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ON VIBRATION OF TWO CIRCULAR CILYNDERS, WHICH ARE IMMERSED IN A WATER REGION—III

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ABSTRACT

This is the continuation of author's study about the same theme as the present one. In the former report, the author has given results of his analytical study with regard to vibration of two circular cylinders, which are immersed in a fluid region of infinite extent. Therein, the vibration was assumed to be of finite amplitudes. And, we treated the case of two dimensional problem of an ideal fluid, merely main analytical formula being given there. In the present paper, the author has made, continuing former study, more detailed account about this problem, and also given some numerical examples. It is to be noted that, here, we are concerned about the case in which No. 1 cylinder is moving, while No. 2 cylinder is kept at stand still.

1. Statement of the Problem

In Fig. 1, two circular cylinders of radii R_1 and R_2 are shown, which represents initial configuration (at time $t=0$). They are immersed in a fluid region of infinite extent, the fluid being considered to be an ideal fluid. Positions of centers of two circular cylinders are here located on the x-axis, and are spaced by a distance of $E_0=D$ each other. Radii R_1 and R_2 are to be kept constant at every subsquent time t $(0\leq t)$. There is no difficulty in extending the following treatment to cases in which R_1 and R_2 varies, (as given function of time t). For $0<t$, positions of two circular cylinders are considered to move in a prescribed manner. Fig. 2 shows us the positions of two circular cylinders at time t. Here, two cylinders have moved by distances a_1 and a_2 in angular directions β_1 and β_2 respectively. In what follows, we take up the case in which a_1 and a_2 are given functions of time t, while β_1 and β_2 are kept at constant values. (However, we can treat the case in which β_1 and β_2 are also given functions of t, if we wish it.) Displacements a_1 and a_2 need not to be taken as small quantities.

The author has made, in the previous report, an analytical study about the fluid motion thus set up, and resulting hydraulic forces acting on walls of two circular cylinders. The surrounding fluid being assumed to be an ideal fluid, we treat the problem as a case of two-dimensional potential flow (but non-stady). In the present paper, which is the continuation of the study, more detailed account

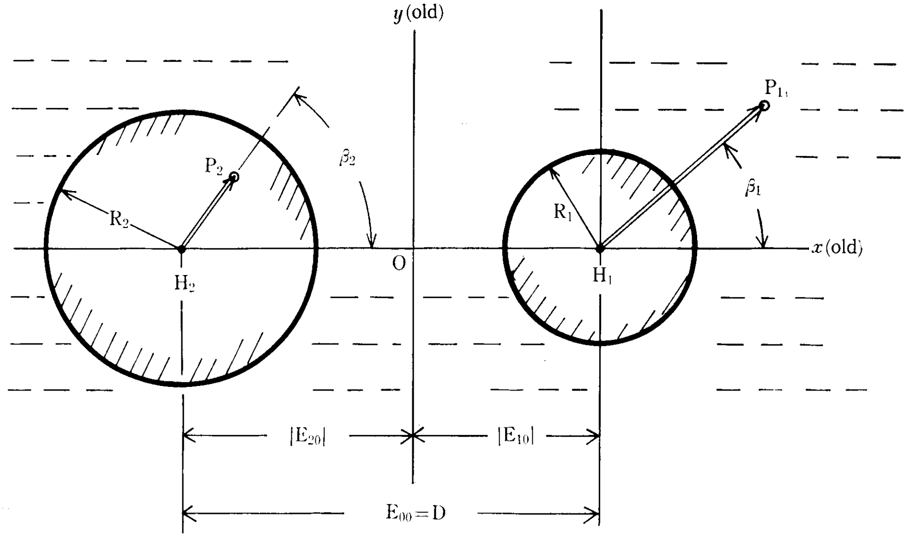

Fig. 1. Position of two Circular Cylinders at initial state. ($t=0$)

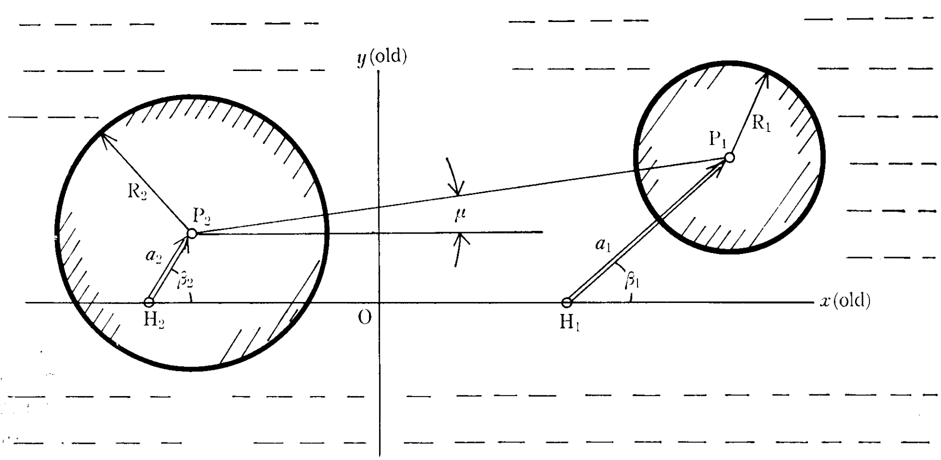

Fig. 2. Position of two Circular Cylinders at displaced state (at time t).

will be given, restricting ourselves to special case in which $a_2 \equiv 0$, while a_1 varies with time t, in a prescribed manner.

2. Notations

We use the following notations which are the same as in author's previous paper, thus ;

x, y=rectangular coodinates of a point in xy plane,
$z=x+iy$, a complex variable,
c=distance from origin 0, of radical centers of system of bi-polar coordinates,
h=coefficient of linear element for the case of bi-polar coordinates (ξ, η),
ξ, η=a system of bi-polar coordinates, representing any point on the xy plane,
R_i=radius of circular cylinder ($i=1, 2$),
E_i=position of center of ditto,
p=fluid pressure, ρ=density of the fluid,
ϕ=velocity potential of fluid motion, giving absolute velocity of flow.

In want follows, several coefficients A_i, B_i, C_i etc., will be used for giving us the solution in form of an infinite series. These coefficients will be defined where they make first appearances. These coefficients A_i, B_i, etc., are independent of variables x, y (also ξ, η), but they may be functions of time t, because we are treating the case of non-stationary fluid motion. Also, guishing individual cases of application.

3. Main Results obtained in the Previous Paper

In what follows, we shall give values of velocity potential ϕ, hydro-dynamic pressure p, and also hydro-dynamic forces F_x, F_y, which we obtained in the previous paper. It may be mentioned here that the starting point of our discussion was based upon a system of bi-polar coordinates as sketched in Fig. 3.

