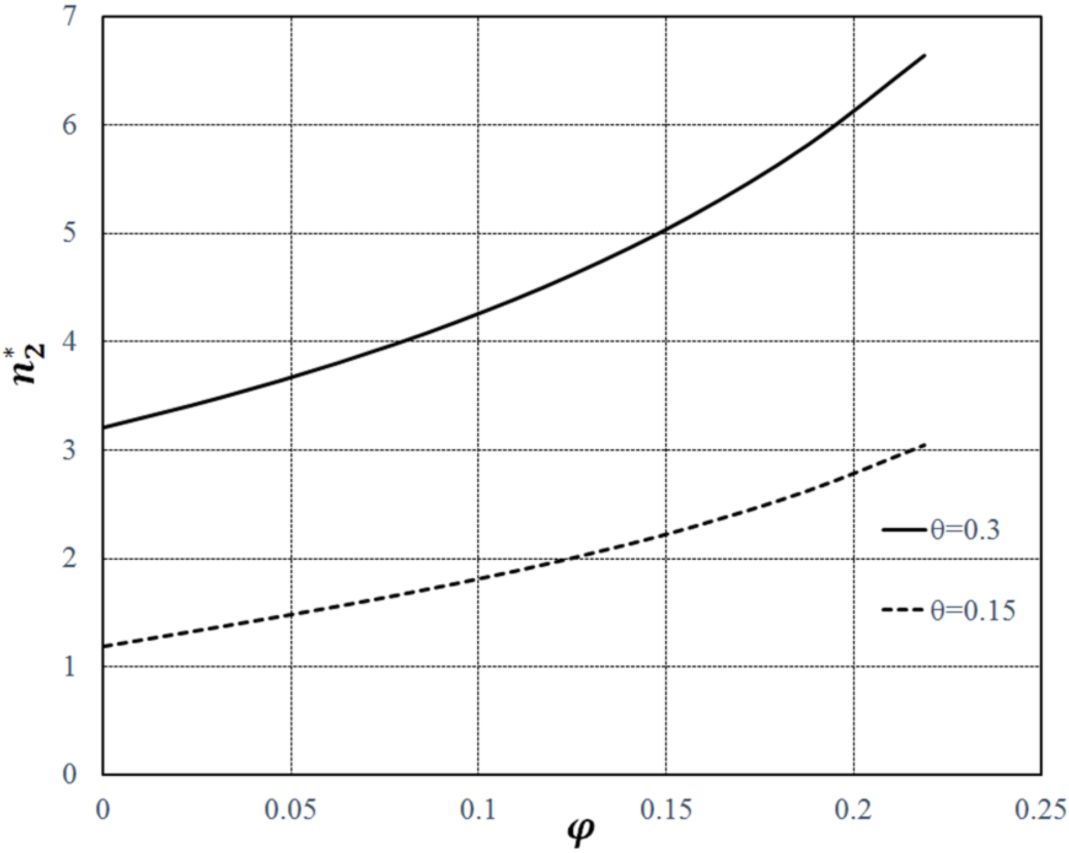

Figure 3.6. Effects of child allowances on fertility (θ is higher)

(Note: $\theta = 0.15, 0.3$)

3.5 Conclusions

This study analyses how endogenous fertility and unemployment are affected by the social security system consisting of the public pension and the child subsidy in an

OLG model in a closed economy. The principal novelties in this chapter are the utilization of public pensions to solve the problem of fertility and unemployment in the meantime, and conclude that it can reverse the negative effect of the conventional use of child subsidy, which has been discussed in Fanti and Gori (2007).

This study finds that for any given minimum wage value, a pension fosters the fertility rate on surpassing some high level. In order to increase the fertility rate and decrease the unemployment rate, a public pension or a combination of a public pension and child allowances may be considered. The former is better at lightening the tax burdens of payers, and the latter is superior at the speed and strength of the improvement in fertility and unemployment. Moreover, pension's level has great impact on people's incentive to have children. How to set the pension's level can guarantee the old-age life should be taken seriously. So if the government would like to increase fertility, ensuring a higher pension for the old-aged or implementing the combination of higher pension and lower child allowances can be alternatives.

Further, there are several problems which can be improved in future studies. First, the assumption that the wage rate is constant with time is hard to justify. Hence, to improve the practicability of our model during long-run analysis, in future study, this can be set as a time variant factors. Second, it can be expected that the utility function of the unemployed and that of those who are employed is totally different; however, in this study, we view them as the same. Therefore, we ignored many factors that differentiate both groups of people, for example, the cost of children in the employed people's families is less than that in the unemployed people's families. In future studies, it is better to take this factor into account in future study in order to raise the feasibility of this model. Third, it is worthwhile to evaluate the scenario of a small open economy, since this situation is closer for most countries of the world.

Acknowledgment

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Chapter 4 Fertility, union wage setting and social security system

4.1 Introduction

Many European countries are plagued with high unemployment during recent decades. Moreover, the aging population is also a common problem faced by many countries. Since unemployment is often viewed as a short-run problem due to business cycle, while fertility is considered as a long-run issue, they are often discussed separately. However, the existing unemployment of Europe is attributed to equilibrium unemployment (Bean, 1994), which does not disappear in the long run (Brauninger, 2000), this thereby brings significant research value to consider fertility and unemployment simultaneously.

To deal with problems of falling fertility, an aging population or increasing unemployment, many policy makers and economists consider social security systems (SSS) as effective tools. However, in-depth analyses reveal many contradictions when SSS is applied: governments' attempts to apply unemployment insurance to improve the living standards of the unemployed may cause the tax burden of social security to increase the unemployment rate; pay-as-you-go (PAYG) pensions have been blamed for declining fertility rates; child allowances used to increase fertility rates can impose a tax burden that reduces capital accumulation, hindering economic growth. Therefore, can we solve the problems of fertility and unemployment by employing the SSS properly?

Previous literatures have attempted to analyze effect of SSS on fertility in an imperfect labor market with unemployment, and put forward policy advices in applying SSS. Fanti and Gori (2007) show that for any given minimum (or union's) wage value, the child subsidy reduces capital accumulation and increases unemployment, and ultimately decreases demand to bear children in a closed economy. Fanti and Gori (2012) examine the effect of child allowances on fertility in a small open economy, and find that in the context of competitive labor market, it acts as a fertility-enhancing device, while in a unionized market the child policy may be ineffective. But both of them failed to suggest how to increase fertility and decrease unemployment by using child policy. To put forward the policy suggestions, Wang (2015) discusses the effect of a SSS on fertility and unemployment by assuming a constant minimum wage, concluding that a pension alone or the combination of a pension and child allowances may positively affect fertility and reduce unemployment. However, the assumption of constant minimum wage is lack of correspondence with reality. Consequently, it motivates this study to extend Wang (2015) by incorporating some institutional features of European countries, thus assuming that wage levels are endogenously set by monopolistic trade unions. This study finds that not only the way to employ the SSS, but also the consideration of wage setting influences the effect of pensions or child allowances on fertility and unemployment.

The model of this current work has several peculiarities. Firstly, wages are set by the monopolistic unions. Unions maximize the expected utility of the representative members (Oswald, 1982; Daveri and Tabellini, 2000; Demmel and Keuschnigg, 2000;

Ono, 2010). Therefore, in a closed economy, the interest rate which is related to wage level decides the future pension's level, and thus determines the final effect of pension. This mechanism implies that wage setting is an important factor to be considered in the effect analysis of SSS on fertility and unemployment. Secondly, SSS are composed of pension, child allowances and unemployment insurance aiming to address problems of fertility and unemployment. Nature of offspring can help explain the origins of the downward tendency of fertility. When progeny is looked upon as a consumption good, child allowances can be used to reduce the costs of childrearing (Becker and Barro, 1988; Galor and Weil, 1996); when child is treated as a capital good, sufficient pension benefits in old age will attenuate the function of offspring, which abates the incentive to rear children (Cigno, 1993); as the more children, the broader the tax base of PAYG-basis transfer, children are often regarded as a public capital good, by which governments encourage higher fertility (Cigno, 1993; Folbre, 1994; Groezen et al., 2003) . Taking into account the demographic features, the social security system include pension and child allowances; to ensure the lives of the unemployed, unemployment insurance are incorporated.

This study finds that, first, increased pension levels lead to a higher fertility rate when wages are higher but a lower rate when wages are lower. Second, an increased child allowances leads to an increased fertility rate when wages are lower but a decreased rate when wages are higher. Moreover, to improve social welfare, it is preferable to increase public pensions or the child allowances tax rate and reduce the unemployment insurance tax rate when wages are lower, while it is preferable to reduce the child allowances or the unemployment insurance level when wages are higher. Therefore, both social security and wage setting should be considered when seeking ways to improve fertility and reduce unemployment. Finally, the results are verified by simulation.

The remainder of this chapter is organized as follows. Section 2 presents the model. Section 3 discusses the equilibrium. Section 4 describes a set of comparative statics, and analyses the effects of pensions, child allowances, and unemployment insurance. Section 5 presents simulation and a welfare analysis, and Section 6 gives conclusions.

4.2 The model

This section constructs the basic model by analyzing the activities of the firms, the households, the government, the unions and capital market equilibrium in a closed economy. A two-period OLG model of general equilibrium is used, and individual lifespans are divided into "youth" and "old" periods. The structure of this model is showed in Figure 4.1.

4.2.1 The firms

Numerous firms in this economy produce homogenous good. For simplicity, the technology is specialized to the Cobb–Douglas form with constant returns to scale. Therefore, the production function is

$$Y_t = {K_t}^{\delta} {L_t}^{1-\delta}.$$

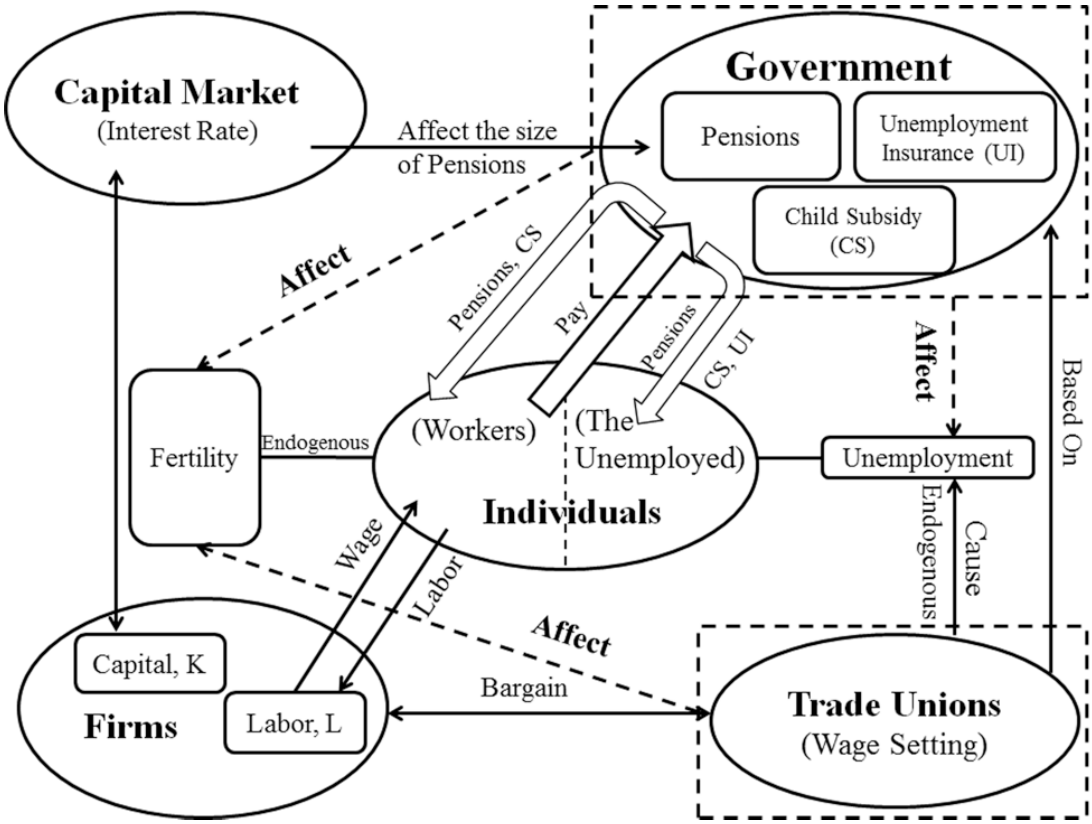

Figure 4.1. Structure of the model

where K_t denotes the capital input, L_t is the labor population, and each worker supplies one unit of labor, so L_t also means the labor input; and δ represents the weight of the capital input. To maximize profits,

$$\omega_t = (1-\delta)\left(\frac{K_t}{L_t}\right)^{\delta}, \tag{1}$$

$$r_t = \delta\left(\frac{K_t}{L_t}\right)^{\delta-1} - 1. \tag{2}$$

As $L_t = N_t(1-u_t)$,

$$\frac{K_t}{L_t} = \frac{k_t}{1-u_t},$$

where $k_t = K_t/N_t$. Thus,

$$\omega_t = (1-\delta)\left(\frac{k_t}{1-u_t}\right)^{\delta}, \tag{1 *}$$

$$r_t = \delta\left(\frac{k_t}{1-u_t}\right)^{\delta-1} - 1. \tag{2 *}$$

Therefore, the wage rate and interest rate of the same period have a relationship expressed as $1 + r_{t+1} = \delta(\omega_{t+1}/1 - \delta)^{\frac{\delta-1}{\delta}}$.

4.2.2 The government

Taxes on workers' income are used to finance the government's public pensions as well as child allowances and unemployment benefits. In this study, separated balanced budgets are assumed (Groezen et.al, 2003; Fenge and Meier, 2005, 2009; Zamac, 2007; Groezen and Meijdam, 2008). In the public pension system, the budget constraint of the government is expressed as $\tau\omega_t N_t(1 - u_t) = \theta_t N_{t-1}$. Rearranging the terms of the equation above produces

$$\tau\omega_t n_{t-1}(1 - u_t) = \theta_t. \tag{3}$$

In the child allowances system, the budget constraint of the government is written as $\rho\omega_t N_t(1 - u_t) = \varphi_t N_{t+1}$. Rearranging the terms of the equation above produces

$$\rho\omega_t(1 - u_t) = \varphi_t n_t. \tag{4}$$

In the unemployment insurance system, the budget constraint of the government is written as $\varepsilon\omega_t N_t(1 - u_t) = \sigma_t N_t u_t$. Rearranging the terms of the equation above produces

$$\varepsilon\omega_t(1 - u_t) = \sigma_t u_t. \tag{5}$$

4.2.3 The individuals

The lifetime utility function of an individual is defined over consumption of youth period ($c_{1,t}$) and old-age period ($c_{2,t+1}$), and the number of children (n_t) (Eckstein and Wolpin, 1985; Eckstein et al. 1988; Galor and Weil, 1996). Therefore,

$$U(c_{1,t}, c_{2,t+1}, n_t) = ln(c_{1,t}) + \beta\, ln(c_{2,t+1}) + \gamma\, ln(n_t).$$

where n_t denotes the endogenous fertility, $N_t n_t = N_{t+1} (n_t > 0)$; β and γ are the weights of $c_{2,t+1}$ and n_t. In this economy, any representative has the possibility u_t to be unemployed, or $1 - u_t$ to be employed (Fanti and Gori, 2011, 2012). In youth, every household in the economy will receive child allowances for each child. The workers receive wages from firms, and pay for consumption, childrearing cost, and social security tax. The rest part is saved. The unemployed receive unemployment insurance from the government to afford consumption, child cost and saving. All proceeds are entirely consumed in the old-age period. The budget constraint is

$$c_{1,t} + s_t = \omega_t(1 - \tau - \rho - \varepsilon)(1 - u_t) + \sigma_t u_t - (m - \varphi_t) n_t,$$

$$c_{2,t+1} = s_t(1 + r_{t+1}) + \theta_t,$$

where τ, ρ, ε are the tax rates for public pensions (θ_t), child allowances (φ_t) and unemployment insurance (σ_t); m is the average childrearing cost (Praag and Warnaar, 1997; Longman, 1998; Groezen et. al, 2003); s_t is savings. To maximize the utility, consumption, savings and fertility are

$$c_{1,t} = \frac{\omega_t(1 - u_t)}{1 + \beta + \gamma}\left[(1 - \tau - \rho) + \frac{\tau n_{t-1}}{1 + r_{t+1}}\right], \tag{6}$$

$$c_{2,t+1} = \frac{\beta(1 + r_{t+1})(1 - u_t)\omega_t}{1 + \beta + \gamma}\left[(1 - \tau - \rho) + \frac{\tau n_{t-1}}{1 + r_{t+1}}\right], \tag{7}$$

$$s_t = \frac{\beta(1 + r_{t+1})(1 - u_t)\omega_t}{1 + \beta + \gamma}\left[(1 - \tau - \rho) + \frac{\tau n_{t-1}}{1 + r_{t+1}}\right], \tag{8}$$

$$n_t = \frac{\omega_t(1 - u_t)}{1 + \beta + \gamma} \cdot \frac{\gamma}{m - \varphi_t}\left[(1 - \tau - \rho) + \frac{\tau n_{t-1}}{1 + r_{t+1}}\right]. \tag{9}$$

