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ON VIBRATION OF TWO CIRCULAR CYLINDERS, WHICH ARE IMMERSED IN A WATER REGION—V

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ABSTRACT

In the previous paper, Reports II—IV, under the same title as the present one, the author has made analytical study (two-dimensional) about vibratory motion of two circular cylinders (of different radii), which are placed in an infinite region of ideal fluid. The present paper is the outcome of continuation of this study, and here we treat the case in which two circular cylinders are situated very close each other, leaving a small gap between them. Our object is to study analytically, about amount of hydrodynamic force for such a case. The result is illustrated by some numerical examples.

1. Introduction

In the previous paper (Reports II to IV) under the same title as the present one, the author has made analytical study about the fluid motion set up in a region of ideal fluid, which extend to infinity, and inside which two circular cylinders of different (or equal) radii are immersed. Taking up the case of two-dimensional potential flow, the author has shown analytical expressions which correspond to the case in which two circular cylinders are making some vibratory motion in prescribed manner. Two circular cylinders were taken to be of different radii, and their motions were not restricted to the case of small motion.

The present report is the outcome of continuation of this study, and here we treat the case in which two circular cylinders are very close each other. Our aim is to study the behavior of fluid motion and hydrodynamic force, when two circular cylinders move each others so close that they nearly collide. The acquired results are illustrated by some numerical examples.

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2. Notations

We use, here again, the following notations which were used by the author in his previous reports, x, y rectangular coordinates of a point in xy plane, $z=x+iy$, a complex variable, c=distance from origin, of radical centers of our system of bi-polar coordinates, h=coefficient of linear element for case of bi-polar coordinates, ξ, η=a system of bi-polaar coordinates, representing any point on xy plane, R_i=radius of circular cylinder ($i=1, 2$), E_i=position of center of ditto, p=fluid pressure, ρ=density of the fluid, ϕ=velocity potential of fluid motion, giving absolute velocity of the flow.

Coefficients A_i, B_i, etc., are used to giving us the solution in form of infinite series.

3. Main Results obtained in previous Reports

In what follows, we shall pick up some results obtained by the author in his previous reports. The analytical results were based on the use of bi-polor coordinates, as shown in Fig. 1.

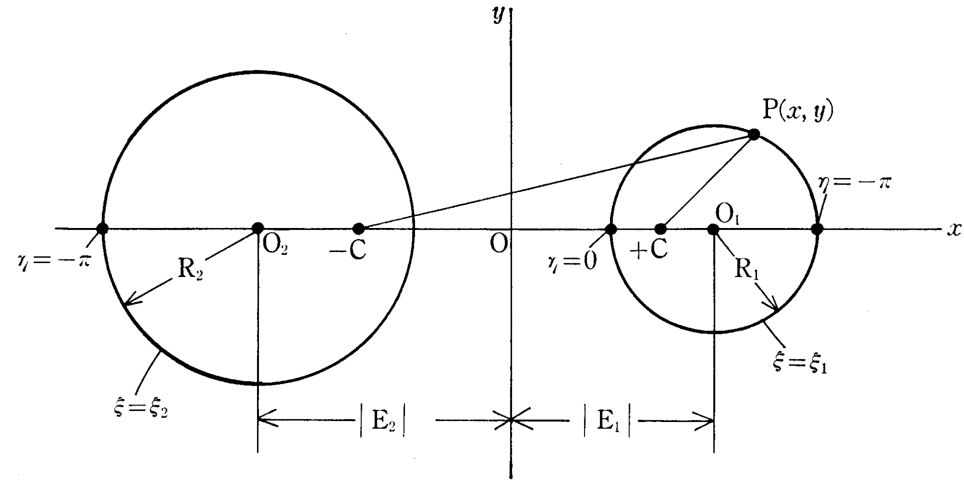

Fig. 1. Configuration of two Circular Cylinders represented by Bipolar Coordinates.

Referring to this Fig. 1, two points $(+c, 0)$ and $(-c, 0)$ lying on the x-axis are taken as radical centers, and we define a system of bi-polar coordinates (ξ, η) by means of the relation

$$\zeta=\xi+i\eta=\log\frac{c+z}{c-z} \tag{1}$$

wherein we put $z=x+iy$. From this eq. (1) we obtain

$$x=\frac{c\,\mathrm{sh}\,\xi}{\mathrm{ch}\,\xi+\cos\eta},\quad y=\frac{c\sin\eta}{\mathrm{ch}\,\xi+\cos\eta} \tag{2}$$

The line element ds is given by

$$(ds)^2=(dx)^2+(dy)^2=h^2[(d\xi)^2+(d\eta)^2] \tag{3}$$

in which we put

$$h=\frac{c}{\mathrm{ch}\,\xi+\cos\eta} \tag{4}$$

The two-dimensional Laplacian of a function ϕ is given by

$$\Delta\phi\equiv\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\equiv\frac{1}{h^2}\left(\frac{\partial^2\phi}{\partial\xi^2}+\frac{\partial^2\phi}{\partial\eta^2}\right)$$

General solution of the eq. $\Delta\phi=0$ may be written in following form of an infinite series

$$\phi=\sum_{n=1}^{\infty}[A_n\sin n\eta+B_n\cos n\eta][\mathrm{sh}\,n\xi+C_n\,\mathrm{ch}\,n\xi] \tag{5}$$

For a two set of circles $\xi=\xi_1$ and $\xi=\xi_2$, we have $(k=1,2)$

$$E_k=\frac{\mathrm{ch}\,\xi_k}{\mathrm{sh}\,\xi_k},\quad R_k=\frac{c}{|\mathrm{sh}\,\xi_k|} \tag{6}$$

