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PHYSICAL THEORY OF MEASURING PROCESS

—A CRITICAL REVIEW—

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ABSTRACT

After a survey of the orthodox interpretation of quantum mechanics and von Neumann's approach to a measurement theory, the measurement theory of Daneri, Loinger and Prosperi is critically reviewed in regard in particular to their aims and assumptions. The review will lead us to see what is still desirable of a physical theory of measurement on quantum mechanical systems.

§ 1. Introduction

Quantum mechanics has many paradoxical aspects. It does not give a self-consistent and intuitively satisfactory view of the physical world. There remains a dualism in the deterministic unitary time-evolution obeying the Schroedinger equation and the probabilistic jump of state at " measurement " (sudden reduction of wave function). One could wish that the latter also be controlled by the Schroedinger equation. In fact, there has been proposed the so-called physical theories of measurement, which however, appear to have many points to be clarified. In this paper, we shall critically examine the physical theory of measurement by Daneri, Loinger and Prosperi. We note that their theory is restricted to the measurement of the first kind, which brings or leaves the state of the system into the eigenstate of the measured quantity.

Before starting our analysis of the physical theory of measurement, we must briefly review the orthodox intrepretation of quantum mechanics in section 2 and von Neumann's theory of measurement in section 3. Then in section 4 the measurement theory of Daneri, Loinger and Prosperi will be analyzed and the paper will be concluded by critical discussions in section 5.

§ 2. Measurement Process within the Orthodox Interpretation of Quantum Mechanics (See ref. N. Bohr 1935 and M. Jammer 1974)

The orthodox interpretation of quantum mechanics. i. e., Copenhagen interpretation of Bohr, treats the measuring process with the concept of complementarity as follows. The orthodox interpretation supposes that the measuring apparatus is described by classical physics and has interaction with microscopic object that brings forth the information of microscopic world. We cannot say anything about the microscopic object itself independently from measuring apparatus, but only can talk about the pair of the apparatus and the object. They are considered to be inseparable one body. Even if objects are identical, one has a different phenomenon when the measuring apparatus are not identical. We see the trace of microscopic objects on the classical measuring apparatus, and it is the way of appearance of this trace that we can discuss.

The concept of complementarity enters here. For example, apparatus which observes wave property leads to wave aspect of the object and apparatus which observes particle property leads to particle aspect of the same object. The wave and the particle aspects are complementary to each other: We cannot observe both of them at the same time, while the description of the object is not complete with just one of them. Microscopic object is something that makes one of such complementary trace depending upon the classical apparatus used. Therefore, interpretation of Heisenberg's uncertainty relation and of wave function cannot be quite like Born's probability interpretation associated with particle image. Thus, in the orthodox interpretation, wave function is not regarded as giving the distribution of measured values which are definite but unknown before measurement. The uncertainty relation is interpreted not as a relation between errors of measurement, but as a relation between degrees of indeterminancy of the complementary aspects of states. Measurement process leads the indetermined state into the more determined one in regard to the measured physical quantity proper to the apparatus used.

Now, measuring apparatus is described by classical physics and microscopic object is described by quantum mechanics. Hence, interaction between them is inevitably uncontrollable, allowing only statistical statements about the results of measurement.

Even in the orthodox interpretation the boundary between the object and the observer can be shifted at one's disposal in so far as the apparatus is classical in nature.

The uncontrollable interaction explains the reduction of wave packets plainly. For example, by such an interaction the state

$$\phi_0 = \sum_n c_n \phi_n \tag{1}$$

with ϕ_n being eigenfunction of measured quantity, is changed into the state

$$\phi = \sum_n c_n \phi_n \exp(i\theta_n) \tag{2}$$

with random phases $\{\theta_n\}$. (2) may be rewritten in the density matrix form

$$|\phi\rangle\langle\phi| = \sum_{nm} c_n c_m^* |\phi_n\rangle\langle\phi_m| \exp[i(\theta_n - \theta_m)]\,. \tag{3}$$

Quantum mechanics is a theory for ensemble. The ensemble average of $\exp[i(\theta_n$

$-\theta_m)]$ is δ_{nm}, so that the density matrix (3) is reduced as

$$|\psi\rangle\langle\psi|\rightarrow\sum_n|c_n|^2|\phi_n\rangle\langle\phi_n|\,. \tag{4}$$

This is the desired form of the reduction of wave packets. If initial state is an eigenstate, then (3) is $|\phi_n\rangle\langle\phi_n|$ and is not changed by the interaction.