4.2.4 The trade unions

Wages are set through bargaining between firms and monopolistic unions. Each firm has one correspondent union, representing benefits of workers. It is assumed that the unions can either set wages or take the interest rate and fiscal policy as given (Daveri and Tabellini 2000; Demmel and Keuschnigg 2000; Booth 1995; Ono, 2010). Therefore, the unions can only affect the members' welfare through lifetime income. The members of the unions may keep employed or become unemployed during their lifetimes, and the possibility is unemployment rate u_t. Therefore, the objective function of the unions is to set wage levels that maximize the lifetime incomes of their members (Dememl and Keuschnigg, 2000; Brauninger, 2005; Ono, 2010). The lifetime income of consistently employed members are wages and public pensions; that of the ones who become unemployed are unemployment insurance and public pensions. Therefore, the objective function of unions is

$$V_t = N_t(1-u_t)\left\{\omega_t(1-\tau-\rho-\varepsilon)+\frac{\theta_t}{1+r_{t+1}}\right\}+N_t u_t\left\{\sigma_t+\frac{\theta_t}{1+r_{t+1}}\right\},$$

where the utility maximization problem for unions is to maximize V_t subject to the labor demand function (1), taking policy variables and interest rate as given (Demmel and Ketchnigg, 2000; Ono, 2010). The first-order condition for maximization is[35]

$$\omega_t = \frac{1}{(1-\delta)(1-\tau-\rho-\varepsilon)}\sigma_t\,. \tag{10}$$

Thus, unions set wages based on the level of unemployment benefits and social

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[35] See Appendix B-1.

security tax rates (Ono, 2010). When the social security tax rate increases, wages also increase; thus, the labor supply will surpass labor demand, causing job loss.

4.2.5 The capital market

The capital market clearing condition is

$$K_{t+1} = S_t$$

Therefore, in per capita

$$k_{t+1} = \frac{s_t}{n_t}$$

From Equations (5) and (6), the capital per capita is

$$k_{t+1} = \frac{\beta(m - \varphi_t)}{\gamma} - \frac{\tau n_{t-1}(1 + \beta + \gamma)}{(1 - \tau - \rho)(1 + r_{t+1}) + \tau n_{t-1}}. \tag{11}$$

Equation (11) shows that the pension and child allowances reduce the income and hinder society's capital accumulation. Thus, when the pension tax rate (τ) increases, the capital per capita decreases (Fanti and Gori 2007, 2010; Wang 2015); and, when the child allowances tax rate (ρ) increases, the capital per capita decreases.

4.3 Equilibrium

The fertility equilibrium can be derived based on the relationships among wage setting, fertility, and capital per capita.

Using equations (3), (4), (5), equations (9), (10), and (11) can be written as follows:

$$\frac{\omega_t(1-u_t)}{mn_t-\rho\omega_t(1-u_t)}\left[(1-\tau-\rho)+\frac{\tau n_{t-1}}{\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}}\right]=\frac{1+\beta+\gamma}{\gamma}, \qquad (6*)$$

$$\frac{\varepsilon(1-u_t)}{u_t}=(1-\delta)(1-\tau-\rho-\varepsilon), \qquad (10*)$$

$$\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{1}{\delta}}(1-u_{t+1}) = \frac{\beta\left(m-\frac{\rho\omega_t(1-u_t)}{n_t}\right)}{\gamma}-\frac{\tau n_{t-1}(1+\beta+\gamma)}{(1-\tau-\rho)\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}+\tau n_{t-1}}. \qquad (11*)$$

From equation (10*)

$$u_t=u=\frac{\varepsilon}{\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)}. \qquad (10**)$$

Therefore, when wage levels are set by bargain between monopolistic trade unions and firms, the social security tax rate level will decide the equilibrium unemployment: if the social security tax increases, unemployment will increase because

$$\frac{\partial u}{\partial \varepsilon}=\frac{(1-\delta)(1-\tau-\rho)}{[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2}>0,$$
$$\frac{\partial u}{\partial \tau}=\frac{\varepsilon(1-\delta)}{[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2}>0,$$
$$\frac{\partial u}{\partial \rho}=\frac{\varepsilon(1-\delta)}{[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2}>0.$$

The relationship between unemployment rates and social security systems are also discussed by Nichell and Layard (1999), Daveri and Tabellini (2000), Demmel and Keuschnigg (2000), and Barr and Diamond (2006). Their findings are consistent with the analysis in Equation (10**). The equilibrium unemployment rate is decided by government social security policies and the wage setting of monopolistic trade unions: when social security tax rates increase, wages are set higher by the trade unions, and unemployment grows. The equilibrium unemployment rate can thus be inferred from the policy parameters.

From Equations $(10 **)$ and $(6 *)$

$$\frac{m(1+\beta+\gamma)}{\gamma(1-u)}\frac{n_t}{\omega_t}-\frac{\gamma(1-\tau)+\rho(1+\beta)}{\gamma}=\frac{\tau n_{t-1}}{\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}}. \tag{6 **}$$

Substituting Equation $(6 **)$ into Equation $(11 *)$ in terms of n_t/ω_t,

$$\frac{\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{1}{\delta}}(1-u)}{1+\beta+\gamma}$$
$$=\frac{m\beta}{\gamma(1+\beta+\gamma)}-\frac{m\beta\rho\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}}{\gamma^2\left\{\tau n_{t-1}+\left[1-\tau+\frac{\rho(1+\beta)}{\gamma}\right]\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}\right\}}$$
$$-\frac{\tau n_{t-1}}{\tau n_{t-1}+(1-\tau-\rho)\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}}. \tag{12}$$

The equation expresses the relationship between endogenous fertility and wages.

Lemma. *From the implicit function theorem, the relationship between fertility and wage levels can be written as the function f:* $n_{t-1}=f(\omega_{t+1})$.

Proof. See Appendix B-2.

Therefore, Equation (12) can be written as the function f as follows: $n_{t-1}=f(\omega_{t+1})(>0)$. From Equation (6**), therefore, the dynamics of wage equilibrium is

$$\frac{m(1+\beta+\gamma)}{\gamma(1-u)}\frac{f(\omega_{t+2})}{\omega_t}-\frac{\gamma(1-\tau)+\rho(1+\beta)}{\gamma}=\frac{\tau f(\omega_{t+1})}{\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}}.$$

In this study, the analysis focuses on the steady state(s):

$$\frac{m(1+\beta+\gamma)}{\gamma(1-u)}\frac{f(\omega)}{\omega}-\frac{\gamma(1-\tau)+\rho(1+\beta)}{\gamma}=\frac{\tau f(\omega)}{\delta\left(\frac{\omega}{1-\delta}\right)^{\frac{\delta-1}{\delta}}}.$$

Thus,

$$f(\omega)\left[\frac{m(1+\beta+\gamma)}{\gamma(1-u)}\frac{1}{\omega}-\frac{\tau}{\delta\left(\frac{\omega}{1-\delta}\right)^{\frac{\delta-1}{\delta}}}\right]=\frac{\gamma(1-\tau)+\rho(1+\beta)}{\gamma}. \qquad (13)$$

As $f(\omega)>0$, $\frac{m(1+\beta+\gamma)}{\gamma(1-u)}\frac{1}{\omega}-\frac{\tau}{\delta\left(\frac{\omega}{1-\delta}\right)^{\frac{\delta-1}{\delta}}}>0$, $\omega<\left[\frac{m\delta(1+\beta+\gamma)}{\gamma\tau(1-u)}\right]^{\delta}(1-\delta)^{1-\delta}$, which ensures interest rates not to fall too low, and a nonnegative capital accumulation. When the wage level is restricted to the upper limit, interest rates will not fall too low, ensuring a nonnegative capital accumulation or continued economic growth. Therefore, when the wage level is set under $\left[\frac{m\delta(1+\beta+\gamma)}{\gamma\tau(1-u)}\right]^{\delta}(1-\delta)^{1-\delta}$, the fertility level can be derived from Equation (13).

4.4 Comparative statics

4.4.1 Public pension's effect on fertility

Differentiating Equation (13) with the public pension tax rate produces

$$\frac{\partial f(\omega)}{\partial\tau}\left[\frac{m(1+\beta+\gamma)}{\gamma(1-u)}\frac{1}{\omega}-\frac{\tau}{\delta\left(\frac{\omega}{1-\delta}\right)^{\frac{\delta-1}{\delta}}}\right]$$

$$=-\frac{M}{\left[m(1+\beta+\gamma)\delta\left(\frac{\omega}{1-\delta}\right)^{\frac{\delta-1}{\delta}}-\tau\omega\gamma(1-u)\right]\gamma(1-u)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^{2}}, \qquad (15)$$

where $M=m(1+\beta+\gamma)\delta\left(\frac{\omega}{1-\delta}\right)^{\frac{\delta-1}{\delta}}\{\gamma(1-u)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^{2}+$

$\varepsilon(1-\delta)[\gamma(1-\tau)+\rho(1+\beta)]]\} - \{\omega(1-u)^2[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2[\gamma^2 + \rho(1+\beta)\gamma]\}$.

Proposition 1.

For the wages endogenously set by trade unions, when the public pension tax rate increases, the fertility rate will be affected as follows:

(i) *If the social security tax rates satisfy* $\tau\varepsilon > (1-\tau-\rho-\varepsilon)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]$, (*i.e.,* $A > B_\tau$), *fertility will decrease when the wage level set by the trade unions is below* B_τ*;*

(ii) *If the social security tax rates satisfy* $\tau\varepsilon < (1-\tau-\rho-\varepsilon)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]$, (*i.e.,,*$A < B_\tau$), *fertility will increase when the wage level is between A and* B_τ*, and decrease when the wage level is below A.*

Here, $A = \left(\frac{m(1+\beta+\gamma)\delta\{\gamma(1-u)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2+\varepsilon(1-\delta)[\gamma(1-\tau)+\rho(1+\beta)]\}}{\gamma[\gamma+\rho(1+\beta)](1-u)^2[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2}\right)^{\delta}(1-\delta)^{1-\delta}, B_\tau = \left[\frac{m\delta(1+\beta+\gamma)}{\gamma\tau(1-u)}\right]^{\delta}(1-\delta)^{1-\delta}$.

Proof. See Appendix B-3.

To substantiate the above proposition, I rewrite Equation (6) as follows:

$$n_t = \frac{\omega_t \overbrace{(1-u_t)}^{\textcircled{3}}}{1+\beta+\gamma} \cdot \frac{\gamma}{m-\varphi_t}\left[\left(1 \overbrace{-\tau}^{\textcircled{1}} - \rho\right) + \frac{\overbrace{\tau n_{t-1}}^{\textcircled{2}}}{1+r_{t+1}}\right]. \tag{16}$$

From Equation (13), the wage upper limit B_τ is set to satisfy a nonnegative capital accumulation. If the wage is set at a higher level but lower than B_τ, the present discounted value of the pension is at a higher level, which will have a positive effect on fertility: the higher the wage, the greater the positive effect. This positive effect can be called the “*income effect*” (part ② in Equation [16]). On the other hand, the pension is also a social security tax, which decreases society’s average income level and thus negatively affects fertility. This negative effect can be called the “*tax effect*” (part ① in Equation [16]). When the income effect is greater than the tax effect $\left(\frac{\tau n_{t-1}}{1+r_{t+1}} > \tau\right)$, the synthesized effect on fertility is positive, and fertility will increase; when the tax effect is stronger $\left(\frac{\tau n_{t-1}}{1+r_{t+1}} < \tau\right)$, the effect on fertility is negative, and fertility will decrease; when the two effects are equal, fertility will be the minimum value of its neighborhood. Moreover, according to Proposition 1, when the public pension tax rate increases, the unemployment rate will also increase, reducing fertility (section 4.1). This negative effect can be called the “*unemployment effect*” (part ③ in Equation [16]), a kind of indirect effect of pension on fertility. Considering the three effects in Equation (16), it can be inferred that, when the synthesized effect of parts ① and ② is positive and large enough, the negative effect from part ③ will be covered, and the final effect on fertility will be positive; otherwise, when the positive synthesized effect is weaker than the effect of part ③ or if the synthesized effect is negative, the final effect will be negative.

Discussion.

Proposition 2 can be understood as follows. **(1)** To ensure a nonnegative capital accumulation, Equation (13) expresses an upper limit of the wage level of B_τ. **(2)** When the government sets the social security tax rates to satisfy $M > 0$ or $M < 0$, the critical value of the wage level is A. **(3)** The social security tax rate will decide which wage level (B_τ or A) is lower: if B_τ is lower, the effect on fertility is negative

(**Proposition. 1 (i)**); if A is lower, the effect can be divided into two parts: **a)** when the wage is set below A (i.e., when the positive effect is smaller than the negative effects), the fertility rate will decrease as the pension tax rate increases; **b)** when the wage is set between A and B_τ (i.e., when the positive effect is bigger than the negative effects), fertility will increase as the pension tax rate increases (**Proposition. 1 (ii)**).[36] Therefore, the wage levels set by the trade unions will affect how public pensions impact fertility, and pensions may have a positive effect on fertility in some cases.

This conclusion differs from that of Wang (2015) because of their different assumptions. Wang (2015) assumed that the minimum wage is constant. First, the capital–labor ratio is constant. Therefore, the capital per capita and unemployment rate are linearly negatively related. When the pension tax increases, the capital per-capita decreases. Hence, the unemployment rate increases while fertility decreases (indirect effect). Second, the interest rate is also constant. When pension tax level becomes higher, its income effect also becomes larger. Therefore, when pension tax is at a higher level, fertility tends to increase as a result. Hence, the two points mentioned indicate that pension's level affects the relative size of pension's income effect, tax effect and indirect effect, and thus fertility in the model of Wang (2015).

In this study, however, the wage is endogenously set by trade unions. On one

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[36] Wages set between 0 to B_τ are discrete to the fertility-pension relationship. Thus, when the wage is set at A, it is ambiguous whether the fertility rate is the lowest in its neighborhood.

side, from Proposition 1, the social security tax rate will decide the equilibrium unemployment rate; thus, fertility rate is negatively affected (indirect effect).[37] On the other side, the endogenous wage level affects the interest rate of the next period $(1 + r_{t+1})$ and thus the present discounted value of old-age pension benefits. Therefore, the wage level affects the size of the income effect, together with the tax effect and the unemployment effect, the pension's final effect on fertility is then decided as a result.