When the circle $\xi=\xi_1$ is moving with linear velocity $\dot{a}_1$ (in angular direction β_1), while the circle $\xi=\xi_2$ is kept at rest, we have for the value of velocity potential ϕ (according to results of previous reports),

$$\phi=\sum_{n=1}^{\infty}[A_{n1}^{(1)}\sin n\eta+B_{n1}^{(1)}\cos n\eta]\frac{\mathrm{ch}\,n(\xi-\xi_2)}{\mathrm{ch}\,n(\xi_1-\xi_2)} \tag{7}$$

in which the coefficients $A_{n1}^{(1)}$, $B_{n1}^{(1)}$ have following values;

$$A_{n1}^{(1)}=\frac{-1}{n}\left[\frac{c^2\dot{a}_1}{R_1}\sin\gamma_1\right]\coth n(\xi_1-\xi_2)[K_{n-1}^{(2)}(\lambda_1)-K_{n+1}^{(2)}(\lambda_1)]$$

$$B_{n1}^{(1)}=\frac{-2}{n}[c\dot{a}_1\cos\gamma_1]\coth n(\xi_1-\xi_2)[\mathrm{sh}^2\,\xi_1K_n^{(2)}(\lambda_1)-\mathrm{ch}\,\xi_1K_n^{(1)}(\lambda_1)]$$

Lastly, we remark that

$$\lambda_1=\mathrm{ch}\,\xi_1,\quad \varepsilon_1=1/[\mathrm{ch}\,\xi_1+|\mathrm{sh}\,\xi_1|]$$

4. Analytical Expression for the Case in which two Circular Cylinders are situated very closely each other

Let us take up the case in which two circular cylinders (of radii R_1, R_2) are situated very closely each other, as shown in Fig. 2. Their centers lie on the real

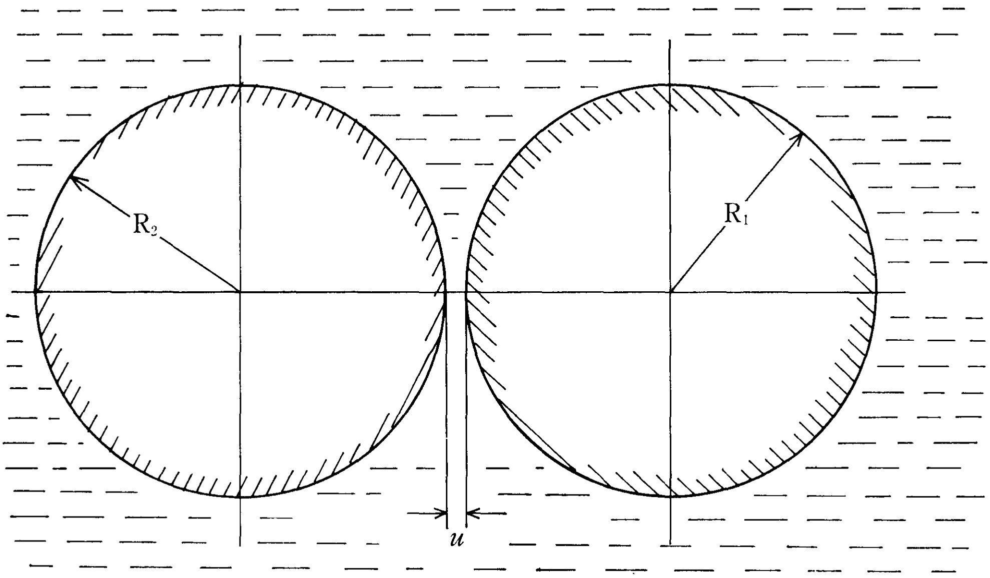

Fig. 2. Configuration of two Circular Cylinders, which is considered in the present Report V.

axis, and are spaced each other, so that a small gap or clearance u is kept between them. Let us assume that No. 1 circular cylinder is moving with linear velocity $\dot{a}_1$ in direction of x-axis. This means that we have to take $\beta_1=0$, $\mu=0$, $\gamma_1=0$, into the previously obtained expressions of Reports II, III.

We first make estimates of quantities c, E_k etc., corresponding to this situation. From the relation $c/R_k=|\mathrm{sh}\,\xi_k|$ we have $(k=1, 2)$.

$$|\xi_k|=\log\left[\frac{c}{R_k}+\left\{\left(\frac{c}{R_k}\right)^2-1\right\}^{1/2}\right]$$

Next, we have

$$\begin{aligned}E_0&=|E_1|+|E_2|=R_1\,\mathrm{ch}\,\xi_1+R_2\,\mathrm{ch}\,\xi_2\\&=(R_1^2+c^2)^{1/2}+(R_2^2+c^2)^{1/2}\end{aligned}$$

Solving this eq. with regard to c, we obtain,

$$c^2=\frac{1}{4E_0^2}[E_0^2-(R_1+R_2)^2][E_0^2-(R_1-R_2)^2]$$

Clearance is given by

$$\begin{aligned}u&=|E_1|+|E_2|-(R_1+R_2)\\&=R_1(\mathrm{ch}\,\xi_1-1)+R_2(\mathrm{ch}\,\xi_2-1)\end{aligned}$$

At initial state at which $t=0$, we put $u=u_0$, $E_0=E_{00}$. At subsequent time t, we assume that the No. 1 circular cylinder has moved by a distance a_1 along the

x-axis, while No. 2 circular cylinder is kept at stand still. This state of configuration can be expressed by assigning ξ_1, ξ_2 and c values proper to it, (R_1 and R_2 being always kept constant) according to the above relations. Thus, we have

$$a_1 = u - u_0, \quad a_2 \equiv 0, \quad E_0 = u + R_1 + R_2$$

from which values of c, ξ_1, ξ_2 are obtained.

