Title	On vibration of two circular cylinders which are immersed in a water region-IV.
Sub Title	
Author	鬼頭, 史城(Kito, Fumiki)
Publisher	慶応義塾大学工学部
Publication year	1980
Jtitle	Keio engineering reports Vol.33, No.9 (1980. 11) ,p.117- 129
JaLC DOI	
Abstract	In the former report, under the same title as the present one, the author has made analytical study about vibratory motion of two circular cylinders, which are immersed in fluid region of infinite extent. The fluid was taken to be an ideal fluid, and we treated the problem as a case of two-dimensional motion, two circular cylinders being of different radii. In the present Report IV, we examined the case in which one circular cylinder is of infinite radius, as a special case to previous study. Thus, we could show here, the case of motion of a circular cylinder, which is immersed in a fluid region which is bounded by a straight rigid wall of infinite extent. And, hereby, for the case of two-dimensional potential flow of an ideal fluid, amount of hydrodynamical force (Fx, Fy) acting on the circular cylinder was obtained. Some numerical examples about this hydrodynamical force, are also given.
Notes	
Genre	Departmental Bulletin Paper
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=KO50001004-00330009-0117

KEIO ENGINEERING REPORTS
VOL. 33, NO. 9, pp. 117-129, 1980

ON VIBRATION OF TWO CIRCULAR CYLINDERS WHICH ARE IMMERSED IN A WATER REGION—IV.

FUMIKI KITO

Department of Mechanical Engineering, Keio University, Hiyoshi, Yokohama 223, Japan

(Received 12 Sept. 1980)

ABSTRACT

In the former report, under the same title as the present one, the author has made analytical study about vibratory motion of two circular cylinders, which are immersed in fluid region of infinite extent. The fluid was taken to be an ideal fluid, and we treated the problem as a case of two-dimensional motion, two circular cylinders being of different radii. In the present Report IV, we examined the case in which one circular cylinder is of infinite radius, as a special case to previous study. Thus, we could show here, the case of motion of a circular cylinder, which is immersed in a fluid region which is bounded by a straight rigid wall of infinite extent. And, hereby, for the case of two-dimensional potential flow of an ideal fluid, amount of hydrodynamical force (F_x, F_y) acting on the circular cylinder was obtained. Some numerical examples about this hydrodynamical force, are also given.

1. Statement of the Problem

As shown in Fig. 2, we assume that at initial state $(t=0)$, the center O_1 of circular cylinder (of radius R_1) is situated at a point (D_{10}, O). There exist a rigid plane wall of infinite extent, which coincide with the y-axis. This plane wall will be assumed to be kept immobile. At time t $(0<t)$, the circular cylinder has moved by a distance a_1 in angular direction β_1. Regarding a_1 to be a given function of time t, we wish to obtain analytical expressions for fluid flow generated in the surrounding fluid. The fluid is regarded to be an ideal fluid (non-viscous, incompressible), and flow to be a two-dimensional potential flow (no vorticity).

In the author's previous paper (Reports II, III), we studied about the case of two circular cylinders of different radii (R_1, R_2). Therefore, it may seem that we may

* Professor Emeritus of Keio University.

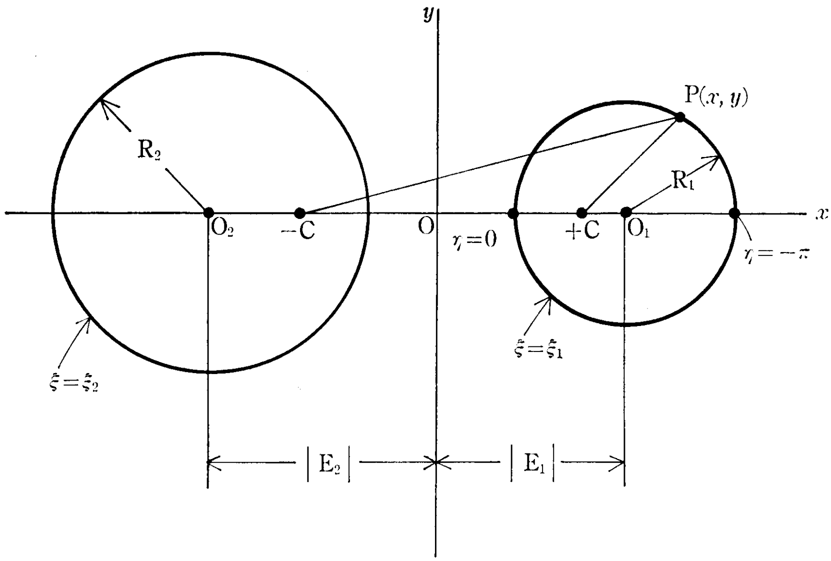

Fig. 1. Configuration of two Circular Cylinders represented by Bipolar Coordinates.

instantly arrive at the desired result, merely by putting $R_2 \to 0$ therein. But, in order to avoid ambiguity, we shall state in what follows, some detailed account about the process of deduction.

2. Notation

We shall use same notations, as we made in previous reports, namely; $x, y=$ rectangular coordinates of a point in x, y plane; $z=x+iy$ a complex variable; $C=$ distance from origin O, of radical centers of bi-polar coordinates; $h=$coefficient of linear element for the case of bi-polar coordinates (ξ, η); $\xi, \eta=$a system of bi-polar coordinates, representig any point on the x, y plane; $R_i=$radius of circular cylinder $(i=1, 2)$; $E_i=$position of center of ditto; $p=$fluid pressure; $\rho=$density of the fluid; $\phi=$velocity potential of fluid motion, giving absolute velocity of flow.

