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Chapter 1

The philosophy of measurement theory

The purpose of this book is to propose "mathematical foundations of measurement theory". The statement:

"There is no science without measurements" (1.1)

is an old famous saying, which of course emphasizes the importance of "measurement". We believe in the saying, i.e., the concept of "measurements" should be the most fundamental in science. However, it is certain that we do not have an authorized "measurement theory" in science yet. Thus, we think that it is worthwhile proposing the mathematical foundations of "measurement theory" ¹:

Chapters 2, 3, 8 ···· Mathematical foundations of measurement theory

Chapter 4 \cdots An application (of measurement theory) to statistical mechanics

Chapters $5\sim 12$ ··· Several theories (e.g., statistics, classical and quantum system theories, etc.) in measurement theory

It should be noted that "measurement theory" and "theoretical physics" are different. In particular, their philosophies are completely different. Although it is a matter of course that it is impossible to understand the philosophy of measurement theory without the complete knowledge of measurements (i.e., the contents of Chapters $2 \sim 12$), the philosophy of measurement theory is also indispensable for the understanding of measurement theory. Therefore, in this first chapter, we devote ourselves to the philosophy of measurement theory.

1.1 How to construct "measurement theory"

It is well known that the dynamical system theory (DST, classical system theory)

¹The measurement theory is proposed in the references [41] \sim [48],[55] in this book. We devote ourselves to the mathematical aspect of "measurement theory". For the other aspects (e.g., practical and general aspects), see [30], which is educational and enlightening.

starts from the following equations:

$$\boxed{\text{DST}} = \begin{cases} \frac{dx(t)}{dt} = f(x(t), u_1(t), t), \ x(0) = x_0 & \cdots \text{((stochastic) state equation)},^2 \\ y(t) = g(x(t), u_2(t), t) & \text{(measurement equation)} \\ \text{where } u_1 \text{ and } u_2 \text{ are external forces (or noises),} \end{cases}$$
(1.2a)

or more precisely,

= "Apply (1.2a) to every phenomenon by an analogy of Newtonian mechanics and the coin-tossing problem".³ (1.2b)

That is, DST is usually believed to be a kind of epistemology called "the mechanical world view", namely, an epistemology to understand and analyze (moreover, control) every phenomenon — economics, psychology, engineering and so on — by an analogy of Newtonian mechanics (and coin-tossing).

Also recall that quantum mechanics is formulated as the following form (cf. von Neumann [84]):

$$\begin{array}{||c|c|c|c|}\hline quantum mechanics &= [measurement] + [the rule of time evolution] \\ & (Born's quantum measurement) & (Schrödinger equation) \\ & (1.3) \end{array}$$

which was discovered by W. Heisenberg, E. Schrödinger, M. Born in between 1924 and 1926.

Here, it should be noted that the term "measurement" appears in both (1.2) and (1.3). Thus, our proposal, i.e., "measurement theory (=MT)", is constructed as follows:

(I₁) Quantum mechanics (1.3) is formulated in B(H), the algebra composed of all bounded linear operators on a Hilbert space H (cf. von Neumann (1932: [84])). Thus it is easy to generalize quantum mechanics in C^{*}-algebra \mathcal{A} ($\subseteq B(H)$, cf. Definition 2.1 in §2.1) such that it includes DST (1.2) as a special case. Namely, $(1.2)+(1.3)\subset$ "MT".

That is, as a kind of generalization of quantum mechanics (1.3), we can propose as follows:

 $^{^{2}}$ A stochastic differential equation (or stochastic difference equation) in dynamical system theory is usually called a *stochastic state equation*.

³That is, DST is, from the mathematical point of view, based on "the theory of differential equations" and "probability theory". Thus, I think that I.Newton (*cf.* [66]) and A.Kolmogorov (*cf.* [56]) are greatest in DST.