§ 3. Von Neumann's Theory of Measurement

Von Neumann treated the problem of measurement in his wellknown book "Die Mathematische Glundlagen der Quantenmechanik" (J. von Neumann 1932). He thought that because of measuring apparatus is composed of microscopic atoms, it has also to be described by quantum mechanics and consequently interaction between apparatus and object also ought to be described quantum mechanically. Whole system of apparatus and object is described by the tensor product of Hilbert spaces of each subsystems.

In the measurement of the first kind, any eigenstate for an object ψ_n of measured observable A proper to the apparatus is never changed by the interaction with the measuring apparatus. On the other hand, for the purpose of our reading out the result of the measurement, the state Φ of the apparatus must be changed into the state Φ_n corresponding to the state of the object after the measurement. Thus, the state vector of the whole system is transformed as

$$\psi_n\otimes\Phi\rightarrow\psi_n\otimes\Phi_n\,. \tag{5}$$

By the linearity of the equation of motion, (5) implies, for a general state $\psi=\sum_n c_n\psi_n$ of the object, that the measurement induces the transformation

$$\sum_n c_n\psi_n\otimes\Phi\rightarrow\sum_n c_n(\psi_n\otimes\Phi_n)\,. \tag{6}$$

This is called the first stage of the measurement because the right-hand side of (6) is still a pure state.

While, mathematically, the transformation (6) exists always as a unitary transformation, it is questionable physically wether or not we can really build such an apparatus, because it is not true that there exist always the correspondence from self-adjoint operators to physical observables. Besides, the transformation (6) is incompatible with some kind of conservation law such as the conservation of angular momentum. This was noticed by Wigner (E. P. Wigner 1952, H. Araki and M. Yanase 1960 and M. Yanase 1961). In general, (6) can hold only in the measurement of the observable which commutes with all the additive conserving observables and otherwise some errors are unavoidable. But if the apparatus is sufficiently large, the errors become smaller, so we may avoid this difficulty in practice by considering a large apparatus (see ref. 3).

Then, after the transformation (6) the second stage of measurement must take place bringing the pure state (6) into a mixture, so that we need another apparatus, which is also described by quantum mechanics. The total system of the object, the first and the second apparatus is again described by a tensor product. Denoting the state of the second apparatus by Θ, the total system is transformed as

$$(\sum_n c_n\psi_n\otimes\Phi_n)\otimes\Theta\to\sum_n c_n\psi_n\otimes\Phi_n\otimes\Theta_n\,. \tag{7}$$

From the right-hand side of (7), we suspect that the statement for the object is unchanged under the shift of the boundary between the object and the observer. That is, regarding $\psi\otimes\Phi$ to be the object and Θ to be the apparatus is the same as regarding ψ to be the object and $\Phi\otimes\Theta$ to be the apparatus, because suffices n of ψ, Φ and Θ are identical. Introduction of the third, fourth···apparatus for reduction of wave packets in the preceding system means that we fall into infinite regression of the observer. But, measuring process must be a finite chain. Therefore, von Neumann postulated the reduction of wave packets as the irreducible element of quantum mechanics. It is called " Projection Postulate ". Thus, the time-evolution of the states in his quantum mechanics has a dual character, namely unitary time-evolution and the projection.

In his view, this projection is ultimately caused by the observer's mind or self-consciousness which has the ability of introspection. This thought to introduce consciousness into the physical world will easily tend to subjectivism or solipsism and causes paradoxes such as " Wigner's friend " (E. P. Wigner 1963).