Several coefficients A_i, B_i, C_i, etc., are used for giving the solution in form of infinite series. These coefficients are independent of variables $(x, y$; also of $\xi, \eta)$, but they may be functions of time t.

3. Main Results obtained in the Previous Paper

We shall extract from the previous paper, the analytical solution for a special case in which the No. 1 circular cylinder moves, while the No. 2 circular cylinder is kept at stand still ($\dot{a}_2 \equiv 0$). The analytical study was based on the use of bi-polar coordinates, as shown in Fig. 1. Referring to this Fig. 1, two points $(+c, 0)$ and

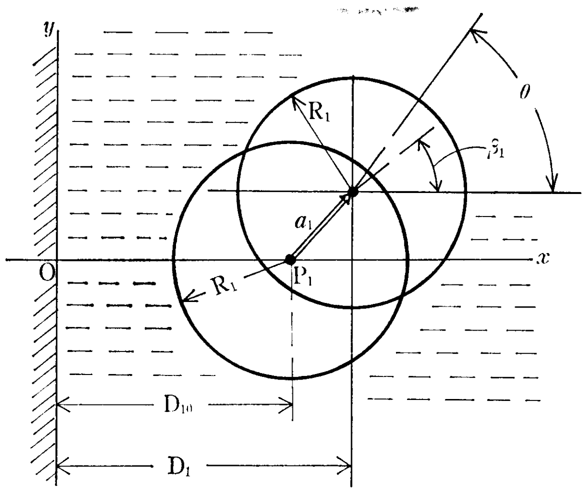

Fig. 2. Positions of a Circular Cylinder, which is immersed in a Water Region.

$(-c, 0)$ lying on the real axis are taken as radical centers, and we define a system of bi-polar coordinates (ξ, η) by means of the relation

$$\zeta = \xi + i\eta = \log \frac{c+z}{c-z} \tag{1}$$

wherein we put $z = x + iy$. From this eq. (1), we have

$$x = \frac{c \operatorname{sh} \xi}{\operatorname{ch} \xi + \cos \eta}, \quad y = \frac{c \sin \eta}{\operatorname{ch} \xi + \cos \eta} \tag{2}$$

The linear element ds is given by

$$(ds)^2 = (dx)^2 + (dx)^2 = h^2[(d\xi)^2 + (d\eta)^2] \tag{3}$$

in which we have put

$$h = \frac{c}{\operatorname{ch} \xi + \cos \eta} \tag{4}$$

The two-dimensional Laplacian $\Delta\phi$ of a function ϕ is given by

$$\Delta\phi \equiv \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \frac{1}{h^2}\left(\frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial \eta^2}\right)$$

General solution of the eq. $\Delta\phi = 0$ may be written in the following form;

$$\phi = \sum_{n=1}^{\infty} [A_n \sin n\eta + B_n \cos n\eta][\operatorname{sh} n\xi + C_n \operatorname{ch} n\xi] \tag{5}$$

For a given set of two circles $\xi = \xi_1$ and $\xi = \xi_2$, we have

$$E_k=\frac{\text{ch}\,\xi_k}{\text{sh}\,\xi_k},\quad R_k=\frac{c}{|\text{sh}\,\xi_k|} \tag{6}$$

When the circle ξ_1 is moving with linear velocity $\dot{a}_1$ (in angular direction β_1), while the circle ξ_2 is kept at rest, we have for the value of velocity potential ϕ, (by results of Reports II, III),

$$\phi=\sum_{n=1}^{\infty}[A_{n1}^{(1)}\sin n\eta+B_{n1}^{(1)}\cos n\eta]\cdot\frac{\text{ch}\,n(\xi-\xi_2)}{\text{ch}\,n(\xi_1-\xi_2)} \tag{7}$$

in which the coefficients $A_{n1}^{(1)}$, $B_{n1}^{(1)}$ have following values;

$$A_{n1}^{(1)}=\frac{-1}{n}\left[\frac{c^2\dot{a}_1}{R_1}\sin\gamma_1\right]\coth n(\xi_1-\xi_2)\cdot[K_{n-1}^{(2)}(\lambda_1)-K_{n+1}^{(2)}(\lambda_1)]$$

$$B_{n1}^{(1)}=\frac{-2}{n}[c\dot{a}_1\cos\gamma_1]\coth n(\xi_1-\xi_2)\cdot[\text{sh}^2\,\xi_1K_n^{(2)}(\lambda_1)-\text{ch}\,\xi_1K_n^{(1)}(\lambda_1)]$$

4. Limiting Case of $\xi_2\to 0(R_2\to\infty)$

Now, let us turn ourselves to the case of Fig. 2, which is the main object of present report. In order to deduce to this case of Fig. 2, from the preceeding result, we have to put $\gamma=\beta_1$, $\xi_2=0$. Thus we obtain

$$A_{n1}^{(1)}=\frac{-1}{n}\left[\frac{c^2\dot{a}_1}{R_1}\sin\beta_1\right]\coth n\xi_1\cdot[K_{n-1}^{(2)}(\lambda_1)-K_{n+1}^{(2)}(\lambda_1)]$$

$$B_{n1}^{(1)}=\frac{-2}{n}[c\dot{a}_1\cos\beta_1]\coth n\xi_1\cdot[\text{sh}^2\,\xi_1K_n^{(2)}(\lambda_1)-\text{ch}\,\xi_1K_n^{(1)}(\lambda_1)]$$