"measurement theory (or in short, MT)"

= [measurement] + ["the rule of the relation among systems"] in C*-algebra A"Axiom 1 (2.37)" "Axiom 2 (3.26)"

(1.4a)

or more precisely,

= "Apply (1.4a) to every phenomenon by an analogy of quantum mechanics" (1.4b)

(For the details, see Chapter 2 [Axiom 1 (2.37)], and Chapter 3 [Axiom 2 (3.26)]). Here it should be noted that MT (= Axiom 1 + Axiom 2) is composed of a few key-words i.e.,

 (I2) system, state, observable, measurement, measured-value, probability, Markov relation, sequential observable, Heisenberg picture, etc.

and Axioms 1 and 2 explain how to use these words. Roughly speaking, Axioms 1 and 2 say "Use these words by analogy of quantum mechanics".⁴

We have the classification of MT as follows:⁵

where a C^* -algebra is either commutative or non-commutative. Also, as mentioned in (I_1) , we consider the following correspondence:

$$\underbrace{\text{``MT''}}_{\text{``MT''}} = \begin{cases}
 \underbrace{\text{``classical MT'' in (1.5)}}_{\text{``quantum MT'' in (1.5)}} \leftrightarrow DST in (1.2) \\ \underbrace{\text{OST in (1.2)}}_{\text{(1.6)}} & (1.6)
 \end{aligned}$$

 4 Thus, our approach is, from the philosophical point of view, characterized as so called *foundational-ism*.

$$MT (="measurement theory") \begin{cases} PMT (="(pure) measurement theory") in Chapters 2 ~ 7\\ SMT (="statistical measurement theory") in Chapters 8 ~) \end{cases}$$

PMT is essential. That is, we can say that there is no SMT without PMT. (Cf. Chapter 8.)

⁵As seen later (i.e., Chapter 8), we also have the classification of MT, i.e., "(pure) measurement theory (= PMT)" and "statistical measurement theory (=SMT)". That is,

1.2 What is measurement theory?

We think that the question "What is measurement theory?" is much more difficult than the question "How is measurement theory constructed?".

As mentioned in (I_1) in §1.1, MT is the mathematical generalization of quantum mechanics (1.3). That is, MT is not quantum mechanics but "something beyond mechanics". Thus, we can assert that

 (I_3) MT is the mathematical representation of the epistemology called "the mechanical world view" (just like DST(1.2) is).

Also, it should be noted that MT is quite a wide theory, that is, we assert:

(I₄) MT is the most fundamental theory of so-called "theoretical informatics", including dynamical system theory, quantum system theory, practical logic, statistics, circuit theory, control theory, chaotic system theory, multivariate analysis, information theory, automata theory, OR, game theory, etc.

This will be discussed in Chapters 5 ~ 12. Also, note that the above (I_4) should be regarded as the same as the following assertion:

 (I_5) The term: "theoretical informatics" is defined as the academic discipline that is composed of all theories understood in MT. That is, "theoretical informatics" = "MT".

We assert that

the most fundamental theory of theoretical physics \implies 'TOE (string theory(?))'⁶

the most fundamental theory of theoretical informatics \implies MT

And therefore, we can present the following table, which indicates where MT is in science.

⁶The string theory (*cf.* [28]) is not necessarily authorized yet. Thus, in this book, the term 'TOE (Theory of Everything)' is used as the symbol of the most fundamental theory of theoretical physics. As emphasized in this section, the philosophy of theoretical physics is different from that of theoretical informatics. And thus, the meanings of "the most fundamental theory" are respectively different in theoretical physics and in theoretical informatics.



That is, the mathematical structures of all theories in (C_3) are common, and thus, they are discussed in the framework of MT.

We add the following remark.

Remark 1.1. (About Table (1.7)).

(a). Note that the class (C_3) (= (I_4)) is usually called "applied mathematics". In this sense, we think that MT is the main part of so-called applied mathematics.