From the analysis of Proposition 1, it is inferred that, if the trade unions set higher wage levels, old-age pensions will increase, and fertility will likely also increase. Therefore, when individuals are guaranteed the old-age benefits through the public pension system, they would have a stronger desire to raise more children, improving fertility rates; otherwise, the desire for children will be suppressed, and people will have to save money for themselves, reducing fertility (Wang, 2015). Therefore, governments seeking to decrease unemployment and increase fertility rates should reduce pension tax rates and wages.

4.4.2 Child allowances' effect on fertility

Differentiating Equation (13) with the child allowances tax rate, the effect of child allowances on fertility is expressed as

$$\frac{\partial f(\omega)}{\partial \rho}\left[\frac{m(1+\beta+\gamma)}{\gamma(1-u)}\frac{1}{\omega}-\frac{\tau}{\delta\left(\frac{\omega}{1-\delta}\right)^{\frac{\delta-1}{\delta}}}\right]$$

[37] This analysis is based on Equation (14).

$$= \frac{N}{\left[m(1+\beta+\gamma)\delta\left(\frac{\omega}{1-\delta}\right)^{\frac{\delta-1}{\delta}} - \omega(1-u)\gamma\tau\right]\gamma(1-u)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2}, \tag{17}$$

where $N = m(1+\beta+\gamma)\delta\left(\frac{\omega}{1-\delta}\right)^{\frac{\delta-1}{\delta}}X - \omega\gamma\tau(1+\beta)(1-u)^2[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2$, $X = (1+\beta)(1-u)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2 - \varepsilon(1-\delta)[\gamma(1-\tau)+\rho(1+\beta)]$.

Proposition 2.

For the wages endogenously set by trade unions, when the child allowances tax rate increases, the fertility rate is affected as follows:

(i) *If the social security tax rates satisfy* $(1+\beta)(1-\tau-\rho-\varepsilon)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)] < \varepsilon[\gamma(1-\tau)+\rho(1+\beta)]$, *fertility will decrease when the wage level set by the trade union is below* B_ρ*;*

(ii) *If the social security tax rates satisfy* $(1+\beta)(1-\tau-\rho-\varepsilon)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)] > \varepsilon[\gamma(1-\tau)+\rho(1+\beta)]$, *fertility will increase when the wage level is set below* C*, and decrease when the wage level is between* C *and* B_ρ*.*

Here, $C = \left(\frac{m(1+\beta+\gamma)\delta}{\tau\gamma(1-u)} - \frac{m(1+\beta+\gamma)\delta\varepsilon(1-\delta)[\gamma(1-\tau)+\rho(1+\beta)]}{\gamma\tau(1-u)^2(1+\beta)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2}\right)^\delta (1-\delta)^{1-\delta}$, $B_\rho = \left[\frac{m(1+\beta+\gamma)\delta}{\tau\gamma(1-u)}\right]^\delta (1-\delta)^{1-\delta}$.

Proof. See Appendix B-4.

To substantiate this proposition, Equation (6) is rewritten as

$$n_t = \frac{\overbrace{-\rho\omega_t(1-u_t)\gamma n_t}^{①} + n_t\gamma\omega_t\overbrace{(1-u_t)\left[(1-\tau)+\frac{\tau n_{t-1}}{1+r_{t+1}}\right]}^{③}}{\underbrace{-\rho\omega_t(1-u_t)(1+\beta+\gamma)}_{②} + mn_t(1+\beta+\gamma)}. \tag{18}$$

The child allowances are a social security tax levied on workers and thus reduces fertility. This negative effect is called the "*tax effect*" (part ② in Equation [18]). It is also a benefit provided for raising children and thus increases fertility. This positive effect is denoted as the "*income effect*" (part ① in Equation [18]). When the tax rate increases, the changes in ① is less than that in ②, which means that the *income effect* is stronger than the *tax effect*.

Proof.

For $n_t \in (0,1)$, | ② |-| ① |= $\rho\omega_t(1-u_t)(1+\beta+\gamma)-\rho\omega_t(1-u_t)\gamma n_t = \rho\omega_t(1-u_t)[1+\beta+\gamma-\gamma n_t] > 0$. Moreover, if there is no unemployment ($u = 0$), Equation (13) is written as

$$f(\omega)\left[\frac{m(1+\beta+\gamma)}{\gamma}\frac{1}{\omega}-\frac{\tau}{\delta\left(\frac{\omega}{1-\delta}\right)^{\frac{\delta-1}{\delta}}}\right]=\frac{\gamma(1-\tau)+\rho(1+\beta)}{\gamma}$$

Then, $\frac{\partial f(\omega)}{\partial \rho}|_{u=0} = \frac{1+\beta}{\gamma} > 0$. Thus, the effect of the child allowances alone on fertility is positive, meaning that the income effect is greater than the tax effect, which is why this subsidy is always implemented to improve fertility. (*Q.E.D*)

Moreover, when the child allowances tax rate increases, the unemployment rate also increases, meaning that the tax burden worsens unemployment, and fertility decreases. This negative effect is called the "*unemployment effect*", a kind of indirect effect of child allowances on fertility. Equation (18) features two kinds of *unemployment effect*: the first is included in parts ① and ② and the other in part ③. As parts ① and ② both contain the same structure in the *unemployment effect*, the synthesized effect formed by the two is not affected by changes in unemployment.

For part ③, however, on one hand, when ρ increases, alterations in the unemployment rate negatively affect fertility; on the other hand, the effect of part ③ also depends on the discounted value of the pension, which can be called the "*dependent effect*" and can be seen as the coefficient of the *unemployment effect*: if wages are set higher by the trade unions ($\in \left(C, B_\rho\right)$), the present discounted value of the pension becomes larger, and the negative part ③ unemployment effect is increased by the higher value of pension benefits, causing the overall effect on fertility to be negative; if wages are set lower ($\in (0, C)$), the negative effect from part ③ will be less, and the final effect on fertility could be positive. Thus, the *unemployment effect* has two parts: one concerns the child allowances, and the other concerns public pensions. As the income effect covers the tax effect of the child allowances, the first part is positive; the synthesized effect of the old-age pension benefit effect (the *dependent effect*) and the unemployment effect, which are both related to the trade union's wage-setting policy, determines the strength of the second part.

Discussion.

Proposition 3 can be understood as follows. **(1)** To ensure a nonnegative capital accumulation, from Equation (13), the upper limit is B_ρ. **(2)** When the government sets social security tax rates to satisfy $X < 0$, the child allowances' effect on fertility must be negative because the numerator is negative ($N < 0$) (**Proposition. 2 (i)**). **(3)** When the government sets social security tax rates to satisfy $N > 0$ or $N < 0$, the critical value is C, which must be lower than B_ρ. Thus, the effect can be divided into two parts: **a)** when wages are set lower than C (when the old-age benefit is lower and the negative effect of unemployment is not great), the final effect on fertility could be positive; **b)** when wages are set between C and B_ρ (when the old-age benefit is higher and the negative effect of unemployment is greater), the final effect on fertility could be negative (**Proposition. 2 (ii)**).[38] Therefore, the effect of child allowances on fertility is related to the wage setting policy of the trade unions.

This conclusion differs from those of Fanti and Gori (2007, 2010) and Wang (2015) because the previous studies assumed a constant minimum wage; in this study, the wage is endogenously set by monopolistic trade unions.

Fanti and Gori (2007) and Wang (2015) state that the assumption of a constant minimum wage ensures the linear relationship between unemployment and capital per capita and thus the negative effects of unemployment on fertility—the indirect effect. On the other hand, the child allowances contains the *tax effect* as a kind of social security tax and the *income effect* as a kind of income—the direct effects. Both the indirect and direct effects are produced by the child allowances alone and can thus be considered independent effects, which are negative. In the two-period OLG model of Wang (2015), who added a pension to the social security system of Fanti and Gori (2007), the child allowances expresses an intra-generational relationship and the public pension an inter-generational relationship; intra-generational alterations cause

[38] Wages set between 0 and B_ρ are discrete to the pension–fertility relationship. Thus, when wages are set at C, it is ambiguous whether the fertility rate is highest in its neighborhood.

inter-generational changes, and the effects of the child allowances are thus linked with those of the pension. This effect was considered dependent effect. According to Wang (2015), when pensions are high, the effect of the child allowances on fertility is positive; when pensions are low or nonexistent, the effect of the child allowances is negative.

In this chapter, as analyzed in Section 4.2.1, the assumption that wages are set by trade unions leads to the following two points. First, the social security tax rate will decide the equilibrium unemployment; thus, changes in the child allowances tax rate affect fertility through unemployment. Second, the inter-generational pension affects the intra-generational child allowances according to how much the size of the pension's present discounted value increases the negative effect caused by changes in the unemployment rate, inspired by variations in the child allowances tax rate. Therefore, the synthesized effect produced by the dependent and unemployment effects offsets the direct effects (tax and income effects), and determines the final effect on fertility.

The analysis of Proposition 3 shows that the equilibrium unemployment rate is decided when wages are set by trade unions aiming to maximize their members' lifetime incomes. When the child allowances tax rate increases, the unemployment rate also increases, meaning that child allowances taxpayers decrease while tax receivers increase. Moreover, the child allowances positively affects fertility (as the tax effect is smaller than the income effect), but both are affected equally by changes in unemployment. Finally, wages that are set higher ($\in (C, B_\rho)$) cause greater losses of future old-age pension income, increasing the negative effect of unemployment on fertility; thus, child allowances increases suppress the desire to have more children. Lower wages ($\in (0, C)$) reduce the losses of old-age pension income, decreasing the negative effect of unemployment on fertility; thus, child allowances increases may heighten the desire to have more children. Therefore, governments seeking to reduce unemployment and improve fertility could reduce the child allowances tax rate and increase wages.

4.4.3 Unemployment insurance's effect on fertility

Differentiating Equation (13) with the unemployment insurance tax rate, the effect of unemployment benefits on fertility is expressed as

$$\frac{\partial f(\omega)}{\partial \varepsilon} = -\frac{m(1+\beta+\gamma)(1-\delta)(1-\tau-\rho)\omega[\gamma(1-\tau)+\rho(1+\beta)]\left[\delta\left(\frac{\omega}{1-\delta}\right)^{\frac{\delta-1}{\delta}}\right]^2}{\left[m(1+\beta+\gamma)\delta\left(\frac{\omega}{1-\delta}\right)^{\frac{\delta-1}{\delta}}-\tau\gamma\omega(1-u)\right]^2[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2}. \quad (19)$$

Unemployment insurance clearly has a negative effect on the fertility rate.

As unemployment insurance is a kind of intra-generational transfer from workers to the unemployed, the insurance level has no effect on the social income level or capital accumulation. It affects fertility via alterations in unemployment. From Equation (10*), when the unemployment benefit tax rate increases, the unemployment rate increases. From Equation (6) and the analysis in 4.1, when the equilibrium unemployment rate increases, fertility decreases. Thus, when the unemployment insurance tax rate increases, fertility decreases.

This result occurs because increases in the unemployment insurance tax rate increases both the workers' tax burden and the revenue of the unemployed; the workers' fertility then decreases, while that of the unemployed increases. Considering the child rearing cost, the average fertility level of the wealthier workers will be higher than that of the poorer unemployed; therefore, the overall fertility rate will decrease with changes in unemployment. Thus, governments seeking to reduce unemployment and increase fertility could also lower the unemployment insurance tax rate.

4.5 Simulation

The parameters of simulation section are set as Table 4.1 shows.

Table 4.1 Parameters set in simulations

	τ	ρ	ε	A	B	C	ω_t
Figure 4.2	0.35~0.49	0.2	0.25		1.18~2.42		0.8,1,1.1
Figure 4.3	0.2~0.34	0.15	0.05	0.55~0.57	1.01~0.86		0.7,0.6,0.5,0.4
Figure 4.4	0.35	0.1~0.24	0.40		1.37~3.13		0.8,1,1.1
Figure 4.5	0.2	0.1~0.24	0.1		1.04~1.07	1.01~0.98	0.8,0.97,1.02,1.03
Figure 4.6	0.2~0.34	0.15	0.05	0.55~0.57	1.01~0.86		0.4~0.8
Figure 4.7	0.2	0.1~0.24	0.1		1.04~1.07	1.01~0.98	0.7~1.04
Figure 4.8	0.2	0.1	0.1~0.24		1.04~1.17		0.7~1.04

4.5.1 Test for pension's effect on fertility

To substantiate Proposition 1 and its intuition, the basic parameters of this model are set as follows: $\{\delta, \beta, \gamma, m\}$= {0.33, 0.20, 0.30, 0.25}. To satisfy **Proposition 1 (i),** the policy parameters are set as follows (see Figure 4.2): $\{\tau = 0.35\sim0.49, \rho = 0.2, \varepsilon = 0.25\}$; to satisfy **Proposition 1 (ii),** the parameters are set as follows (see Figure 4.3): $\{\tau = 0.2\sim0.34,\ \rho = 0.15, \varepsilon = 0.05\}$.[39]

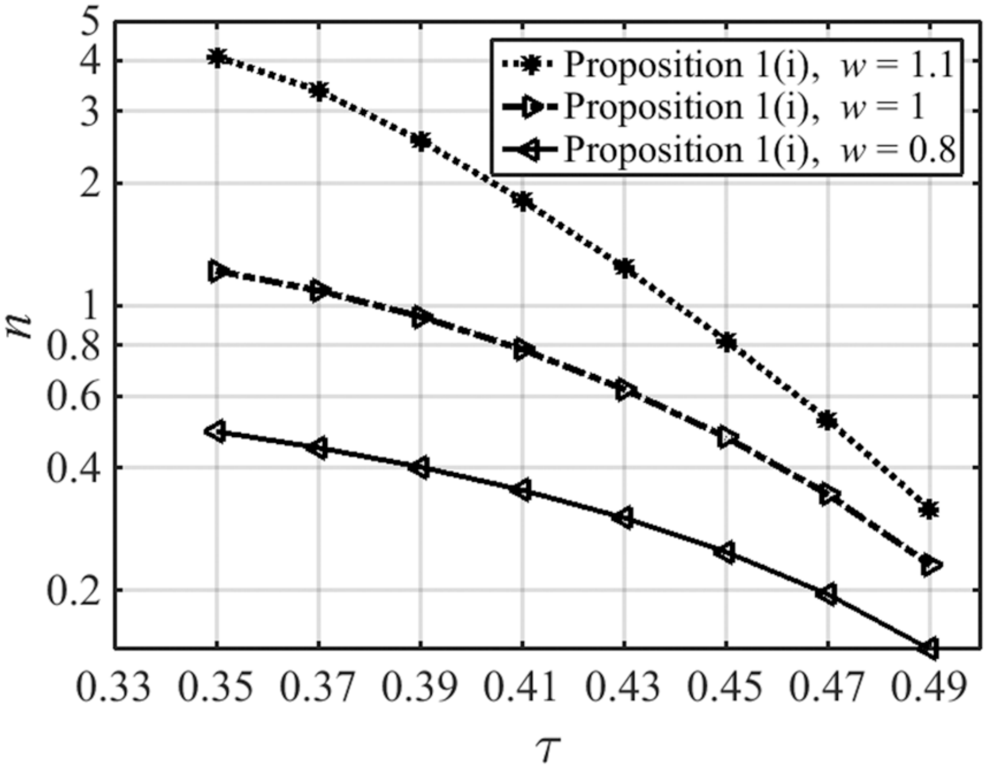

Figure 4.2. Public pension's effect on fertility (1)

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[39] According to 'General Welfare and Labor, No. 22-International Comparison of National Burden Ratio (http://www.mhlw.go.jp/english/wp/wp-hw4/dl/general_welfare_and_labour/P24.pdf), as I do not consider income or consumption taxes, I set the total security tax level at around 0.4. The parameter set of Proposition 3 **(i)** also considers the condition that $\tau\varepsilon > (1-\tau-\rho-\varepsilon)[\varepsilon + (1-\delta)(1-\tau-\rho-\varepsilon)]$, which seems difficult to satisfy in reality.