Firstly, the velocity potential ϕ can be written,

$$\phi=\phi_1+\phi_2 \tag{1}$$

where ϕ_1 satisfies the boundary condition at wall of No. 1 cylinder ($\xi=\xi_1$), while the No. 2 cylinder stands still. On the other hand, ϕ_2 satisfies the boundary

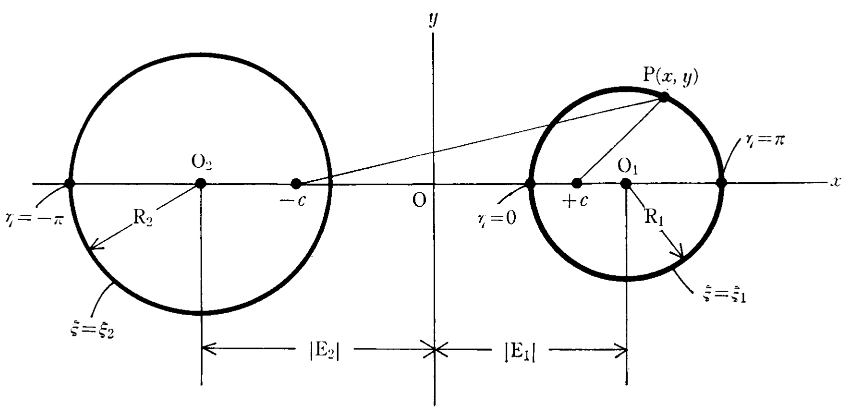

Fig. 3. Configuration of two circular cylinders, represented by bipolar coordinates

condition at wall of No. 2 cylinder ($\xi=\xi_2$), while the No. 1 cylinder is kept at stand still. Thus, we have, ($k=1,2$),

$$\phi_k=\sum_{n=1}^{\infty}(A_{nk}\sin n\eta+B_{nk}\cos n\eta)\cdot(\text{sh}\, n\xi+C_{nk}\,\text{ch}\, n\xi) \tag{2}$$

Especially, for the values of ϕ_s ($s=1,2$) at two cylindrical walls (at which $p=1,2$) we have

$$\phi_s^{(p)}=\sum_{n=1}^{\infty}[A_{ns}^{(p)}\sin n\eta+B_{ns}^{(p)}\cos n\eta] \tag{3}$$

Actual values of coefficients $A_{ns}^{(p)}$ and $B_{ns}^{(p)}$ are as follows;

$$A_{n1}^{(1)}=\frac{-1}{n}\left[\frac{c^2\dot{a}_1}{R_1}\sin\gamma_1\right]\coth n(\xi_1-\xi_2)\cdot[K_{n-1}^{(2)}(\lambda_1)-K_{n+1}^{(2)}(\lambda_1)]$$

$$B_{n1}^{(1)}=\frac{-2}{n}[ca_1\cos\gamma_1]\coth n(\xi_1-\xi_2)\cdot[\text{sh}^2\,\xi_1K_n^{(2)}(\lambda_1)-\text{ch}\,\xi_1K_n^{(1)}(\lambda_1)]$$

$$A_{n1}^{(2)}=\frac{-1}{n}\left[\frac{c^2\dot{a}_1}{R_1}\sin\gamma_1\right]\frac{1}{\text{sh}\, n(\xi_1-\xi_2)}\cdot[K_{n-1}^{(2)}(\lambda_1)-K_{n+1}^{(2)}(\lambda_1)]$$

$$B_{n1}^{(2)}=\frac{-2}{n}[c\dot{a}\cos\gamma_1]\frac{1}{\text{sh}\, n(\xi_1-\xi_2)}\cdot[\text{sh}^2\,\xi_1K_n^{(2)}(\lambda_1)-\text{ch}\,\xi_1K_n^{(1)}(\lambda_1)]$$

$$A_{n2}^{(1)}=\frac{-1}{n}\left[\frac{c^2\dot{a}_2}{R_2}\sin\gamma_2\right]\frac{1}{\text{sh}\, n(\xi_1-\xi_2)}\cdot[K_{n-1}^{(2)}(\lambda_2)-K_{n+1}^{(2)}(\lambda_2)]$$

$$B_{n2}^{(1)}=\frac{-2}{n}[c\dot{a}_2\cos\gamma_2]\frac{1}{\text{sh}\, n(\xi_1-\xi_2)}[-(\text{sh}\,\xi_2)^2K_n^{(2)}(\lambda_2)+\text{ch}\,\xi_2K_n^{(1)}(\lambda_2)]$$

$$A_{n2}^{(2)}=\frac{-1}{n}\left[\frac{c^2\dot{a}_2\sin\gamma_2}{R_2}\right]\coth n(\xi_1-\xi_2)\cdot[K_{n-1}^{(2)}(\lambda_2)-K_{n+1}^{(2)}(\lambda_2)]$$

$$B_{n2}^{(2)}=\frac{-2}{n}[c\dot{a}_2\cos\gamma_2]\coth n(\xi_1-\xi_2)\cdot[-(\text{sh}\,\xi_2)^2K_n^{(2)}(\lambda_2)+\text{ch}\,\xi_2K_n^{(1)}(\lambda_2)]$$

It will be observed that, infinite solutions (2) and (3) are absolutely convergent. After numerical estimation about several practical cases, we find that the convergence is fairly good, and we may stop at $n=1,2,\cdots,6$, at least for practical purposes.

In the following treatment, in which we take the case of $a_2\equiv0$, we have

$$A_{n2}^{(1)}=0,\qquad A_{n2}^{(2)}=0,\qquad B_{n2}^{(1)}=0,\qquad B_{n2}^{(2)}=0.$$

As to the hydrodynamic pressure p, it is to be obtained from the formula (for the case of moving axes of coordinates),

$$-\frac{1}{\rho}p+U=\frac{\partial\phi}{\partial t}+\frac{1}{2}\left\{\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2\right\}-(c_x-\Omega y)\frac{\partial\phi}{\partial x}-(c_y+\Omega x)\frac{\partial\phi}{\partial y}+C \tag{4}$$

Here we denote by (c_x, c_y) linear velocity of motion of origin of our moving axes, while Ω is angular velocity of rotation of this frame of axes. U is the potential of field of gravitation, but here we take $U \equiv 0$.