(b). For example, if electromagnetic theory and relativity theory can not be unified, we must consider two categories (e.g., "Theoretical physics (I)" and "Theoretical physics (II)") in theoretical physics. However, most physicists believe that physics consists of only one category, that is, the theories in (C_2) must be unified in the most fundamental theory (= 'TOE'). The purpose of this book is, of course, to show that the theories in (C_3) are

mathematically understood in MT. Also, in this book, "Newtonian mechanics" [resp. "quantum mechanics"] in MT is called "classical system theory (= dynamical system theory)" [resp. "quantum system theory"] (though the addition of "measurement equation" to DTS(1.2a) should be regarded as the act of genius (since there is no concept of "measurement" in Newtonian mechanics)). That is, the two (i.e., Newtonian mechanics and quantum mechanics) are common in both "theoretical physics" and "theoretical informatics" (*cf.* §10.5).

(c). The purpose of theoretical physics is to represent "natural forces" in terms of mathematics. On the other hand, as mentioned in (I_3) , MT is a kind of epistemology called "the mechanical world view", namely, an epistemology to understand and analyze (moreover, control) every phenomenon — economics, psychology, engineering and so on — by an analogy of mechanics. That is, MT is the mathematical representation of "the mechanical world view". Or, precisely speaking, the definition of "the mechanical world view" is given by MT.

(d). From the mathematical point of view, the difference between "theoretical physics" and "theoretical informatics" is that of "differential geometry" and "the theory of Hilbert spaces (or operator algebras)". Cf. Remark 8.26.

(e). It is a matter of course that the theories in theoretical physics (= (C_2) in (1.7)) should be tested by experiments. For example, the question: "Is electromagnetic theory experimentally true or not?" is meaningful. In fact, serious experiments have been often conducted as big projects (such as SERN, Kamioka Observatory (Japan), etc.) in theoretical physics. On the other hand, the experimental tests of the theories in theoretical informatics (= (C_3)) are nonsense. For example, the experimental test of statistics is meaningless just like that of mathematics (e.g., linear algebra) is obviously meaningless.⁷ Thus, we think that the question: "Is statistics experimentally true or not?" is meaningless. However, it should be noted that the question: "Is statistics convenient (= useful)?" is meaningful.

(f). We hope that some will find and propose "The third mathematical scientific theory in (C_4) ".

⁷There may be some truth in the assertion that statistics is a kind of mathematics. However, as mentioned in Table (1.7), we think that "statistics" = "mathematics + something".

Summing up, we assert the following table:

Table

(1	8	a)

	Theoretical Physics	Theoretical Informatics
(1). the theories	classical and quantum mechanics,	dynamical system theory, statistics,
in this field	electromagnetic theory,	logic, quantum system theory,
(<i>cf.</i> Remark 1.1 (b))	Weinberg-Salam theory, etc. $(cf. (C_2))$	information theory, etc. $(cf. (C_3))$
(2). the most fundamental theory	'TOE (Theory of Everything)'	MT (measurement theory)
(<i>cf.</i> Remark 1.1 (b))	(will be proposed in the future)	(proposed in this book <i>cf.</i> $[41] \sim [48], [55]$)
(3). the purpose	the mathematical representation	the mathematical representation
(<i>cf.</i> Remark 1.1 (c))	of "force"	of "the mechanical world view"
(4). mathematical language	differential geometry (gauge theory)	operator algebra
(cf. Remark 1.1 (d))		(functional analysis, real analysis)
(5). experimentally	meaningful	meaningless
true or false		
(<i>cf.</i> Remark 1.1 (e))		

Next let us consider the following problem.

Problem 1.2. ("experimentally true or false" and "theoretically true or false"). Consider the following problems (i) and (ii).

- (i) Assume that someone proposes "psychokinetic theory" as a theory of theoretical physics. Determine whether his/her theory is true or false.
- (ii) In [93], Zadeh proposed "the fuzzy sets theory" as a theory of theoretical informatics.
 Determine whether his theory is true or false.⁸

[Answer (i)]. The problem (i) is solved by two methods. One is the experimental test. If it is OK (i.e., if it is experimentally true), "psychokinetic theory" should be accepted as a physical theory. Also, if we have the most fundamental theory (= 'TOE'), we can determine whether "psychokinetic theory" is theoretically true in 'TOE'. If it is OK (i.e., if it can be understood in 'TOE'), the "psychokinetic theory" should be accepted as a physical theory. Of course, it always holds that "experimentally true" = "theoretically true".