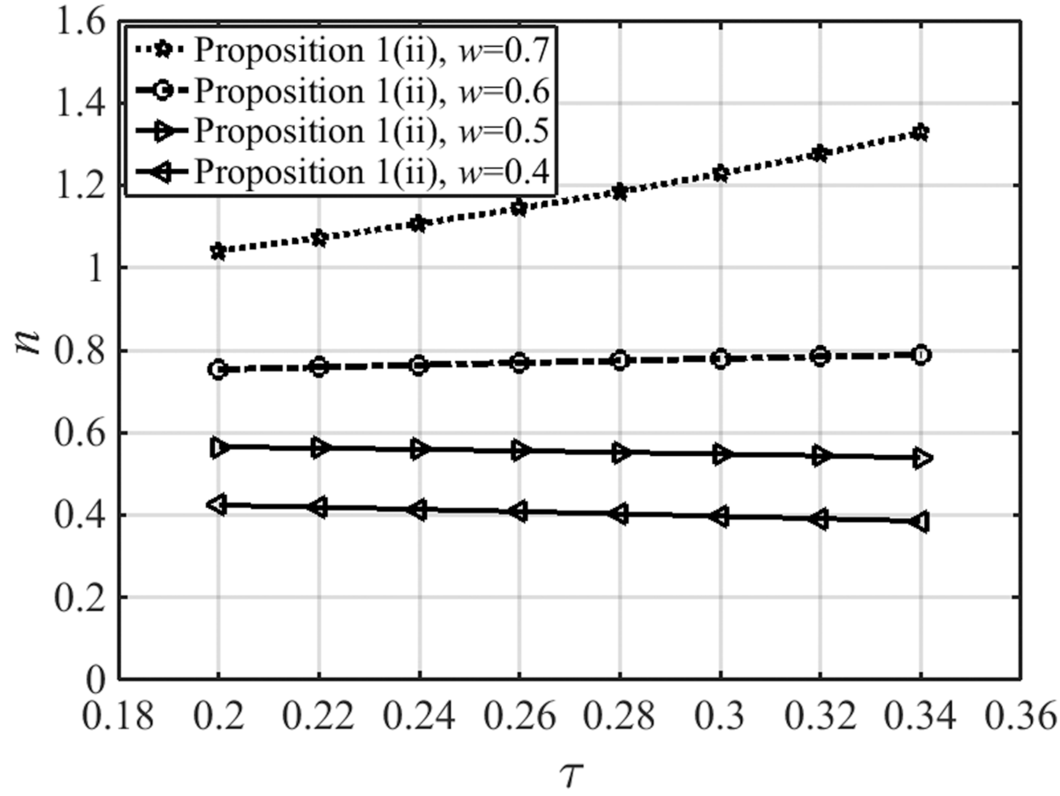

Figure 4.3. Public pension's effect on fertility (2)

The two figures above show that the simulation result accords with Proposition 1.

4.5.2 Test for the child allowances' effect on fertility

To substantiate Proposition 2, the basic parameters of this model are set as in section 5.1. To satisfy **Proposition 2 (i),** the policy parameters are set as follows (see Figure 4.4): $\{\tau = 0.35, \rho = 0.1{\sim}0.24, \varepsilon = 0.4\}$; to satisfy **Proposition 2 (ii),** the parameters are set as follows (see Figure 4.5): $\{\tau = 0.2,\ \rho = 0.1{\sim}0.24, \varepsilon = 0.1\}$.

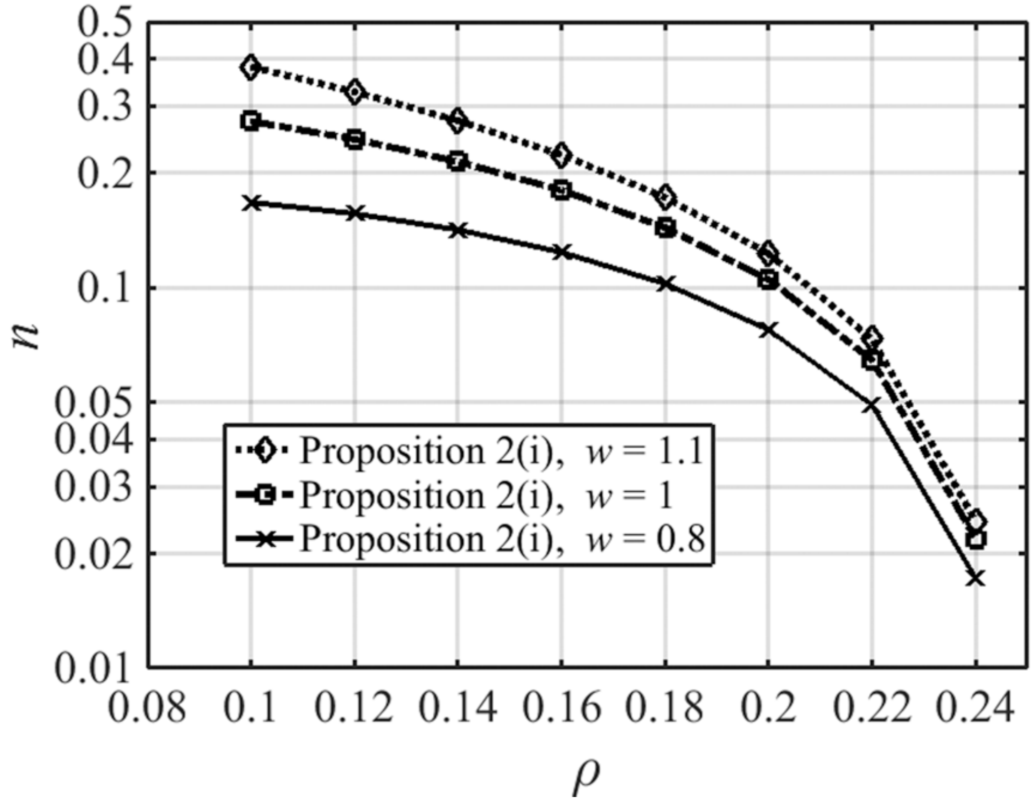

Figure 4.4. Child allowances' effect on fertility (1)

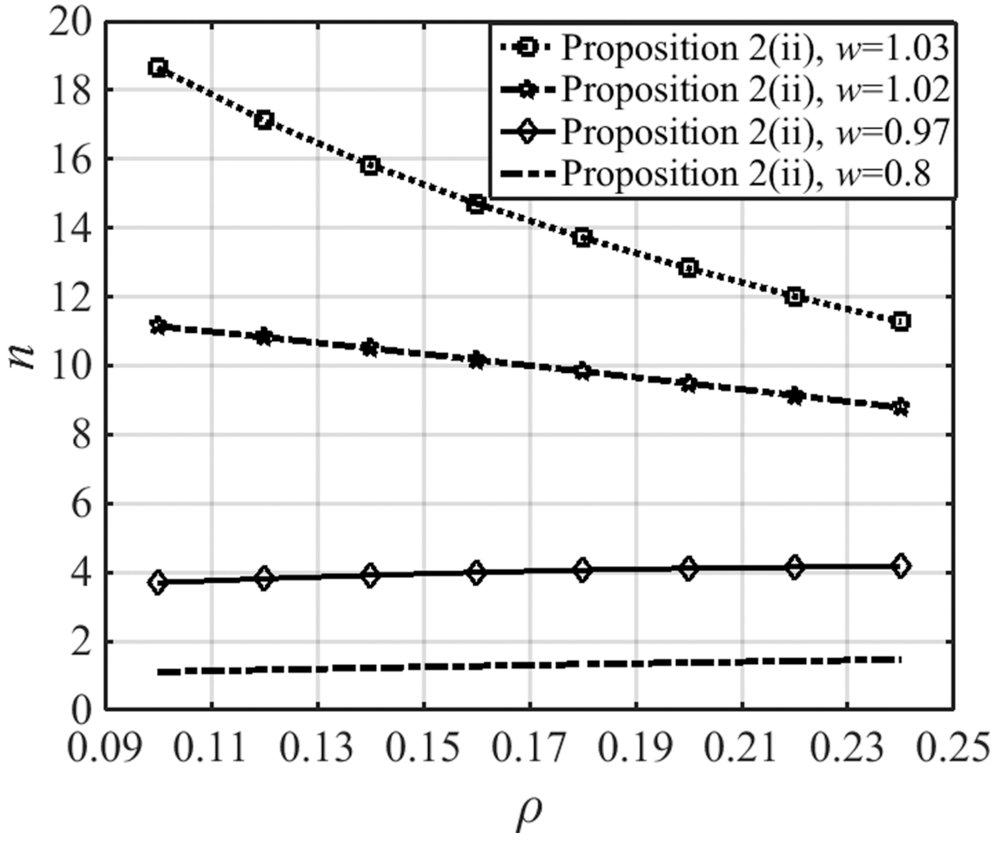

Figure 4.5. Child allowances' effect on fertility (2)

The two figures above show that the simulation result accords with Proposition 2.

4.5.3 Welfare analysis

Since firms aim at profit maximization, social welfare is sum of the individuals' lifetime consumption and fertility level:

$$U\left(c_{1,t}, c_{2,t+1}, n_t\right) = ln\left(c_{1,t}\right) + \beta\, ln\left(c_{2,t+1}\right) + \gamma\, ln(n_t),$$

From Eq. (6), (7), and (9), we derive the welfare utility function as

$$\begin{aligned} U(c_1, c_2, n) = & \, ln\left\{\frac{\omega_t(1-u_t)}{1+\beta+\gamma}\left[(1-\tau-\rho)+\frac{\tau n_{t-1}}{1+r_{t+1}}\right]\right\} \\ & + \beta ln\left\{\frac{\beta(1+r_{t+1})\omega_t(1-u_t)}{1+\beta+\gamma}\left[(1-\tau-\rho)+\frac{\tau n_{t-1}}{1+r_{t+1}}\right]\right\} \\ & + \gamma\, ln\left\{\frac{\omega_t(1-u_t)}{1+\beta+\gamma} \cdot \frac{\gamma}{m-\varphi_t}\left[(1-\tau-\rho)+\frac{\tau n_{t-1}}{1+r_{t+1}}\right]\right\}. \end{aligned}$$

The effects of the social security system on social welfare are shown in Figures 6~8. The three figures above show that, the effect on social welfare of the tax rates of pensions and child allowances similarly with that on fertility. The effect can be summarized as the two propositions described. The reason is that in the utility function of both welfare and fertility, the deterministic and common part is $\omega_t(1-u_t)\left[(1-\tau-\rho)+\frac{\tau n_{t-1}}{1+r_{t+1}}\right]$, where the pension's effect is ambiguous.