By the way, we shall notice that unitary time-evolution will never bring a pure state into a mixture in the exact sense. The final mixture must be derived from a mixture initial state. Wigner pointed out that assuming an initial state of the apparatus to be a mixture is incompatible with the equation of motion (E. P. Wigner 1963). Let an initial state of the object be $\psi=\sum_\nu \alpha_\nu\psi^\nu$ and the apparatus' mixture to be $\sum_\rho p_\rho|\Phi^{\rho\nu}><\Phi^{\rho\nu}|$. The suffix ρ of the state vector for the apparatus represents degeneracy of microscopic state of the apparatus for the same macroscopic reading and this reading itself is represented by suffix ν, p_ρ is mixing probability. Then by the first stage of measurement, whole system evolves as

$$|\psi><\psi|\otimes\sum_\rho p_\rho|\Phi^{\rho k}><\Phi^{\rho k}|=\sum_{\nu\mu} \alpha_\nu\alpha_\mu{}^*|\psi^\nu><\psi^\mu|\otimes\sum_\rho p_\rho|\Phi^{\rho k}><\Phi^{\rho k}|$$

$$\to\sum_\rho p_\rho|\Theta^\rho><\Theta^\rho|\,, \tag{8}$$

with

$$\Theta^\rho=\sum_\nu \alpha_\nu[\psi^\nu\otimes\Phi^{\rho\nu}]\,. \tag{9}$$

This Θ^ρ is a superposition of the states $[\psi^\nu\otimes\Phi^{\rho\nu}]$ by suffix ν. So Θ^ρ has various ψ^ν as components. On the other hand, it is demanded for completing measuring process that final state is a mixture of vectors which are superposition within each macroscopic component, i. e., mixture of $\{\Xi^\mu\}$ which are superposition of the states $\{\psi^\mu\otimes\Phi^{\rho\mu}\}$ by suffix ρ so that the apparatus part of Ξ^μ is made of one macroscopic state Φ^μ. We cannot, however, rewrite (8) into the desired final mixture as mentioned above (E. P. Wigner 1963). Hence it is inconsistent with the law of motion to assume initial state of measuring apparatus as mixture.

This conclusion is unavoidable if the mixture is taken in the exact sense of words, and it may not be forbidden to assume the initial state of the apparatus to be a mixture approximately so that it can approximately evolve into a mixture as desired.

Another important remark concerns the so-called "Reduction Formula" (J. M. Jauch 1964 and 1968). We describe object-apparatus system with a tensor product. From this follows that, even if the total system is a pure state, the state of each subsystem is not necessarily pure. In fact, consider a system consisting of two subsystems I and II, and let W and W_I be density matrices for the total system and the subsystem I respectively. Then,

$$W_I = \mathrm{tr}_{II}\, W \tag{10}$$

and this is a mixture in general; here, tr_{II} denotes trace operation in the Hilbert space for subsystem II. For example, if W is a pure state represented by a linear combination of tensor products, $\psi = \sum_r c_r \phi_r \otimes \psi_r$, i. e.,

$$W = |\psi\rangle\langle\psi| = \sum_{rs} c_r c_s^* |\phi_r\rangle\langle\phi_s| \otimes |\psi_r\rangle\langle\psi_s|\,, \tag{11}$$

then subsystem I is a mixture

$$W_I = \mathrm{tr}_{II}\, W = \sum_r |c_r|^2 |\phi_r\rangle\langle\phi_r|\,. \tag{12}$$

If we apply this result on the measuring process, the reduction of wave packets appears to be achieved without the second stage of measurement if the system consists of a pair of subsystems of which only one is subjected to observation.

The mixture obtained by observing a subsystem only is called the improper mixture (B. d'Espagnat 1965 and 1976). On the other hand the proper mixture is a real mixture composed of objectively definite vector states. In the case of (12) the subsystem I cannot have the corresponding vector. But, when we calculate the average of a physical quantity belonging solely to the subsystem I, the formula has the same form as proper mixture of subsystem I has. We cannot distinguish them from each other by any experiment on a subsystem only. We can distinguish them only by observing two or more quantities pertaining to both of the subsystems; that a measurement of any single quantity does not help will be explained in the next section.

§ 4. Physical Theory of Quantum Measuring Process

Both the orthodox interpretation and von Neumann's theory can be called the orthodox in a wide sense, in which the basic formulation of quantum mechanics is not altered. The orthodox interpretation uses another view of the world, classical mechanics, as a premise in explanation of foundation of quantum mechanics. This attitude would not satisfy the majority of people. On the other hand, in von Neumann's theory, the problem of infinite regression of observer or the reaction of consciousness to physical world remains perhaps as a philosophical problem.