After these preliminary remarks, we turn now to apply the solution obtained previously, to the present case. The velocity potential ϕ is obtained by

$$\phi = \sum_{n=1}^{\infty} B_{n1}^{(1)} \cos n\eta \cdot \frac{\text{ch}\, n(\xi - \xi_2)}{\text{ch}\, n(\xi_1 - \xi_2)}$$

in which we have

$$B_{n1}^{(1)} = \frac{-2c\dot{a}_1}{n} \coth n(\xi_1 - \xi_2) \cdot [\text{sh}^2 \xi_1 K_n^{(2)}(\lambda_1) - \text{ch}\, \xi_1 K_2^{(1)}(\lambda_1)]$$

Hydrodynamic pressure p was previously been given by

$$-\frac{1}{\rho} p = \frac{\partial \phi}{\partial t} + \frac{1}{2}\left[\left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial y}\right)^2\right] - (c_x - \Omega y)\frac{\partial \phi}{\partial x} - (c_y + \Omega x)\frac{\partial \phi}{\partial y}$$

where c_x, c_y represent linear velocity of translation of instantaneous frame of reference (coordinate axes), and Ω its angular velocity of rotation. In the present case, we have $c_y = 0$, $\Omega = 0$, and thus we have

$$-\frac{1}{\rho} p = \frac{\partial \phi}{\partial t} + \frac{1}{2}\left[\left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial y}\right)^2\right] - c_x \frac{\partial \phi}{\partial x}$$

5. Hydrodynamic Force (F_x, F_y) acting on No. 1 circular cylinder

Hydrodynamic force (F_x, F_y) acting on No. 1 circular cylinder, which is caused by action of fluid pressure p, is given by

$$F_x = \int_0^{2\pi} (-p)\left[\left(\frac{c}{R_1}\right)^2 \frac{1}{\text{ch}\, \xi_1 + \cos \eta} - \text{ch}\, \xi_1\right] \cdot \left[\frac{c}{\text{ch}\, \xi_1 + \cos \eta}\right] d\eta$$

$$F_y = \int_0^{2\pi} (-p)\left[\frac{c^2}{R_1} \frac{\sin \eta}{(\text{ch}\, \xi_1 + \cos \eta)^2}\right] d\eta$$

The evaluation of this force (F_x, F_y) will be made in three steps, as follows;

(a) Coutribution by term in $\partial\phi/\partial t$. We put, for convenience,

$$B_{n1}^{(1)} = [D_1 \dot{a}_1] b_{n1}^{(1)} \quad (n = 1, 2, \cdots)$$

in which $D_1 = 2R_1$ (diameter of No. 1 circular cylinder), $\dot{a}_1 = u_1$. $b_{n1}^{(1)}$ ($n = 1, 2, \cdots$) are numerical coefficients of no dimension, and their values are

$$b_{n1}^{(1)}=-\frac{1}{n}\operatorname{sh}\xi_1\coth n(\xi_1-\xi_2)\cdot[\operatorname{sh}^2\xi_1K_{n1}^{(2)}(\lambda_1)-\operatorname{ch}\xi_1K_n^{(1)}(\lambda_1)]$$

in which we have put

$$\lambda_1=\operatorname{ch}\xi_1,\quad \varepsilon_1=1/[|\operatorname{sh}\xi_1|+\operatorname{ch}\xi_1]$$

In passing, it may be noted that we have, (which is verified by comparing actual expressions for $K_n^{(1)}(\lambda_1)$ and $K_n^{(2)}(\lambda_1)$;

$$\operatorname{sh}^2\xi_1K_n^{(2)}(\lambda_1)-\operatorname{ch}\xi_1K_n^{(1)}(\lambda_1)=[K_{n-1}^{(2)}(\lambda_1)-K_{n+1}^{(2)}(\lambda_1)]\left(-\frac{1}{2}\operatorname{sh}\xi_1\right)$$

In our case of non-stationary fluid flow, constants $b_{n1}^{(1)}$, ξ_1, c, etc., involved therein are functions of a_1, and in turn, a_1 is a function of time t. Keeping this fact in mind, we have

$$\frac{d}{dt}[B_{n1}^{(1)}]=[D_1\ddot{a}_1]b_{n1}^{(1)}+[D_1(\dot{a}_1)^2]\frac{db_{n1}^{(1)}}{da_1}$$

Thus the part of hydrodynamic force which is contributed by the term in $\partial\phi/\partial t$, is given by;

$$\frac{1}{\rho}F_x=\int_0^{2\pi}\left[\Sigma\frac{d}{dt}(B_{n1}^{(1)})\cos n\eta\right]\cdot\left[\frac{\operatorname{sh}^2\xi_1}{\operatorname{ch}\xi_1+\cos\eta}-\operatorname{ch}\xi_1\right]\cdot\frac{c\,d\eta}{\operatorname{ch}\xi_1+\cos\eta}$$
$$=\Sigma\frac{d}{dt}[B_{n1}^{(1)}]\cdot v_{n1}^{(1)}$$

in which we have put, for shortness;

$$v_{n1}^{(1)}=2\pi R_1\operatorname{sh}\xi_1[\operatorname{sh}^2\xi_1K_n^{(2)}(\lambda_1)-\operatorname{ch}\xi_1K_n^{(1)}(\lambda_1)]$$
$$=-\pi nD_1\tanh n(\xi_1-\xi_2)[b_{n1}^{(1)}]$$

Also we have

$$\frac{1}{\rho}F_y=0\,.$$

(b) Contribution by the term in $1/2[(\partial\phi/\partial x)^2+(\partial\phi/\partial y)^2]$
Here we have

$$\frac{1}{2}\left[\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2\right]=\frac{1}{2h^2}\left[\left(\frac{\partial\phi}{\partial\xi}\right)^2+\left(\frac{\partial\phi}{\partial\eta}\right)^2\right]$$

