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ON VIBRATION OF TWO CIRCULAR CYLINDERS WHICH ARE IMMERSED IN A WATER REGION—II.

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ABSTRACT

In former report, under the same title, which appeared about ten years ago, by the same author, we made an analytical study with regard to vibration of two circular cylinders which are immersed in a fluid region of infinite extent. Therein, the vibration was assumed to be of infinitesimally small magnitude. The present paper is the continuation of analytical study of the same problem, and here we treat the case of vibration of finite amplitudes. As before, we take the case of two dimensional problem about an ideal fluid. In the present study, (Part II), only main analytical formulae about this problem is given, more detailed account together with numerical example being to be given in subsequent paper.

1. Statement of the Problem.

In Fig. 1, two circular cylinders are shown, which represents the initial configuration, at time $t=0$. Radii of two circular cylinders are R_1 and R_2. They are immersed in water region of infinite extent, which is considered to be an ideal fluid. Positions of centers of two circular cylinders are located on the x-axis, and are spaced by a distance $D=E_0$ each other. Radii R_1 and R_2 are to be kept constant at every subsequent time t $(0\leq t)$.

Positions of centers of two circular cylinders are considered to move afterwards (at $0\leq t$), in a prescribed manner. Fig. 2 shows us positions of two circular cylinders at time t. The two cylinders have moved by distances a_1 and a_2, in angular directions β_1 and β_2 respectively. In what follows, we take up the case in which a_1 and a_2 are given functions of time t, while angles β_1 and β_2 are kept

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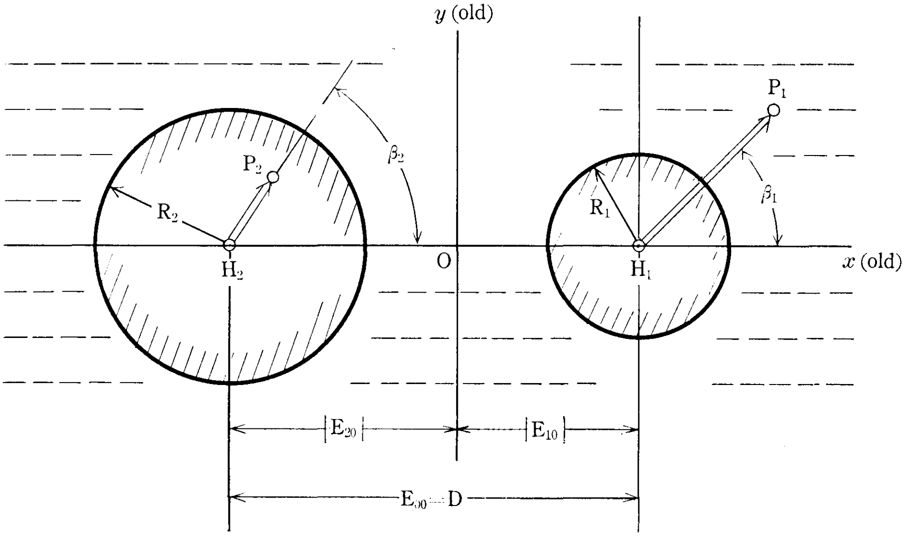

Fig. 1. Position of two Circular Cylinders at initial State. ($t=0$)

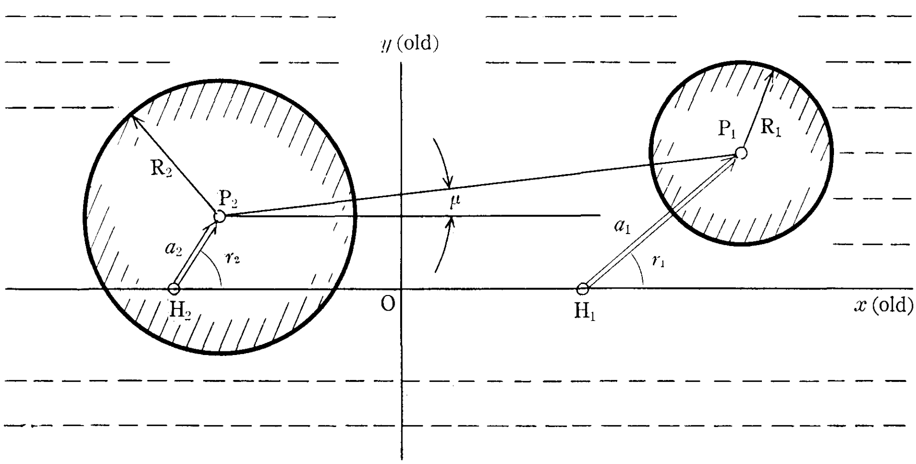

Fig. 2. Position of two Circular Cylinders at displaced State (at time t)

at constant values. Displacements of centers a_1 and a_2 need not to be taken as of small amount.