Moreover, we have, referring to Fig. 2;

$$E_1=D_1=D_{10}+a_1\cos\beta_1$$

in which D_1 is the distance from the straight wall (y-axis), of center of circle $\xi=\xi_1$ (radius R_1), D_{10} being value at initial state ($t=0$). Next, from the relation $E_1/R_1=\text{ch}\,\xi_1$, $c=E_1(\text{sh}\,\xi_1/\text{ch}\,\xi_1)$ we obtain;

$$\xi_1=\log\left[\frac{D_1}{R_1}+\sqrt{\left(\frac{D_1}{R_1}\right)^2-1}\right],$$

$$\lambda_1=\text{ch}\,\xi_1=D_1/R_1,\quad \text{sh}\,\xi_1=\sqrt{(D_1/R_1)^2-1}$$

$$D/R_1=(D_{10}/R_1)+(a_1/R_1)\cos\beta_1,$$

$$\varepsilon_1=1/[\text{ch}\,\xi_1+|\text{sh}\,\xi_1|],$$

$$c=E_1(\text{sh}\,\xi_1/\text{ch}\,\xi_1)=D_1(\text{sh}\,\xi_1/\text{ch}\,\xi_1)$$

It is convenient to use following numerical coefficients (non-dimensional)

$$a_{n1}^{(1)}=A_{n1}^{(1)}/[D_{10}\dot{a}_1]\,,\quad b_{n1}^{(1)}=B_{n1}^{(1)}/[D_{10}\dot{a}_1]$$

Their actnal values are as given below;

$$a_{n1}^{(1)}=\frac{-1}{n}\left(\frac{R_1}{D_{10}}\right)\mathrm{sh}^2\,\xi_1\coth n\xi_1\sin\beta_1[K_{n-1}^{(2)}(\lambda_1)-K_{n+1}^{(2)}(\lambda_1)]$$

$$b_{n1}^{(1)}=\frac{-2}{n}\left(\frac{R_1}{D_{10}}\right)\mathrm{sh}\,\xi_1\coth n\xi_1\cos\beta_1[\mathrm{sh}^2\,\xi_1K_n^{(2)}(\lambda_1)-\mathrm{ch}\,\xi_1K_n^{(1)}(\lambda_1)]$$

Although $A_{n1}^{(1)}$, $B_{n1}^{(1)}$ and $a_{n1}^{(1)}$, $b_{n1}^{(1)}$ are constants with regard to (ξ,η), they are functions of time t. So that we have

$$\frac{d}{dt}[A_{n1}^{(1)}]=[D_{10}\ddot{a}_1]a_{n1}^{(1)}+[D_{10}\dot{a}_1]\frac{da_{n1}^{(1)}}{da_1}\dot{a}_1$$

$$\frac{d}{dt}[B_{n1}^{(1)}]=[D_{10}\ddot{a}_1]b_{n1}^{(1)}+[D_{10}\dot{a}_1]\frac{db_{n1}^{(1)}}{da_1}\dot{a}_1\,.$$

It may be noted here, that we have used following coefficients

$$K_n^{(1)}(\lambda_1)=\frac{1}{2\pi}\int_0^{2\pi}\frac{\cos n\eta}{\lambda_1+\cos\eta}d\eta$$
$$=(-)^n\frac{2\varepsilon_1{}^{n+1}}{(1-\varepsilon_1{}^2)}$$

$$K_n^{(2)}(\lambda_1)=\int_0^{2\pi}\frac{\cos n\eta}{(\lambda_1+\cos\eta)^2}d\eta$$
$$=\frac{(-)^n4\varepsilon_1{}^{n+2}}{(1-\varepsilon_1{}^2)^2}\left[n+1+\frac{2\varepsilon_1{}^2}{1-\varepsilon_1{}^2}\right]$$

5. Estimation of Hydrodynamic Force (F_x, F_y)

Value of hydrodynamic force, exerted by surrounding fluid upon circular cylinder of radius R_1 (Fig. 2) is given by.

$$F_x=2\int_{\eta=0}^{\eta=\pi}(-p)\cos\theta ds$$
$$=2\int_{\eta=0}^{\eta=\pi}(-p)\left[\frac{\mathrm{sh}^2\,\xi_1}{\mathrm{ch}\,\xi_1+\cos\eta}-\mathrm{ch}\,\xi_1\right]\frac{cd\eta}{\mathrm{ch}\,\xi_1+\cos\eta}\,,$$

$$F_y=2\int_{\eta=0}^{\eta=\pi}(-p)\sin\theta ds$$
$$=2\int_{\eta=0}^{\eta=\pi}(-p)\left[\frac{\mathrm{sh}\,\xi_1\sin\eta}{\mathrm{ch}\,\xi_1+\cos\eta}\right]\frac{cd\eta}{\mathrm{ch}\,\xi_1+\cos\eta}\,.$$