(1) The resultant force (F_x, F_y), exerted by pressure p, upon wall surface of No. 1 circular cyliner is given by;

$$F_x=\int_0^{2\pi}[-p]\left[\left(\frac{c}{R_1}\right)^2\frac{1}{\operatorname{ch}\xi_1+\cos\eta}-\operatorname{ch}\xi_1\right]\cdot\frac{c}{\operatorname{ch}\xi_1+\cos\eta}d\eta \tag{5}$$

$$F_y=\int_0^{2\pi}[-p]\left[\frac{c^2}{R_1}\frac{\sin\eta}{(\operatorname{ch}\xi_1+\cos\eta)^2}\right]d\eta \tag{6}$$

(2) Hydrodynamic Force acting upon will of No. 2 circular cylinder, is given by;

$$F_x=\int_0^{2\pi}[+p]\left[\left(\frac{c}{R_2}\right)^2\frac{1}{(\operatorname{ch}\xi_2+\cos\eta)}-\operatorname{ch}\xi_2\right]\cdot\left[\frac{c}{\operatorname{ch}\xi_2+\cos\eta}\right]d\eta \tag{7}$$

$$F_y=\int_0^{2\pi}[-p]\left[\frac{c^2}{R_2}\frac{\sin\eta}{(\operatorname{ch}\xi_2+\cos\eta)^2}\right]d\eta \tag{8}$$

4. Detailed Discussion for the Case in which $a_2 \equiv 0$

In what follows, we shall show actual exoressions of hydrodynamic forces F_x, F_y for the case in which $a_1 \neq 0$, $a_2 \equiv 0$, this being done for presenting simpler expressions. Task of evaluation of F_x, F_y may be done conveniently, in following four steps.

(A) Effect of the Term $\partial\phi/\partial t$

As we see from eq. (4), we must first evaluate the effect of term $\partial\phi/\partial t$ upon F_x and F_y. This is done, for No. 1 cylinder, by evaluating following definite integrals;

$$(F_x)=\rho\int_0^{2\pi}\frac{\partial\phi}{\partial t}\left[\frac{c}{R_1}\frac{\operatorname{sh}\xi_1}{\operatorname{ch}\xi_1+\cos\eta}-\frac{E_1}{R_1}\right]\cdot\left[\frac{c}{\operatorname{ch}\xi_1+\cos\eta}\right]d\eta \tag{9}$$

$$(F_y)=\rho\int_0^{2\pi}\frac{\partial\phi}{\partial t}\left[\frac{c^2}{R_1}\frac{\sin\eta}{(\operatorname{ch}\xi_1+\cos\eta)^2}\right]d\eta \tag{10}$$

Value of velocity potential ϕ, contained in these definite integrals, being the value at $\xi=\xi_1$, we take it in form of eq. (3), in which we have $s=1$, $p=1$. Moreover, differentiation $\partial/\partial t$, with respect to time t, must be carried out in two ways. The first one is with regard to $\dot{a}_1$, and write

$$\frac{d}{dt}(\dot{a}_1)=\ddot{a}_1$$

The second is with respect to a_1 which is contained implicitly in coefficients $A_{ns}^{(p)}$ and $B_{ns}^{(p)}$ (which contain such factors as c, E_1, etc.). And, this term may conveniently be expressed formally, as

$$\frac{\partial\phi}{\partial a_1}\frac{da_1}{dt}=\frac{\partial\phi}{\partial a_1}\dot{a}_1$$

We may express the above-mentioned fact by writing;

$$\frac{d\phi}{dt}=\frac{\partial\phi}{\partial\dot{a}_1}\ddot{a}_1+\frac{\partial\phi}{\partial a_1}\dot{a}_1$$

The author finds it more convenient to put

$$A_{n1}^{(1)}=[D\dot{a}_1]a_{n1}^{(1)}, \qquad B_{n1}^{(1)}=[D\dot{a}_1]b_{n1}^{(1)}$$

etc., etc., where $a_{n1}^{(1)}$, $b_{n1}^{(1)}$ are numerical (non-dimensional) coefficients, which depend implicitly on a_1. Further, we have

$$\frac{\partial}{\partial t}[a_{n1}^{(1)}]=\frac{\partial}{\partial a_1}[a_{n1}^{(1)}]\dot{a}_1=\alpha_{n1}^{(1)}\dot{a}_1$$

$$\frac{\partial}{\partial t}[b_{n1}^{(1)}]=\frac{\partial}{\partial a_1}[b_{n1}^{(1)}]\dot{a}_1=\beta_{n1}^{(1)}\dot{a}_1$$

Strictly speaking, values of coefficients $\alpha_{n1}^{(1)}$, $\beta_{n1}^{(1)}$, etc., are to be obtained by actuating process of differentiation, which is very complicated. However, the author has found a practical method of estimating them, approximately, which will be reported, in what follows. Under these preparatory consideration, we obtain;

$$F_x=\sum_n \rho n\pi\left(\frac{c}{D}\right)\left(\frac{D}{R_1}\right)\left(\frac{D}{\cos\gamma_1}\right)\cdot\frac{\tanh n(\xi_1-\xi_2)}{\mathrm{sh}\,\xi_1}\cdot[b_{n1}^{(1)}][dB_{n1}^{(1)}/dt] \tag{11}$$

$$F_y=\sum_n \frac{\rho n\pi D}{\sin\gamma_1}\cdot\tanh n(\xi_1-\xi_2)\cdot[a_{n1}^{(1)}][dA_{n1}^{(1)}/dt] \tag{12}$$

Therein, we have,

$$\frac{dB_{n1}^{(1)}}{dt}=D\ddot{a}_1b_{n1}^{(1)}+D\frac{\partial b_{n1}^{(1)}}{\partial a_1}(\dot{a}_1)^2$$

$$\frac{dA_{n1}^{(1)}}{dt}=D\ddot{a}_1a_{n1}^{(1)}+D\frac{\partial a_{n1}^{(1)}}{\partial a_1}(\dot{a}_1)^2$$

Summing up these results, we see that hydrodynamical forces F_x, F_y may be expressed in following form;

$$\left.\begin{aligned}F_x&=K_x\ddot{a}_1+S_x(\dot{a}_1)^2\\F_y&=K_y\ddot{a}_1+S_y(\dot{a}_1)^2\end{aligned}\right\} \tag{13}$$

We observe that, coefficients K_x, K_y of acceleration term $\ddot{a}_1$ just coincide with those values obtained (in Report I) for the case of small vibration. Terms $S_x(\dot{a}_1)^2$, $S_y(\dot{a}_1)^2$ are newly added terms representing non-linearity for the case of vibration at finite applitudes.

