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STRESS ANALYSIS OF TWO ELASTIC ANNULAR PLATES, CONNECTED EACH OTHER BY ELASTIC RODS

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ABSTRACT

Here we take up the case of two elastic plates of annular form, which are placed horintally, kept at a vertical distance apart, and placed concentrically. These plates are fixed along inner and outer boundary edge-lines respectively. Moreover, these two annular elastic plates are mutually connected by means of a number of elastic straight rods which are fixed to elastic plates at both ends. The points of fixation of rods are arranged equidistantly along a circumference of a concentric circle. The state of stress caused by the action of elastic rods (such as originating from thermal expansion due to external heating, or by shrinkage of elastic rods, etc.) in the elastic plate, is studied analytically.

Furthermore, study is made about the case in which two elastic plates are in form of infinite strip of rectangular elastic plates, that is, annular elastic plates in which the radius R is made infinitely large.

The results obtained by these series of analytical studies are explained by graphs, which were obtained by applying numerical estimation to our analytical results.

As to practical meaning of the present analytical study, we may mention that it represents an idealized case of structures of once-through boilers. Also, we may mention the case of structiral form of inlet ring of hydraulic turbines, also as an idealized case about them.

1. Introduction

In the present paper, we take up the case of two elastic plates which are placed horizonyally, and kept at a vertical distance l, with each other. These two elastic plates are taken to be of annular forms (with outer radius R and inner radius cR), and to be placed concentrically each other. These two annular elastic plates are taken to be connected each other by a number of vertical elastic rods of lengths l. These elastic bars, in number N, are attached to the elastic plates

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at a row of points which lie along the circle of radius bR ($c<b<1$), and arranged equidistantly, keeping at angular distance of $2\pi/N$.

Our purpose is to study analytically, the state of deformation and stress, which takes place when these elastic bars exert transverse forces upon annular elastic plates. These transverse forces are thought to be caused due to their thermal expansion, owing probably to the effect of external heating.

The mode of fixation of elastic annular plates are here taken to be that of clamped edges, both along inner and outer boundary circumferences.

About the practical meaning of our study, we may mention that it represents, in an idealized form, the problem of strength of supporting plates (top and bottom bed-plates) of once-through boilers. We may also mention that it represents in an idealized form, the problem of strength of inlet-rings of hydraulic turbines.

The author has, in a previous occation, made analytical study about similar problem in which the lower plate is assumed to be a rigid body. (Ref. (2)) The present study may be regarded to be an extension of author's previous study.

Moreover, when the outer radius R of annular plates are made infinitely large, while keeping their breadth $(1-c)R$ at a fixed value, we arrive at the case of two infinite strips of rectangular elastic plates, which are connected each other by a row of elastic rods. This problem of rectangular elastic plates is also treated here, by a separate analysis.

Results of these analytical studies are shown in graphs, which were obtained by numerical evaluation about cases A and B, which is thought likely in practice. Comparison among them, and concluding remarks about acquired resultes are also given.

2. Notations

In the present paper, following notations are used; (r, θ)=polar coordinates of any point on the middle plane of our elastic annular plate; h=thickness (uniform) of the plate; (R, cR)=outer and inner radius of annular plate; bR=radius of circular cotour-line, along which concentrated loads (by means of elastic bars) are imposed; (E, ν)=Young's modulus and Poisson's ratio of the plate material; w= transverse displacement of middle plane of the elastic plate (infinitesimally small, and taken positive downwards); w_1=those values of w, which belongs to the region $cR \leq r \leq bR$ ($c \leqq x \leq b$); $D=Eh^3/[12(1-\nu^2)]$=flexural rigidity of the elastic plate; $\Delta \equiv$two dimensional Laplace operator; $x=r/R$ no dimensional variable, expressing radial position of any point of the middle plane; P=transverse concentrated load, which is assumed to act on the plate (due to pushing action of elastic bar, and taken as positive downwards).

Numerical coefficients R_0, R_1, $\cdots$, S_0 S_1, $\cdots$ (these being functions of the variable x) are used to express the displacement w, in form of infinite series. Indices, such as $D^{(k)}$, $(k=1, 2)$ are used in order to discriminate to which one of the elastic plates ($k=1$ upper, $k=2$ lower) it refers to.

3. Statement of our Problem, and Fundamental Equations which represent it

In the present paper, we take up the case of two annular elastic plates, as shown in Fig. 1 (a) and (b), and arranged in following manners. Each one of them have outer radius R and inner radius cR, and are concentrically placed, keeping at a vertical distance l apart between them. These two elastic plates (No. 1 upper, and No. 2 lower) are interconnected each other, by a number N of elastic bars of lengths l. The elastic bars are taken to be fixed to plates at N points of circumference of radius bR, keeping equal distantly, respectively. Of course, we have $0<c<b<1$.

To fix our ideas, we consider the case in which inner and outer edges of each annular plate are fixed to rigid wall (in manner of so called clamped edges). The

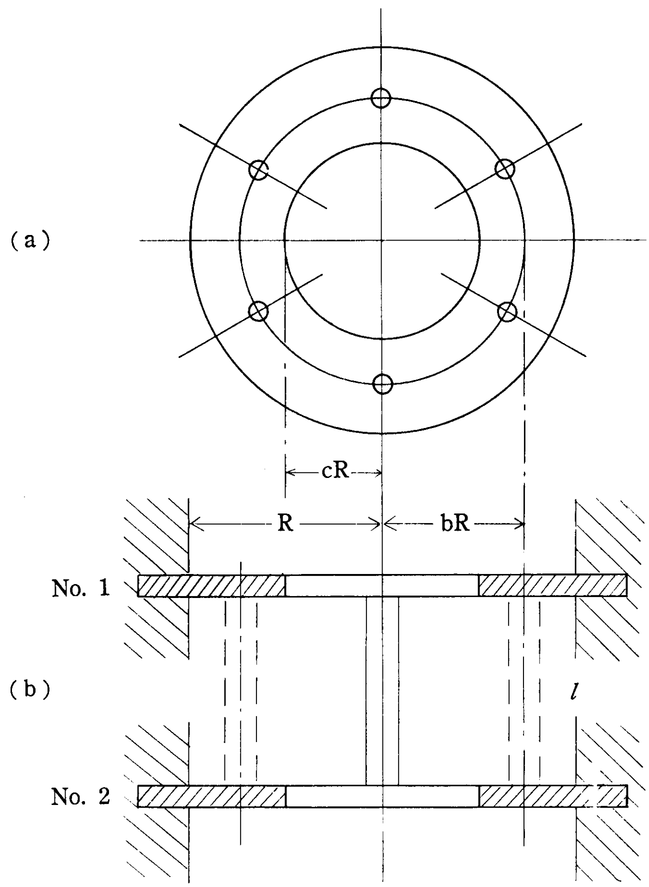

Fig. 1. Two annular elastic plates, Connected each other by Several elastic bars.

author has, in previous occation (Reference (2)), made a study about similar problem, in which the lower plate is supposed to be a rigid body instead of an elastic plate. The content of the present paper is result of revision and extension of the author's previous study. Among the various cases possible to occur, the author has confined himself to the case of which the connecting rods suffer a kind of thermal expansion, thus generating axial force which act upon two (upper and lower) annular elastic plates.

The fundamental differential equation of an elastic plate can be written (Ref. (1));

$$\Delta\Delta w \equiv \left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)^2 w=0 \tag{1}$$

This equation may also be rewritten in the form:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{1}{x}\frac{\partial}{\partial x}+\frac{1}{x^2}\frac{\partial^2}{\partial\theta^2}\right)^2 w=0 \tag{2}$$

by using the non-dimensional variable $x=r/R$. These equations (1) and (2) are given with regard to the case of infinitesimally small displacement w, under the assumption that no external load act upon the surface of elastic plate under consideration.