As other approaches to the measurement problem, there are proposed many theories of hidden variables and the many-world interpretation (H. Everette 1957 and B. S. De Witt and N. Graham 1973). The hidden variable theories aim at recovering the deterministic picture of microscopic world in sub-quantum level. The many-world interpretation alters the orthodox interpretation in such a way that the states are never reduced but our world splits into parallel worlds corresponding to each eigenvalue.

Within the framework of the orthodox interpretation in a wide sense, there is a room for conceiving a physical theories of the measuring process. They are designed to make, in some sense and with some physically appropriate condition, the unitary time-evolution during measuring process and the reduction of wave packets give the same prediction in regard to probability interpretation.

Among such theories, the theory of Daneri, Loinger and Prosperi (A. Daneri, A. Loinger and G. M. Prosperi 1962), (D. L. P.), is probably most well-known. While the von Neumann theory did not make use of the macroscopic nature of measuring apparatus, D. L. P. does thereby terminating the regression before measuring apparatus so that the measured data acquire the objectivity.

Their apparatus is composed of a detector which directly interacts with microscopic object and an amplifier or a memory, whose state changes upon the interaction from the initial metastable state to a state having impression of the information of the object, and then goes over to a state of thermal equilibrium. The irreversible part of the process prepares the apparatus in such a state that we can read off the record classically.

The position of the pointer of a measuring apparatus that we can read is a macroscopic state which is enormously degenerate in regard to the microscopic states; such microscopic states cannot be distinguished by any observations of macroscopic observables. Similar indistinguishability exists also between some pure state and mixture.

In order to formulate this concept of indistinguishability put forward by D. L. P. more precisely we follow Jauch (J. M. Jauch 1964 and 1968) to define equivalence relation among the density matrices of measuring apparatus in regard to some set of macroscopic observables. We say that the state W_1 and W_2 are equivalent under the set of observables S if

$$\mathrm{tr}\, AW_1 = \mathrm{tr}\, AW_2\,, \tag{13}$$

for all A belonging to S. This equivalence is denoted by $W_1 \overset{(S)}{\sim} W_2$. With this definition we can classify (microscopic) states of the measuring apparatus into equivalence classes, which we call the macroscopic state. Namely, the macroscopic state [W] is a set of those microscopic states which are equivalent to a microscopic state W. The condition that {[W]} are not changed by measurement of all observables in S (classical measurement) is that S is abelian. The concept of the classical quantity cannot be defined without specifying the set of observables S (See ref. 5). In particular, it is the same as classical measurement to discuss only one quantity.

One may wonder that the set of all macroscopic observables is not abelian. It is true that some macroscopic observables do not commute each other, but the commutator of the macroscopic observables (each of order N, the particle number) is often of the order lower than N.

Now, if we assume that only the macroscopic measurements S are all commutable with each other, then there exists at least one mixture equivalent to a given pure state. Namely

$$\sum_{rs} c_r c_s^* |\Phi_r> <\Phi_s| \overset{(S)}{\sim} \sum_r |c_r|^2 |\Phi_r> <\Phi_r| , \tag{14}$$

where $\{\Phi_r\}$ are simultaneous eigenvector of S. However, one cannot be satisfied with this result, because the equivalence relation (14) is broken down by the unitary time-evolution after measurement.

$$\begin{array}{ccccc} \text{superposition} & \rightarrow & \text{superposition} & & \\ \wr & \text{time-evolution} & \not\wr & \begin{array}{c}\sim \\ \not\sim\end{array} & \text{another mixture} \\ \text{mixture} & \rightarrow & \text{mixture} & & \end{array}$$

At this point, statistical mechanical property of the apparatus should be taken into consideration.

In the D. L. P. theory, the macroscopic nature of apparatus leads to the expression

$$\sum_r c_r \psi_r \otimes \sum_j \alpha_j \Phi_{rj} \tag{15}$$

for the state of the whole system, the object and the apparatus, after the first stage of measurement, where $\sum_r c_r \psi_r$ is the initial state of the object and $\{\Phi_{rj}\}_{j=1,2,\dots,n_r}$ are the bases for the r-th macroscopic state of the apparatus. D. L. P. call this set of the macroscopic states (except the difference of the value of non-conservative macroscopic quantities) the r-th channel.