The effect of the term in $(1/2h^2)(\partial\phi/\partial\xi)^2$ is null (as pointed out in Reports II, III). So that we need only to estimate the effect of term in $(1/2h^2)(\partial\phi/\partial\eta)^2$. Thus we obtain

$$\frac{1}{\rho}F_x=\int_0^{2\pi}[\Sigma nB_{n1}^{(1)}\sin n\eta]^2\cdot\left[\frac{\operatorname{sh}^2\xi_1}{\operatorname{ch}\xi_1+\cos\eta}-\operatorname{ch}\xi_1\right]$$
$$\cdot\frac{1}{2}\left[\frac{\operatorname{ch}\xi_1+\cos\eta}{c}\right]^2\frac{c\,d\eta}{\operatorname{ch}\xi_1+\cos\eta}$$

$$=\int_0^{2\pi}[\Sigma nB_{n1}^{(1)}\sin n\eta]^2\frac{1}{2c}[\mathrm{sh}^2\xi_1-\mathrm{ch}\,\xi_1(\mathrm{ch}\,\xi_1+\cos\eta)]d\eta$$
$$=I$$

where we put

$$I=\pi M\Sigma\{nB_{n1}^{(1)}\}^2-\pi N\Sigma\{nB_{n1}^{(1)}\}\{(n+1)B_{n+11}^{(1)}\}$$

with

$$M=-\frac{1}{2c},\quad N=\frac{1}{2c}\mathrm{ch}\,\xi_1,\quad c=R_1\,\mathrm{sh}\,\xi_1$$

Also, we have

$$\frac{1}{\rho}F_y=0\,.$$

(c) Effect of term with c_x.
Firstly, we have

$$(-1)\rho c_x\frac{\partial\phi}{\partial x}=(-1)\rho\frac{c_x}{c}\left[(1+\mathrm{ch}\,\xi_1\cos\eta)\frac{\partial\phi}{\partial\xi}+\mathrm{sh}\,\xi_1\sin\eta\frac{\partial\phi}{\partial\eta}\right]=\rho U$$

say. For values of $\partial\phi/\partial\xi$, $\partial\phi/\partial\eta$ at the circumference of No. 1 cylinder, that is, at $\xi=\xi_1$, we have

$$\frac{\partial\phi}{\partial\xi}=\sum_n nB_{n1}^{(1)}\tanh n(\xi_1-\xi_2)\cos n\eta$$

$$\frac{\partial\phi}{\partial\eta}=-\sum_n nB_{n1}^{(1)}\sin n\eta$$

Hence, we have

$$\frac{1}{\rho}F_x=\int_0^{2\pi}U\left[\frac{\mathrm{sh}\,\xi_1}{\mathrm{ch}\,\xi_1+\cos\eta}-ch\,\xi_1\right]\frac{cd\eta}{\mathrm{ch}\,\xi_1+\cos\eta}$$
$$=-\int_0^{2\pi}cU\frac{1+\mathrm{ch}\,\xi_1\cos\eta}{(\mathrm{ch}\,\xi_1+\cos\eta)^2}d\eta$$

This integral may be regarded to consist of following two parts;

(1) First part, which is contributed by the term in $\partial\phi/\partial\xi$ of the expression U,

$$\int_0^{2\pi}c_x\frac{\partial\phi}{\partial\xi}\frac{(1+\mathrm{ch}\,\xi_1\cos\eta)^2}{(\mathrm{ch}\,\xi_1+\cos\eta)^2}d\eta$$
$$=\sum_n B_{n1}^{(1)}\int_0^{2\pi}c_xn\tanh n(\xi_1-\xi_2)\cdot\frac{(1+\mathrm{ch}\,\xi_1\cos\eta)^2\cos n\eta}{(\mathrm{ch}\,\xi_1+\cos\eta)^2}d\eta$$
$$=\sum_n[nc_x\tanh n(\xi_1-\xi_2)]B_{n1}^{(1)}\cdot[L_{3n}+2\,\mathrm{ch}\,\xi_1I_{3n}+(\mathrm{ch}\,\xi_1)^2J_{3n}]\qquad\text{(I)}$$

(2) Second part, which is contributed by the term in $\partial\phi/\partial\eta$ of the expression

U,

$$\int_0^{2\pi} c_x \frac{\partial \phi}{\partial \eta} \operatorname{sh} \xi_1 \frac{(1+\operatorname{ch} \xi_1 \cos \eta) \sin \eta}{(\operatorname{ch} \xi_1+\cos \eta)^2} d\eta$$

$$=\sum_n B_{n1}^{(1)} \int_0^{2\pi} c_x[-n \operatorname{sh} \xi_1] \cdot \frac{\sin \eta \sin n\eta (1+\operatorname{ch} \xi_1 \cos \eta)}{(\operatorname{ch} \xi_1+\cos \eta)^2} d\eta$$

$$=-\sum_n n \operatorname{sh} \xi_1 c_x B_{n1}^{(1)} [I_{1n}+\operatorname{ch} \xi_1 J_{2n}] \qquad \text{(II)}$$