The stress resultant forces and moments are given by;

$$Q_x=-\frac{D}{R}\frac{\partial}{\partial x}(\Delta w), \qquad Q_t=-\frac{D}{R}\frac{\partial}{x\partial\theta}(\Delta w),$$

$$M_r=-\frac{D}{R^2}\left[\frac{\partial^2 w}{\partial x^2}+\nu\left(\frac{1}{x}\frac{\partial w}{\partial x}+\frac{1}{x^2}\frac{\partial}{x\partial\theta}\right)\right]$$

$$M_t=-\frac{D}{R^2}\left[\frac{1}{x}\frac{\partial w}{\partial x}+\frac{1}{x^2}\frac{\partial^2 w}{\partial\theta^2}+\nu\frac{\partial^2 w}{\partial x^2}\right]$$

$$M_{rt}=(1-\nu)\frac{D}{R^2}\left[\frac{1}{x}\frac{\partial^2 w}{\partial x\partial\theta}-\frac{1}{x^2}\frac{\partial^2 w}{\partial\theta^2}\right]$$

In order to obtain the solution of our problem, we assume that the following form for the displacement w exist,

$$w=R_0+\sum_{m=1}^{\infty} R_m \cos m\theta \tag{3}$$

where R_0, R_1, $\cdots$ are functions of x. The boundary conditions to be satisfied by displacement w are;

(a) Along $r=R$, $(x=1)$, which is fixed edge line,

$$w=0, \qquad \partial w/\partial x=0$$

(b) Along $r=cR$ $(x=c)$, which also is fixed edge-line

$$w_1=0, \qquad \partial w_1/\partial x=0$$

It is to be noted that, we denote by w_1, the value of w which belongs to the region of $cR \leq r \leq bR$ ($c \leq x \leq b$).

(c) Along the circle $r=bR$ ($x=b$), two functions of x (w and w_1), must be continuous each other, up to derivatives of second order, thus;

$$w=w_1 , \qquad \partial w/\partial x=\partial w_1/\partial x , \qquad \partial^2 w/\partial x^2=\partial^2 w_1/\partial x^2$$

(d) Assuming that, a concentrated load of amount P is acting on a point ($x=b$, $\theta=0$), we have

$$D\left|\frac{\partial}{\partial r}(\Delta w)\right|^{x=b}-D\left|\frac{\partial}{\partial r}(\Delta w_1)\right|^{x=b}=\frac{P}{\pi bR}\left[\frac{1}{2}+\sum_{m=1}^{\infty}\cos m\theta\right]$$

It is to be understood that the r. h. s. of this equation, being used in the sense of distribution (generalized function) of L. Schwartz.

The general solution of our problem is to be given in a form of expression (3), where $R_0, \cdots, R_m, \cdots$ are functions of x. For convenience, we divide the region $c \leq x \leq 1$ into two parts; namely the outer part $b \leq x \leq 1$ and inner part $c \leq x \leq b$.

(A) For outer part

$$w=R_0+\sum_{m=1}^{\infty} R_m \cos m\theta$$

with

$$R_0=A_0+B_0x^2+C_0 \log x+D_0x^2 \log x ,$$
$$R_1=A_1x+B_1x^3+C_1x^{-1}+D_1x \log x ,$$
$$R_m=A_mx^m+B_mx^{-m}+C_mx^{m+2}+D_mx^{-m+2}$$

for $m=2, 3, \cdots$. A_i, B_i, C_i and D_i ($i=0, 1, 2, \cdots$) are numerical constants to be determined by above boundary conditions (a), (b), (c) and (d).

(B) For inner part $c \leq x \leq b$,

$$w_1=S_0+\sum_{m=1}^{\infty} S_m \cos m\theta$$

with

$$S_0=E_0+F_0x^2+G_0 \log x+H_0x^2 \log x$$
$$S_1=E_1x+F_1x^3+G_1x^{-1}+H_1x \log x$$
$$S_m=E_mx^m+F_mx^{-m}+G_mx^{m+2}+H_mx^{-m+2}$$

for $m=2, 3, \cdots$. E_i, F_i and G_i are numerical constants, also to be determined by the boundary conditions.

4. The Solution for the Case of a single Annular Elastic Plate

It being thought to be fundamental to solution of our problem, we present here the analytical study about the case in which there exist only single annular elastic plate, which is under the action of a single load P at the point $(bR, 0)$. When we make calculations for determining unknown constants $A_i, \cdots, E_i, \cdots$, under above boundary conditions (a), (b), (c) and (d), we obtain systems of linear algebraic equations, which may be solved by known method. The result of this process of estimation is, as given below;

(1) The solution for the case of $m=0$

$$A_0=-B_0\,,\qquad B_0=-\frac{1}{2}(C_0+D_0)\,,$$

$$C_0=\frac{\Delta_C}{\Delta}H_0\,,\qquad D_0=\frac{\Delta_D}{\Delta}H_0\,,\qquad G_0=\frac{\Delta_G}{\Delta}H_0\,,$$

$$\frac{4}{bR^3}(D_0-H_0)=\frac{P}{2\pi bRD}$$

wherein we have,

$$\Delta=\begin{vmatrix}(1,1) & (2,1) & (3,1)\\ (1,2) & (2,2) & (3,2)\\ (1,3) & (2,3) & (3,3)\end{vmatrix},\qquad \Delta_C=\begin{vmatrix}X & (2,1) & (3,1)\\ Y & (2,2) & (3,2)\\ Z & (2,3) & (3,3)\end{vmatrix},$$

$$\Delta_D=\begin{vmatrix}(1,1) & X & (3,1)\\ (1,2) & Y & (3,2)\\ (1,3) & Z & (3,3)\end{vmatrix},\qquad \Delta_G=\begin{vmatrix}(1,1) & (2,1) & X\\ (1,2) & (2,2) & Y\\ (1,3) & (2,3) & Z\end{vmatrix}$$

Values of elements (1, 1), etc., are as follows:—

$$(1,1)=\frac{1}{2}(1-b^2)+\log b\,,\qquad (2,1)=\frac{1}{2}(1-b^2)+b^2\log b\,,$$

$$(3,1)=-\frac{1}{2}\left(1-\frac{b^2}{c^2}\right)-\log\frac{b}{c}\,,\qquad (1,2)=\frac{1}{b}-b\,,$$

$$(2,2)=2b\log b\,,\qquad (3,2)=-\frac{1}{b}+\frac{b}{c^2}\,,\qquad (1,3)=-\left(1+\frac{1}{b^2}\right),$$

$$(2,3)=2(1+\log b)\,,\qquad (3,3)=\frac{1}{b^2}+\frac{1}{c^2}\,,$$

$$X=\frac{1}{2}(c^2-b^2)+b^2\log\frac{b}{c}\,,\qquad Y=2b\log\frac{b}{c}\,,\qquad Z=2\left(1+\log\frac{b}{c}\right)$$

The value of w at $r=bR$ $(x=b)$, is given by

$$w_b=\frac{\Delta_C}{\Delta}\left(\frac{1}{2}-\frac{1}{2}b^2+\log b\right)H_0+\frac{\Delta_D}{\Delta}\left(\frac{1}{2}-\frac{1}{2}b^2+b^2\log b\right)H_0$$

(2) The solution for the case of $m=1$.