Our problem of the present paper is to make an analytical study about the fluid motion set up, and also about the resulting hydrodynamic forces set up on walls of two cylinders. The surrounding fluid being assumed to be an ideal fluid of infinite extent, we are to treat the problem as a case of two-dimensional potential flow (which necessarily contain time t as another independent variable.)

In the present report, only general expressions corresponding to our aim is given, more detailed account being left for a future report.

2. Notations used.

x, y=rectangular coordinates of a point in xy plane
$z=x+iy$ a complex variable
c=distance from origin 0, of radical centers of system of bipolar coordinates
h=coefficient of linear element for the case of bipolar coordinates (ξ, η)
ξ, η=a system of bipolar coordinates representing any point on xy plane
R_i=radius of circular cylinder $(i=1, 2)$
E_i=position of center of ditto
p=fluid pressure, ρ=density of the fluid
ϕ=velocity potential of fluid motion, giving absolute velocity of flow

In what follows, several coefficients A_i, B_i, C_i etc., will be used for giving us the solution in form of an infinite series. These coefficients will be defined where they make first appearances. These coefficients are independent of variables x, y (also ξ, η), but they may be functions of time t, because we are treating the case of non-stationary fluid motion. Also, suffixes will be attached to them for distinguishing individual cases of application.

3. Fundamental Formulae.

The main formula used in the present paper will be (as in previous report, Ref. (1)) those relating to a system of bipolar coordinates, and is given below.

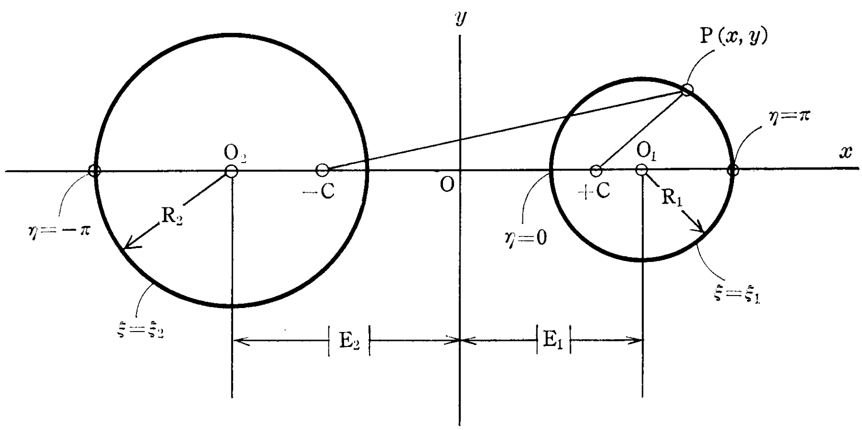

Fig. 3. Configuration of two circular Cylinders, represented by bipolar Coordinates

Referring to Fig. 3, two points $(+c, 0)$ and $(-c, 0)$ lying on the real axis are taken as radical centers, and we define a system of bipolar coordinates (ξ, η) by means of the relation

$$\xi+i\eta=\log\frac{c+z}{c-z} \tag{1}$$

wherein we have $z=x+iy$.

From this eq. (1), we obtain

$$x=\frac{c\,\mathrm{sh}\,\xi}{\mathrm{ch}\,\xi+\cos\eta},\ y=\frac{c\sin\eta}{\mathrm{ch}\,\xi+\cos\eta} \tag{2}$$

The linear element ds is given by

$$(ds)^2=(dx)^2+(dy)^2=h^2[(d\xi)^2+(d\eta)^2] \tag{3}$$

in which we have put

$$h=\frac{\mathrm{ch}\,\xi+\cos\eta}{c} \tag{4}$$

Therefore, two-dimensional Laplacian $\Delta\phi$ of the function ϕ is given by

$$\Delta\phi\equiv\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\equiv\frac{1}{h^2}\left(\frac{\partial^2\phi}{\partial\xi^2}+\frac{\partial^2\phi}{\partial\eta^2}\right)$$

For the case of an ideal fluid, we have to put $\Delta\phi=0$, whose general solution may be given in following form,

$$\phi=\sum_{n=1}^{\infty}[A_n\sin n\eta+B_n\cos n\eta]\cdot$$

$$[\mathrm{sh}\,n\xi+C_n\,\mathrm{ch}\,n\xi] \tag{5}$$

A_n and B_n, C_n are constants independent of variables ξ, η. But in our case of non-steady fluid motion, they may be functions of time t.