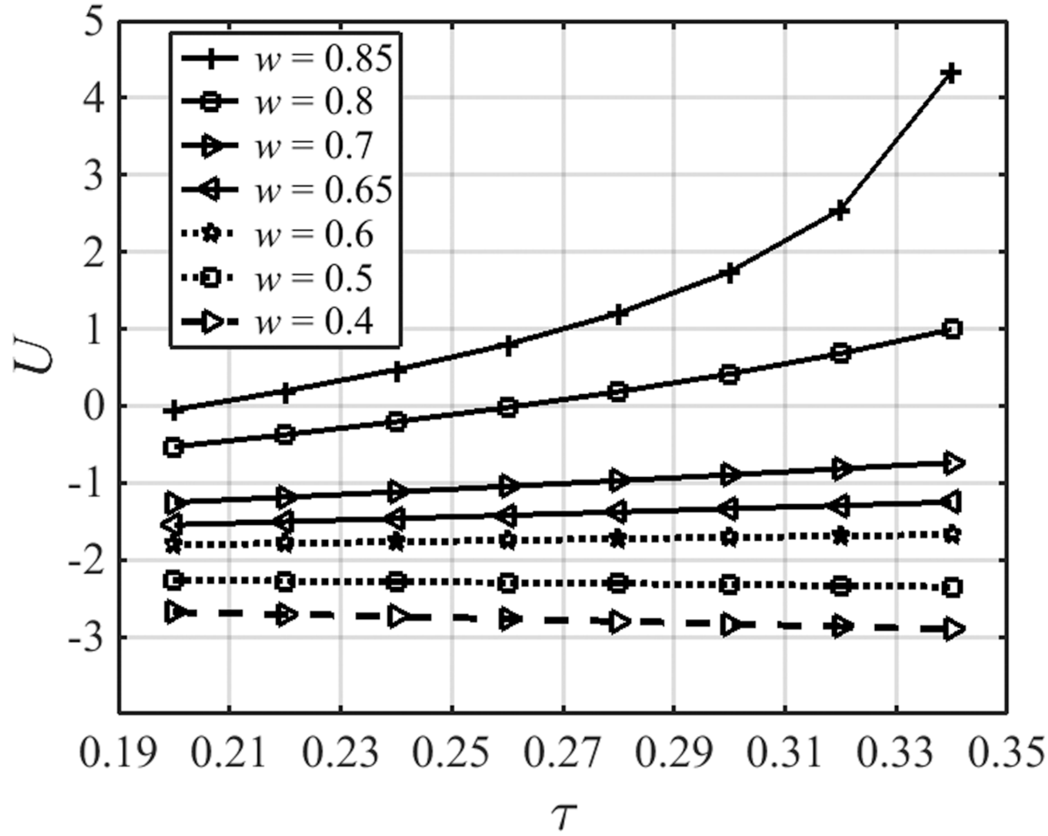

Figure 4.6. Public pension's effect on social welfare

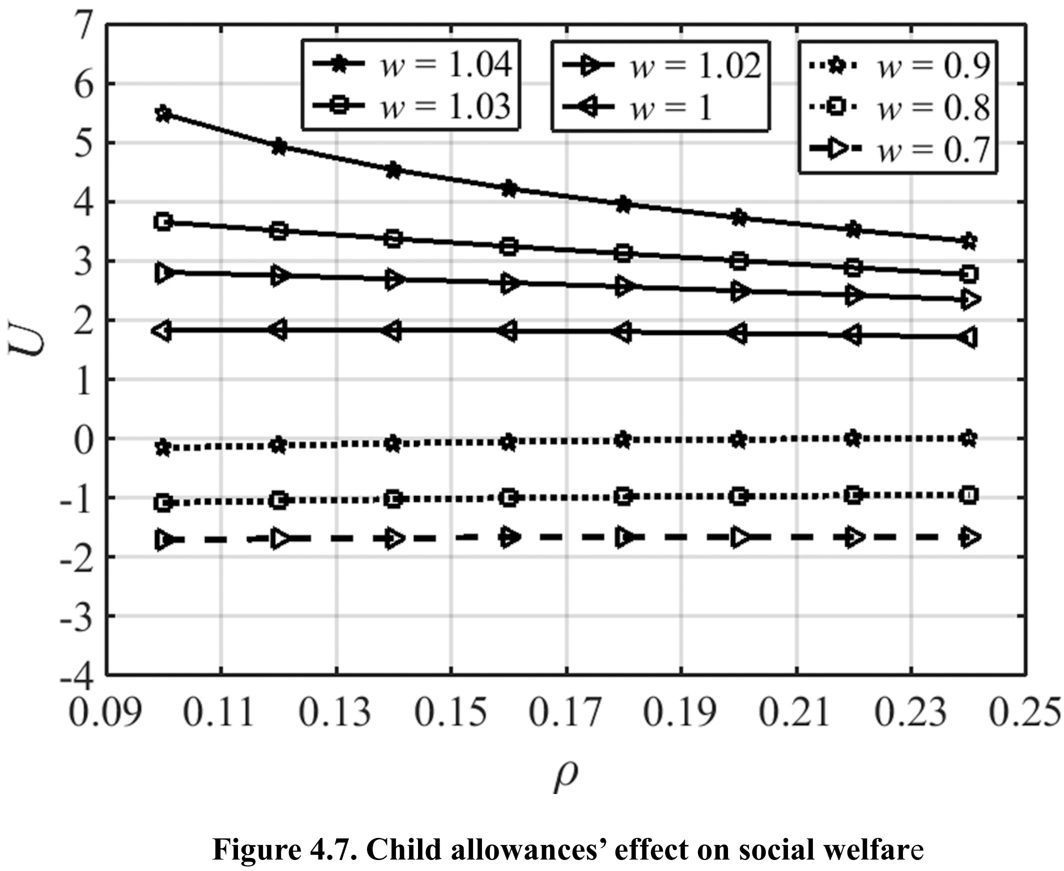

Figure 4.7. Child allowances' effect on social welfare

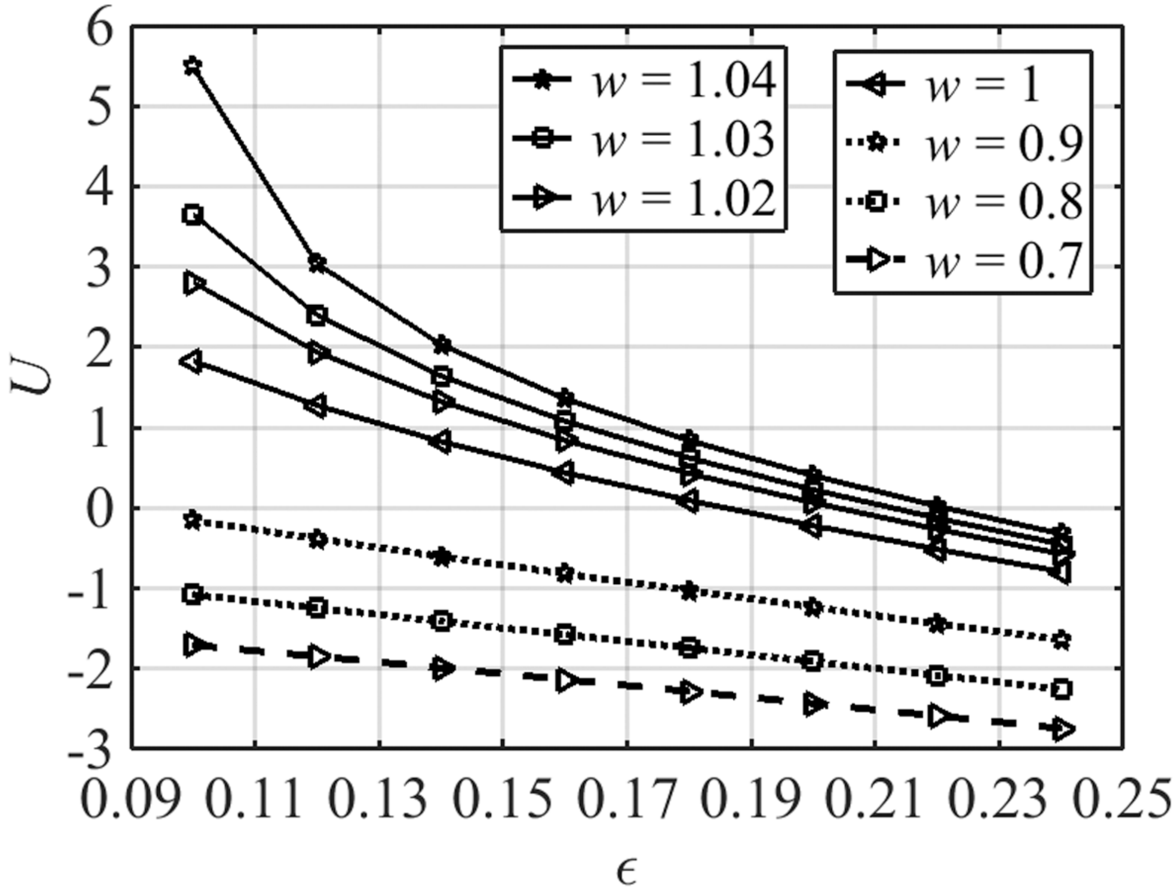

Figure 4.8. Unemployment insurance's effect on social welfare

Since unemployment deteriorates when social security tax rates increases. Therefore, taking into account the unemployment state, at least one kind of social security tax rates should be decreased: if unemployment insurance decreases and public pensions or the child allowances increases, the welfare and the fertility will become better when wages are set at (A, B_τ); the welfare and the fertility will become worse when wages are set at $(0, A)$ and (C, B_ρ); If unemployment insurance decreases and the child allowances or pension decreases, the welfare and the fertility will become better when wages are set at (C, B_ρ) or $(0, A)$; or the welfare and the fertility will become worse when wages are set at $(0, C)$.

4.6 Conclusion

This study examines how a social security system consisting of a public pension, child allowances, and unemployment insurance affects endogenous fertility and unemployment when wage is endogenously set by monopolistic trade unions in an OLG model. The trade unions set wages based on the government's social security policy, bargaining with firms to maximize their members' lifetime incomes.

If the pension tax rate increases, when wages are higher ($\in (A, B_\tau)$), the pensions received in old age increased, and fertility increases as a result. Thus, when the individuals are guaranteed old-age benefits through a public pension system, they will prefer to raise more children, improving fertility. Otherwise, when wage are lower ($\in (0, A)$), the old-age pension decreases, the desire for raising children is suppressed and people have to save money for themselves, decreasing fertility.

If the child allowances tax rate increases, it brings two kinds of effects. On one hand, the child allowances increases fertility because it brings more revenue (income effect) than the payment (tax effect) on fertility. On the other hand, unemployment increases. Higher wages ($\in (C, B_\rho)$) cause more losses of future old-age pension income, increasing the negative effect of unemployment on fertility and reducing people's desire for more children. Lower wages ($\in (0, C)$) cause smaller losses of old-age pension income, reducing the negative effect of unemployment on fertility. Therefore, the ultimate effect of increases in child allowances tax rates involves wage-setting policies.

To improve social welfare, it is preferable to increase public pensions or the child allowances tax rate and reduce the unemployment insurance tax rate when wages are lower ($\in (A, B_\tau)$), while it is preferable to reduce the child allowances or the unemployment insurance level when wages are higher ($\in (C, B_\rho)$).

This study therefore finds that, when pensions increase, fertility also increases if wages are set higher ($\in (A, B_\tau)$) but decreases if wages are set lower ($\in (0, A)$). When the child allowances increases, fertility also increases if wages are set lower ($\in (0, C)$) but decreases if wages are set higher ($\in (C, B_\rho)$). Moreover, the equilibrium unemployment resulting from the wage-setting policy is determined by the social security tax level. Therefore, to reduce unemployment and improve both fertility and social welfare, governments should reduce the sum of the social security tax rates and increase the tax rates for public pensions or child allowances and reduce the unemployment insurance tax rate when wages are set at the lower level ($\in (A, B_\tau)$); alternatively, they should reduce the child allowances or the unemployment insurance level and not employ a pension scheme when wages are set at the higher level ($\in (C, B_\rho)$).

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Chapter 5 Fertility, efficiency wages and social security in an OLG economy

5.1 Introduction

Many industrialized countries have faced a population-aging crisis due to declining fertility rates and increasing life expectancy. As expected, the decreasing labor force taxed to fund the PAYG social security system may bear the burden of an increasing population and living standard of beneficiaries. Numerous studies have discussed the effect of social security systems (SSS) on fertility in competitive labor markets. However, long-run unemployment has become a significant feature of contemporary society, affecting the income and thus the desire to raise children. Although the relationship between unemployment and growth has attracted the attention of many economists (Bräuninger, 2000; Corneo and Marquardt, 2000), the literature on population aging analysis in an economy with unemployment remains limited (Fanti and Gori, 2007; Wang, 2015).

Regarding long-term unemployment, it is often asked why the high unemployment in society cannot be self-corrective through falling wages. The efficiency wage model provides an explanation. First, reducing income reduces morale (gift exchange model by Akerlof, 1982) and phases out trained workers (labor turnover model by Salop, 1979), and the loss of productivity may exceed the savings made through lower wages. Second, the inability to monitor workers' efforts requires involuntary unemployment to be a penalty for misdemeanor (shirking model by Shapiro and Stiglitz, 1984). Third, firms raise wages to attract better applicants (adverse selection model by Weiss, 1980). As rearing children is income-related, it is expected that the effects of SSS on fertility should vary with the wage setting.[40]

This study analyzes the effects in an OLG model of a small open economy where firms set wages higher than the market-clearing level under the no-shirking condition, which states that painstaking workers have higher utilities than shirkers, causing shirkers to phase out of the labor force. SSS consists of public pensions and unemployment insurance. The results show that the effects of SSS on fertility depend on childrearing costs. Higher effort or labor efficiency leads to higher productivity and income, which makes it easier to balance the SSS tax payment and childrearing costs, facilitating parents to raise children.

5.2 Model

5.2.1 The individuals

Consider a two-period OLG in a small open economy. We assume that everyone decides whether to shirk or not at the beginning of their youth. Specifically, if an individual decides to be a non-shirker, he(she) will either stay employed until

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[40] Theoretical and practical evidence shows that, besides childrearing costs, parental time is an essential factor for childrearing. We ignore that here to focus on childrearing costs for simplicity.

retirement or become unemployed due to an exogenous shock(σ). If he(she) decides to be a shirker, then the probability of becoming unemployed is higher than that for non-shirkers, because when the shirking is detected(ρ), he(she) will be fired. Both the exogenous shock and detection by firms will occur once the decisions are made. Therefore, there are three possible states individuals may stay during their youth: work diligently, shirk, or become unemployed. The utility functions of the three states are assumed to be logarithmic, and comprise consumption during youth, old age, and number of children:

$$U_i = \alpha ln c_{1,t}^i + \beta ln c_{2,t+1}^i + \gamma ln n_t^i,$$

where $\alpha + \beta + \gamma = 1$ for convenience, and $i = a, b \ or \ c,$ denoting the three states respectively. The lifetime utilities are

$$\phi_N = (1-\sigma)U_a + \sigma U_c,$$

And

$$\phi_S = (1-\rho-\sigma)U_b + (\rho+\sigma)U_c,$$

where N and S represent no shirking and shirking respectively.

The utility maximization problem of non-shirkers is presented as follows:

$$max U_a = \alpha ln c_{1,t}^a + \beta ln c_{2,t+1}^a + \gamma ln n_t^a$$

$$s.t. \ c_{1,t}^a + s_t^a = \omega_t(1-\tau-\eta) - e - m n_t^a,$$

$$c_{2,t+1}^a = s_t^a R_{t+1} + \theta_t.$$

During youth, the employee receives a wage ω_t, and incurs a social security tax for public pension (θ_t) and unemployment insurance (b_t). The tax rates are τ and η, respectively. As assumed by Shapiro and Stiglitz (1984), e is the effort level of risk-neutral workers. The rest is used for childrearing costs (m), consumption during youth $(c_{1,t}^a)$ and savings (s_t^a). s_t^a and θ_t are used for consumption during the retirement period. Therefore, maximizing the utility of state a gives

$$c_{1,t}^a = \alpha\left[\omega_t(1-\tau-\eta) - e + \frac{\theta_t}{R_{t+1}}\right]$$
$$c_{2,t+1}^a = \beta R_{t+1}\left[\omega_t(1-\tau-\eta) - e + \frac{\theta_t}{R_{t+1}}\right]$$
$$s_t^a = \beta[\omega_t(1-\tau-\eta) - e] - (1-\beta)\frac{\theta_t}{R_{t+1}},$$
$$n_t^a = \frac{\gamma}{m}\left[\omega_t(1-\tau-\eta) - e + \frac{\theta_t}{R_{t+1}}\right]. \tag{1 *}$$

The utility of this state is

$$U_a = \alpha ln\alpha\left[\omega_t(1-\tau-\eta) - e + \frac{\theta_t}{R_{t+1}}\right] + \beta ln\beta R_{t+1}\left[\omega_t(1-\tau-\eta) - e + \frac{\theta_t}{R_{t+1}}\right] + \gamma ln\frac{\gamma}{m}\left[\omega_t(1-\tau-\eta) - e + \frac{\theta_t}{R_{t+1}}\right]. \tag{1}$$

The effort level of shirkers differentiates their budget constraint from non-shirkers. For simplification, we assume the effort is 0.

$$maxU_b = \alpha lnc_{1,t}^b + \beta lnc_{2,t+1}^b + \gamma lnn_t^b$$

$$s.t.\ c_{1,t}^b + s_t^b = \omega_t(1-\tau-\eta) - mn_t^b,$$

$$c_{2,t+1}^b = s_t^b R_{t+1} + \theta_t.$$

Solving this utility maximization problem gives

$$c_{1,t}^{b} = \alpha\left[\omega_t(1-\tau-\eta)+\frac{\theta_t}{R_{t+1}}\right],$$

$$c_{2,t+1}^{b} = \beta R_{t+1}\left[\omega_t(1-\tau-\eta)+\frac{\theta_t}{R_{t+1}}\right],$$

$$s_t^{b} = \beta\omega_t(1-\tau-\eta)-(1-\beta)\frac{\theta_t}{R_{t+1}}$$

$$n_t^{b} = \frac{\gamma}{m}\left[\omega_t(1-\tau-\eta)+\frac{\theta_t}{R_{t+1}}\right].$$

The utility of this state is

$$U_b = \alpha ln\alpha\left[\omega_t(1-\tau-\eta)+\frac{\theta_t}{R_{t+1}}\right]+\beta ln\beta R_{t+1}\left[\omega_t(1-\tau-\eta)+\frac{\theta_t}{R_{t+1}}\right] + \gamma ln\frac{\gamma}{m}\left[\omega_t(1-\tau-\eta) + \frac{\theta_t}{R_{t+1}}\right]. \quad (2)$$

For the unemployed, the living expenses during youth are funded by b_t, and the consumption during the retirement period is funded by s_t^a and θ_t^u. The pensions are contributed by the unemployed, with a lower tax rate than the employed, $\xi\tau$ ($0 < \xi < 1$), on a pay-as-you-go basis.[41]

$$maxU_c = \alpha lnc_{1,t}^{c}+\beta lnc_{2,t+1}^{c}+\gamma lnn_t^{c}$$

$$s.t.\ c_{1,t}^{c}+s_t^{c} = b_t(1-\xi\tau)-mn_t^{c},$$

$$c_{2,t+1}^{c} = s_t^{c}R_{t+1}+\theta_t^{u}.$$

Therefore,

[41] Unemployment compensation is taxable income and can be used to contribute to personal pension accounts. In this study we assume that the unemployed can contribute to the PAYG pension scheme.

$$c_{1,t}^{c} = \alpha\left[b_t(1-\xi\tau) + \frac{\theta_t^u}{R_{t+1}}\right],$$

$$c_{2,t+1}^{c} = \beta R_{t+1}\left[b_t(1-\xi\tau) + \frac{\theta_t^u}{R_{t+1}}\right]$$

$$s_t^c = \beta b_t(1-\xi\tau) - (1-\beta)\frac{\theta_t^u}{R_{t+1}}$$

$$n_t^c = \frac{\gamma}{m}\left[b_t(1-\xi\tau) + \frac{\theta_t^u}{R_{t+1}}\right]. \tag{3 *}$$

The utility of this state is

$$U_c = \alpha ln\alpha\left[b_t(1-\xi\tau) + \frac{\theta_t^u}{R_{t+1}}\right] + \beta ln\beta R_{t+1}\left[b_t(1-\xi\tau) + \frac{\theta_t^u}{R_{t+1}}\right] + \gamma ln\frac{\gamma}{m}\left[b_t(1-\xi\tau) + \frac{\theta_t^u}{R_{t+1}}\right]. \tag{3}$$