For the case in which $m=1$, we have;

$$A_1=-2C_1+\frac{1}{2}D_1\,,\qquad B_1=C_1-\frac{1}{2}D_1\,,\qquad C_1=\frac{\Delta_C}{\Delta}H_1\,,$$

$$D_1=\frac{\Delta_D}{\Delta}H_1, \qquad F_1=\frac{1}{c^4}G_1-\frac{1}{2c^2}H_1, \qquad G_1=\frac{\Delta_G}{\Delta}H_1$$

Δ, Δ_C, Δ_D and Δ_G are determinants as mentioned above, but their elements have following values:—

$$(1,1)=\frac{1}{b}(b^4-2b^2+1), \qquad (2,1)=\frac{1}{b}\left(\frac{1}{2}-\frac{1}{2}b^4+b^2\log b\right),$$

$$(3,1)=-\frac{1}{b}\left(-2\frac{b^2}{c^2}+\frac{b^4}{c^4}+1\right), \qquad (1,2)=\frac{1}{b^2}(3b^4-2b^2-1),$$

$$(2,2)=\frac{3}{2}-\frac{3}{2}b^2+\log b, \qquad (3,2)=-\frac{1}{b^2}\left(-1+3\frac{b^4}{c^4}-2\frac{b^2}{c^2}\right),$$

$$(1,3)=\frac{2}{b^3}(3b^4+1), \qquad (2,3)=\frac{1}{b}(1-3b^2),$$

$$(3,3)=-\frac{2}{b^3}\left(3\frac{b^4}{c^4}+1\right), \qquad X=\frac{1}{b}\left(1-\frac{b^2}{c^2}\right)+b\log\frac{b}{c},$$

$$Y=\frac{3}{2}\left(1-\frac{b^2}{c^2}\right)+\log\frac{b}{c}, \qquad Z=-\frac{3b}{c^2}+\frac{1}{b}$$

Also, we have

$$(B_1-F_1)-\frac{2}{b^2}(D_1-H_1)=\frac{R^2P}{8\pi bD},$$

$$w_b \text{ (for part of } m=1)=\left(\frac{1}{2}-\frac{1}{2}b^2+\log b\right)H_1$$
$$+\frac{\Delta_D}{\Delta}\left(\frac{1}{2}-\frac{1}{2}b^2+b^2\log b\right)H_1$$

(3) The solution for the case in which $2\leq m$.

For the case in which the index number m is equal to, or greater than 2, we have;

$$A_m=-\frac{m+1}{m}C_m-\frac{1}{m}D_m, \qquad B_m=\frac{1}{m}C_m-\frac{m-1}{m}D_m$$

$$F_m=\frac{1}{m}c^{2m+2}G_m-\frac{m-1}{m}c^2H_m, \qquad E_m=-\frac{m+1}{m}c^2G_m-\frac{1}{m}c^{-2m+2}H_m$$

$$C_m=\frac{\Delta_C}{\Delta}H_m, \qquad D_m=\frac{\Delta_D}{\Delta}H_m, \qquad G_m=\frac{\Delta_G}{\Delta}H_m$$

The elements $(1,1)$, $\cdots$, of determinants Δ, Δ_C, $\cdots$ have following values:

$$(1,1)=-\frac{m+1}{m}b^m+\frac{1}{m}b^{-m}+b^{m+2}$$

$$(2,1)=-\frac{1}{m}b^m-\frac{m-1}{m}b^{-m}+b^{-m+2}$$

$$(3,1)=\frac{m+1}{m}b^mc^2-\frac{1}{m}c^{2m+2}b^{-m}-b^{m+2}$$

$$(1, 2)=-(m+1)b^{m-1}-b^{-m-1}+(m+2)b^{m+1}$$
$$(2, 2)=-b^{m-1}+(m-1)b^{-m-1}-(m-2)b^{-m+1}$$
$$(3, 2)=(m+1)c^2b^{m-1}+c^{2m+2}b^{-m-1}-(m+2)b^{m+1}$$
$$(1, 3)=-(m-1)(m+1)b^{m-2}+(m+1)b^{-m-2}+(m+2)(m-1)b^{m}$$
$$(2, 3)=-(m-1)b^{m-2}-(m+1)(m-1)b^{-m-2}+(m-2)(m-1)b^{-m}$$
$$(3, 3)=(m-1)(m+1)b^{m-2}c^2-(m+1)b^{-m-2}c^{2m+2}-(m+2)(m+1)b^{m}$$
$$X=-\frac{1}{m}c^{-2m+2}b^{m}-\frac{m-1}{m}c^2b^{-m}+b^{-m+2}$$
$$Y=-b^{m-1}c^{-2m+2}+(m-1)b^{-m-1}c^2-(m-2)b^{-m+1}$$
$$Z=-(m-1)b^{m-2}c^{-2m+2}-(m+1)(m-1)b^{-m+2}c^2+(m-2)(m-1)b^{-m}$$

Also, we have,

$$4m(m+1)b^{m-1}(C_m-G_m)+4m(m-1)(D_m-H_m)=\frac{R^2P}{\pi bD},$$

$$w_b \text{ (part for } m)=\frac{\Delta_C}{\Delta}\left(-\frac{m+1}{m}b^{m}+\frac{1}{m}b^{-m}+b^{m+2}\right)H_m$$
$$+\frac{\Delta_D}{\Delta}\left(-b^{m}-\frac{m-1}{m}b^{-m}+b^{-m+2}\right)H_m$$

(4) Bending moments M_r acting along the edge-lines.

Bending moment M_r is given by the formula,

$$M_r=-\frac{D}{R^2}\left[\frac{\partial^2 w}{\partial x^2}+\nu\left(\frac{1}{x}\frac{\partial w}{\partial x}+\frac{1}{x^2}\frac{\partial^2 w}{\partial \theta^2}\right)\right]$$

but we notice that the term containing the factor ν vanishes at both boundaries. Thus, we have;

For $m=0$,

at outer boundary,

$$M_r=-\frac{D}{R^2}[2B_0-C_0+3D_0]$$

at inner boundary,

$$M_r=-\frac{D}{R^2}\left[2F_0-\frac{1}{c^2}G_0+H_0(3+2\log c)\right]$$

For $m=1$,

at outer boundary,

$$M_r=-\frac{D}{R^2}[6B_1+2C_1+D_1]$$

at inner boundary,

$$M_r=-\frac{D}{R^2}\left[6cF_1+\frac{2}{c^3}G_1+\frac{1}{c}H_1\right]$$

For $2 \leq m$,
at outer boundary,

$$-\frac{R^2}{D}M_r = m(m-1)A_m + m(m+1)B_m + (m+2)(m+1)C_m + (m-2)(m-1)D_m$$

at inner boundary,

$$-\frac{R^2}{D}M_r = m(m-1)c^{m-2}E_m + m(m+1)c^{-m-2}F_m + (m+2)(m+1)c^m G_m + (m-2)(m-1)c^{-m}H_m$$

(5) Case of a number N of bars which are arranged equi-distantly along the circle $r = bR$.

For this case we can obtain analytical expressions, by linear combination of above mentioned solution for a single bar, thus;

$$w_s = \sum_{i=1}^{N}\left[\sum_{m=1}^{\infty} R_m \cos\left\{m\left(\theta - \frac{2\pi i}{N}\right)\right\}\right]$$

Nothing that, for $1 \leq m$ we have

$$\sum_{i=1}^{N} \cos\left(\frac{m}{N}\cdot \pi i\right) = N \qquad \text{if } m = Ns,$$
$$= 0 \qquad \text{if } m \neq Ns,$$

we observe that, for the case of a number N of bars,

$$w_s = NR_0 + \Sigma NR_m \cos m\theta$$

wherein we take, $m = Ns$ $(s = 1, 2, 3, \cdots)$.