The hydrodynamic pressure p in these expressions, is given by,

$$-\frac{1}{\rho}p=\frac{\partial\phi}{\partial t}+\frac{1}{2}\left[\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2\right]+(c_x-\Omega y)\frac{\partial\phi}{\partial x}$$
$$-(c_y+\Omega x)\frac{\partial\phi}{\partial y}+C$$

where (c_x, c_y) is the linear velocity of origin of our moving axis, Ω being instantaneous angular velocity of rotation of frame of axes. In the present instance we have $c_x=0$, $\Omega=0$. Also, we have

$$\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2=\frac{1}{h^2}\left[\left(\frac{\partial\phi}{\partial\xi}\right)^2+\left(\frac{\partial\phi}{\partial\eta}\right)^2\right]$$

The task of estimating the value of F_x, F_y may conveniently be carried out, by making it in three steps, thus;

(a) Part contributed by the term in $\partial\phi/\partial t$. For this part, we have,

$$\frac{1}{\rho}F_x=\int_0^{2\pi}\left[\Sigma\frac{d}{dt}(A_{n1}^{(1)})\sin n\eta+\Sigma\frac{d}{dt}(B_{n1}^{(1)})\cos n\eta\right]$$
$$\times\left[\frac{\text{sh}^2\,\xi_1}{\text{ch}\,\xi_1+\cos\eta}-\text{ch}\,\xi_1\right]\frac{cd\eta}{\text{ch}\,\xi_1+\cos\eta}$$
$$=\Sigma\frac{d}{dt}(B_{n1}^{(1)})\cdot v_{n1}^{(1)}$$

where we have put, for shortness,

$$v_{n1}^{(1)}=2\pi[\text{sh}^2\,\xi_1K_n^{(2)}(\lambda_1)-\text{ch}\,\xi_1K_n^{(1)}(\lambda_1)]\cdot R_1\,\text{sh}\,\xi_1$$
$$=-\frac{\pi nD_{10}}{\cos\beta_1}\tanh n\xi_1\cdot b_{n1}^{(1)}$$

$$\frac{1}{\rho}F_y=\int_0^{2\pi}\left[\Sigma\frac{d}{dt}(A_{n1}^{(1)})\sin n\eta+\Sigma\frac{d}{dt}(B_{n1}^{(1)})\cos n\eta\right]$$
$$\cdot\left[\frac{\text{sh}\,\xi_1\sin\eta}{\text{ch}\,\xi_1+\cos\eta}\right]\frac{cd\eta}{\text{ch}\,\xi_1+\cos\eta}$$
$$=\Sigma\frac{d}{dt}(A_{n1}^{(1)})\cdot u_{n1}^{(1)}$$

where we have put, for shortness,

$$u_{n1}^{(1)}=\pi c\,\text{sh}\,\xi_1[K_{n-1}^{(2)}(\lambda_1)-K_{n+1}^{(2)}(\lambda_1)]$$
$$=-\frac{\pi nD_{10}}{\sin\beta_1}\tanh n\xi_1\cdot a_{n1}^{(1)}$$

(b) Contribution by term $\frac{1}{2}[(\partial\phi/\partial\eta)^2]$.

In passing, it is to be noted that the term in $\frac{1}{2}[(\partial\phi/\partial\xi)^2]$ has no effect on F_x, F_y, as was stated in the previous paper. As for contribution by term $\frac{1}{2}[(\partial\phi/\partial\eta)^2]$, we have,

$$\frac{1}{\rho}F_x=\int_0^{2\pi}\left[\Sigma nA_{n1}^{(1)}\cos n\eta-\Sigma\, nB_{n1}^{(1)}\sin n\eta\right]^2$$

$$\cdot\left[\frac{\text{sh}^2\,\xi_1}{\text{ch}\,\xi_1+\cos\eta}-\text{ch}\,\xi_1\right]\cdot\frac{1}{2}\left[\frac{\text{ch}\,\xi_1+\cos\eta}{c}\right]^2\cdot\frac{cd\eta}{\text{ch}\,\xi_1+\cos\eta}$$

$$=\int_0^{2\pi}[\Sigma nA_{n1}^{(1)}\cos n\eta-\Sigma nB_{n1}^{(1)}\sin n\eta]^2\cdot\frac{1}{2c}[-1-\text{ch}\,\xi_1\cos\eta]d\eta$$

$$=I,$$

say. Then we have

$$I=\pi M[\Sigma\{nA_{n1}^{(1)}\}^2+\Sigma\{nB_{n1}^{(1)}\}^2]-\pi N[\Sigma\{nA_{n1}^{(1)}\}\{(n+1)A_{n+11}^{(1)}\}]$$
$$-\pi N[\Sigma\{nB_{n1}^{(1)}\}\{(n+1)B_{n+11}^{(1)}\}]$$

in which, we put,

$$M=-\frac{1}{2c},\quad N=\frac{1}{2c}\,\text{ch}\,\xi_1,\quad c=R_1\,\text{sh}\,\xi_1$$

Similarly, for the force component F_y, we have,

$$\frac{1}{\rho}F_y=\int_0^{2\pi}[\Sigma nA_{n1}^{(1)}\cos n\eta-\Sigma nB_{n1}^{(1)}\sin n\eta]^2$$

$$\cdot\frac{c\sin\eta}{R_1(\text{ch}\,\xi_1+\cos\eta)}\cdot\frac{1}{2}\left[\frac{\text{ch}\,\xi_1+\cos\eta}{c}\right]^2\cdot\frac{cd\eta}{\text{ch}\,\xi_1+\cos\eta}$$

$$=\int_0^{2\pi}\frac{1}{2R_1}[\Sigma nA_{n1}^{(1)}\cos n\eta-\Sigma nB_{n1}^{(1)}\sin n\eta]^2\cdot\sin\eta d\eta$$

$$=\frac{1}{2}J$$

say. Then, we have,

$$J=\pi M'[\Sigma\{nA_{n1}^{(1)}\}\{(n+1)B_{n+11}^{(1)}\}-\Sigma\{nB_{n1}^{(1)}\}\{(n+1)A_{n+11}^{(1)}\}]$$

with $M'=1/R_1$.