Their theory is based upon the following assumptions:

1] One considers some macroscopic measurements upon the apparatus which are commutable each other.

2] The channel is conserved under the time-evolution due to the Schroedinger equation.

3] The microscopic states behave ergodically in each channel. More precisely, there exists in each channel k a shell C_{ke_k} corresponding to the thermal equilibrium state.

Assumption 1] means that the S in (13) is abelian. A set of some macroscopic quantities is not abelian, but we can often approximate them by an abelian set, and it is appropriate to think one macroscopic quantity which couples with and represents certain aspect of complementarity of the object in the spirit of the orthodox interpretation.

From this assumption interference terms do not arise. In fact, as in the classical system, there is no room for measurements requiring superposition of states from different channels.

Assumption 2] guarantees that the absence of the interference terms at the instant of the interaction remains even after the time-evolution due to the Schroedinger equation; the breaking down of the equivalence (13) due to the time-evolution is thus avoided.

D. L. P. use ergodic property to eliminate the possibility that the result of the measurement should depend upon the microscopic details of the apparatus. They divide the Hilbert space of the states of the apparatus into subspaces each corresponding to the macroscopic quantum numbers. First, they consider the energy shells C_a which are the subspaces spanned by all the energy eigenstates with the eigenvalues belonging to interval $(E_a, E_{a+1}=E_a+\varDelta E)$; the dimension of C_a is denoted by S_a. Then similarly in regard to other macroscopic conserved observables, they subdivide the energy shell C_a into the subspaces $\{C_{ak}\}_k$, each of dimension S_{ak}. The set C_{ak} defines the channels. Further, C_{ak} is subdivided into $\{C_{ak\nu}\}_\nu$ in regard to the other macroscopic but non-conserving observables. Then finally the basis vectors in the $C_{ak\nu}$ are the microscopic vectors $\{\Omega_{ak\nu i}\}_i$. The quantum number ν is used for describing the change of the state into the thermal equilibrium.

Now, we restrict our attention to an energy shell and drop the suffix a. Let

$$M[\cdots]\equiv\lim_{T\to\infty}(1/T)\int_0^T dt[\cdots] \tag{16}$$

be the time average and let $\mathfrak{A}$ denote the ensemble average with respect to the initial states of the apparatus. Then, the ergodicity condition

$$\mathfrak{A}M\{\sum_{i=1}^{s_{k\nu}}|<\Omega_{k\nu i},\ \exp(-iHt/\hbar)\cdot\Omega_{k\mu j}>|^2\simeq s_{k\nu}/S_k\,, \tag{17}$$

where $s_{k\nu}$ and S_k are dimensions of $C_{k\nu}$ and C_k respectively, has been established in several cases (L. van Hove 1959, G. M. Prosperi and A. Scotti 1960 and P. Bocchieri and A. Loinger 1959). The left-hand side of (17) is the transition probability into the subspace $C_{k\nu}$ as averaged all over the initial states in channel k. The ergodicity (17) says that the averaged transition probability is given simply by the ratio of the dimensions of channel k and the final subspace $C_{k\nu}$ independently from initial state. We notice that the description of apparatus by mixture, as was criticized by Wigner (J. M. Jauch, E. P. Wigner and M. M. Yanase 1967), is assumed. Nevertheless, the D. L. P. theory can be accepted as an approximate theory, although Wigner was right in the rigorous sense of the term.

The subspace C_{ke_k}, that corresponds to thermal equilibrium state, has a dimension enormously larger than that of subspaces $C_{k\nu}$ $(\nu\neq e_k)$, so that the probability $u_{k\nu}(t)$ for the state of apparatus to be in $C_{k\nu}$, becomes

$$u_{k\nu}(t)\simeq\delta_{\nu e_k}\,, \tag{18}$$

after sufficiently long time t irrespective of the initial value $u_{k\nu}(t_0)$, if the initial state belonged to one channel. Thus, the apparatus spends almost all its time in a macroscopic state C_{ke_k}. If the initial state ψ_0 is a superposition of the states from different channels, i e., $\psi_0=\sum_{k\mu i}<\Omega_{k\mu i},\psi_0>\Omega_{k\mu i}$, then (18) must be replaced by

$$u_{k\nu}(t)\simeq\sum_{\mu i}|<\Omega_{k\mu i},\psi_0>|^2\delta_{\nu e_k}\,. \tag{19}$$

Having made the assumptions of D. L. P. clear, we now wish to go into their theory of measurement process. After the first stage of measurement, the interaction between object and apparatus is cut off. If this is followed by a measurement of another observables, non-commutable with the previous one, the result will in general be different depending upon whether the first measurement has resulted in a pure state or a mixture. But with the above assumptions 1] and 2], interference terms vanish and they give the same prediction for second measurement, where dependence on microscopic state is wiped out by 3].