5. The Case of two Annular Elastic Plates which are interconnected by a number N of Elastic Bars

After these preliminary considerations have been given, we proceed to the case of two annular elastic plates, which are interconnected each other by a number N of elastic rods. Among the various possibilities of mode of interaction between the bars and plates, we take the case in which elastic bars suffer thermal expansion, due to external heating. The axial force will be given by

$$P_B = \alpha + \beta \Delta w_b$$

in which we put

$$\alpha = \lambda \vartheta E_B A_B, \qquad \beta = E_B A_B / l$$

A_B = cross sectional area of the bar
E_B = its Young's modulus

ϑ=temperature rise
λ=coefficient of thermal expansion
l=initial length of the bar
Δw_b=difference of displacements of top and bottom plates, at the point of connection to the bar

In order to distinguish between two annular elastic plates, we use index notations. Thus $w^{(1)}$ refers to displacement of the upper plate, while $w^{(2)}$ represents the value for lower plate. Coefficients A_i, B_i, $\cdots$ are also written $A_i^{(k)}$, $B_i^{(k)}$, $\cdots$ (k=1 for upper, k=2 for lower, plates) to discriminate among them. The thrust forces which act upon the plates are, thus, expressed by;

$$P^{(k)}=(-)^k[\alpha+\beta\{w_b^{(1)}-w_b^{(2)}\}] \quad (k=1, 2)$$

Using these expressions, and also making use of expressions obtained in previous sections, and which apply to upper and lower annular elastic plates, we arrive at the following expressions;

$$S_0[H_0^{(1)}-H_0^{(2)}]=S_1[H_1^{(1)}-H_1^{(2)}]=\cdots=S_m[H_m^{(1)}-H_m^{(2)}] \quad (2\leq m)$$
$$=-\frac{R^2}{\pi b}\left[\frac{1}{D^{(1)}}+\frac{1}{D^{(2)}}\right]\cdot[\alpha+\beta\{w_b^{(1)}-w_b^{(2)}\}]$$

where we have put,

$$S_0=\frac{8}{b}\left[\frac{\Delta_D}{\Delta}-1\right], \qquad S_1=8\frac{\Delta_C}{\Delta}-\left[4+\frac{2}{b^2}\right]\frac{\Delta_D}{\Delta}-\frac{8}{c^4}\frac{\Delta_G}{\Delta}$$
$$S_m=4m(m+1)b^{m-1}\left[\frac{\Delta_C}{\Delta}-\frac{\Delta_G}{\Delta}\right]+4m(m-1)b^{-m-1}\left[\frac{\Delta_D}{\Delta}-1\right]$$

On the other hand, we have, for the value of difference of displacements $w_b^{(1)}-w_b^{(2)}$ at the location of elastic bar,

$$\{w_b^{(1)}-w_b^{(2)}\}=K_0[H_0^{(1)}-H_0^{(2)}]+K_1[H_0^{(1)}-H_1^{(2)}]+\cdots+K_m[H_m^{(1)}-H_m^{(2)}]+\cdots,$$

in which we have put,

$$K_0=\frac{\Delta_C}{\Delta}\left(\frac{1}{2}-\frac{1}{2}b^2+\log b\right)$$
$$K_1=\left[-2b+b^3+\frac{1}{b}\right]\frac{\Delta_C}{\Delta}+\left[\frac{1}{2}b-\frac{1}{2}b^3+b\log b\right]\frac{\Delta_D}{\Delta}$$
$$K_m=\left[-\frac{m+1}{m}b^m+\frac{1}{m}b^{-m}+b^{m+2}\right]\frac{\Delta_C}{\Delta}$$
$$+\left[-b^m-\frac{m-1}{m}b^{-m}+b^{-m+2}\right]\frac{\Delta_D}{\Delta} \quad (2\leq m)$$

Combining these results, we arrive at the following equation;

$$\{w_b^{(1)}-w_b^{(2)}\}=-U(HZ)[\alpha+\beta\{w_b^{(1)}-w_b^{(2)}\}]$$

where we put, for shortness,

$$U=\frac{R^2N}{\pi bD^{(1)}}\left[\sum_{m=0}^{\infty}(K_m/S_m)\right], \qquad Z=D^{(1)}/D^{(2)}$$

Thus finally, we obtain the following value of $\{w_b^{(1)}-w_b^{(2)}\}$ which is difference of displacements of the elastic bar at both ends, in the following form;

$$\{w_b^{(1)}-w_b^{(2)}\}=\frac{-\alpha(1+Z)U}{1+\beta(1+Z)U},$$

and consequently,

$$\alpha+\beta\{w_b^{(1)}-w_b^{(2)}\}=\frac{\alpha}{1+\beta(1+Z)U}=J_Z, \quad \text{say.}$$

It may be noted that, r.h.s. of these formulae are expressed in terms of known constants, b, c, R, $D^{(k)}$, etc. It is also to be noted that, for the case of a number N of elastic bars arranged along the circumference of a circle of radius bR $(x=b)$, we have to pick up solely those terms which is such that $m=Ns$ $(s=0,1,2,\cdots)$.

6. Approximate Formulae, when the index *m* is a large Whole Number

When the whole number m is fairly large, so that we may neglect b^m in comparison with b^{-m}, it is found that the following approximate (abridged) formulae may be used conveniently.

Thus, we have

$$X=X_1+X_2, \qquad Y=Y_1+Y_2, \qquad Z=Z_1+Z_2$$

where we have for part 1 item,

$$X_1=\left[b^2-\left(\frac{m-1}{m}\right)c^2\right]b^{-m}$$

$$Y_1=\frac{1}{b}[-(m-2)b^2+(m-1)c^2]b^{-m}$$

$$Z_1=\frac{1}{b^2}[(m-2)(m-1)b^2-(m+1)(m-1)c^2]b^{-m}$$

Thus term (X_1, Y_1, Z_1) is of rather secondary importance.

$$X_2=-\frac{b}{m}\left(\frac{b}{c^2}\right)^{m-1}, \qquad Y_2=-\left(\frac{b}{c^2}\right)^{m-1}$$

$$Z_2=-(m-1)\frac{1}{b}\left(\frac{b}{c^2}\right)^{m-1}$$

This term (part 2 item) is of primary importance. The following approximative expressions were also obtained.

(a) For part 1 and part 2, commouly

$$\Delta \doteqdot -4b^2[(c^2+b^2)+(2m-1)(b^2-c^2)]$$

(b) For part 1,

$$\Delta_D \doteqdot [-8b^2\{mb^2-(m-1)c^2\}]\cdot\left(\frac{m+1}{m}\right)[b^{-m-3}]$$

$$\Delta_C \doteqdot \left[-\left(\frac{m}{m+1}\right)(m^2+2)b^4+b^2c^2(m^2-m+2)+(m+1)(m+2)(b^2-c^2)\right]$$

$$\Delta_G \doteqdot 0$$

(c) For part 2,

$$\Delta_C \doteqdot [-8b^2\{1+m(1-b^2)\}]\left(\frac{m-1}{m}\right)\cdot\left[-\left(\frac{b}{c^2}\right)^{m-1}\right]\left(\frac{1}{b^2}\right)$$

$$\Delta_D \doteqdot [-8b^2\{1+m(1-b^2)\}]\left(\frac{m-1}{m}\right)\cdot\left[-\left(\frac{b}{c^2}\right)^{m-1}\right]\left[\frac{1}{b^2}\right]$$

$$\Delta_G \doteqdot [+8b^2]\left(\frac{m-1}{m}\right)\left[-\left(\frac{b}{c^2}\right)^{m-1}\right][b^{-2m-2}]$$

Thus, we have for the second (most important) part,

$$\frac{\Delta_C}{\Delta} \doteqdot \left(\frac{m-1}{m+1}\right)\left[\left(\frac{b^2}{c^2}\right)^{m-1}\left[\frac{-\Lambda}{4\Omega}\right]\right]$$

$$\frac{\Delta_D}{\Delta} \doteqdot \left[\frac{-2b^2}{\Omega}\right]\left[\left(\frac{b^2}{c^2}\right)^{m-1}\right]$$

$$\frac{\Delta_G}{\Delta} \doteqdot \left(\frac{m-1}{m}\right)\left(\frac{2b^4}{\Omega}\right)\left[\left(\frac{b^2}{c^2}\right)^{m-1}\right][b^{-2m-2}],$$

where we have put, for shortness,

$$\Omega=(b^2+c^2)+(2m-1)(b^2-c^2)$$

$$\Lambda=8b^2[1+m(1-b^2)]$$

7. Case of an Infinite Strip of Rectangular Elastic Plates

When we make the radius R increase to infinity, while keeping the breadth $B_s=(1-c)R$ at a constant value, we arrive at the case of an infinite strip of rectangular (elastic) flat plate. Here, we shall try to treat the case of infinite rectangular plate, directly from an analysis in terms of rectangular coordinates.