Therefore, the lifetime utilities are

$$\phi_N = \alpha ln\alpha\left[\omega_t(1-\tau-\eta) - e + \frac{\theta_t}{R_{t+1}}\right] + \beta ln\beta R_{t+1}\left[\omega_t(1-\tau-\eta) - e + \frac{\theta_t}{R_{t+1}}\right] + \gamma ln\frac{\gamma}{m}\left[\omega_t(1-\tau-\eta) - e + \frac{\theta_t}{R_{t+1}}\right] - \sigma ln\frac{\omega_t(1-\tau-\eta)-e+\frac{\theta_t}{R_{t+1}}}{b_t(1-\xi\tau)+\frac{\theta_t^u}{R_{t+1}}}, \tag{4}$$

$$\phi_S = \alpha ln\alpha\left[\omega_t(1-\tau-\eta) + \frac{\theta_t}{R_{t+1}}\right] + \beta ln\beta R_{t+1}\left[\omega_t(1-\tau-\eta) + \frac{\theta_t}{R_{t+1}}\right] + \gamma ln\frac{\gamma}{m}\left[\omega_t(1-\tau-\eta) + \frac{\theta_t}{R_{t+1}}\right] - (\rho+\sigma)ln\frac{\omega_t(1-\tau-\eta)+\frac{\theta_t}{R_{t+1}}}{b_t(1-\xi\tau)+\frac{\theta_t^u}{R_{t+1}}} \tag{5}$$

We assume that the non-shirkers have a higher utility than the shirkers because of the risks involved in being unemployed; this is referred to as the *no-shirking condition (NSC)*: $\phi_N \geq \phi_S$ (Shapiro and Stiglitz, 1984). Substituting Eq. (4) and (5) into this inequality,

$$\rho ln\frac{\omega_t(1-\tau-\eta)+\frac{\theta_t}{R_{t+1}}}{b_t(1-\xi\tau)+\frac{\theta_t^u}{R_{t+1}}} \geq (1-\sigma)ln\frac{\omega_t(1-\tau-\eta)+\frac{\theta_t}{R_{t+1}}}{\omega_t(1-\tau-\eta)-e+\frac{\theta_t}{R_{t+1}}} \tag{6}$$

This implies that no employee will shirk as far as this condition is satisfied. Individuals will either be employed or become unemployed. The unemployment rate is expressed as

$$u=\frac{N_s(\rho+\sigma)+N_N\sigma}{N_s+N_N}, \tag{7}$$

where N_N and N_s are the number of non-shirkers and shirkers of this economy, respectively. However, by *NSC*, $N_s=0, u=\sigma.$

5.2.2 The government

Incomes of the employed are taxed to finance the public pension and unemployment insurance on a pay-as-you-go basis. $N_{t+1}=N_t n_t.$ For the public pension of workers,

$$\tau\omega_t L_t=\theta_t L_{t-1}. \tag{8}$$

For the unemployment insurance and pension of the unemployed,

$$\eta\omega_t L_t=b_t(N_t-L_t), \tag{9}$$

$$\xi\tau b_t(N_t-L_t)=\theta_t^u(N_{t-1}-L_{t-1}), \tag{10}$$

where by the definition of unemployment,

$$\frac{N_t-L_t}{N_t}=u.$$

Therefore,

$$\theta_t=\tau\omega_t n_{t-1}, \tag{8 *}$$

$$b_t=\frac{\eta\omega_t(1-u)}{u}, \tag{9 *}$$

$$\theta_t^u = \xi\tau b_t n_{t-1} = \frac{\xi\tau n_{t-1}\eta\omega_t(1-u)}{u}. \qquad (10*)$$

Substituting the three equations into Eq. (6),

$$\omega_t \geq \left[\frac{e}{\left(\frac{M}{\frac{\eta(1-u)}{u}N}\right)^{\frac{\rho}{1-\sigma}} - 1} + e\right]\frac{1}{M} \equiv \widetilde{\omega_S}, \qquad (11)$$

where $\widetilde{\omega_S}$ is defined as no-shirking wage, and the subscript S represents labor supply, and $M = (1-\tau-\eta) + \frac{\tau n_{t-1}}{R}, N = 1 - \xi\tau + \frac{\xi\tau n_{t-1}}{R}$. When the wage is set over the no-shirking wage, nobody will shirk. As this equation reveals how the workers would supply their labor according to wages, this wage determines the wage curve of labor supply.

5.2.3 The production and the capital market

Aggregate output is determined by the Cobb-Douglas production function:

$$Y_t = AK_t^{\delta}(eL_t)^{1-\delta},$$

where A is a time-independent technology parameter, K_t denotes the amount of capital, and eL_t represents the efficient labor, in which e is the effort level of workers.

The profit maximization problem of firms gives

$$max\pi_t = Y_t - (1+r_t)K_t - \omega_t L_t.$$

Therefore,

$$R_t = 1 + r_t = \delta A e^{1-\delta} \left(\frac{k_t}{1-u}\right)^{\delta-1}, \tag{12}$$

$$\omega_t = (1-\delta) A e^{1-\delta} \left(\frac{k_t}{1-u}\right)^{\delta}, \tag{13}$$

where $k_t = \frac{K_t}{N_t}$. We consider a small open economy with perfect capital mobility both from domestic and foreign markets. Thus, the interest rate is

$$R = 1 + r = \delta A e^{1-\delta} \left(\frac{k}{1-u}\right)^{\delta-1}. \tag{12 *}$$

Eq. (13) becomes

$$\omega = e(1-\delta)\delta^{\frac{\delta}{1-\delta}} A^{\frac{1}{1-\delta}} R^{\frac{\delta}{\delta-1}} \equiv \widetilde{\omega_D}\,. \tag{13 *}$$

The profit-maximizing wage is defined as the wage of labor demand.

5.3 Equilibrium

5.3.1 Labor market equilibrium

Labor market equilibrium occurs when wage of labor demand $(\widetilde{\omega_D})$ intersects the wage of labor supply $(\widetilde{\omega_S})$:

$$eP = \frac{eM^{\frac{\rho+\sigma-1}{1-\sigma}}}{M^{\frac{\rho}{1-\sigma}} - \left[\frac{\eta(1-u)}{u} N\right]^{\frac{\rho}{1-\sigma}}}, \tag{14}$$

where $P = (1-\delta)\delta^{\frac{\delta}{1-\delta}} A^{\frac{1}{1-\delta}} R^{\frac{\delta}{\delta-1}}$.

5.3.2 Steady-state equilibrium of dynamic fertility

To understand how the aggregate fertility is affected by the policy parameters, we analyze the dynamics of fertility when the economy is in equilibrium. As above, $\widetilde{\omega_S} = eP$. The aggregate fertility is the sum of fertility of the employed and unemployed:

$$n_t = n_t^a(1-u) + n_t^c u.$$

Substituting Eq. (1*) and (3*) into above,

$$n_t = \frac{\gamma}{m}(1-u)\left[\widetilde{\omega_S}\,(1-\tau-\eta) - e + \frac{\tau\widetilde{\omega_S}n_{t-1}}{R_{t+1}}\right]$$
$$+\frac{\gamma}{m}\widetilde{\omega_S}\,(1-u)\eta\left[(1-\xi\tau) + \frac{\xi\tau n_{t-1}}{R_{t+1}}\right]. \quad (15)$$

Substituting the wage level of the equilibrium into above,

$$n_t = \frac{\gamma}{m}(1-u)eP\left[\,(1-\tau-\eta) + \frac{\tau n_{t-1}}{R}\right] + \frac{\gamma}{m}(1-u)eP\eta\left[(1-\xi\tau) + \frac{\xi\tau n_{t-1}}{R}\right]$$
$$-\frac{\gamma}{m}(1-u)e. \quad (16)$$

In the steady state,

$$n^*\left[m - \gamma(1-u)eP\frac{\tau}{R}(1+\xi\eta)\right]$$
$$= \gamma(1-u)eP[(1-\tau-\eta) + \eta(1-\xi\tau)] - \gamma(1-u)e$$

The right-hand side is the after-tax income of individuals, which is positive. On the left-hand side, the coefficient denotes the difference between childrearing costs and pension's the present discounted value. Hence, the steady-state fertility is

$$n^* = \frac{\gamma(1-\sigma)e[P-1-P\tau(1+\xi\eta)]}{m-\gamma(1-\sigma)eP\frac{\tau}{R}(1+\xi\eta)}\,, \quad (17)$$

where $m > \gamma(1-\sigma)eP\frac{\tau}{R}(1+\xi\eta) \equiv X$, ensuring a non-negative fertility.

5.4 Effects of social security system on fertility

Proposition.

When the childrearing costs are higher than Y, the increase in the pension (unemployment insurance) tax rate will decrease the fertility; when the costs are higher than X, but lower than Y, the increase in the pension (unemployment insurance) tax rate will increase the fertility.

Here, $X = \gamma(1-\sigma)eP\frac{\tau}{R}(1+\xi\eta)$, $Y = \frac{\gamma e(P-1)(1-\sigma)}{R}$.

Proof.

Differentiating steady-state fertility with respect to τ and η gives

$$\frac{\partial n^*}{\partial \tau} = \frac{-\gamma eP(1+\xi\eta)(1-\sigma)\left[m - \frac{\gamma e(P-1)(1-\sigma)}{R}\right]}{\left[m - \gamma(1-\sigma)eP\frac{\tau}{R}(1+\xi\eta)\right]^2}, \tag{18}$$

$$\frac{\partial n^*}{\partial \eta} = \frac{-\gamma eP\tau\xi(1-\sigma)\left[m - \frac{\gamma e(P-1)(1-\sigma)}{R}\right]}{\left[m - \gamma(1-\sigma)eP\frac{\tau}{R}(1+\xi\eta)\right]^2}. \tag{19}$$

From Eq. (17) we know that the lower limit of m is X. Since $Y - X = \frac{\gamma e(1-\sigma)}{R}[P - 1 - P\tau(1+\xi\eta)] > 0$, when $X < m < Y$, the derivative is positive; when $m > Y$, it is negative. (Q.E.D)

This indicates that the effect of social security on fertility depends on the childrearing costs. As unemployment insurance has a similar effect to pension, we use pension to illustrate. If the childrearing costs are not high, the families could balance them with an increasing pension burden; therefore, the fertility rate would increase.

However, if childrearing costs are too high, when the pension burden increases, the desire to have children will be suppressed. Moreover, the critical value involving effort implies that if individuals exert greater effort, childrearing costs will less likely reach the breakthrough point, which implies higher productivity efficiency, making the childrearing costs more likely to be affordable.[42] Hence, even when faced with the tax burden of pensions, individuals can balance pension payments and childrearing costs.

This implies placing emphasis on moderating the pressure of households with children and encouraging the efficiency of employee contributes to increasing fertility in the imperfect labor market.

5.5 Simulation

To illustrate the effects of SSS on fertility, simulation parameters are set as in Tables 5.1 and 5.2. Figures 5.1 and 5.2 show the results of the proposition.[43]

Table 5.1. Basic Parameters

α	β	γ	σ	ρ	ξ	δ	A	r	e
0.5	0.2	0.3	0.06	0.3	0.2	0.33	3	0.005	1

[42] $\frac{\partial n^*}{\partial e} = \frac{m\gamma(1-\sigma)[P-1-P\tau(1+\xi\eta)]}{\left[m-\gamma(1-\sigma)eP\frac{\tau}{R}(1+\xi\eta)\right]^2} > 0.$

[43] Unemployment rate is set at 0.06 according to World Bank data. The interest rate is based on that of US in 2015.

Table 5.2. Social Security Parameters

	τ	η	X	Y
Figure 5.1	0.2~0.27	0.1	0.11~0.15	0.28
Figure 5.2	0.3	0.08~0.22	0.17	0.27

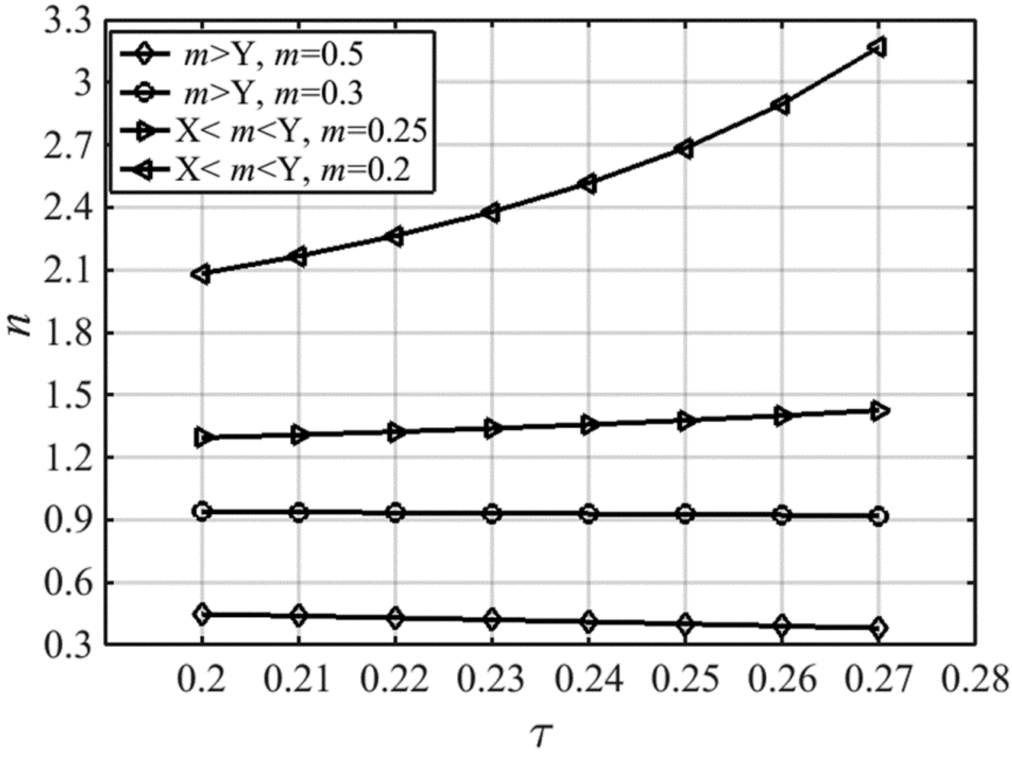

Figure 5.1. Pension's effect on fertility

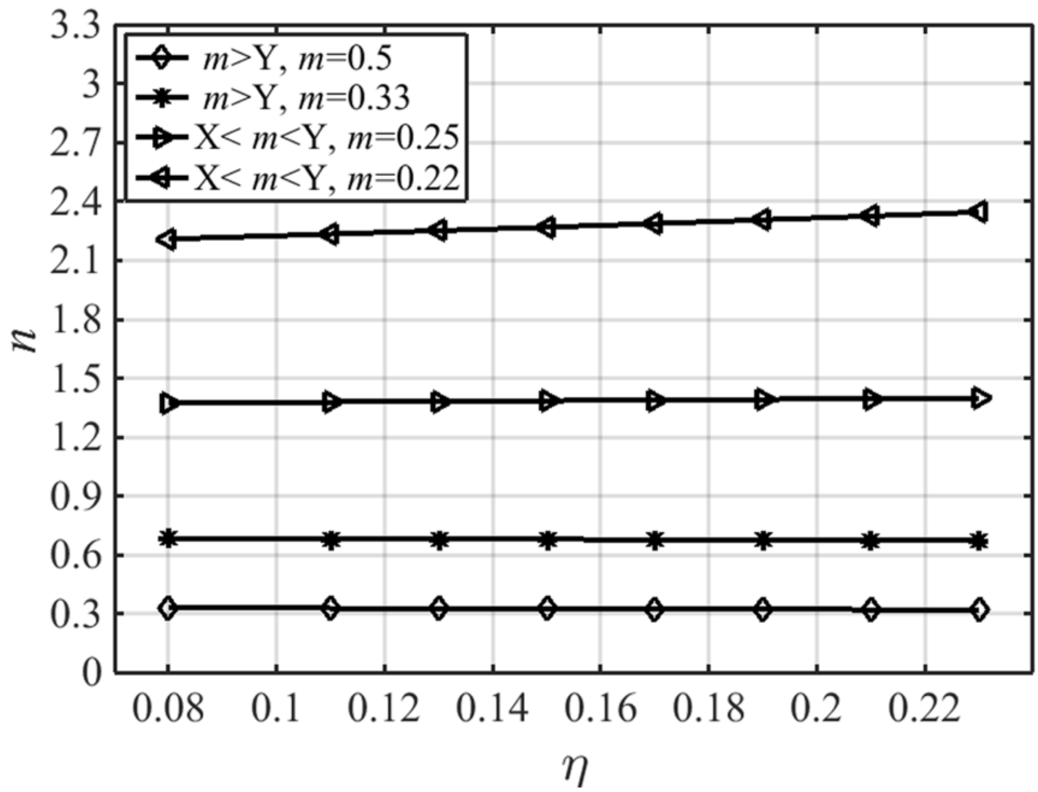

Figure 5.2. Unemployment insurance's effect on fertility

5.6 Conclusion

This study provides theoretical foundation for that offering financial support either from government or by encouraging parents to work with higher efficiency, is helpful to increase fertility, and thus mitigate population aging.