(B) Contribution by the Term $[(\partial\phi/\partial x)^2+(\partial\phi/\partial y)^2]$
Here we have

$$\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2=\frac{1}{h^2}\left[\left(\frac{\partial\phi}{\partial\xi}\right)^2+\left(\frac{\partial\phi}{\partial\eta}\right)^2\right]$$

Also we have, on the wall surface,

$$-\frac{1}{h}\frac{\partial\phi}{\partial\xi}=V_\nu$$

And, total effect of the term $(1/h^2)[(\partial\phi/\partial\xi)^2]$ on F_x, F_y, is null, due to consideration of symmetry. So that we need only to put

$$\frac{1}{2}\rho\frac{1}{h^2}\left[\left(\frac{\partial\phi}{\partial\eta}\right)^2\right]$$

into formulae (5) and (6), instead of p there. We thus obtain the following values; for force (F_x, F_y) which act on wall of No. 1 cylinder, $(k=1)$,

$$F_x=\frac{\rho}{2c}\int_0^{2\pi}[\sum_n\{nA_n\cos n\eta-nB_n\sin n\eta\}$$
$$\cdot\{\mathrm{sh}\, n\xi_k+C_n\,\mathrm{ch}\, n\xi_k\}]^2\left[\frac{c}{R_k}\,\mathrm{sh}\,\xi_k-\frac{E_k}{R_k}(\mathrm{ch}\,\xi_k+\cos\eta)\right]d\eta \tag{14}$$

$$F_y=\frac{\rho}{2}\int_0^{2\pi}[\sum_n\{nA_n\cos n\eta-nB_n\sin n\eta\}\cdot\{\mathrm{sh}\, n\xi_k+C_n\,\mathrm{ch}\, n\xi_k\}]^2\frac{\sin\eta}{R_k}d\eta \tag{15}$$

Thus, actual values of F_x, F_y can be obtained in form of infinite series, consisting of each term in form of squares and products of $A_{n1}^{(1)}$ and $B_{n1}^{(1)}$. We observe that this component part of forces F_x, F_y are proportional to $(\dot{a}_1)^2$. Carrying out this estimation of infinite series expansion, we obtain the result as given below, which is the value for No. 1 cylinder wall;

$$F_x=\frac{\rho}{2c}\int_0^{2\pi}[\sum_n\{nA_{n1}^{(1)}\cos n\eta-nB_{n1}^{(1)}\sin n\eta\}]^2$$
$$\cdot\left[\frac{c}{R_1}\,\mathrm{sh}\,\xi_1-\frac{E_1}{R_1}(\mathrm{ch}\,\xi_1+\cos\eta)\right]d\eta \tag{16}$$
$$=\frac{\rho}{2c}I,$$

where we have

$$I=\pi M[\sum_n\{nA_{n1}^{(1)}\}^2+\sum_n\{nB_{n1}^{(1)}\}^2]$$
$$-\pi N[\sum_n\{nA_{n1}^{(1)}\}\{(n+1)A_{n+11}^{(1)}\}]$$
$$-\pi N[\sum_n\{nB_{n1}^{(1)}\}\{(n+1)B_{n+11}^{(1)}\}]$$

with

$$M=-\frac{1}{2c}, \qquad N=\frac{1}{2c}\operatorname{ch}\xi_1$$

The summations $\sum$ are to be made for $n=1, 2, \cdots$.

Also, we have;

$$F_y=\frac{\rho}{2}\int_0^{2\pi}\Big[\sum_n\{nA_{n1}^{(1)}\cos n\eta-nB_{n1}^{(1)}\sin n\eta\}\Big]^2\cdot\frac{\sin\eta}{R_1}d\eta \tag{17}$$

$$=\frac{\rho}{2}J,$$

in which we put,

$$J=\pi M'\Big[\sum_n\{nA_{n1}^{(1)}\}\{(n+1)B_{n1}^{(1)}\}\Big]+M'\Big[\sum_n\{nB_n^{(1)}\}\{(n+1)A_{n+11}^{(1)}\}\Big] \tag{17}$$

with

$$M'=\frac{|\operatorname{sh}\xi_1|}{c}$$

(C) Effect of Term which contain the Factor Ω in Eq. (4).

For this purpose, we have to put

$$\rho\Omega\left[y\frac{\partial\phi}{\partial x}-x\frac{\partial\phi}{\partial y}\right]=\rho\Omega\left[\frac{\partial\phi}{\partial\xi}\operatorname{ch}\xi_k\sin\eta-\frac{\partial\phi}{\partial\eta}\operatorname{sh}\xi_k\cos\eta\right]$$

in place of $(-p)$ in eqs. (5) and (6). Firstly, we note that, for $\xi=\xi_1$ we have

$$\frac{\partial\phi}{\partial\xi}=\sum_n[A_{n1}\sin n\eta+B_{n1}\cos n\eta]\cdot n[\operatorname{ch}n\xi_1+C_{n1}\operatorname{sh}n\xi_1]$$

$$=\sum_n n[A_{n1}^{(1)}\sin n\eta+B_{n1}^{(1)}\cos n\eta] \tag{18}$$

$$\frac{\partial\phi}{\partial\eta}=\sum_n[A_{n1}\cos n\eta-B_{n1}\sin n\eta]\cdot n[\operatorname{sh}n\xi_1+C_{n1}\operatorname{ch}n\xi_1]$$

$$=\sum_n n\left(\frac{E_{n1}}{D_{n1}}\right)[A_{n1}^{(1)}\cos n\eta-B_{n1}^{(1)}\sin n\eta] \tag{19}$$

Putting these values into eq. (5) and (6), we find:—

(a) For F_x, aggregate of terms, which consist of; as multiplier of $A_{n1}^{(1)}$,

$$-\rho\left(\frac{c}{R_1}\right)^2(cn)\left[\operatorname{ch}\xi_1 I_{1n}-\left(\frac{E_{n1}}{D_{n1}}\right)\operatorname{sh}\xi_1 I_{3n}\right]$$

$$-\rho(\operatorname{ch}\xi_1)(cn)\left[\operatorname{ch}\xi_1 I_{2n}-\left(\frac{E_{n1}}{D_{n1}}\right)\operatorname{sh}\xi_1 I_{4n}\right] \tag{20}$$

while we have, for factor of $B_{n1}^{(1)}=0$.

(b) For, F_y, aggregate of terms, which consist of; as multiplier of $A_{n1}^{(1)}=0$, while we have, for multiplier of $B_{n1}^{(1)}$,

$$\rho\left(\frac{c^2}{R_1}\right)(n)\left[\operatorname{ch}\xi_1 J_{1n}+\left(\frac{E_{n1}}{D_{n1}}\right)\operatorname{sh}\xi_1 J_{2n}\right] \tag{21}$$