(c) Effect of term with c_y.

For the values of ϕ, $\partial\phi/\partial\eta$, $\partial\phi/\partial\xi$ along the circumference $\xi=\xi_1$, of our circular cylinder, we have,

$$\phi=\Sigma[A_{n1}^{(1)}\sin n\eta+B_{n1}^{(1)}\cos n\eta]$$

$$\frac{\partial\phi}{\partial\xi}=\Sigma[nA_{n1}^{(1)}\sin n\eta+nB_{n1}^{(1)}\cos n\eta]\tanh n\xi_1$$

$$\frac{\partial\phi}{\partial\eta}=\Sigma[nA_{n1}^{(1)}\cos n\eta-nB_{n1}^{(1)}\sin n\eta]$$

Also, we have, for $\xi=\xi_1$,

$$-\rho c_y\frac{\partial\phi}{\partial y}=\left(\rho\frac{c_y}{c}\right)\left[\frac{\partial\phi}{\partial\xi}\,\text{sh}\,\xi_1\sin\eta+\frac{\partial\phi}{\partial\eta}(1+\text{ch}\,\xi_1\cos\eta)\right]=\rho U$$

say. Using these values, we obtain,

$$\frac{1}{\rho}F_x=\int_0^{2\pi}U\left[\frac{\mathrm{sh}^2\,\xi_1}{\mathrm{ch}\,\xi_1+\cos\eta}-\mathrm{ch}\,\xi_1\right]\frac{cd\eta}{\mathrm{ch}\,\xi_1+\cos\eta}$$
$$=\int_0^{2\pi}[-cU]\frac{1+\mathrm{ch}\,\xi_1\cos\eta}{(\mathrm{ch}\,\xi_1+\cos\eta)^2}\,dy$$

As first part of this expression, we shall take up the term in $\partial\phi/\partial\xi$. For which, we have,

$$-\int_0^{2\pi}c_y\frac{\partial\phi}{\partial\xi}\frac{(1+\mathrm{ch}\,\xi_1\cos\eta)}{(\mathrm{ch}\,\xi_1+\cos\eta)^2}\cdot\mathrm{sh}\,\xi_1\sin\eta d\eta \tag{A}$$

Putting the value of $\partial\phi/\partial\xi$ into this expression (A), we find that it consists of sum of terms whose coefficients of $A_{n1}^{(1)}$ are,

$$-c_y\int_0^{2\pi}[n\sin n\eta\cdot\tanh n\xi_1]\cdot\frac{(1+\mathrm{ch}\,\xi_1\cos\eta)\,\mathrm{sh}\,\xi_1\sin\eta}{(\mathrm{ch}\,\xi_1+\cos\eta)^2}d\eta$$
$$=[-n\ \mathrm{tonh}\ n\xi_1\,\mathrm{sh}\,\xi_1\cdot c_y][I_{1n}+\mathrm{ch}\,\xi_1J_{2n}]$$

while the terms which contain $B_{n1}^{(1)}$ are null.

Next, as the second term, we take,

$$-\int_0^{2\pi}\left(c_y\frac{\partial\phi}{\partial\eta}\right)\frac{(1+\mathrm{ch}\,\xi_1\cos\eta)^2}{(\mathrm{ch}\,\xi_1+\cos\eta)^2}d\eta \tag{B}$$

Putting the value of $\partial\phi/\partial\eta$ into this expression (B), we observe that it consists of sum of terms whose coefficients of $A_{n1}^{(1)}$ are,

$$-nc_y\int_0^{2\pi}\frac{(1+\mathrm{ch}\,\xi_1\cos\eta)^2}{(\mathrm{ch}\,\xi_1+\cos\eta)^2}\cos n\eta d\eta=nc_y[L_{3n}+2\,\mathrm{ch}\,\xi_1I_{3n}+\mathrm{ch}^2\,\xi_1J_{3n}]$$

while the terms which contain $B_{n1}^{(1)}$ are null. Summarizing these inferences, we obtain;

$$\frac{1}{\rho}F_x=\Sigma(nc_y)[-\mathrm{sh}\,\xi_1\cdot\tanh n\xi_1(I_{1n}+\mathrm{ch}\,\xi_1J_{2n})$$
$$+(L_{3n}+2\,\mathrm{ch}\,\xi_1I_{3n}+\mathrm{ch}^2\,\xi_1J_{3n}]\cdot A_{n1}^{(1)}$$

Finally, by repeating similar process of evaluation, we obtain;

$$\frac{1}{\rho}F_y=\int_0^{2\pi}U\frac{\mathrm{sh}\,\xi_1\sin\eta}{\mathrm{ch}\,\xi_1+\cos\eta}\cdot\frac{cd\eta}{\mathrm{ch}\,\xi_1+\cos\eta}$$
$$=\int_0^{2\pi}\frac{\mathrm{sh}\,\xi_1\sin\eta}{(\mathrm{ch}\,\xi_1+\cos\eta)^2}[cU]d\eta$$
$$=\Sigma(nc_y)[\mathrm{sh}^2\,\xi_1\cdot\tanh n\xi_1J_{4n}-\mathrm{sh}\,\xi_1(I_{1n}+\mathrm{ch}\,\xi_1J_{2n})]$$