⁸One of our motivations for this research may be inspired by the fashion of Zadeh's fuzzy sets theory (*cf.* [93], which is the most cited paper in all fields of 20th century science) in 1980s ~ 1990s. We had a lot of arguments about "Is Zadeh's fuzzy sets theory true or false?" or "Can it be justified?" However, these arguments may be fruitless. That is because all controversies were engaged without the understanding of the meaning of "true" (or "justification"). It should be noted that we do not only have the answer to the question: "Is Zadeh's fuzzy sets theory (theoretical) true or false?" but also "Is Fisher's statistics (theoretically) true or false?". (These will be respectively answered in Chapter 5~7.) In this sense, we can say that the purpose of this book is to introduce the criterion: "theoretically true or false" into theoretical informatics. (Cf. Declaration (1.11) later). Here, two criterions of "theoretically true or false," and "useful or not (in informatics-related engineering)" should not be confused. Throughout this book we are not concerned with "useful or not" but "theoretically true or false", though we, of course, know that the criterion "useful or not" is also quite important.

[Answer (ii)]. On the other hand, the problem (ii) is solved by one method. If we have the most fundamental theory (='measurement theory'), we can determine whether "Zadeh's fuzzy sets theory" is theoretically true or false in the most fundamental theory. If it can be understood in the most fundamental theory, "Zadeh's fuzzy sets theory" should be accepted as a theory of theoretical informatics. Our answer will be presented in Chapter 7. However, as mentioned in Remark 1.1 (e), it should be noted that the question: "Is Zadeh's fuzzy sets theory experimentally true or not?" is nonsense.

Remark 1.3. (What should measurement theory be applied to?). Recall that MT is a kind of epistemology called "the mechanical world view", namely, an epistemology to understand and analyze (moreover, control) *every* phenomenon by an analogy of mechanics. In this sense, MT may be applied to everything. However, it is certain that some problems (or phenomena) are fit for "the mechanical world view", but others are not. Thus, we have the following question.

 (I_6) What phenomenon should measurement theory be applied to?

The following fields are generally believed to be fit for "the mechanical world view" to some degree.

(I₇) the fields in informatics-related engineering, e.g., information engineering, administration engineering, mathematical psychology, statistical medicine, mathematical economics, financial engineering, cognitive engineering, quality control engineering, chaotic engineering, electrical circuit engineering ⁹, etc.

And further, we add

 (I_8) statistical mechanics, fluid mechanics, etc.¹⁰

though the two are usually believed to belong to theoretical physics. As mentioned later (i.e., the footnote under (I_{13})), the theories in (C_5) in Table (1.7) should be studied by several methods (and not only by "the mechanical world view" (= MT)). Also, we say

⁹For example, the distinction between "electrical circuit engineering" in (I_7) and "circuit theory" in (I_4) may be ambiguous. However, we want to say "MT itself is not engineering but the mathematical representation of "the mechanical world view".

¹⁰Boltzmann's statistical mechanics will be discussed as one of applications (of MT) in Chapter 4. Therefore, there is a reason to call "theoretical physics" [resp. "theoretical informatics"] "the first physics" [resp. "the second physics"].

 (I_9) It is too optimistic to consider that the completely precise theory exists in (I_7) and (I_8) . However, the theories in (I_7) and (I_8) may be "almost experimentally true" to such a degree that they are assured to be "useful". That is, every theory in (I_7) and (I_8) is, more or less, ambiguous. Although the challenge to make a precise theory should be worthy of praise, what is most important is not "precise" but "useful" in engineering.

Remark 1.4. (Aristotles and Plato). As mentioned before, theoretical physics must be always checked by experimental tests. That is, it is based on realism (i.e., the Aristotles spirit). On the other hand, recall that the experimental test for MT is nonsense. Therefore, we can not deny MT by any experimental tests.¹¹ Thus, we may agree to the opinion that

"MT is self-righteous".