It is to be noted that, we use here (and hereafter) following temporary notations;

$$I_{1n}=\int_0^{2\pi}\frac{\sin\eta\sin n\eta}{N^2}d\eta$$

$$I_{2n}=\int_0^{2\pi}\frac{\sin\eta\sin n\eta}{N}d\eta$$

$$I_{3n}=\int_0^{2\pi}\frac{\cos\eta\cos n\eta}{N^2}d\eta$$

$$I_{4n}=\int_0^{2\pi}\frac{\cos\eta\cos n\eta}{N}d\eta$$

$$J_{1n}=\int_0^{2\pi}\frac{\sin^2\eta\cos n\eta}{N}d\eta$$

$$J_{2n}=\int_0^{2\pi}\frac{\sin\eta\cos\eta\sin n\eta}{N^2}d\eta$$

$$L_{3n}=\int_0^{2\pi}\frac{\cos n\eta}{N^2}d\eta$$

$$L_{4n}=\int_0^{2\pi}\frac{\cos n\eta}{N}$$

In these formulae, we have put, for shortness,

$$N=\operatorname{ch}\xi_1+\cos\eta$$

It will be seen that these values of definite integrals may easily be expressed as linear combinations of following definite integrals ($m=0, 1, 2, \cdots$), which were used in author's previous papers.

$$K_m^{(s)}=\frac{1}{2\pi}\int_0^{2\pi}\frac{\cos m\eta}{N^s}d\eta,$$

$$K_m^{(1)}=\frac{(-)^m 2\varepsilon^{m+1}}{(1-\varepsilon^2)},$$

$$K_m^{(2)}=\frac{(-)^m 4(m+1)\varepsilon^{m+2}}{(1-\varepsilon^2)^2}+\frac{(-)^m 8\varepsilon^{m+4}}{(1-\varepsilon^2)^3}$$

(D) Effect of Term which contain c_x, c_y in eq. (4).

For this effect, we put

$$\rho\left[c_x\frac{\partial\phi}{\partial x}+c_y\frac{\partial\phi}{\partial y}\right](-1)$$

$$=\rho\frac{\partial\phi}{\partial\xi}\left[\frac{c_x}{c}(1+\mathrm{ch}\,\xi_1\cos\eta)-\frac{c_y}{c}\,\mathrm{sh}\,\xi_1\sin\eta\right](-1)$$

$$+\rho\frac{\partial\phi}{\partial\eta}\left[\frac{c_x}{c}\,\mathrm{sh}\,\xi_1\sin\eta-\frac{c_y}{c}\,(1+\mathrm{ch}\,\xi_1\cos\eta)\right](-1)$$

in place of $(-p)$ in eq. (5) and (6).

For $\partial\phi/\partial\xi$ and $\partial\phi/\partial\eta$ the values as given by eqs. (18) and (19) are to be used. Actuating this process of estimation, we obtain following results;

(a) For F_x, it consists of aggregates of terms of following four kinds;

A multipler of $A_{n1}^{(1)}$,

$$+\rho n\frac{c_y}{c}\,\mathrm{sh}\,\xi_1\left[\left(\frac{c}{R_1}\right)^2 cI_{1n}-c\,\mathrm{ch}\,\xi_1 I_{2n}\right]$$

As multiplier of $B_{n1}^{(1)}$,

$$-\rho n\frac{c_x}{c}\left[\left(\frac{c}{R_1}\right)^2 cL_{3n}-c\,\mathrm{ch}\,\xi_1 L_{4n}\right]-\rho n\frac{c_x}{c}\left[\left(\frac{c}{R_1}\right)^2 cI_{3n}-c\,\mathrm{ch}\,\xi_1 I_{4n}\right](\mathrm{ch}\,\xi_1)$$

As multiplier of $(E_{n1}/D_{n1})A_{n1}^{(1)}$,

$$+\rho\frac{c_y}{c}(nc)\left[\left(\frac{c}{R_1}\right)^2 L_{3n}-\mathrm{ch}\,\xi_1 L_{4n}+\mathrm{ch}\,\xi_1\left\{\left(\frac{c}{R_1}\right)^2 I_{3n}-\mathrm{ch}\,\xi_1 I_{4n}\right\}\right]$$

and, as pultiplier of $(E_{n1}/D_{n1})B_{n1}^{(1)}$,

$$nc_x\,\mathrm{sh}\,\xi_1\left[\left(\frac{c}{R_1}\right)^2 I_{1n}-\mathrm{ch}\,\xi_1 I_{2n}\right]$$

(b) For F_y it is made up of aggregate of terms of following three kinds;

As multiplier of $A_{n1}^{(1)}$,

$$-\rho\left(\frac{c_x}{c}\right)\left(\frac{c^2 n}{R_1}\right)[I_{1n}+\mathrm{ch}\,\xi_1 J_{2n}]$$

As multiplier to $B_{n1}^{(1)}$,

$$+\rho\frac{c_y}{c}\,\mathrm{sh}\,\xi_1\left(\frac{nc^2}{R_1}\right)J_{1n}$$

Lastly, as multiplier to $A_{n1}^{(1)}$,

$$-\rho\frac{c_x}{c}\,\mathrm{sh}\,\xi_1\left[n\frac{E_{n1}}{D_{n1}}\right]\left(\frac{c^2}{R_1}\right)J_{1n}$$

(E) Values of c_x, c_y and Ω.

As we see, by referring to Figs. 1 and 2, and by geometrical consideration, we have following expressions for c_x, c_y and Ω. The coordinates of origin of frame of axes (at time t), as referred to initial frame of axes (at time $t=0$), are given by

$$\alpha_x = H_1 + a_1 \cos\beta_1 - |E_1| \cos\mu$$

$$\alpha_y = a_1 \sin\beta_1 - |E_1| \sin\mu$$

in which we have

$$\tan\mu = \frac{a_1 \sin\beta_1}{D + a_1 \cos\beta_1}$$

(as we are taking the case of $a_2 \equiv 0$).

From these values of α_x, α_y we obtain, for linear velocity c_x, c_y of motion of origin of moving axes as referred to initial frame of axes;

$$c_x = \frac{d\alpha_x}{da_1}\dot{a}_1, \qquad c_y = \frac{d\alpha_y}{da_1}\dot{a}_1$$

in which the differentiation d/da_1 is to be understood as the differentiation with respect to the variable a_1 which are contained both explicitly and implicitly, therein. Moreover, we obtain by differentiation of the eq.

$$\tan\mu = \frac{a_1 \sin\beta_1}{D + a_1 \cos\beta_1}$$

the value of Ω (angular velocity of rotation of moving axes, as referred to initial frame of axes),

$$\Omega = \frac{d\mu}{dt}$$

$$= \dot{a}_1 \frac{D \sin\beta_1}{(D + a_1 \cos\beta_1)^2 + (a_1 \sin\beta_1)^2}$$

5. Numerical Example

In order to show us the application of above-mentioned analytical theory, let us take up following cases about relative configurations of two circular cylinders under consideration;