In these expressions, we have used following notations (as in the previous report);

$$J_{2n}=\int_0^{2\pi}\frac{\sin\eta\cos\eta\sin n\eta}{N^2}d\eta$$

$$J_{3n}=\int_0^{2\pi}\frac{\cos^2\eta\cos n\eta}{N^2}d\eta$$

$$J_{4n}=\int_0^{2\pi}\frac{\sin^2\eta\cos n\eta}{N^2}d\eta$$

$$I_{1n}=\int_0^{2\pi}\frac{\sin\eta\sin n\eta}{N^2}d\eta$$

$$I_{3n}=\int_0^{2\pi}\frac{\cos\eta\cos n\eta}{N^2}d\eta$$

$$L_{3n}=\int_0^{2\pi}\frac{\cos n\eta}{N^2}d\eta$$

$$(L_{3n}=J_{3n}+J_{4n};\quad N=\text{ch}\,\xi_1+\cos\eta)$$

All of these coefficients are expressible with the factor $K_n^{(2)}(\lambda_1)$, which we have introduced before. The velocity of advance c_y of our frame of reference is given by;

$$c_y=\dot{a}_1\sin\beta_1$$

referring to Fig. 2.

By summing up the above mentioned three items (a), (b) and (c), we are enabled to obtain actual value of hydrodynamic force (F_x, F_y) acting upon the circular cylinder of radius R_1.

6. Numerical Example

In order to illustrate the results of analytical study which was mentioned above, let us take up three cases, as mentioned below; (A), $\beta_1=0$, (B) $\beta_1=\pi/4$, (C) $\beta_1=\pi/2$. For each case we take $R_1/D_{10}=1/2$, and choose five values of $a_1/R_1=0$, 1/4, 1/2, 3/4 and 1. Values of numerical coefficients $a_{n1}^{(1)}$, $b_{n1}^{(1)}$ which gives coefficients $A_{n1}^{(1)}$, $B_{n1}^{(1)}$ of

Table 1. Values of $b_{n1}^{(1)}$ for Case A $[a_{n1}^{(1)}=0]$.

$a_1/R_1=$	0	1/4	1/2	3/4	1
$n=1$	+0.53589820	+0.52748000	+0.52178046	+0.51772122	+0.51471873
$n=2$	−0.12564432	−0.11144674	−0.09981033	−0.09102321	−0.08340570
$n=3$	+0.03334566	+0.02597835	+0.02083496	+0.01709093	+0.01428612
$n=4$	−0.00892834	−0.00608822	−0.00434786	−0.00321806	−0.00245101
$n=5$	+0.00239233	+0.00142734	+0.00090749	+0.00060590	+0.00042040

Table 2. Values of $a_{n1}^{(1)}$ for Case B $[b_{n1}^{(1)}=-a_{n1}^{(1)}]$

$a_1/R_1=$	0	1/4	1/2	3/4	1
$n=1$	−0.37893737	−0.37448086	−0.37113694	−0.36855485	−0.36651499
$n=2$	+0.08884396	+0.08149485	+0.07529634	+0.06998882	+0.06538871
$n=3$	−0.02357894	−0.01968873	−0.01671312	−0.01436720	−0.01248637
$n=4$	+0.00631329	+0.00479014	+0.00372627	+0.00295901	+0.00236804
$n=5$	−0.00169164	−0.00116540	−0.00083099	−0.00060951	−0.00045777

Table 3. Values of $a_{n1}^{(1)}$ for the Case C. (for every values of a_1/R_1)

	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
$a_{n1}^{(1)}$	−0.53589837	+0.12564433	−0.03334566	+0.00892834	−0.00239234
$b_{n1}^{(1)}$	0	0	0	0	0

Table 4. Values of Numerical Coefficients k_x, k_y, f_{xa} and f_{ya}.

Case	k_x	k_y	f_{xa}	f_{yb}
A_1	−0.89108480	0	0.18103491	0
A_2	−0.86745146	0	0.15110533	0
A_3	−0.85060578	0	0.09911495	0
A_4	−0.83922654	0	0.07503460	0
A_5	−0.83085295	0	0.05282927	0
B_1	−0.63009250	−0.63009250	0.10921783	0.10921783
B_2	−0.61761916	−0.61761916	0.08222989	0.08222989
B_3	−0.60810073	−0.60810073	0.06372090	0.06372090
B_4	−0.60064494	−0.60064494	0.05057822	0.05057822
B_5	−0.59558382	−0.59558382	0.03820658	0.03820658
C	0	−0.89108529	0	0

Table 5. Values of Numerical Coefficients f_{xb}, f_{yb}, f_{xc} and f_{yc}.