We take up the case of a rectangular elastic plate of breadth a and of infinite length, as shown in Fig 2(a). We assume that, along the straight line $y=b$, concentrated transverse loads P are applied, which are spaced by the distance $2L$, successively. Upper edge $(y=a)$ and lower edge $(y=0)$ are supposed to be fixed to rigid walls (in state of clamped edges).

Moreover, we consider two such rectangular elastic plates of infinite lengths

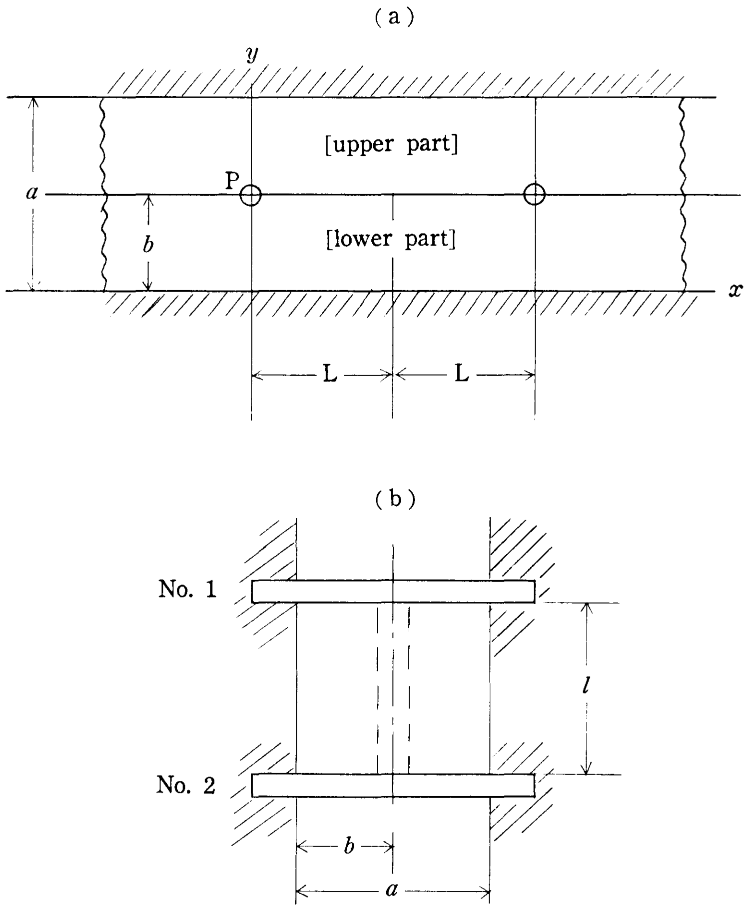

Fig. 2. Elastic Two Flat Plates which are interconnected each other by several elastic bars.

are placed, one upon another, keeping at a distance l between them, as shown in Fig. 2(b).

Firstly, we shall consider the case of single flat plate of elastic material. The equation of such a plate is given by

$$\Delta\Delta w \equiv \left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2 w=0$$

using the rectangular coordinates (x, y). In this equation w is, as before, the transverse displacement (which is considered to be infinitesimally small). The boundary conditions to be satisfied by this displacement w are;

(a) Along the upper edge line $y=a$

$$w=0\,, \qquad \partial w/\partial y=0$$

(b) Along the lower edge line $y=0$

$$w_1=0\,, \qquad \partial w_1/\partial y=0$$

where w_1 means the value of w for a point in lower region, that is $0\leq y\leq b$, of the flat plate.

(c) Along the line $y=b$, along which applied (concentrated) loads P are arranged in a row,

$$w=w_1\,, \qquad \partial w/\partial y=\partial w_1/\partial y\,, \qquad \partial^2 w/\partial y^2=\partial^2 w_1/\partial y^2\,.$$

(d) To represent the action of the row of forces P for $y=b$,

$$|Q_y|^{y=b}-|Q_{1y}|^{y=b}=Q_b$$

or

$$D\left[\frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)\right]^{y=b}-D\left[\frac{\partial}{\partial y}\left(\frac{\partial^2 w_1}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)^{y=b}=Q_b$$

$$=\frac{P}{L}\left[\frac{1}{2}+\sum_{m=1}^{\infty}\cos\frac{m\pi x}{L}\right]$$

For the solution of this problem, we put (1) for upper region

$$Y_0=A_0+B_0 y+C_0 y^2+D_0 y^3$$

$$Y_m=A_m\operatorname{sh}\frac{m\pi y}{L}+B_m\operatorname{ch}\frac{m\pi y}{L}+C_m\frac{m\pi y}{L}\operatorname{sh}\frac{m\pi y}{L}+D_m\frac{m\pi y}{L}\operatorname{ch}\frac{m\pi y}{L} \quad (1\leq m)$$

and (2) for the lower region

$$Y_0=E_0+F_0 y+G_0 y^2+H_0 y^3$$

$$Y_m=E_m\operatorname{sh}\frac{m\pi y}{L}+F_m\operatorname{ch}\frac{m\pi y}{L}+G_m\frac{m\pi y}{L}\operatorname{sh}\frac{m\pi y}{L}+H_m\frac{m\pi y}{L}\operatorname{ch}\frac{m\pi y}{L} \quad (1\leq m)$$

$$w(w_1)=\sum_{m=1}^{\infty}Y_m\cos\frac{m\pi x}{L}$$

For the case of two elastic flat plates which are interconnected each other, as shown in Fig. 2(b), we shall use indices such as $A_m^{(k)}$ ($k=1, 2$), in order to discriminate between them.

Solving a system of linear algebraic equations, about unknown constants A_m, B_m, $\cdots$, we obtain values of constants A_m, B_m, etc., which are shown below.

$$A_0=-\frac{1}{3}b^3 S_0\,, \qquad B_0=b^2 S_0\,, \qquad C_0=b\left[\left(\frac{a-b}{a}\right)^2-1\right]S_0$$

$$D_0=\frac{1}{3}\left[1-\left(\frac{a-b}{a}\right)^2\left\{1+2\left(\frac{b}{a}\right)\right\}\right]S_0\,, \qquad E_0=0\,, \qquad F_0=0$$

$$G_0=b\left(\frac{a-b}{a}\right)^2 S_0\,, \qquad H_0=-\frac{1}{3}\left(\frac{a-b}{a}\right)\cdot\left\{1+2\left(\frac{b}{a}\right)\right\}S_0$$

where we put

$$S_0=\frac{P}{4DL}=S/L\,; \qquad S=\frac{PL}{4D}\ \text{(non-dimensional factor)}$$

Also, for $1\leq m$,

$$A_m=E_m\left[1-\frac{m\pi b}{L}\tanh\frac{m\pi b}{L}\right]\text{ch}\,\frac{m\pi b}{L}\cdot S_m$$

$$B_m=-\left[\frac{m\pi b}{L}-\tanh\frac{m\pi b}{L}\right]\text{ch}\,\frac{m\pi b}{L}\cdot S_m$$

$$C_m=G_m-\text{sh}\,\frac{m\pi b}{L}\cdot S_m\,, \qquad D_m=H_m+\text{ch}\,\frac{m\pi b}{L}\cdot S_m$$

$$E_m=-H_m\,, \qquad F_m=0$$

$$E_m=S\begin{vmatrix}C_p & B_p\\ F_p & E_p\end{vmatrix}\div\begin{vmatrix}A_p & B_p\\ D_p & E_p\end{vmatrix}, \qquad G_m=S_m\begin{vmatrix}A_p & C_p\\ D_p & E_p\end{vmatrix}\div\begin{vmatrix}A_p & B_p\\ D_p & E_p\end{vmatrix}$$

where we have put

$$S_m=\frac{P}{2LD}\left(\frac{L}{m\pi}\right)^3=\frac{2L}{(m\pi)^3}S$$

$$A_p=\tanh\frac{m\pi a}{L}-\left(\frac{m\pi a}{L}\right), \qquad B_p=\frac{m\pi a}{L}\tanh\frac{m\pi a}{L}$$