(1) "A", $R_1 = R_2 = \frac{1}{4}D$

(2) "B", $R_1 = \frac{1}{5}D$, $R_2 = \frac{2}{5}D$

(3) "C", $R_1 = \frac{1}{6}D$, $R_2 = \frac{1}{2}D$

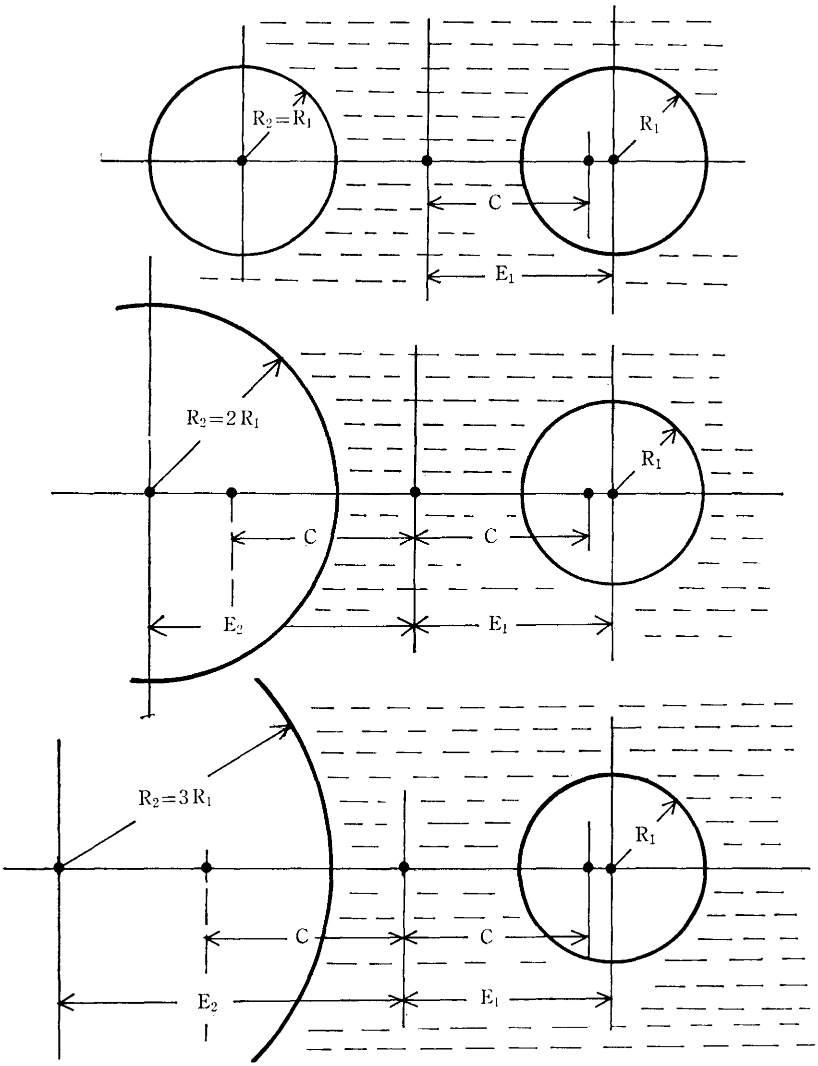

Fig. 4. Configurations of two Circular Cylinders.

For every one of these series, we take $a_2 \equiv 0$, $\beta_1 = \pi/4$, and make a_1 vary in five steps such that $a_1/R = 0, 1/4, 1/2, 3/4$ and 1. Rough sketch of these three configurations A, B and C at the initial state at which $a_1 = 0$, $a_2 = 0$ are shown in Fig. 4.

Fundamental factors for our treatments are coefficients $A_{n1}^{(1)}$ and $B_{n1}^{(1)}$ which appear in infinite series expansion of the solution. Numerical values of these coefficients $A_{n1}^{(1)}$ and $B_{n1}^{(1)}$ obtained by numerical estimation are shown in Table 1 and 2, whose general tendency of variation are shown as a graph in Fig. 5.

As to coordinates (α_x, α_y) of origin of our moving frame of axes, as referred to original coordinate axes, are shown in Fig 6. We could deduce values (c_x, c_y) of linear velocity of motion of origin of moving axes, by the relation

Table 1. Values of $a_{n1}^{(1)}=A_{n1}^{(1)}/[D\dot{a}_1]$

	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
$A-0$	−0.16578502	+0.04391565	−0.01178075	+0.00315664	−0.00084582
$A-1$	−0.15793956	+0.04163701	−0.01102305	+0.00291826	−0.00077259
$A-2$	−0.15220776	+0.03949451	−0.01034394	+0.00270929	−0.00071198
$A-3$	−0.14590565	+0.03751422	−0.00973239	+0.00252500	−0.00065509
$A-4$	−0.12747997	+0.03538793	−0.00912753	+0.00235166	−0.00060554

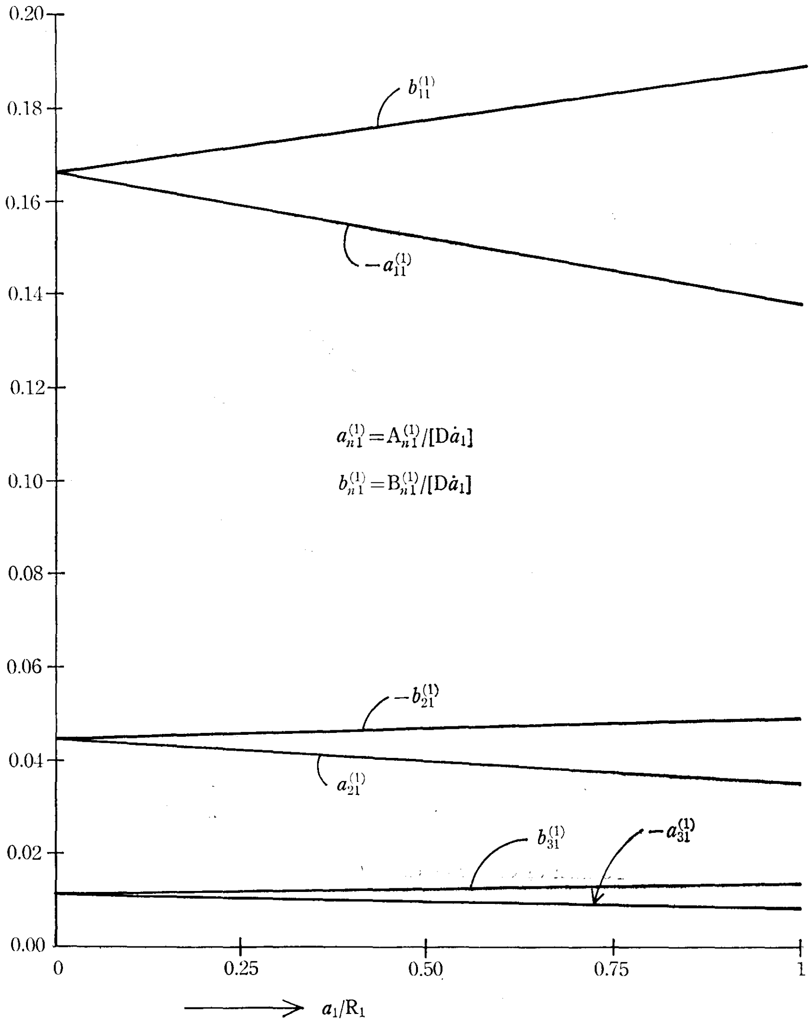

Fig. 5. Values of coefficients $A_{n1}^{(1)}$ and $B_{n1}^{(1)}$.