And, the hydrodynamic force is obtainable by adding these two terms, thus;

$$\frac{1}{\rho} F_x=\text{(I)}+\text{(II)}$$

On the other hand, we have

$$\frac{1}{\rho} F_y=0$$

In the above expressions we used the numerical coefficients I_{1n}, J_{2n}, etc. These are defined as follows;

$$I_{1n}=\int_0^{2\pi} \frac{\sin \eta \sin n\eta}{N^2} d\eta$$

$$I_{2n}=\int_0^{2\pi} \frac{\sin \eta \sin n\eta}{N} d\eta$$

$$I_{3n}=\int_0^{2\pi} \frac{\cos \eta \cos n\eta}{N^2} d\eta$$

$$I_{4n}=\int_0^{2\pi} \frac{\cos \eta \cos n\eta}{N} d\eta$$

$$J_{1n}=\int_0^{2\pi} \frac{\sin^2 \eta \cos n\eta}{N} d\eta$$

$$J_{2n}=\int_0^{2\pi} \frac{\sin \eta \cos \eta \sin n\eta}{N^2} d\eta$$

$$L_{3n}=\int_0^{2\pi} \frac{\cos n\eta}{N^2} d\eta, \quad L_{4n}=\int_0^{2\pi} \frac{\cos n\eta}{N} d\eta$$

in which we have put,

$$N=\operatorname{ch} \xi_1+\cos \eta$$

These values of definite integrals may be expressed in terms of following definite integrals ($n=0, 1, 2, \cdots$), which were used in author's previous papers. (Convenient forms for them are given below.)

$$K_n^{(s)}=\frac{1}{2\pi}\int_0^{2\pi}\frac{\cos n\eta}{N^s}d\eta$$

$$K_n^{(1)}=(-)^n\frac{2\varepsilon_1^{n+1}}{(1-\varepsilon_1^2)}$$

$$K_n^{(2)}=(-)^n\frac{4(n+1)\varepsilon_1^{n+2}}{(1-\varepsilon_1^2)^2}+(-)^n\frac{8\varepsilon_1^{n+4}}{(1-\varepsilon_1^2)^3}$$

(d) Value of c_x.

Referring to Fig. 2, we have $\overline{O_2O}=E_2=\sqrt{R_2^2+c^2}$ denoting by O_2 the center of No. 2 circular cylinder. When No. 2 cylinder is kept at stand still, while No. 1 cylinder moves in x-direction, the origin of coordinate axes, that is O, will also move, with linear velocity c_x. Now we have

$$c_x=\frac{d}{dt}\sqrt{R_2^2+c^2}=\frac{c}{\sqrt{R_2^2+c^2}}\frac{dc}{dE_0}\dot{u}$$

By differentiation of equation

$$4E_0^2c^2=[E_0^2-(R_1+R_2)^2][E_0^2-(R_1-R_2)^2]$$

we obtain

$$2\left[c^2+E_0c\frac{dc}{dE_0}\right]=E_0^2-(R_1^2+R_2^2)$$

and, we have

$$\frac{dc}{dE_0}=\frac{E_1E_2}{cE_0}$$

Thus, we obtain following value c_x for translational velocity c_x of our instantaneous coordinate axes;

$$c_x=(E_1/E_0)\dot{u}$$

It is to be noted that here we have

$$E_0=u+R_1+R_2\,,\quad E_k=R_k\operatorname{ch}\xi_k\quad(k=1,2)$$

6. Additional Remarks about Numerical Evaluation

In order to make evaluation of numerical values about the obove mentioned analytical expressions, the author took following considerations;

Firstly, after rearrangement, we use the following more concise expressions;

$$\operatorname{sh}\xi_1=\frac{1-\varepsilon_1^2}{2\varepsilon_1},\quad \operatorname{ch}\xi_1=\frac{1+\varepsilon_1^2}{2\varepsilon_1},\quad \varepsilon_1=1/[\operatorname{ch}\xi_1+|\operatorname{sh}\xi_1|$$

$$[\operatorname{sh}^2 \xi_1 K_n^{(2)}(\lambda_1) - \operatorname{ch} \xi_1 K_n^{(1)}(\lambda_1)] = (-)^n n \varepsilon_1{}^n$$

$$[K_{n-1}^{(2)}(\lambda_1) - K_{n+1}^{(2)}(\lambda_1) = (-)^{n+1} \frac{4n\varepsilon_1{}^{n+1}}{(1-\varepsilon_1{}^2)}$$

Secondly, we obtained more concise expressions for numerical constants I'_{1n}, J'_{3n}, L'_{3n}, etc., as follows;

$$I'_{1n} = -8\pi n^2 \varepsilon_1{}^{2n+1} \frac{1}{1-\varepsilon_1{}^2}$$

$$I'_{3n} = -\frac{8\pi n}{(1-\varepsilon_1{}^2)^2} \varepsilon_1{}^{2n+1} \left[(1+\varepsilon_1{}^2)n + \frac{4\varepsilon_1{}^2}{(1-\varepsilon_1{}^2)} \right]$$

$$J'_{2n} = 4\pi n \frac{1}{(1-\varepsilon_1{}^2)} \left[n(1+\varepsilon_1{}^4) + 3\varepsilon_1{}^4 - 1 + \frac{2\varepsilon_1{}^2(1+\varepsilon_1{}^4)}{(1-\varepsilon_1{}^2)} \right]$$

$$J'_{3n} = \pi n \left(\frac{1+\varepsilon_1{}^2}{1-\varepsilon_1{}^2} \right)^2 \varepsilon_1{}^{2n} \left[n + 2\varepsilon_1{}^2 + \frac{3\varepsilon_1{}^2}{1+\varepsilon_1{}^2} \right]$$

$$L'_{3n} = \frac{16\pi n}{(1-\varepsilon_1{}^2)^2} \varepsilon_1{}^{2n+2} \left[n + 1 + \frac{2\varepsilon_1{}^2}{1-\varepsilon_1{}^2} \right]$$

Also, we have, for the part of hydrodynamic force, which correspond to effect of the factor c_x,

$$\frac{1}{\rho} F_x = c_x [D_1 \dot{a}_1] \sum_{n=1}^{\infty} H_n$$

where we put, for shortness,

$$\begin{aligned} H_n &= (-)^n 2n\varepsilon_1{}^n [L_{3n} + 2 \operatorname{ch} \xi_1 I_{3n} + (\operatorname{ch} \xi_1)^2 J_{3n}] \\ &\quad - \coth n(\xi_1 - \xi_2) \operatorname{sh} \xi_1 (I_{1n} + \operatorname{ch} \xi_1 J_{2n}] \\ &= [L'_{3n} + 2 \operatorname{ch} \xi_1 I'_{3n} + (\operatorname{ch} \xi_1)^2 J'_{3n}] \\ &\quad - \coth n(\xi_1 - \xi_2) \operatorname{sh} \xi_1 [I'_{1n} + \operatorname{ch} \xi_1 J'_{2n}] \end{aligned}$$