Chapter 6 Closing remarks

6.1 Conclusions

This dissertation explores how the social security system composed of pension, unemployment insurance, and child allowances affects fertility in the labor market with unemployment. Since unemployment rises due to several reasons such as minimum wages, union wage setting, or efficiency wages, I focus on the analysis of whether population aging can be affected by social security, and whether considering unemployment is critical to the final result.

The study "Fertility and unemployment in a social security system" gives the implications that for any given minimum wage value, a pension fosters the fertility rate on surpassing some high level. In order to increase the fertility rate and decrease the unemployment rate, a public pension or a combination of a public pension and child allowances may be considered. Moreover, pension's level has great impact on people's incentive to have children. How to set the pension's level can guarantee the old-age life should be taken seriously. So if the government would like to increase fertility, ensuring a higher pension for the old-aged or implementing the combination of higher pension and lower child allowances can be alternatives.

The study "Fertility, union wage setting and social security system" gives the implication that first, if the pension tax rate increases, when wages are higher, the pensions received in old age increased, and fertility increases as a result. Thus, when the individuals are guaranteed old-age benefits through a public pension system, they will prefer to raise more children, improving fertility, which accords with the conclusion with the first study. If the child allowances tax rate increases, it brings two kinds of effects. On one hand, the child allowances increases fertility because it reduces the childrearing cost for parents. On the other hand, unemployment increases. Higher wage results more losses of future old-age pension income, increasing the negative effect of unemployment on fertility and reducing people's desire for more children. Lower wage causes smaller losses of old-age pension income, reducing the negative effect of unemployment on fertility. Therefore, the ultimate effect of increases in child allowances tax rates involves wage-setting policies. Therefore, both social security and wage setting should be considered when seeking ways to improve fertility and reduce unemployment.

The study "Fertility, efficiency wages and social security in an OLG economy" reveals that when the childrearing costs are at higher level, both pensions and unemployment insurances will decrease the fertility; while when the costs are at lower level the effects are opposite. On the other hand, the higher effort or labor efficiency level leads to higher ability to raise children, improving fertility. This implies placing emphasis on moderating the pressure of households with children and encouraging the efficiency of employee contributes to increasing fertility in the imperfect labor market.

By using micro-founded general-equilibrium macro OLG models, this dissertation includes studies that discuss how to increase fertility by using the social security system in an imperfect labor market with unemployment, and suggest that it is necessary to consider in both macroscopic (social) and microscopic (household) contexts. For the former, both the social security reform and the wage-setting strategy are to be taken seriously. For the latter, all that reduce childrearing cost are to be encouraged.

6.2 Future research plan

For future research, I would like to focus on, but not be confined to, exploring population aging and the social security system, especially public pensions, taking into consideration the factors of labor market.

6.2.1 Research questions

Based on the research I have performed, some further research questions, although without too much deliberation, are to be investigated:

1) Does the reform of social security to deal with population aging affect the labor market performance?
2) If the shift in labor migration or immigration is taken into account, how does the social security reform affect population aging?
3) From a microeconomic perspective, are investing human capital of the young and promoting lifelong learning effective in eliminating the stagnation of the growth of the aging society?

6.2.2 Model Construction

All of my studies have employed the two-period overlapping generations model. The two-period model may show the simplest mechanism and simplify the calculation, but may ignore many details or cannot involve realistic features of the economy. For instance, the childhood and the youth cannot be differentiated, or retirement age cannot be considered. Stochastic-aging lifecycle model is the model with higher feasibility, and can be the a better alternative.

6.2.3 Unemployment causes

This research has taken unemployment as a circumstance, not considering one of the most popular topic related to unemployment, the job search friction. The future studies will involve this factor and analyze the mechanism of its effect on population aging.

6.2.4 Methodology

Although I have conducted simple simulations based on the experience data, the shortage of empirical verification limits the research to a merely theoretical margin, which cannot be accepted extensively. Therefore, the methodology involving simulation, calibration, and empirical study should be employed in the future.

APPENDIX A

Appendix A-1

The stable-equilibrium fertility rate is $n_2^* = \frac{\left[\frac{\beta(m-\varphi)(1-\delta)}{\gamma\varepsilon}+\frac{\theta\varepsilon}{\delta}\right]+\sqrt{\Delta}}{2\frac{m-\varphi+\gamma\varphi}{\gamma}}$, where

$\Delta = \left[\frac{\beta(m-\varphi)(1-\delta)}{\gamma\varepsilon}+\frac{\theta\varepsilon}{\delta}\right]^2 - 4\frac{m-\varphi+\gamma\varphi}{\gamma}\frac{\theta}{\delta}$ $(\partial\,\Delta/\partial\theta > 0)$. Differentiating n_2^* with respect to pension θ yields

$$\frac{\partial n_2^*}{\partial\theta} = \frac{\left[\sqrt{\Delta}\,\gamma\varepsilon\delta + \beta(m-\varphi)(1-\delta)\delta + \theta\gamma\varepsilon^2\right] - 2(m-\varphi+\gamma\varphi)\delta}{2(m-\varphi+\gamma\varphi)\sqrt{\Delta}\,\delta^2}. \tag{14}$$

Let $\Delta \geq 0$,

$$\left[\frac{\varepsilon}{\delta}\theta - \frac{1}{\gamma\varepsilon}\left(2A - B + 2\sqrt{A(A-B)}\right)\right]\left[\frac{\varepsilon}{\delta}\theta - \frac{1}{\gamma\varepsilon}\left(2A - B - 2\sqrt{A(A-B)}\right)\right] \geq 0.$$

Then,

$$\theta \geq \theta_1 = \frac{\delta}{\gamma\varepsilon^2}\left(2A - B + 2\sqrt{A(A-B)}\right)$$

$$or\ \theta \leq \theta_2 = \frac{\delta}{\gamma\varepsilon^2}\left(2A - B - 2\sqrt{A(A-B)}\right),$$

where $A = m - \varphi + \gamma\varphi, B = \beta(m-\varphi)(1-\delta)$.

When the numerator of Equation (14) $\left[\sqrt{\Delta}\,\gamma\varepsilon\delta + \beta(m-\varphi)(1-\delta)\delta + \theta\gamma\varepsilon^2\right] - 2(m-\varphi+\gamma\varphi)\delta \gtreqless 0$,

$$\theta \gtreqless \theta_3 = \frac{\delta}{\gamma\varepsilon^2}\cdot\frac{2A(A-B)}{A+B}.$$

As $\theta \geq \theta_1 > \theta_3$, $\partial n^*/\partial\theta > 0$;[44]

If $\theta \leq \theta_2 < \theta_3$ (when $B^2 - 3AB + 2(A+B)\sqrt{A(A-B)} > 0$), $\partial n^*/\partial\theta < 0$;

If $\theta < \theta_3 < \theta_2$ (when $B^2 - 3AB + 2(A+B)\sqrt{A(A-B)} < 0$), $\partial n^*/\partial\theta < 0$.

Equations (9) and (11) imply that when the fertility rate increases (decreases), the unemployment rate decreases (increases).[45]

Appendix A-2

Differentiating n_2^* with respect to the child allowance φ yields

$$\frac{\partial n_2^*}{\partial\varphi} = \frac{XY + 2Z}{2W^2}, \tag{16}$$

where $W = \frac{m-\varphi+\gamma\varphi}{\gamma}, X = \frac{\beta(m-\varphi)(1-\delta)}{\gamma\varepsilon} + \frac{\theta\varepsilon}{\delta} + \sqrt{\Delta}, Y = \frac{1-\gamma}{\gamma} - \frac{\beta(1-\delta)(m-\varphi+\gamma\varphi)}{\gamma^2\varepsilon\sqrt{\Delta}}$, $Z = \frac{\theta(1-\gamma)(m-\varphi+\gamma\varphi)}{\gamma^2\delta\sqrt{\Delta}}$.

When $XY + 2Z > 0, \theta > \theta_4$; when $XY + 2Z < 0$, $\theta < \theta_5$. θ_4 and θ_5 are the two solutions of $XY + 2Z = 0$ and $\theta_4 > \theta_5$.

If $\theta > \theta_4 > \theta_1$ or $\theta > \theta_1 > \theta_4$, $\partial n^*/\partial\varphi > 0$; If $\theta < \theta_2 < \theta_5$ or $\theta < \theta_5 < \theta_2$, $\partial n^*/\partial\varphi < 0$.

Equations (9) and (11) show that when the fertility rate increases (decreases), the

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44 $\theta_1 - \theta_3 = [B^2 - 3AB - 2(A+B)\sqrt{A(A-B)}]/(A+B) < 0$, because $B^2 - 3AB = \beta(m-\varphi)(1-\delta)[\beta(m-\varphi)(1-\delta) - 3\,(m-\varphi+\gamma\varphi)] < 0$.

45 $\frac{dk^*}{d\theta} = \underbrace{\frac{\partial k^*}{\partial n^*}}_{+}\underbrace{\frac{dn^*}{d\theta}}_{+or-} + \underbrace{\frac{\partial k^*}{\partial\theta}}_{-} = \frac{\theta\varepsilon}{\delta(n_2^*)^2}\frac{dn_2^*}{d\theta} - \frac{\varepsilon}{\delta n_2^*}$; $\frac{\partial u^*}{\partial\theta} = \underbrace{\frac{\partial u^*}{\partial k^*}}_{-}\underbrace{\frac{dk^*}{d\theta}}_{+or-} = -\left(\frac{1-\delta}{\square}\right)^{\frac{1}{\delta}}\frac{dk^*}{d\theta}$.

unemployment rate decreases (increases).[46]

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[46] $\frac{dk^*}{d\varphi} = \underbrace{\frac{\partial k^*}{\partial \varphi}}_{-} + \underbrace{\frac{\partial k^*}{\partial n^*}}_{+} \underbrace{\frac{dn^*}{d\varphi}}_{+or-} = -\frac{\beta}{\gamma} + \frac{\theta\varepsilon}{\delta(n^*)^2}\frac{dn^*}{d\varphi}$; $\frac{\partial u^*}{\partial \varphi} = \underbrace{\frac{\partial u^*}{\partial k^*}}_{-} \underbrace{\frac{dk^*}{d\varphi}}_{+or-} = -\left(\frac{(1-\delta)}{\omega}\right)^{\frac{1}{\delta}} \frac{dk^*}{d\varphi}$.

APPENDIX B

Appendix B-1

$$V_t = L_t\left\{\omega_t(1-\tau-\rho-\varepsilon)+\frac{\theta_t}{1+r_{t+1}}\right\}+(N_t-L_t)\left\{\sigma_t+\frac{\theta_t}{1+r_{t+1}}\right\}$$

From equation (1), the labor demand can be derived as

$$L_t = K_t\left(\frac{1-\delta}{\omega_t}\right)^{\frac{1}{\delta}}.$$

Substituting the equation above into the objective equation of the trade union produces

$$V_t = K_t\left(\frac{1-\delta}{\omega_t}\right)^{\frac{1}{\delta}}\left\{\omega_t(1-\tau-\rho-\varepsilon)+\frac{\theta_t}{1+r_{t+1}}\right\}$$
$$+\left(N_t-K_t\left(\frac{1-\delta}{\omega_t}\right)^{\frac{1}{\delta}}\right)\left\{\sigma_t+\frac{\theta_t}{1+r_{t+1}}\right\},$$

$$\frac{\partial V_t}{\partial \omega_t} = K_t(1-\delta)^{\frac{1}{\delta}}\left(-\frac{1}{\delta}\right)\omega_t^{-\frac{1}{\delta}-1}\left[\omega_t(1-\tau-\rho-\varepsilon)+\frac{\theta_t}{1+r_{t+1}}\right]$$
$$+K_t(1-\delta)^{\frac{1}{\delta}}\omega_t^{-\frac{1}{\delta}}(1-\tau-\rho-\varepsilon)$$
$$+(-K_t)(1-\delta)^{\frac{1}{\delta}}\left(-\frac{1}{\delta}\right)\omega_t^{-\frac{1}{\delta}-1}\left[\sigma_t+\frac{\theta_t}{1+r_{t+1}}\right]$$
$$=K_t(1-\delta)^{\frac{1}{\delta}}\left(1-\frac{1}{\delta}\right)\omega_t^{-\frac{1}{\delta}}(1-\tau-\rho-\varepsilon)$$
$$+K_t(1-\delta)^{\frac{1}{\delta}}\frac{1}{\delta}\omega_t^{-\frac{1}{\delta}-1}\left(\sigma_t+\frac{\theta_t}{1+r_{t+1}}-\frac{\theta_t}{1+r_{t+1}}\right),$$

To maximize the lifetime incomes of their members,

$$\frac{\partial V_t}{\partial \omega_t}=0$$

Therefore,

$$\omega_t = \frac{1}{(1-\delta)(1-\tau-\rho-\varepsilon)}\sigma_t.$$

Appendix B-2

Based on the Implicit Function Theorem, we have to make sure if the Equation (12) is continuously differentiable and the differentiation of the equation with n_{t-1} is not equal to 0, and then verify the two conditions measured.