In this sense, we cay say that MT is based on idealism (i.e., the Plato spirit)¹² However, it does not imply "unfair". That is because, if some want to deny MT, it suffices to propose another fundamental theory *better* than MT. Here,

 (I_{10}) the question: "Which is better?" is decided by majority (or popularity).

Here it should be noted that to win popularity is as difficult as to find the truth. Also, as mentioned in (I_9) , we can expect that every theory in (I_7) and (I_8) is "almost experimentally true". That is because, if it is not "almost experimentally true", it can never win popularity.

 $^{^{11}}$ Thus, I assume that MT itself is a kind of *metaphysics* (and not science in the sense of Popper [70], "falsifiability").

¹²If I were familiar with the history of philosophy, I could stress the correspondences: "theoretical physics (realism) \leftrightarrow Aristotles" and "theoretical informatics (idealism) \leftrightarrow Plato".

Summing up, we assert the following table:

	Table	(1.8b)
	Theoretical Physics	Theoretical Informatics
(6). important criterion	experimentally true or false	useful or not, likes or dislikes
(cf. Remark 1.1 (e), Remark 1.4)	objective	popularity, economical, subjective
(7). theoretically	meaningful in 'TOE'	meaningful in MT
true or false		
(<i>cf.</i> Problem 1.2 (i),(ii))		
(8). what to be applied to	physical phenomena	all phenomena (particularly,
(<i>cf.</i> Remark 1.3)		appearing in (I_7) and (I_8))
(9). fundamental spirit	realism (due to Aristotles)	idealism (due to Plato)
(<i>cf.</i> Remark 1.4)	Theory is dominated by experiment.	Theory is free from experiment.

1.3Measurement theory in engineering

As mentioned in (I_7) in §1.2, MT plays an important role in engineering. The theoretical physics (= 'TOE') itself may be worthy even if it has no applications. However, MT is not so. Thus, in this section, we consider the relation between engineering and MT. Here, engineering is usually considered to be composed of physics-related engineering (e.g., laser engineering, etc.), chemistry-related engineering (e.g., chemistry engineering, etc.), informatics-related engineering (e.g., financial engineering, etc.), etc.

The area of physics-related engineering is clear. That is because the physics-related engineering is generally believed to be supported by "physics" as the theoretical backbone. We studied physics as one of the important subjects in high-school, and therefore, we believe that theoretical physics is only one discipline, i.e., classical mechanics, relativity theory, electromagnetic theory and so on that should be unified. That is, physics-related engineering has the authorized root (= physics). Also, note that the circumstance of chemistry-related engineering is similar to that of physics-related engineering.

On the other hand, the area of informatics-related engineering may be vague. This is due to the fact that we do not know the most fundamental root in informatics-related engineering. Note that there is a possibility that informatics-related engineering has two (or more than two) fundamental roots. If it is so, we must consider "informaticsrelated engineering (I)" and "informatics-related engineering (II)" (cf. Remark 1.1 (b)). Therefore, we must answer the following question:

(Or, is (I_{11}) What subject is the most fundamental in informatics-related engineering? theoretical informatics the only one?)

Of course, our answer is

 (I_{12}) MT is the most fundamental theoretical backbone in informatics-related engineering.

MT (or, theoretical informatics) is not studied in high-school. However, statistics and differential equations (which are closely related to MT (i.e., Axioms 1 and 2 in (1.4a))) are studied as mathematics in high-school. In this sense, theoretical informatics is not underestimated in high-school education.

Thus we have the following table.

Table

(1.9)

	fundamental subject	area (applications)
physics-related	physics (mathematical)	semiconductor engineering,
engineering	experimental test is possible	laser engineering, etc.
chemistry-related	chemistry (non-mathematical)	chemical engineering
engineering	experimental test is possible	
informatics-related	MT (mathematical)	Cf. (I_7) and (I_8)
engineering	experimental test is meaningless	

Thus we conclude that

 (I_{13}) The area of informatics-related engineering is roughly¹³ determined by MT (= "theoretical informatics"), just like the area of physics-related engineering is roughly determined by physics. Also, recall (I_5) .