Case	f_{xb}	f_{yb}	f_{xc}	f_{yc}
A_1	−0.06206656	0	0	0
A_2	−0.04014837	0	0	0
A_3	−0.02749631	0	0	0
A_4	−0.02109768	0	0	0
A_5	−0.01721753	0	0	0
B_1	−0.06194593	0	−0.03770697	0.02658908
B_2	−0.04603408	0	−0.02029300	0.02030487
B_3	−0.03526371	0	−0.01606071	0.01606076
B_4	−0.02770056	0	−0.01267389	0.01267692
B_5	−0.02222053	0	−0.01028972	0.01028985
C	−0.06206656	0	−0.05332545	0

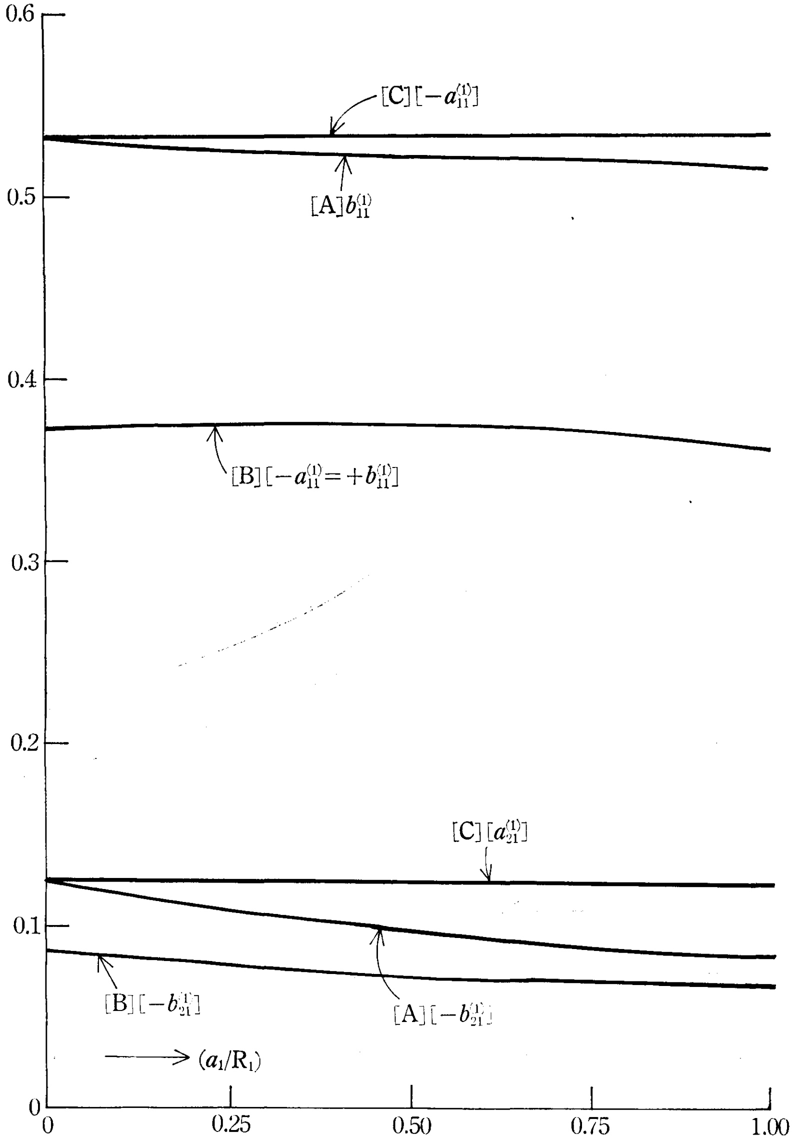

Fig. 3. Coefficients $a_{n1}^{(1)}$ and $b_{n1}^{(1)}$.

infinite series solution of velocity potential ϕ, are shown in Tables 1—3. Also, a rough graph is shown in Fig. 3, showing us that the mode of variation of these coefficients with the value of a_1/R_1 is comparatively slow. It will be noted from Fig 3, that the values of $a_{n1}^{(1)}$ and $b_{n1}^{(1)}$ varies very slowly and nearly in straight lines, at least within the range of variation of the variable a_1/R_1 shown here.

As to the values of hydrodynamical force (F_x, F_y) acting on the circular cylinder, we may account it in three steps, thus;