Table 2. Values of $b_{n1}^{(1)}=B_{n1}^{(1)}/[D\dot{a}_1]$

	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
$A-0$	+0.16578491	−0.04396866	+0.01178074	−0.00315664	+0.00084582
$A-1$	+0.17188157	−0.04531499	+0.01199742	−0.00371622	+0.00084087
$A-2$	+0.17911725	−0.04647658	+0.01217257	−0.00318824	+0.00083506
$A-3$	+0.18459473	−0.04746099	+0.01231305	−0.00319453	+0.00082880
$A-4$	+0.18608650	−0.04789946	+0.01235460	−0.00318310	+0.00088695

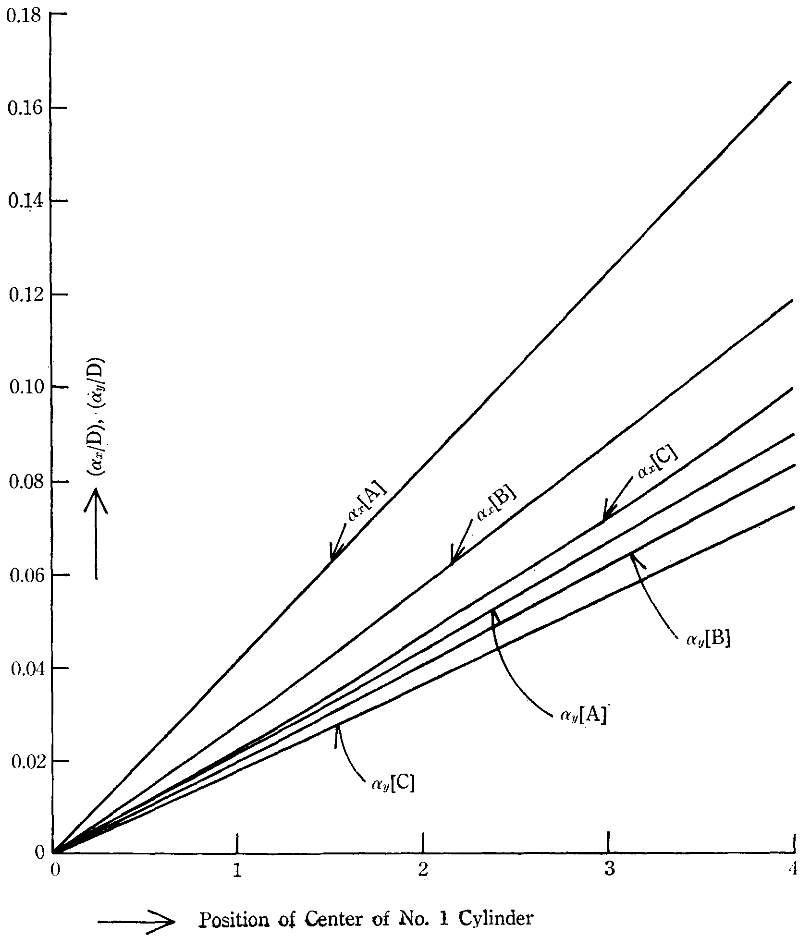

Fig. 6. Position of Origin of Moving Axes (Values of α_x/D, α_y/D).

$$c_x=\frac{d\alpha_x}{dt}=\frac{d\alpha_x}{da_1}\dot{a}_1, \qquad c_y=\frac{d\alpha_y}{dt}=\frac{d\alpha_y}{da_1}\dot{a}_1$$

Values of angular velocity Ω of frame of moving axes has been obtained, whose

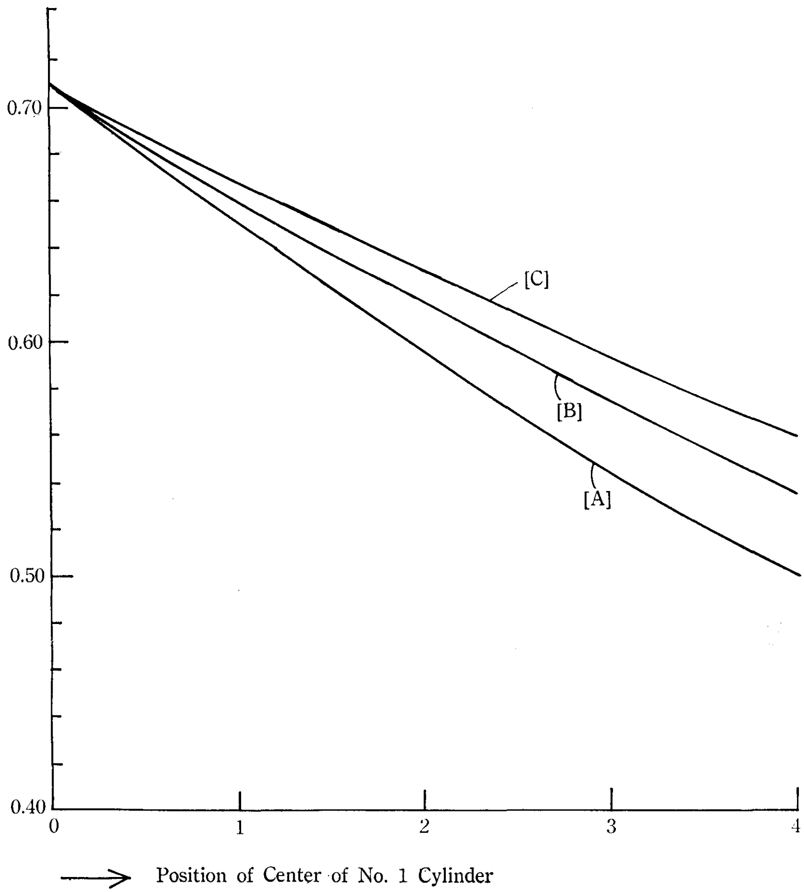

Fig. 7. Angular Velocity of Rotation Moving Axes. (Values of $\Omega/[a_1/D]$)

values are shown as graph in Fig. 7.

Using the set of values of these numerical factors, we could obtain the value of forces (F_x, F_y) exerted by fluid upon the circular cylinders. Summation of infinite series was made for $n=1$ to $n=5$. In what follows, we shall list up the result of numerical estimation for the case of series "A".