$$C_p=\tanh\frac{m\pi a}{L}\left[1+\frac{m\pi(a-b)}{L}\tanh\frac{m\pi b}{L}\right]X_m$$

$$-\left[\frac{m\pi(a-b)}{L}+\tanh\frac{m\pi b}{L}\right]X_m$$

$$D_p=-B_p\,, \qquad E_p=\tanh\frac{m\pi a}{L}+\frac{m\pi a}{L}$$

$$F_p=\frac{m\pi(a-b)}{L}\left[\tanh\frac{m\pi b}{L}-\tanh\frac{m\pi a}{L}\right]X_m$$

$$X_m=\text{ch}\,\frac{m\pi b}{L}$$

Following concise form, for the values of above written determinants, may be found to be convenient. Thus, using the notation,

$$T_m(a)=\tanh\frac{m\pi a}{L}\,, \qquad T_m(b)=\tanh\frac{m\pi b}{L}$$

we have;

$$\begin{vmatrix}A_p & B_p\\ D_p & E_p\end{vmatrix}=[T_m(a)]^2-\left(\frac{m\pi a}{L}\right)^2\{1-(T_m(a))^2\}$$

$$\begin{vmatrix} C_p & B_p \\ F_p & E_p \end{vmatrix} = [T_m(a)]^2\left[1+\left(\frac{m\pi a}{L}\right)\left(\frac{m\pi(a-b)}{L}\right)+\frac{m\pi(a-b)}{L}(T_m(b))\right]X_m$$

$$+[T_m(a)]\left[\frac{m\pi b}{L}-T_m(b)\right]X_m$$

$$-\left(\frac{m\pi a}{L}\right)\left[\frac{m\pi(a-b)}{L}+T_m(b)\right]X_m$$

$$\begin{vmatrix} A_p & C_p \\ D_p & F_p \end{vmatrix} = [T_m(a)]^2\left[\frac{m\pi b}{L}+\left(\frac{m\pi a}{L}\right)\left(\frac{m\pi(a-b)}{L}\right)\cdot T_m(b)\right]X_m$$

$$-[T_m(a)]\left[\frac{m\pi b}{L}T_m(b)\right]X_m$$

$$-\left(\frac{m\pi a}{L}\right)\left(\frac{m\pi(a-b)}{L}\right)T_m(b)X_m$$

Also, for a sufficiently large value of integer m, we may put approximately,

$$T_m(a) \doteqdot 1-2t_m(a)\,, \qquad T_m(b) \doteqdot 1-2t_m(b)\,,$$

$$t_m(a)=\exp\left[-\frac{2m\pi a}{L}\right], \qquad t_m(b)=\exp\left[-\frac{2m\pi b}{L}\right].$$

For such case, we may put,

$$\begin{vmatrix} A_p & B_p \\ D_p & E_p \end{vmatrix} \doteqdot 1-4\left[1+\left(\frac{m\pi a}{L}\right)^2\right]t_m(a)+4\{t_m(a)\}^2\left[1+\left(\frac{m\pi a}{L}\right)^2\right]$$

$$\begin{vmatrix} C_p & B_p \\ F_p & E_p \end{vmatrix} \doteqdot -2t_m(a)\left[1+3\left(\frac{m\pi(a-b)}{L}\right)\right.$$

$$\left.+\left(\frac{m\pi b}{L}\right)+2\left(\frac{m\pi a}{L}\right)\left(\frac{m\pi(a-b)}{L}\right)\right]X_m$$

$$+2t_m(b)\left[1+\frac{m\pi a}{L}\right]X_m$$

$$\begin{vmatrix} A_p & C_p \\ D_p & F_p \end{vmatrix} \doteqdot [-2t_m(a)]\left[2\left(\frac{m\pi a}{L}\right)\left(\frac{m\pi(a-b)}{L}\right)+\left(\frac{m\pi b}{L}\right)\right]X_m$$

$$+[2t_m(b)]\left[\frac{m\pi b}{L}\right]X_m$$

The bending moment M_y is given by the formula

$$M_y=-D\left[\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right]$$

Its value for the edge line $y=0$ is biven by

$$M_y \text{ (part for } m=0)=-2b\left(\frac{a-b}{a}\right)^2 DS_0$$

$$M_y \text{ (part for } m)=-\left(\frac{m\pi}{L}\right)^2 2D\cos\frac{m\pi x}{L}$$

Bending moment M_y along the upper edge line $y=a$ is also to be obtained. But it may, other-wise, be obtained from the above formula for the case of $y=0$, provided that we make interchange of b and $(a-b)$, about all expressions comprised therein.

8. Case of two infinite Strip of Rectangular Elastic Plates, which are interconnected each other by a number of Elastic Bars

For the case in which there exist two infinite strips of rectangular elastic plates which are interconnected each other by a row of elastic straight bars (as shown in Fig. 2(b)), we can make, about analytical study of stressed state, in quite similar way as for the case of two annular elastic plates, which was mentioned above. We shall, in what follows, give brief description about it.

We shall express the displacements of two elastic plate at points on the line $x=b$ (location of the row of bars), in following form;

$$w_b^{(k)}=S_0^{(k)}U_0+\sum_{m=1}^{\infty}S_m^{(k)}U_m \quad (k=1,2)$$

in which we put,

$$U_0=(G_0/S_0)b^2+(H_0/S_0)b^3$$

$$U_m=\frac{E_m}{S_m}\,\mathrm{sh}\,\frac{m\pi b}{L}+\frac{G_m}{S_m}\,\frac{m\pi b}{L}\,\mathrm{sh}\,\frac{m\pi b}{L}+\frac{H_m}{S_m}\,\frac{m\pi b}{L}\,\mathrm{ch}\,\frac{m\pi b}{L}$$

On the other hand, we have

$$S_0^{(k)}=\frac{P^{(k)}}{4D^{(k)}L}=\frac{(-)^k}{4D^{(k)}L}[\alpha+\beta\{w_b^{(1)}-w_b^{(2)}\}]$$

$$S_m^{(k)}=\frac{P^{(k)}}{4D^{(k)}}\left[\frac{2L}{(m\pi)^3}\right]=\frac{(-)^k}{4D^{(k)}}\left[\frac{2L}{(m\pi)^3}\right][\alpha+\beta\{w_b^{(1)}-w_b^{(2)}\}]$$

Combining these results, we obtain

$$[w_b^{(1)}-w_b^{(2)}]=-[\alpha+\beta\{w_b^{(1)}-w_b^{(2)}\}]U_f(1+Z)$$

in which we have put

$$U_f=\frac{1}{4LD^{(1)}}\left[U_0+\sum_{m=1}^{\infty}\left(\frac{2L^3}{(m\pi)^3}\right)U_m\right]$$

$$Z=D^{(1)}/D^{(2)}$$

And, we have, finally

$$\{w_b^{(1)}-w_b^{(2)}\}=\frac{-\alpha(1+Z)U_f}{1+\beta(1+Z)U_f} \quad \cdots\cdots\cdots\cdots(\mathrm{a})$$

and also,

$$\alpha+\beta\{w_b^{(1)}-w_b^{(2)}\}=\frac{\alpha}{1+\beta(1+Z)U_f}=J_{fZ}\,,\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\text{(b)}$$

say.

The equation (a) gives us the value of difference of displacements (at both ends) of elastic bar, as combined effect of thermal expansion and elastic strain. The equation (b) gives us the absolute value of bar-thrust, caused by the deformation given by eq. (a).

9. Numerical Examles

In order to explain the results of our analytical study, we shall take up following two numerical cases, namely.

Case A; $c=0.500$, $b=0.750$

Case B; $c=0.700$, $b=0.850$

Furthermore, to fix ideas, we shall take, for numerical values of R, l, h, etc., the following specific values;

(a) About the annular elastic plate,

Outer radius $R=50$ cm, Thickness $h=1.00$ cm,

Young's modulus of its material $E=2.0\times10^6$ kg/cm^2.