Lastly, we note that the estimation depend mainly on numerical calculation of infinite series of following form;

$$S = \sum_{n=1}^{\infty} n^s \varepsilon_1{}^{2n} \coth n(\xi_1 - \xi_2) \quad (s = 1, 2)$$

Although these infinite series are absolutely convergent, so long as $0 < \xi_1 < 1$, $0 < (\xi_1 - \xi_2)$ yet it requires to take many terms (up to $n = 50$, for an instance), for obtaining fairly good approximate values. In order to avoid this inconvenience, we put

$$S_1 = \sum_{n=1}^{N-1} n^s \varepsilon_1{}^{2n} \coth n(\xi_1 - \xi_2)$$

$$S_2 = \sum_{n=N}^{\infty} n^s \varepsilon_1{}^{2n} \coth n(\xi_1 - \xi_2)$$

For a suitably taken value of integer ($N=6$, for an instance), we find that it may be put S_2' instead of S_2, where we put,

$$S_2^1=\sum_{n=N}^{\infty} n^s \varepsilon_1^{2n}[1+2\exp\{-2n(\xi_1-\xi_2)\}]$$

with a fair degree of approximation, at least for range of numerical values which appear in this Report V. This latter infinite power series is one that admits of obtaining its value by known formula of summation of power series, thus;

$$\sum_{n=N}^{\infty} nX^n=\frac{X^N}{(1-X)^2}[X+N(1-X)]$$

$$\sum_{n=N}^{\infty} n^2X^n=\frac{X^N}{(1-X)^3}[X+\{X+N(1-X)\}^2] \quad \text{for } |X|<1$$

7. Numerical Example

In order to illustrate the above-mentioned analytical results, we took up following three series of cases A, B, and C, and made numerical estimations about them, in accordance with the above mentioned method of approximation.

A-Series $(R_2/R_1=1)$, $(u_0=R_1/4)$
$A_1(u/u_0=1)$, $A_2(u/u_0=3/4)$, $A_3(u/u_0=1/2)$, $A_4(u/u_0=3/8)$, $A_5(u/u_0=1/4)$

B-Series $(R_2/R^0=5)$, $(u_0=R_1/4)$
$B_0(u/u_0=1)$, $B_2(u/u_0=3/4)$, $B_3(u/u_0=2/1)$, $B_4(u/u_0=3/8)$, $B_5(u/u_0=1/4)$

C-Series $(R_2/R_1\rightarrow\infty)$, $(u_0=R_1/4)$
$C_1(u/u_0=1)$, $C_2(u/u_0=3/4)$, $C_3(u/u_0=1/2)$, $C_4(u/u_0=3/8)$, $C_5(u/u_0=1/4)$

Values of numerical coefficients $b_{n1}^{(1)}$ about them are shown in Tables 1 to 3.

Table 1. Values of $b_{n1}^{(1)}$, for Case A

	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
A_1	0.41287470	−0.19898032	0.11757400	−0.07111698	0.04339553
A_2	0.41424332	−0.20005242	0.12348247	−0.7959419	0.05170457
A_3	0.41638912	−0.20825448	0.12871268	−0.08855854	0.06196036
A_4	0.41961601	−0.20044367	0.13061368	−0.09144764	0.06771105
A_5	0.42581314	−0.20119557	0.13183967	−0.09642982	0.07341271

Table 2. Values of $b^{(1)}_{n1}$, for Case B

	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
B_1	0.53613432	−0.20739593	0.10336537	−0.05431700	0.02885294
B_2	0.57350092	−0.22154932	0.11539962	−0.06486441	0.03720074
B_3	0.59895768	−0.21425277	0.13017556	−0.07883495	0.04940232
B_4	0.61420365	−0.24816893	0.13933112	−0.08771499	0.05769114
B_5	0.64358262	−0.26509507	0.15195455	−0.09909728	0.06936208

Table 3. Values of $b^{(1)}_{n1}$, for Case C.

	$n=1$	$n=2$	$n=3$	$n=4$	$n-5$
C_1	0.625000	−0.2125000	0.09672619	−0.04724265	0.02348332
C_2	0.64793375	−0.23013475	0.11063233	−0.05829090	0.03153241
C_3	0.68581329	−0.25291252	0.12939372	−0.07394637	0.04401068
C_4	0.71265211	−0.26955011	0.14212238	−0.08470553	0.05311165
C_5	0.74743139	−0.29291510	0.15945764	−0.09914231	0.06563476

Table 4. Values of Numerical Coefficients for Case A

	k_x	f_{xa}	f_{xb}	f_{xc}	$c_x/\dot{a}_1$
A_1	−0.88547105	1.28718491	−0.26706061	24.53505543	0.500000
A_2	−0.90146821	1.52983879	−0.38679180	42.13976203	0.500000
A_3	−0.93035140	2.78751646	−0.59841021	87.33488515	0.500000
A_4	−0.94095644	4.39922972	−0.79107070	170.3477044	0.500000
A_5	−0.95156148	10.38291536	−1.10690526	348.3222708	0.500000