Rewrite the Equation (12) as

$$F(\omega_{t+1}, n_{t-1};\ \beta, \gamma, \delta, u, \tau) = 0\,. \tag{13 $*$}$$

$$\frac{m\beta}{\gamma(1+\beta+\gamma)} - \frac{m\beta\rho}{\gamma^2\left[\frac{\tau n_{t-1}}{\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}} + \frac{\gamma(1-\tau)+\rho(1+\beta)}{\gamma}\right]}$$

$$- \frac{\tau n_{t-1}}{\tau n_{t-1} + (1-\tau-\rho)\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}} - \frac{\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{1}{\delta}}(1-u)}{1+\beta+\gamma} = 0$$

$$\frac{\partial F}{\partial \omega_{t+1}} = F_\omega = -\frac{0 - m\beta\rho\gamma^2 \frac{-\tau n_{t-1}\delta(1-\delta)^{\frac{1-\delta}{\delta}}\frac{\delta-1}{\delta}\omega_{t+1}^{\frac{-1}{\delta}}}{\delta^2(1-\delta)^{\frac{2(1-\delta)}{\delta}}\omega_{t+1}^{\frac{2(\delta-1)}{\delta}}}}{\gamma^4\left[\frac{\tau n_{t-1}}{\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}} + \frac{\gamma(1-\tau)+\rho(1+\beta)}{\gamma}\right]^2}$$

$$- \frac{0 - \tau n_{t-1}(1-\tau-\rho)\delta(1-\delta)^{\frac{1-\delta}{\delta}}\frac{\delta-1}{\delta}\omega_{t+1}^{\frac{-1}{\delta}}}{\tau n_{t-1} + (1-\tau-\rho)\delta\omega_{t+1}^{\frac{\delta-1}{\delta}}(1-\delta)^{\frac{1-\delta}{\delta}}}$$

$$- \frac{\omega_{t+1}^{\frac{1-\delta}{\delta}}(1-\delta)^{\frac{-1}{\delta}}(1-u)}{\delta(1+\beta+\gamma)}$$

$$= \frac{m\beta\rho \frac{\tau n_{t-1}(1-\delta)^{\frac{1}{\delta}}\omega_{t+1}{}^{\frac{-1}{\delta}}}{\delta^2(1-\delta)^{\frac{2(1-\delta)}{\delta}}\omega_{t+1}{}^{\frac{2(\delta-1)}{\delta}}}}{\gamma^2\left[\frac{\tau n_{t-1}}{\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}} + \frac{\gamma(1-\tau)+\rho(1+\beta)}{\gamma}\right]^2}$$

$$- \frac{\tau n_{t-1}(1-\tau-\rho)(1-\delta)^{\frac{1}{\delta}}\omega_{t+1}{}^{\frac{-1}{\delta}}}{\tau n_{t-1} + (1-\tau-\rho)\delta\omega_{t+1}{}^{\frac{\delta-1}{\delta}}(1-\delta)^{\frac{1-\delta}{\delta}}}$$

$$- \frac{\omega_{t+1}{}^{\frac{1-\delta}{\delta}}(1-\delta)^{\frac{-1}{\delta}}(1-u)}{\delta(1+\beta+\gamma)}$$

$$= \frac{m\beta\rho\tau n_{t-1}(1-\delta)^{\frac{2\delta-1}{\delta}}\omega_{t+1}{}^{\frac{1-2\delta}{\delta}}}{\gamma^2\delta^2\left[\frac{\tau n_{t-1}}{\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}} + \frac{\gamma(1-\tau)+\rho(1+\beta)}{\gamma}\right]^2}$$

$$- \frac{\tau n_{t-1}(1-\tau-\rho)(1-\delta)^{\frac{1}{\delta}}\omega_{t+1}{}^{\frac{-1}{\delta}}}{\tau n_{t-1} + (1-\tau-\rho)\delta\omega_{t+1}{}^{\frac{\delta-1}{\delta}}(1-\delta)^{\frac{1-\delta}{\delta}}}$$

$$- \frac{\omega_{t+1}{}^{\frac{1-\delta}{\delta}}(1-\delta)^{\frac{-1}{\delta}}(1-u)}{\delta(1+\beta+\gamma)} \quad (B1.1)$$

$$\frac{\partial F}{\partial n_{t-1}} = F_n = \frac{m\beta\rho\tau\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}}{\gamma^2\left[\tau n_{t-1} + \frac{\gamma(1-\tau)+\rho(1+\beta)}{\gamma}\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}\right]^2}$$

$$- \frac{(1-\tau-\rho)\tau\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}}{\left[\tau n_{t-1} + (1-\tau-\rho)\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}\right]^2}$$

$$= \tau\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}\left\{\frac{m\beta\rho}{\gamma^2\left\{\tau n_{t-1}+\left[(1-\tau)+\frac{\rho(1+\beta)}{\gamma}\right]\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}\right\}^2} - \frac{(1-\tau-\rho)}{\left[\tau n_{t-1}+(1-\tau-\rho)\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}\right]^2}\right\}$$

$$= \tau\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}\frac{m\beta\rho\left[\tau n_{t-1}+(1-\tau-\rho)\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}\right]^2-(1-\tau-\rho)\gamma^2\left\{\tau n_{t-1}+\left[(1-\tau)+\frac{\rho(1+\beta)}{\gamma}\right]\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}\right\}^2}{\gamma^2\left[\tau n_{t-1}+(1-\tau-\rho)\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}\right]^2\left\{\tau n_{t-1}+\left[(1-\tau)+\frac{\rho(1+\beta)}{\gamma}\right]\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}\right\}^2} \quad (B1.2)$$

From equation (B1.1) and (B1.2), we can say that the Equation (13*) is continuously differentiable.

Since $m\beta\rho > (1-\tau-\rho)\gamma^2, \tau n_{t-1}+(1-\tau-\rho)\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}} < \tau n_{t-1} + \left[(1-\tau)+\frac{\rho(1+\beta)}{\gamma}\right]\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}$,

when $m\beta\rho\left[\tau n_{t-1}+(1-\tau-\rho)\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}\right]^2 \neq (1-\tau-\rho)\gamma^2\left\{\tau n_{t-1} + \left[(1-\tau)+\frac{\rho(1+\beta)}{\gamma}\right]\delta\left(\frac{\omega_{t+1}}{1-\delta}\right)^{\frac{\delta-1}{\delta}}\right\}^2$, which guarantee the $F_n \neq 0$, we will derive the implicit function as

$$n_{t-1} = f(\omega_{t+1}).$$

Appendix B-3

Differentiating Equation (13) with pension tax rate produces

$$\frac{\partial f(\omega)}{\partial\tau}\left[\frac{m(1+\beta+\gamma)}{\gamma(1-u)}\frac{1}{\omega} - \frac{\tau}{\delta\left(\frac{\omega}{1-\delta}\right)^{\frac{\delta-1}{\delta}}}\right]$$

$$= -1 - f(\omega)\left[\frac{m(1+\beta+\gamma)}{\gamma\omega}(1-u)^{-2}\frac{\partial u}{\partial \tau} - \frac{1}{\delta\left(\frac{\omega}{1-\delta}\right)^{\frac{\delta-1}{\delta}}}\right]. \qquad (B2)$$

From Equation (10**),

$$\frac{\partial u}{\partial \tau} = \frac{\varepsilon(1-\delta)}{[\varepsilon + (1-\delta)(1-\tau-\rho-\varepsilon)]^2},$$

Therefore, the right-hand side of Equation $(B2)$ is

$$-1 - f(\omega)\left[\frac{m(1+\beta+\gamma)}{\gamma\omega}(1-u)^{-2}\frac{\partial u}{\partial \tau} - \frac{1}{\delta\left(\frac{\omega}{1-\delta}\right)^{\frac{\delta-1}{\delta}}}\right]$$

$$= -\frac{M}{\left[m(1+\beta+\gamma)\delta\left(\frac{\omega}{1-\delta}\right)^{\frac{\delta-1}{\delta}} - \tau\omega\gamma(1-u)\right]\gamma(1-u)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2}. \quad (15)$$

where $M = m(1+\beta+\gamma)\delta\left(\frac{\omega}{1-\delta}\right)^{\frac{\delta-1}{\delta}}\{\gamma(1-u)[\varepsilon + (1-\delta)(1-\tau-\rho-\varepsilon)]^2 + \varepsilon(1-\delta)[\gamma(1-\tau)+\rho(1+\beta)]\} - \{\omega(1-u)^2[\varepsilon + (1-\delta)(1-\tau-\rho-\varepsilon)]^2[\gamma^2 + \rho(1+\beta)\gamma]\}$.

1. When $M > 0$, then the wage level has to satisfy

$$\omega < A,$$

where $A = \left(\frac{m(1+\beta+\gamma)\delta\{\gamma(1-u)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2+\varepsilon(1-\delta)[\gamma(1-\tau)+\rho(1+\beta)]\}}{\gamma[\gamma+\rho(1+\beta)](1-u)^2[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2}\right)^{\delta}(1-\delta)^{1-\delta}$. Therefore, if $\omega < A$, $\frac{\partial f(\omega)}{\partial \tau} < 0$; if $\omega > A$, $\frac{\partial f(\omega)}{\partial \tau} > 0$.

2. From Equation (13),

$$\omega < B_\tau,$$

where $B_\tau = \left[\frac{m(1+\beta+\gamma)\delta}{\tau\gamma(1-u)}\right]^{\delta}(1-\delta)^{1-\delta}$;

3. When

$$\frac{A}{B_\tau} = \frac{\tau\gamma(1-u)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2 + \tau\varepsilon(1-\delta)[\gamma(1-\tau)+\rho(1+\beta)]}{(1-u)[\gamma+\rho(1+\beta)][\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2}$$

$$> 1,$$

ie. $\tau\varepsilon(1-\delta) > (1-u)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2$. Substituting u from Equation (10**) to this expression, then when $\tau\varepsilon > (1-\tau-\rho-\varepsilon)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]$, $A > B_\tau$; when $\tau\varepsilon < (1-\tau-\rho-\varepsilon)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]$, $A < B_\tau$.

In summary,

a. If $\tau\varepsilon > (1-\tau-\rho-\varepsilon)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]$,

when $\omega < \left[\frac{m(1+\beta+\gamma)\delta}{\tau\gamma(1-u)}\right]^\delta (1-\delta)^{1-\delta}$, $\frac{\partial f(\omega)}{\partial \tau} < 0$;

b. If $\tau\varepsilon < (1-\tau-\rho-\varepsilon)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]$,

when $\left(\frac{m(1+\beta+\gamma)\delta\{\gamma(1-u)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2+\varepsilon(1-\delta)[\gamma(1-\tau)+\rho(1+\beta)]\}}{\gamma[\gamma+\rho(1+\beta)](1-u)^2[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2}\right)^\delta (1-\delta)^{1-\delta} < \omega < \left[\frac{m(1+\beta+\gamma)\delta}{\tau\gamma(1-u)}\right]^\delta (1-\delta)^{1-\delta}$, $\frac{\partial f(\omega)}{\partial \tau} > 0$;

when $\omega < \left(\frac{m(1+\beta+\gamma)\delta\{\gamma(1-u)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2+\varepsilon(1-\delta)[\gamma(1-\tau)+\rho(1+\beta)]\}}{\gamma[\gamma+\rho(1+\beta)](1-u)^2[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2}\right)^\delta (1-\delta)^{1-\delta}$, $\frac{\partial f(\omega)}{\partial \tau} < 0$.

Appendix B-4

$$\frac{\partial f(\omega)}{\partial \rho}\left[\frac{m(1+\beta+\gamma)}{\gamma(1-u)}\frac{1}{\omega} - \frac{\tau}{\delta\left(\frac{\omega}{1-\delta}\right)^{\frac{\delta-1}{\delta}}}\right] = \frac{1+\beta}{\gamma} - f(\omega)\frac{m(1+\beta+\gamma)}{\omega\gamma(1-u)^2}\frac{\partial u}{\partial \rho}$$

The right-hand side of the equation above is

$$\frac{1+\beta}{\gamma} - f(\omega)\frac{m(1+\beta+\gamma)}{\omega\gamma(1-u)^2}\frac{\partial u}{\partial \rho}$$

$$= \frac{N}{\left[m(1+\beta+\gamma)\delta\left(\frac{\omega}{1-\delta}\right)^{\frac{\delta-1}{\delta}} - \omega(1-u)\gamma\tau\right]\gamma(1-u)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2}. \quad (17)$$

where $N = m(1+\beta+\gamma)\delta\left(\frac{\omega}{1-\delta}\right)^{\frac{\delta-1}{\delta}}X - \omega\gamma\tau(1+\beta)(1-u)^2[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2$, $X = (1+\beta)(1-u)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2 - \varepsilon(1-\delta)[\gamma(1-\tau)+\rho(1+\beta)]$.

1. If $X < 0$, ie. $(1+\beta)(1-\tau-\rho-\varepsilon)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)] < \varepsilon[\gamma(1-\tau)+\rho(1+\beta)]$, then

$$\frac{\partial f(\omega)}{\partial \rho}\left[\frac{m(1+\beta+\gamma)}{\gamma(1-u)}\frac{1}{\omega} - \frac{\tau}{\delta\left(\frac{\omega}{1-\delta}\right)^{\frac{\delta-1}{\delta}}}\right] < 0,$$

that is, when $\omega < B_\rho, \frac{\partial f(\omega)}{\partial \rho} < 0$;

2. If $X > 0$,

(1) If $N > 0$, then the wage level set by the trade union has to satisfy

$$\omega < C,$$

where $C = \left(\frac{m(1+\beta+\gamma)\delta}{\tau\gamma(1-u)} - \frac{m(1+\beta+\gamma)\delta\varepsilon(1-\delta)[\gamma(1-\tau)+\rho(1+\beta)]}{\gamma\tau(1+\beta)(1-u)^2[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2}\right)^\delta (1-\delta)^{1-\delta}$;

From Equation (17),

$$\omega < B_\rho,$$

where $B_\rho = \left(\frac{m(1+\beta+\gamma)\delta}{\tau\gamma(1-u)}\right)^\delta (1-\delta)^{1-\delta}$;

And

$$C < B_\rho.$$

Therefore, when $\omega < \left(\frac{m(1+\beta+\gamma)\delta}{\tau\gamma(1-u)} - \frac{m(1+\beta+\gamma)\delta\varepsilon(1-\delta)[\gamma(1-\tau)+\rho(1+\beta)]}{\gamma\tau(1+\beta)(1-u)^2[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2}\right)^\delta (1-\delta)^{1-\delta}, \frac{\partial f(\omega)}{\partial \rho} > 0$.

(2) If $N < 0$, then the wage level has to satisfy

$$\omega > C;$$

From Equation (17),

$$\omega < B_\rho.$$

Therefore, when $\left(\frac{m(1+\beta+\gamma)\delta}{\tau\gamma(1-u)} - \frac{m(1+\beta+\gamma)\delta\varepsilon(1-\delta)[\gamma(1-\tau)+\rho(1+\beta)]}{\gamma\tau(1+\beta)(1-u)^2[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2}\right)^\delta (1-\delta)^{1-\delta} < \omega < \left(\frac{m(1+\beta+\gamma)\delta}{\tau\gamma(1-u)}\right)^\delta (1-\delta)^{1-\delta}$, $\frac{\partial f(\omega)}{\partial\rho} < 0$.

In summary,

a. If $(1+\beta)(1-\tau-\rho-\varepsilon)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)] < \varepsilon[\gamma(1-\tau)+\rho(1+\beta)]$, then $\frac{\partial f(\omega)}{\partial\rho} < 0$;

b. If $(1+\beta)(1-\tau-\rho-\varepsilon)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)] > \varepsilon[\gamma(1-\tau)+\rho(1+\beta)]$,

(1) If $m(1+\beta+\gamma)\delta\left(\frac{\omega}{1-\delta}\right)^{\frac{\delta-1}{\delta}}\{(1+\beta)(1-u)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2 - \varepsilon(1-\delta)[\gamma(1-\tau)+\rho(1+\beta)]\} - \omega(1-u)^2\gamma\tau(1+\beta)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2 > 0$,

i.e. $\omega < \left(\frac{m(1+\beta+\gamma)\delta}{\tau\gamma(1-u)} - \frac{m(1+\beta+\gamma)\delta\varepsilon(1-\delta)[\gamma(1-\tau)+\rho(1+\beta)]}{\gamma\tau(1+\beta)(1-u)^2[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2}\right)^\delta (1-\delta)^{1-\delta}$, $\frac{\partial f(\omega)}{\partial\rho} > 0$;

(2) If $m(1+\beta+\gamma)\delta\left(\frac{\omega}{1-\delta}\right)^{\frac{\delta-1}{\delta}}\{(1+\beta)(1-u)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2 - \varepsilon(1-\delta)[\gamma(1-\tau)+\rho(1+\beta)]\} - \omega(1-u)^2\gamma\tau(1+\beta)[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2 < 0$,

i.e. $\left(\frac{m(1+\beta+\gamma)\delta}{\tau\gamma(1-u)} - \frac{m(1+\beta+\gamma)\delta\varepsilon(1-\delta)[\gamma(1-\tau)+\rho(1+\beta)]}{\gamma\tau(1+\beta)(1-u)^2[\varepsilon+(1-\delta)(1-\tau-\rho-\varepsilon)]^2}\right)^\delta (1-\delta)^{1-\delta} < \omega <$

$\left(\frac{m(1+\beta+\gamma)\delta}{\tau\gamma(1-u)}\right)^{\delta}(1-\delta)^{1-\delta}$, $\frac{\partial f(\omega)}{\partial \rho} < 0$.

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