(b) About the elastic straight rod, we take it to be made up of a hollow steel tube, having sectional area of $A_b=4.805$ cm (pipe dia$=5.10$ cm, wall thickness $=0.32$ cm), $E_b=2.0\times10^6$ kg/cm^2, coefficient of thermal expansion $\lambda=1.23\times10^{-5}$/°C, temperature rise $\vartheta=300$°C.

From these fundamental values, it follows that we have

$D^{(1)}=0.183150\times10^6$ kg cm, $\alpha=35.40090\times10^3$,

$\beta=6.40667\times10^4$

Using these values, we made numerical evaluations about results of our analytical study. Firstly, we obtained values of the thrust force P, which give rise to pushing action of elastic rods upon the annular elastic plates, caused by thermal expansion of elastic rods due to external heating. They are shown as graphs in Fig. 3 (for the case A) and Fig. 4 (for the case B).

From these graphs, we may observe, in what manner the value of thrust force P is affected by the ratio $Z=D^{(1)}/D^{(2)}$ of flexural rigidities of top and bottom bed plates. The case of $Z=0$ corresponds to the case in which the bottom bedplate consist of rigid material and such that $D^{(2)}\to\infty$, and the value of P is the largest. The value of thrust P decreases as the value of Z is increased. If we make $Z=\infty$, that is, if the bottom plate is very soft or non-existent, we shall have $P=0$. The dotted lines in Figs. 3 and 4 shows us the value of thrust force P_N, which were obtained for the imaginary case in which the effect of elastic bars were uniformly distributed along the circumference of radius $bR=r$, instead of

Case A

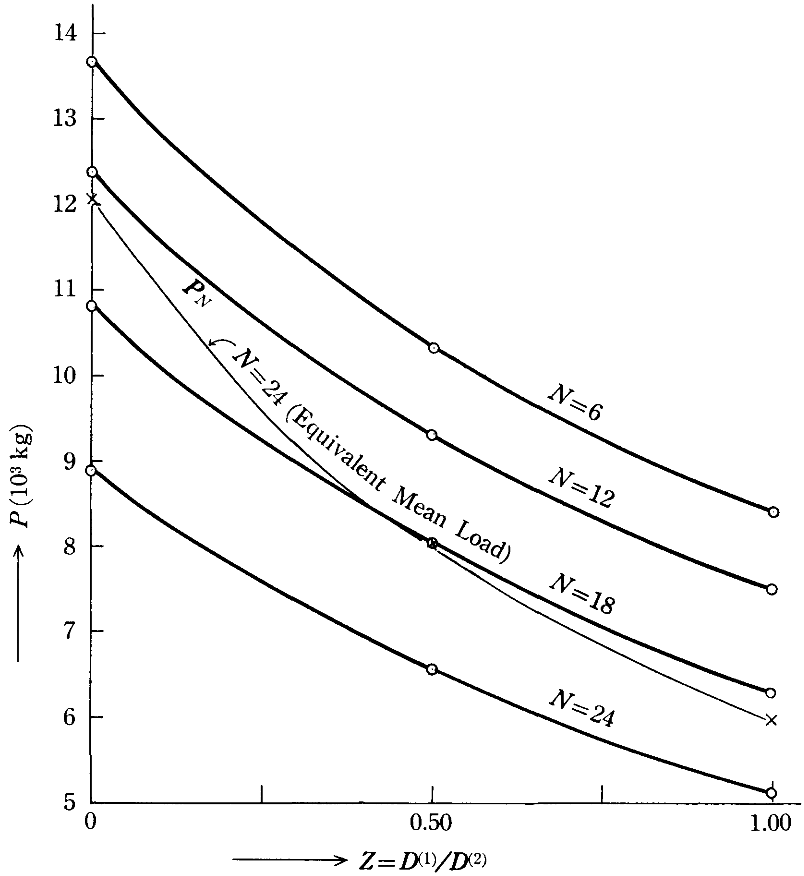

Fig. 3. Value of Acting Force P, which is caused by thermal expansion of Connecting Rods.

acting spotwise, that is, acting in form of a cylindrical supporting wall. The value of this force P_N is given referring to each one panel (of angular space $2\pi/N$), for the case of $N=24$.

Moreover, Fig. 5 shows us the graph of values of thrust force P, for the case of an infinite strip of rectangular elastic plate, which, as previously been noted, may be regarded to represent the case of an annular elastic plate of very large radius, while keeping its breads $a=(1-c)R$ at a given fixed value.

The value of bending moment M_r, which act along boundary lines (both outer and inside) have been evaluated numerically, by using the results of above-mentioned analytical study. The results are shown as graphs in Fig. 6.

In this graph of Fig. 6, the ordinate represents the value of M_r (in cm kg/cm), which is the value of M_r at the angular position of $\theta=0$ (where it attains its maximum value around the circumference). Here, we have shown the cases of annular elastic plates and the case of an infinite strip of rectangular plates simul-

Case B

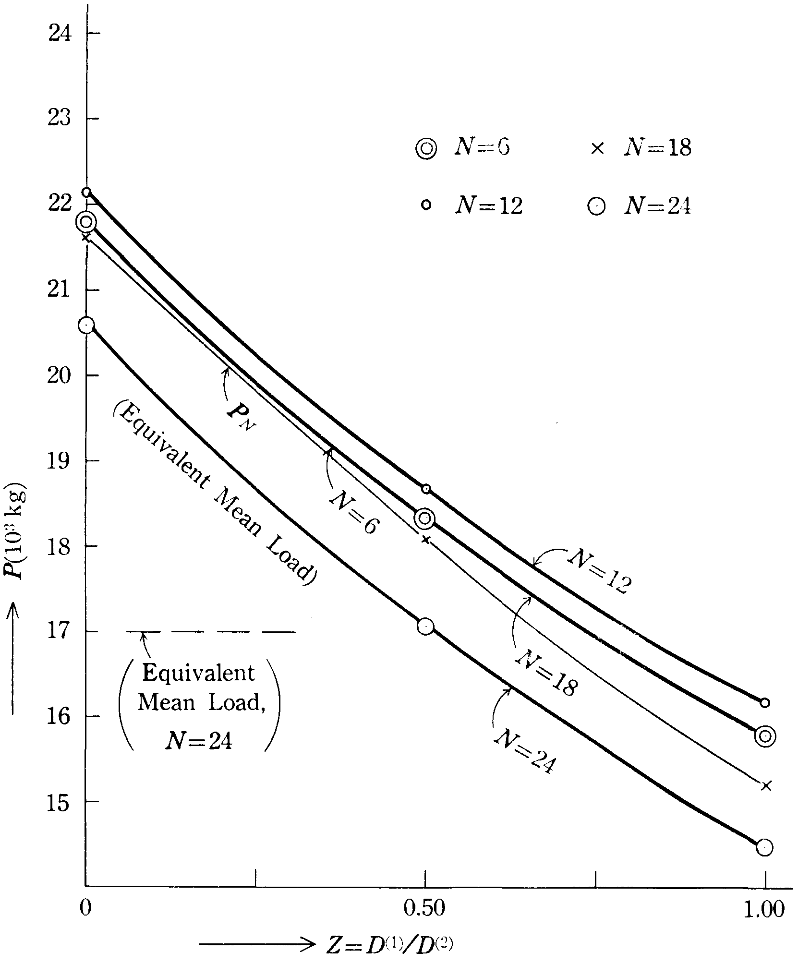

Fig. 4. Value of Acting Force P, which is caused by thermal expansion of Connecting Rods.

taneously in one plane. The abscissa, for the case of an infinite strip of rectangular plate is taken to be $\xi=2L/a$. For the case of annular plate, the abscissa is taken to be

$$\frac{2L}{a}:\frac{2\pi R}{N}\cdot\frac{1}{(1-cR)}=\frac{2\pi}{(1-c)NR}$$

which is done for mere convenience, in order to facilitate comparison with the case of infinite strip of rectangular plates. Moreover, the values of M_r (for the case of annular elastic plates) are connected by smooth curves. This also, is done for mere convenience. Rigorously speaking, the value of M_r (for the case of annular plates), is the function of discrete whole number N, and not a continuous