Table 5. Values of Numerical Constants for Case B

	k_x	f_{xb}	f_{xc}	$c_x/\dot{a}_1$
B_1	−1.00433299	−0.35976379	4.22216129	0.1928000
B_2	−1.07138820	−0.58930646	6.26867555	0.18656260
B_3	−1.09759165	−1.10105246	12.75202957	0.18013328
B_4	−1.15617398	−1.25117012	22.55790665	0.17684386
B_5	−1.19928380	−1.87914753	45.09874912	0.17350409

Table 6. Values of Numerical Constants for Case *C*.

	k_x	f_{xa}	f_{xb}	f_{xc}	c_x/a_1
C_1	−1.11041589	2.32963895	−0.50010997	0	0
C_2	−1.14971303	2.54758849	−0.63430337	0	0
C_3	−1.17387715	3.95240321	−1.13451910	0	0
C_4	−1.19661510	6.40164143	−1.54180526	0	0
C_5	−1.33901436	9.93287945	−2.28265717	0	0

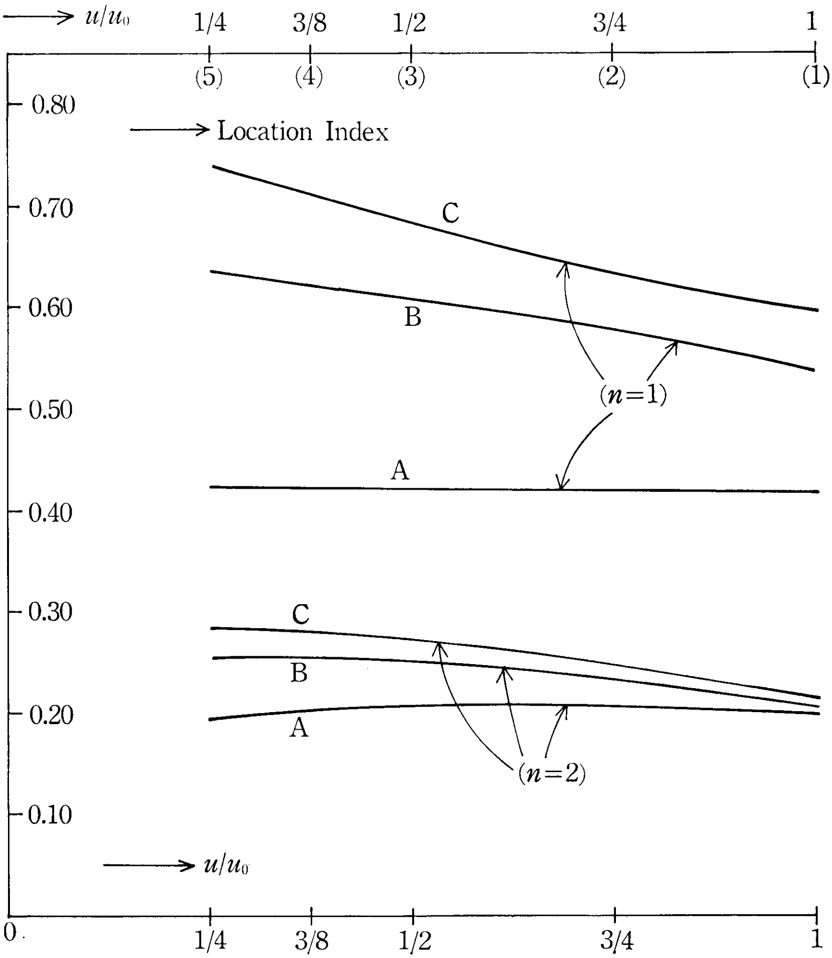

Fig. 3. Grophs for $|b_{n1}^{(1)}|$.

Also, graphs of these factors $b_{n1}^{(1)}$ are given as Fig. 3.

The hydrodynamic force F_x, acting on No. 1 circular cylinder, is expressed in the form of