In fact, if the interaction time for the measurement of an observable A is t_1, then the measurement changes the initial state at $t=0$ into some other state at t_1. Namely,

$$\sum_r c_r\psi_r \otimes \sum_{\nu i} \alpha^0_{\nu i}\Phi_{0\nu i} \rightarrow \sum_r c_r\psi_r \otimes \sum_{\nu i} \alpha^{t_1}_{\nu i}\Phi_{r\nu i}\,; \tag{20}$$

note that the initial state of the apparatus is assumed to be in the channel 0. What value do we get now if, upon the state (20), we make the measurement of B which is non-commutable with A? The probability of obtaining the eigenvalve b_s of B under the condition that previous observation of A gave the value a_k is given by

$$\text{Prob. } (\text{B}=b_s|\text{A}=a_k)=\sum_{j=1}^{s_{k_\nu}} |<\phi_s\otimes\Phi_{k_\nu j},\ \sum_r c_r \text{U}(t_2-t_1)\psi_r\otimes\sum_{\mu i}\alpha^{t_2}_{\mu i}\Phi_{r\mu i}>|^2$$

$$=\sum_{j=1}^{s_{k_\nu}} |\sum_r c_r<\phi_s,\ \text{U}(t_2-t_1)\psi_r><\Phi_{k_\nu j},\ \sum_{\mu i}\alpha^{t_2}_{\mu i}\Phi_{r\mu i}>|^2\,, \tag{21}$$

where $\text{U}(t_2-t_1)$ and α^{t_2} represent time-evolution, from the time of the interaction to the time of the second measurement, of the object and apparatus respectively. Because of the orthogonality $<\Phi_{k_\nu j}, \Phi_{r_\mu j}>=0,\ k\neq r$, (21) becomes

$$|c_k|^2|<\phi_s,\ \text{U}(t_2-t_1)\psi_k>|^2\sum_{j=1}^{s_{k_\nu}}|<\Phi_{k_\nu j},\ \sum_{\mu i}\alpha^{t_2}_{\mu i}\Phi_{k\mu i}>|^2\,, \tag{22}$$

which depends still upon the microscopic details of the state of the apparatus. By the ergodicity assumption 3], however, the probability (21) tends in a sufficiently long, but not macroscopically long time after the interaction for the measurement of A to

$$|c_k|^2|<\phi_s,\ \text{U}(t_2-t_1)\psi_k>|^2\delta_{\nu e_k}\,. \tag{23}$$

This result is in accord with the view of the orthodox interpretation that the state of the object undergoes the reduction of wave packets, going thereby over to

$$\sum_k |c_k|^2|\psi_k><\psi_k|\,, \tag{24}$$

in that both (23) and (24) give the same prediction about the measurement of B.

§ 5. Discussion

As we have seen in the previous section, the D. L. P. theory as well as other physical theories of measurement shows only that the prediction about the second measurement is the same whether or not the reduction of wave packets is assumed for the first measurement. That is, they never deduce the reduction of wave packets from some other principles of the quantum mechanics in any sense. Within the framework of quantum mechanics, it seems to be inevitable to use the probability interpretation in some form. But, it causes a problem because it demands measurement process for itself. We can shift the use of probability interpretation for defining the equivalence between the pure state and the mixture to the later stage by introducing an additional measurement which is to be carried out after the measurements we are interested in and to which the probability interpretation is applied. But, in order to analyse the last measurement, we must have one more measurement. This is a kind of infinite regression like von Neumann's case. We must reflect on our method of recognition of our world here.