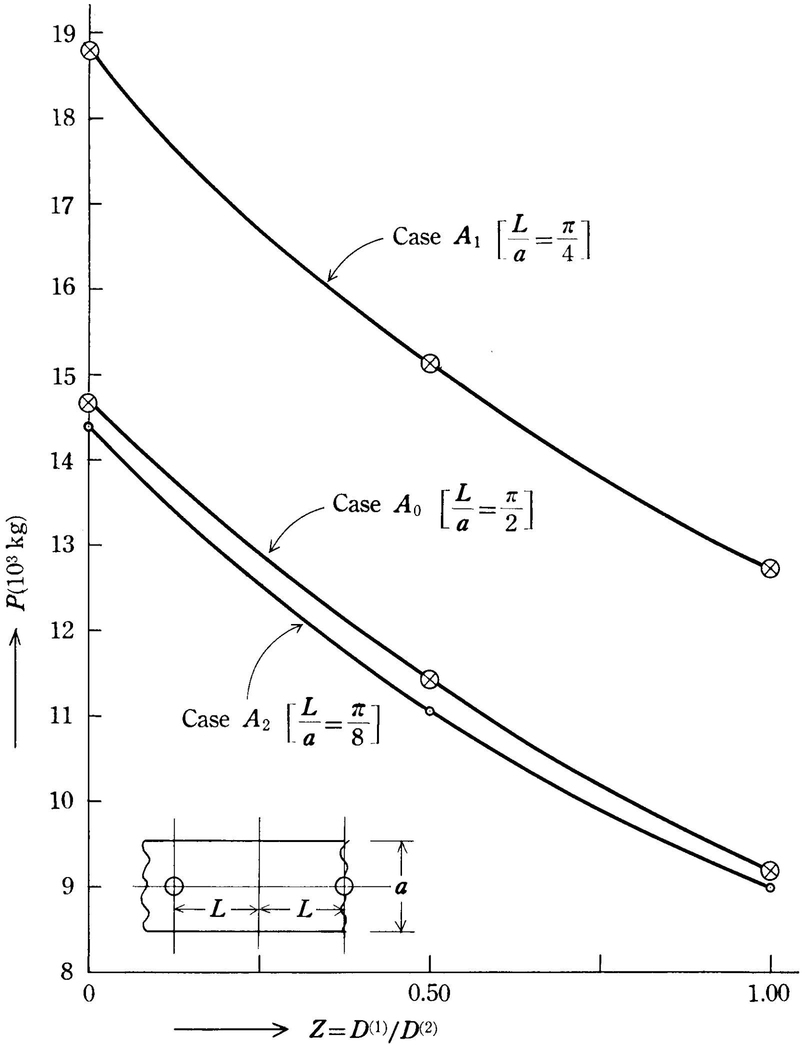

Fig. 5. Value of Acting Force P, which is caused by thermal Expansion of Connecting Rods. [Infinite Strip of Rectangular Plate]

Numerical Values: $a=25$ cm.
We take other dimensions, as the same values for the Case of annular elastic plates.

function of the variable N.

Observing the graph of M_r, as given by Fig. 6, we may make following inferences;

(a) For the range of values of ξ such that $1.50<\xi$ we observe that values of M_r are nearly constant, which means that mutual interference of each neighbouring rod is relatively small. For values of ξ in the range of $0.50<\xi<1.50$ we

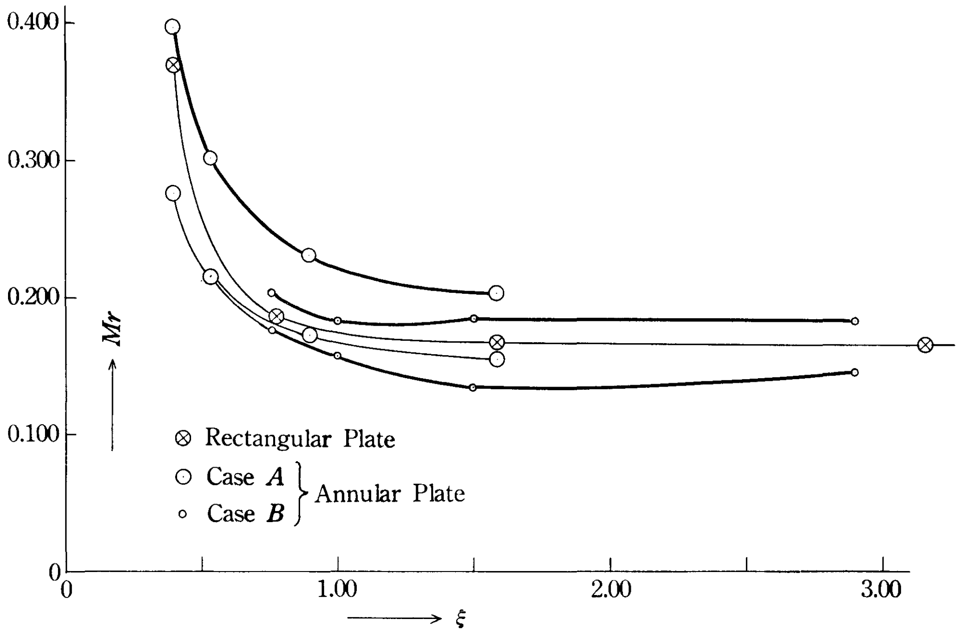

Fig. 6.

$\xi=2L/a$ for the case of infinite strip of rectangular plates
$\xi=2\pi/[(1-c)NR]$ for the Case of annular elastic plates

observe rapid change of values of M_r, when ξ is varied, indicating that the mutual effect of adjacent rod is gradually appearing. Lastly, for the range of $0<\xi<0.50$ the rapid change is seen, and the value of M_r increases approximately in proportion with $1/\xi$ as we make ξ tend to zero. This is the range in which the bars are very close each other, and the value of M_r depend upon the number of bars per cm of circumference.

(b) About the values of bending moment M_r, along the circumferences, for the case of annular elastic plates, we observe that their values along inner edge lines are larger than those along outer edge lines. This fact is in accordance with usual case of stress analysis, along curved boundaries such as in case of curved beams.

(c) As we have mentioned above, when the radius R of an annular elastic plate is very large, while keeping its breadth $(1-c)R$ at a fixed value, we are led to the case of an infinite strip of rectangular elastic plate. So far as we see from Fig. 6, values of M_r for the Case A ($c=0.50$) and Case B ($c=0.70$) are remarkable different from these values of M_r for the case of rectangular elastic plates. We note that, in order that an annular ring may approximately be regarded as an infinite strip of rectangular elastic plate, we must have, roughly speaking, $0.80<c<1.00$. And, this is not the common case which we meet in the formation of once-through boilers, nor in formation of inlet rings of hyraulic turbines.

10. Concluding Remarks

In the present paper, the author has reported the result of his analytical study about the state of stress in elastic plates of annular forms. Especially, we considered two such elastic plates which are interconnected each other by a number of elastic straight rods. When these elastic rods make elongation in axial direction, possibly due to external heating, they will push both top and bottom (annular) elastic plates. Boundary edge-lines of both (inner- and outer) circumferences were assumed to be fixed in state of clamped edge-lines. But we may treat the cases of other modes of fixation, in quite similar manner.

From the view point of analytical theory of elastic bars and plates, we have shown here a case of inter-action between several number of elastic bars and elastic plates, which is caused (for an instance) by the effect of external heating.

REFERENCES

(1) S. TIMOSHENKO: Theory of Plates and Shells, MacGraw-Hill Co., (Section 5), 1940.

(2) F. KITO: Elastic Plates in Form of Circular Ring, under Concentrated Loads, Journ. Coll. Engng., Nihon Univ., (1) Vol. 20, Mar. 1979, (2) Vol. 21, Mar., 1980, (3) Vol. 22, Mar. 1981.