Dissertation

For the Doctoral Degree of Engineering

Vortex Particle Redistribution and Regularisation

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Thesis Abstract

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Thesis Title

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Thesis Summary

Vortex methods are particle schemes that were first introduced as a tool to solve inviscid, unbounded flow problems in two- and three-dimensional space. In this thesis we briefly introduce the mathematical framework necessary to understand vortex methods and their benefits. When one tries to extend the method to viscous or bounded flows, however, one faces several additional problems. Our main contributions to the field address three of the problems encountered.

Our first contribution is a new scheme to handle unbounded, viscous flows. Based on the vorticity redistribution method, this scheme requires neither a frequent regridding of the particle field, nor makes use of the viscous splitting. It can thus be used with higher order time-stepping methods. Its consistency, stability, and conservation properties are proven. Together with the new heuristics of reduced operators and small neighbourhoods we demonstrate in numerical experiments that the method can be implemented efficiently on computers. The numerical results are in good agreement with the analysis.

As a second contribution we propose a new scheme to tackle the particle regularisation problem in bounded domains. This problem refers to the task of obtaining a smooth approximation of a function from a given particle field. To this end we construct a new class of globally smooth finite element spaces and prove their approximation properties. The global smoothness of these spaces will allow us to use them as test-functions for particle approximations. The regularisation problem is then modelled as a perturbation to a stabilized L^2-projection onto fictitious domains. After proving consistency, stability, and convergence of the method we show that optimal results are obtained when choosing σ proportional to h, where σ refers to the smoothing length and h to the particle spacing. As a consequence the complexity of the velocity computation can be reduced from $O(h^{-d})$ to $O(h^{-d/2})$. Numerical experiments confirm the analysis and that the derived error bounds are sharp.

Our third contribution are simple and efficient formulae to evaluate the Biot–Savart integral on three-dimensional domains using tetrahedral meshes. The derived formulae are exact for piecewise linear functions. Compared to the previously published formulae by Suh, the presented approach is numerically more stable and reduces the number of required arctangent evaluations from twelve to four. The increased stability is demonstrated in a simple numerical example.

We finish this thesis with concluding remarks on the results and possible interesting topics for future research.