(a) Contribution by the term in $d\phi/dt$:

$(A-0)$

$$F_x=0.13946223\,[\rho D^2\ddot{a}_1]+0.06558306\,[\rho D(\dot{a}_1)^2]$$

$$F_y=0.13998789\,[\rho D^2\ddot{a}_1]-0.0975099\,[\rho D(\dot{a}_1)^2]$$

$(A-1)$

$$F_x=0.14411681\,[\rho D^2\ddot{a}_1]+0.06539150\,[\rho D(\dot{a}_1)^2]$$

$$F_y=0.13267401\,[\rho D^2\ddot{a}_1]-0.09703973\,[\rho D(\dot{a}_1)^2]$$

$(A-2)$

$$F_x=0.15085330\,[\rho D^2\ddot{a}_1]+0.06577557\,[\rho D(\dot{a}_1)^2]$$

$$F_y=0.12818901\,[\rho D^2\ddot{a}_1]-0.09749165\,[\rho D(\dot{a}_1)^2]$$

$(A-3)$

$$F_x=0.15652941\,[\rho D^2\ddot{a}_1]+0.06608083\,[\rho D(\dot{a}_1)^2]$$

$$F_y=0.12271835\,[\rho D^2\ddot{a}_1]-0.09747292\,[\rho D(\dot{a}_1)^2]$$

$(A-4)$

$$F_x=0.15396191\,[\rho D^2\ddot{a}_1]+0.06472541\,[\rho D(\dot{a}_1)^2]$$

$$F_y=0.11374446\,[\rho D^2\ddot{a}_1]-0.09587596\,[\rho D(\dot{a}_1)^2]$$

(b) Contribution by the term in $[(\partial\phi/\partial x)^2+(\partial\phi/\partial y)^2]$

$(A-0)$

$$F_x=-0.00214436\,[\rho D(\dot{a}_1)^2]$$

$$F_y=0$$

$(A-1)$

$$F_x=-0.00182738\,[\rho D(\dot{a}_1)^2]$$

$$F_y=+0.00000799\,[\rho D(\dot{a}_1)^2]$$

$(A-2)$

$$F_x=-0.00186137\,[\rho D(\dot{a}_1)^2]$$

$$F_y=+0.00004147\,[\rho D(\dot{a}_1)^2]$$

$(A-3)$

$$F_x=-0.00188155\,[\rho D(\dot{a}_1)^2]$$

$$F_y=-0.00004136\,[\rho D(\dot{a}_1)^2]$$

$(A-4)$

$$F_x=+0.00156060\,[\rho D(\dot{a}_1)^2]$$

$$F_y=+0.00003984\,[\rho D(\dot{a}_1)^2]$$

(c) Contribution by the term in Ω.

$(A-0)$

$$F_x=+0.09945096\,[\rho D(\dot{a}_1)^2]$$

$$F_y=-0\ 09946432\,[\rho D(\dot{a}_1)^2]$$

$(A-1)$

$$F_x = +0.08664210\,[\rho D(\dot{a}_1)^2]$$

$$F_y = -0.09652688\,[\rho D(\dot{a}_1)^2]$$

$(A-2)$

$$F_x = +0.07619551\,[\rho D(\dot{a}_1)^2]$$

$$F_y = -0.089679112\,[\rho D(\dot{a}_1)^2]$$

$(A-3)$

$$F_x = +0.07151467\,[\rho D(\dot{a}_1)^2]$$

$$F_y = -0.08474334\,[\rho D(\dot{a}_1)^2]$$

$(A-4)$

$$F_x = +0.05792686\,[\rho D(\dot{a}_1)^2]$$

$$F_y = -0.07937163\,[\rho D(\dot{a}_1)^2]$$

(d) Contribution by the term in (c_x, c_y)

$(A-0)$

$$F_x = -0.00016004\,[\rho D(\dot{a}_1)^2]$$

$$F_y = +0.00123553\,[\rho D(\dot{a}_1)^2]$$

$(A-1)$

$$F_x = -0.00026714\,[\rho D(\dot{a}_1)^2]$$

$$F_y = +0.00143589\,[\rho D(\dot{a}_1)^2]$$

$(A-2)$

$$F_x = +0.00028822\,[\rho D(\dot{a}_1)^2]$$

$$F_y = +0.00115134\,[\rho D(\dot{a}_1)^2]$$

$(A-3)$

$$F_x = +0.00019341\,[\rho D(\dot{a}_1)^2]$$

$$F_y = +0.00110641\,[\rho D(\dot{a}_1)^2]$$

$(A-4)$

$$F_x = +0.00054768\,[\rho D(\dot{a}_1)^2]$$

$$F_y = +0.00101246\,[\rho D(\dot{a}_1)^2]$$

(e) The total algebraic sum of the above four items:

$(A-0)$

$$F_x=0.13946223\,[\rho D^2\ddot{a}_1]+0.16272962\,[\rho D(\dot{a}_1)^2]$$

$$F_y=0.13998789\,[\rho D^2\ddot{a}_1]-0.19573869\,[\rho D(\dot{a}_1)^2]$$

$(A-1)$

$$F_x=0.14411681\,[\rho D^2\ddot{a}_1]+0.14993908\,[\rho D(\dot{a}_1)^2]$$

$$F_y=0.13267401\,[\rho D^2\ddot{a}_1]-0.19212282\,[\rho D(\dot{a}_1)^2]$$

$(A-2)$

$$F_x=0.15085330\,[\rho D^2\ddot{a}_1]+0.14039793\,[\rho D(\dot{a}_1)^2]$$

$$F_y=0.12818901\,[\rho D^2\ddot{a}_1]-0.18597795\,[\rho D(\dot{a}_1)^2]$$

$(A-3)$

$$F_x=0.15652941\,[\rho D^2\ddot{a}_1]+0.13590736\,[\rho D(\dot{a}_1)^2]$$

$$F_y=0.12271835\,[\rho D^2\ddot{a}_1]-0.18115121\,[\rho D(\dot{a}_1)^2]$$

$(A-4)$

$$F_x=0.15396191\,[\rho D^2\ddot{a}_1]+0.12476055\,[\rho D(\dot{a}_1)^2]$$

$$F_y=0.1137446\,[\rho D^2\ddot{a}_1]-0.17419529\,[\rho D(\dot{a}_1)^2]$$

It is observed that item (a) (which gives effect of term in $d\phi/dt$), and item (c) which relates to term in Ω, play more important part, while other two items (b) and (d) are of rather minor importance, so far as we are concerned about present range of estimation.

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[2] KITO, F. (1979): Ditto-II, Keio Engineering Reports, 32, No. 5, pp. 45-56.