Even if we accept the probability interpretation, the consequence of the D. L. P. theory or other physical theories of measurement are only approximate, so that they do not fill up the gap between the concepts of classical and quantum physics. If we do not regard the state vector as merely a carrier of information, or find it as real, approximate identification does not make much sense. If the state vector is given a reality, it ought to evolve continuously during measuring process into some eigenvector even though its selection is probabilistic (See ref. 12). Pursuit of this line of thought would inevitably lead to theories of hidden variables.

Let us now critically examine the assumptions of the D. L. P. theory and try to see if this theory can resolve the measurement problem.

On the assumption 1]: It is quite natural to assume that apparatus has a macroscopic scale. But, is there a clear-cut boundary between macroscopic and microscopic systems? No, the boundary may be shifted continuously to some extent, and in fact there are macroscopic quantum phenomena such as superconductivity. Even if we could define a boundary we cannot prohibit the superposition of different macroscopic states by any principles of quantum mechanics. If we cannot find a reason for prohibition of superposition, we fall into von Neumann's infinite regression again. On this point D. L. P.'s view seems to partially resemble with Bohr's which presupposes classical apparatus for objectivity.

Moreover, from assumption 1], we cannot substitute for the left-hand member of (21) an eigenstate of total angular momentum of the apparatus, if the state of the apparatus carrying the information of the object is an eigenstate of the total momentum of the apparatus which is not commutative with the total angular momentum. That the substitution in (21) is not allowed implies that (23) is no longer valid, and hence interference terms arise in regard to the state of the apparatus and consequently to those of the object, if the measurement is made of the angular momentum state of the apparatus despite the fact that the object has interacted with the macroscopic apparatus.

On the assumption 2]: Preservation of channels is satisfied during a measurement process so that we can read out the information of the object by the channels. As other quantum mechanical theories, the D. L. P. theory does not describe the whole universe with one state vector. They take account of only those systems which are coupled physically with the very measuring apparatus. So if there exists some system which interacts with the apparatus after the measurement and destroys the channels before our second measurement on the apparatus, the consequence of the D. L. P. theory cannot hold any longer, for vanishing of the interference terms is due to the macroscopic commuting observation on the apparatus and not due to any affair on the side of the object itself. Thus, it is still to be desired that reduction of wave packets of the object is not affected by whatever happens to the apparatus after the cut off of interaction (See ref. 13).

On the assumption 3]: On the basis of ergodic behaviour, the states can interfere again at the Poincaré recurrence time. However, the recurrence time is much too longer than the time scale of human sense.

The ergodicity condition is not used in the D. L. P. theory in accounting for vanishing of the interference terms; it is used only to guarantee the wiping out of the dependence on the microscopic states. But, it is more desirable that the interference terms are eliminated by the ergodic or the irreversible processes.

Further, the ensemble average and the time average is taken in (17). The time average is reasonable if we take into account the time duration of our reading the pointer position of the apparatus. However, the ensemble average means to describe the initial state of the apparatus with mixture, i. e., our ignorance. However, it should not be allowed in the fundamental theory treating the irreducible probability that the probabilities due to our ignorance are mingled (See ref. 14).

In general, probabilistic description seems to have limitation that it cannot describe the whole universe because it takes an outer world which picks a sample world up. We cannot pick a sample up because we are also described by the theory itself. In other words, the probabilistic description is for the open systems.

In conclusion, we have seen that the D. L. P. theory explains measuring process under the peculiar assumption 1] which contains a grave problem. Moreover the D. L. P. theory has the aspect of infinite regression in being a theory which use the probability interpretation. The measurement process in quantum mechanics are still to be studied more deeply and thoroughly.

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12. Many-Worlds interpretation (See ref. 9) is complete monoism of the Schrödinger equation and it can lead out probability interpretation from the theory itself. States are always evolved continuously under the Schrödinger equation in the theory.
13. It resembles the Einstein-Podolsky-Rosen paradox (See ref. 7). Another difficulty about this is that the states of the system on the earth would be reduced by a process in other galaxy, if we wanted to shift the initial state to the beginning of the universe, the big-bang. Of course, usually initial state is taken at the beginning of the measuring process in question.
14. It is not cyclic to deduce the reduction of wave packets from irreversibility which, by the principle of quantum mechanics, arises only at the reduction of wave packet itself, because this irreversibility is not irreducible but caused by the lack of information or the coarse grained description.