$$\frac{1}{\rho}F_x = k_x[D_1{}^2\ddot{a}_1] + f_{xs}[D_1(\dot{a}_1)^2]$$

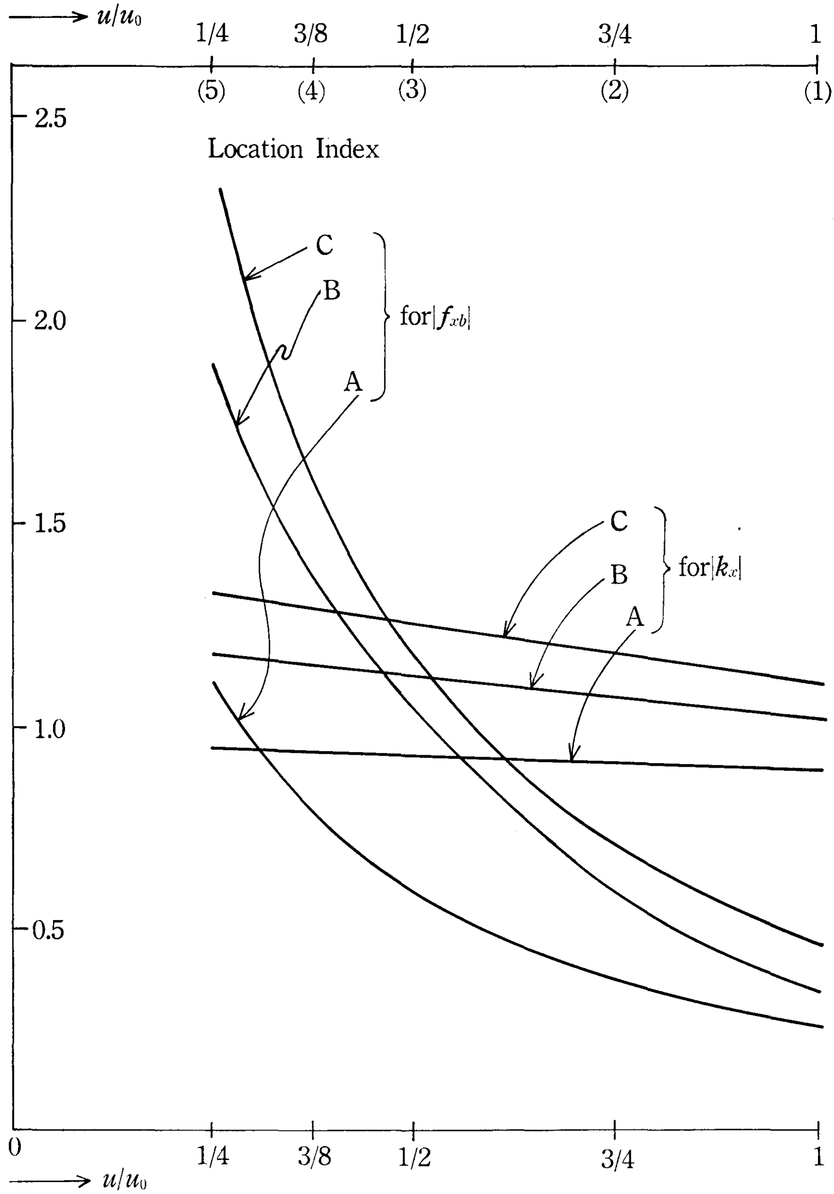

Fig. 4. Graphs for $|k_x|$ and $|f_{xb}|$.

which may also be written in following form;

$$\frac{1}{\rho}F_x = k_x D_1{}^3 \frac{d^2}{dt^2}\left[\frac{a_1}{D_1}\right] + f_{xs} D_1{}^3\left[\frac{d}{dt}\left(\frac{a_1}{D_1}\right)\right]^2$$

The numerical coefficient f_{xs} consists of three parts, thus;

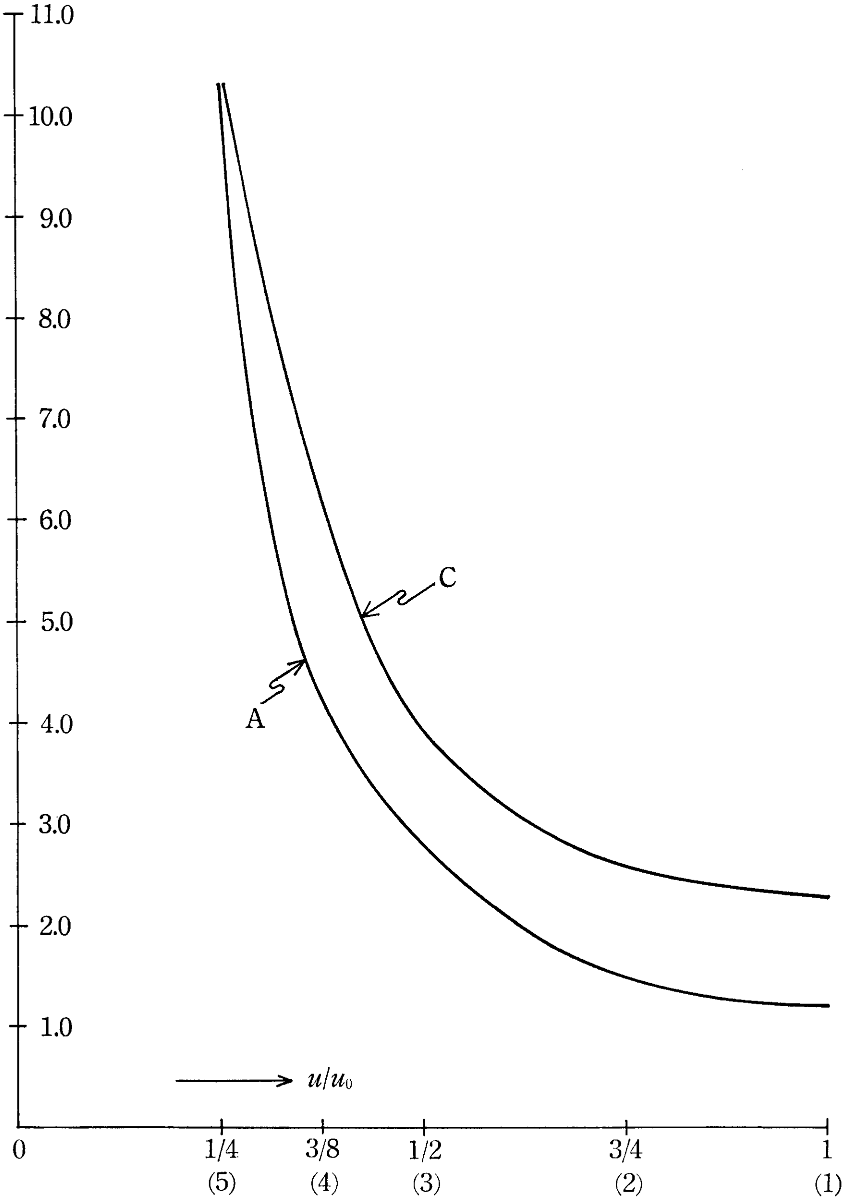

Fig. 5. Graphs for $|f_{xa}|$.

$$f_{xs}=f_{xa}+f_{xb}+f_{xc}$$

Numerical values of these constants are as given below, and their graphs are shown as Fig. 4 to 5.

CORRECTION to previous Report III. Values of numerical coefficients (page 71) for $[\rho D^2 \ddot{a}_1]$ should be multiplied by (-2π).

Reference

[1] KITO, F.: On Vibration of two Circular Cylinders which are immersed in a Water Region, Keio Engineering Reports, Rep. II (1979), 32, No. 5, pp. 45-56, Rep. III, (1979), 32, No. 6, pp. 57-74, Rep. IV (1980), 33, No. 9, pp. 117-129.