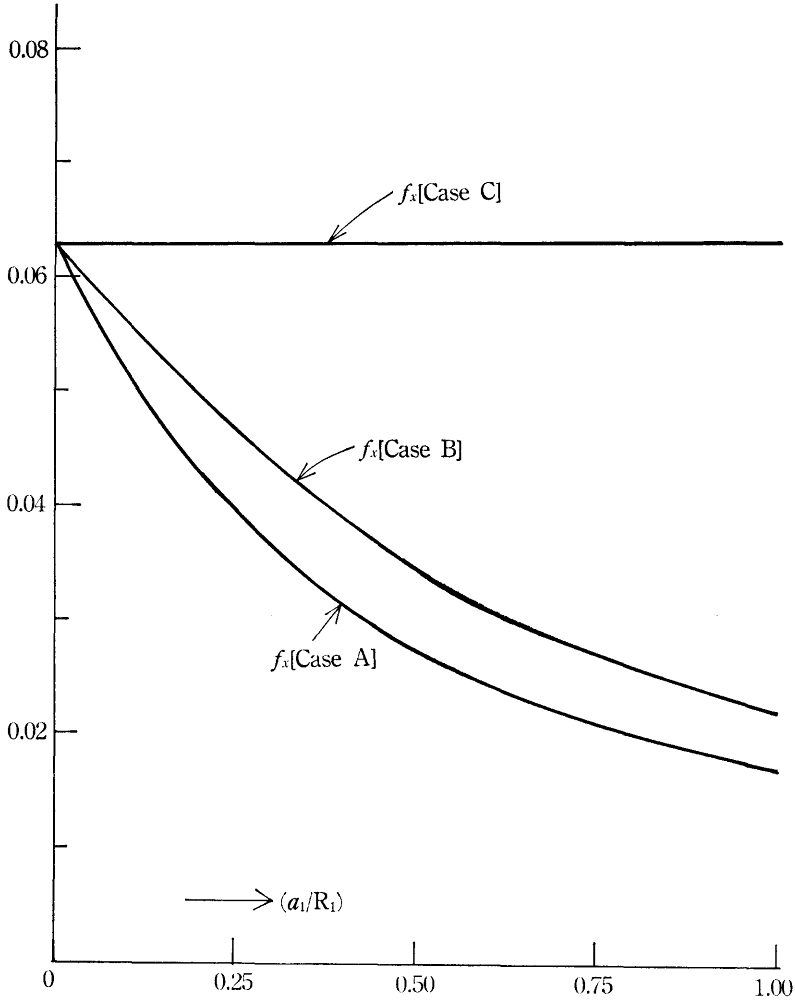

Fig. 4. Contribution by term in $\frac{1}{2}[(\partial\phi/\partial x)^2+(\partial\phi/\partial y)^2]$, $F_x=-f_x[\rho D_{10}(a_1)^2]$.

(a) Contribution by the term in $\partial\phi/\partial t$.

This was done, by taking, approximately $\Delta a_{n1}^{(1)}/\Delta a_1$, in place of $da_{n1}^{(1)}/da_1$. The result is given in the form,

$$F_x=k_x[\rho D_{10}^2\ddot{a}_1]+f_{xa}[\rho D_{10}(\dot{a}_1)^2]$$

$$F_y=k_y[\rho D_{10}^2\ddot{a}_1]+f_{ya}[\rho D_{10}(\dot{a}_1)^2]$$

(b) Contribution by term in $1/2[(\partial\phi/\partial x)^2+(\partial\phi/\partial y)^2]$. This part was obtained in the form of,

$$F_x=f_{xb}[\rho D_{10}(\dot{a}_1)^2], \quad F_y=f_{yb}[\rho D_{10}(\dot{a}_1)^2]$$

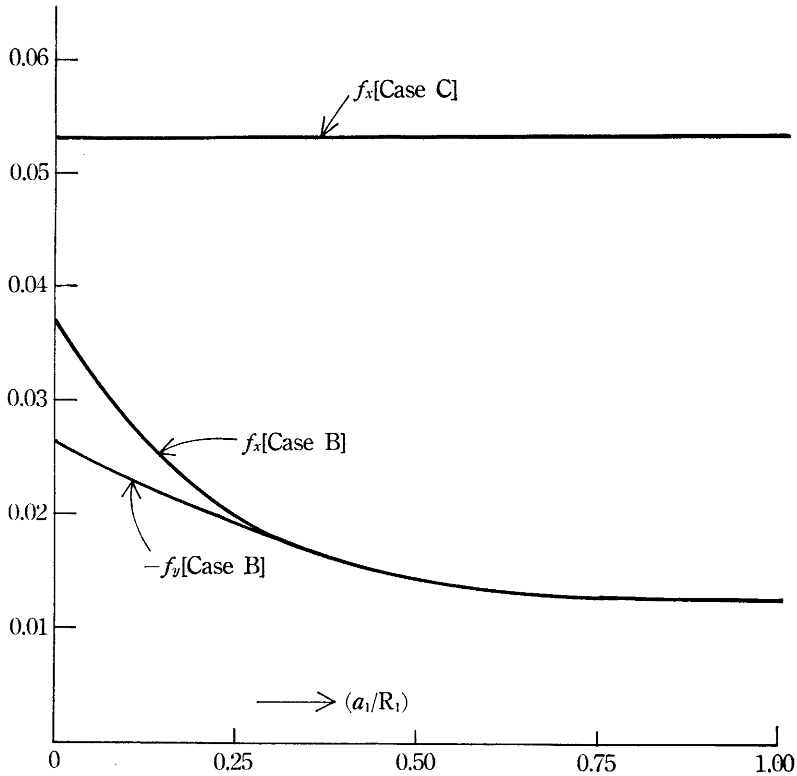

Fig. 5. Effect of term with Cy.

$$F_x = -f_x[\rho D_{10}(a_1)^2]$$
$$F_y = -f_y[\rho D_{10}(a_1)^2]$$

(c) Contribution by term in c_y.
This part as obtained in the form of

$$F_x = f_{xc}[\rho D_{10}(\dot{a}_1)^2], \quad F_y = f_{yc}[\rho D_{10}(\dot{a}_1)^2]$$

Coefficients k_x, k_y; f_{xa}, f_{ya}, etc., are numerical constants, whose values, as obtained by numerical calculation, are shown in Tables 4 and 5. It will be seen that, the effect of non-linear term in $(\dot{a}_1)^2$ is rather small, in so far as we are concerned in the range of configurations taken up in our numerical estimations.

REFERENCES

[1] KITO, F. (1967): On Vibration of two Circular Cylinders, which are immersed in Water Region of infinite Extent, Soc. Naval Architects of Japan, pp. 1~9.
[2] KITO, F. (1979): On Vibration of two Circular Cylinders, which are immersed in a Water Region-II, Keio Engineering Reports, **32**, No. 5, pp. 45-56.
[3] KITO, F., Dito- III, **32**, No. 6, pp. 57-74.