That is, we say:



where "M.R." = "mathematical representation", "ET" = "experimental test" (*cf.* Problem 1.2 (i)), "Appl" = "Applications" (*cf.* Table (1.9)), "PP" = "popularity" (*cf.* (I_{10})), "AET" = "almost experimentally true" (*cf.* (I_9) in §1.3).

¹³For example, mechanical engineering is closely related to physics. However, control theory (in (C_3) of Table (1.7)) plays an important role in robot engineering (which is a kind of mechanical engineering). Also, electrical circuit engineering may be close to electromagnetic theory as well as dynamical system theory. Thus, such a classification of engineering (presented in Table (1.9)) is somewhat forcible. That is because "Use everything available" is the engineer's spirit. Thus we must say that physics (as well as measurement theory) is more or less influential to every field in (I_7) and (I_8) . However we can, at least, assert that physics, chemistry and MT are the most fundamental subjects in the faculty of engineering.

Here again note that

 (I_{14}) Theoretical physics has to be precise. On the other hand, engineering has to be useful rather than precise. Since ambiguous statements can not be tested "exactly", we use the term : "AET (= almost experimentally true)" in the above (1.10).

Thus we see

 (I_{15}) There is a possibility that a phenomenon has two (or, more than two) explanations in MT. And moreover, in this case, we may not choose one from the two by experimental tests but a sense of beauty (\approx like or dislike).

1.4 The spirit of "the mechanical world view"

We think that "measurement", "its philosophy" and "its applications (\approx informaticsrelated engineering)" should be regarded as "the Trinity". And we assert the following declaration, which was essentially proposed in [Ishikawa, 2002, [48]].

Declaration (1.11)

We assert the following (i) \sim (iii), which should be understood as the different representations of the same thing:

- (i) MT is the most fundamental theory of theoretical informatics, which is regarded as the theoretical backbone of informatics-related engineering.
- (ii) MT is the ultimately generalized form of the dynamical system theory (1.2). Thus, MT is regarded as the mathematical representation of the epistemology called "the mechanical world view". And thus, MT is sometimes called the general dynamical system theory (or in short, GDST).
- (iii) MT is entitled to check all theories in theoretical informatics. In other words, we can, by using MT, introduce the criterion: "(theoretically) true or false" into theoretical informatics.

Here, note that:

• in this book, "the mechanical world view" means "the quantum mechanical world view" and not "the Newtonian mechanical world view".

We might say too much in this chapter. It may suffice to say

The spirit of "the mechanical world view" (1.12)

• Mind Declaration (1.11) and Tables (1.7) and (1.8). And further, at any rate (= setting aside the reason), study every (physical or non-physical) problem in the framework of MT.¹⁴

Summing up, we have "the Trinity" as follows:



Here, again note that the philosophy of "theoretical informatics" is completely different from that of "theoretical physics", Although it is a matter of course that it is impossible to understand the philosophy of measurement theory without the complete knowledge of measurements (i.e., the contents of Chapters $2 \sim 12$), the philosophy of measurement theory is also indispensable for the understanding of measurement theory.

Remark 1.5 (Another important problem) The problem:

 (I_{16}) "Propose The third mathematical scientific theory in (C_4) of Table (1.7)"

¹⁴As mentioned in Remark 1.4, we do not necessarily need a perfect reason in theoretical informatics. In this sense, the term: "extensive interpretation" is one of the most important terms in theoretical informatics.

may be the most important. I think that the above problem (I_{16}) is so difficult. Thus I may prefer waiting the appearance of a genius to doing it ourselves!⁵

¹⁵As mentioned in this chapter, our purpose may be, briefly speaking, to study all fields which can be understood in terms of "measurement (i.e., Axioms 1 and 2)". In this sense, Frieden's challenge [24] is also interesting. His purpose seems to study all fields (of physics) which can be understood in terms of "Fisher information". Although we do not completely understand his theory, we expect that his theory may be one of the candidates of The third mathematical scientific theory]. We never hope that MT is the only one mathematical theory that belongs to the category of "idealism".