In order to apply these fundamental formula to our problem, we take two circular cylinders to be represented by $\xi=\xi_1$ and $\xi=\xi_2$ respectively. ξ_1 is positive, but ξ_2 has a negative value.

For a given set (ξ_1, ξ_2) of values for two circles, we have $(k=1, 2)$

$$E_k=c\frac{\mathrm{ch}\,\xi_k}{\mathrm{sh}\,\xi_k},\quad R_k=c\frac{1}{|\mathrm{sh}\,\xi_k|} \tag{6}$$

Corresponding to the instantaneous state of flow given by eq. (5), value of absolute velocity of flow normal to wall surfaces of circular cylinders are given by

$$V_n=-\frac{\partial\phi}{h\partial\xi}$$

$$=(-)^{k+1}\frac{1}{c}(\mathrm{ch}\,\xi_k+\cos\eta)\times$$

$$\times \sum_n n(A_n \sin n\eta + B_n \cos n\eta) \cdot (\mathrm{ch}\, n\xi_k + C_n \,\mathrm{sh}\, n\xi_k) \quad (7)$$

at $\xi=\xi_k$ $(k=1, 2)$. The sense of V_n is taken as positive in outward direction with regard to each one of cylinder wall. Also, we note that ξ_1 is positive, while ξ_2 is negative. If values of V_n along two circumferences $\xi=\xi_1$ and $\xi=\xi_2$ (as functions of η) are given by hydrodynamic consideration, we can obtain values of coefficients A_i, B_i and C_i from the eq. (7), by means of so-called Fourier analysis.

4. Value of Velocity Potential for Instantaneous State of Fluid Motion.

In order to apply the fundamental formula as given above to obtain value of velocity potential ϕ which gives us instantaneous state of fluid motion at time t, we take ϕ in such way that it represents the state of matters as shown in Fig. 2. Thus, in order to conform the state of Fig. 3 to instantaneous configuration of Fig. 2, we take the line P_1P_2 of Fig. 2 to correspond to x-axis of Fig. 3. Also, we take

$$\begin{aligned}[][|E_1|+|E_2|]^2 &= E_0^2 \\ &= (D+a_1\cos\beta_1-a_2\cos\beta_2)^2+(a_1\sin\beta_1-a_2\sin\beta_2)^2 \end{aligned} \quad (8)$$

Moreover, we have

$$\left.\begin{aligned} E_0 &= |E_1|+|E_2| = R_1\,\mathrm{ch}\,\xi_1 + R_2\,\mathrm{ch}\,\xi_2 \\ c &= \frac{1}{2E_0}[\{E_0^2-(R_1-R_2)^2\}\{E_0^2-(R_1+R_2)^2\}]^{1/2} \\ \xi_1 &= \log\frac{c+\sqrt{c^2+R_1^2}}{R_1} \\ |\xi_2| &= \log\frac{c+\sqrt{c^2+R_2^2}}{R_2} \end{aligned}\right\} \quad (9)$$

Angle μ of inclination of new x-axis P_1P_2 to old x-axis is given by

$$\tan\mu = \frac{a_1\sin\beta_1-a_2\sin\beta_2}{D+a_1\cos\beta_1-a_2\cos\beta_2} \quad (10)$$

When two cylinders are moving with velocities $\dot{a}_k$ $(k=1, 2)$ in directions of β_k respectively (relative to old axis), normal velocities (taken outwardly) of their wall surfaces are given by $(k=1, 2)$

$$V_\nu = \frac{d}{dt}(a_k)\cos(\nu, \gamma_k) = \dot{a}_k\cos(\theta-\gamma_k) \quad (11)$$

in which we have

$$\cos\theta=(x-E_k)/R_k,\ \sin\theta=y/R_k \tag{12}$$

$$x=\frac{c\,\mathrm{sh}\,\xi_k}{\mathrm{ch}\,\xi_k+\cos\eta},\quad y=\frac{c\sin\eta}{\mathrm{ch}\,\xi_k+\cos\eta}$$

It is to be noted that directions of motion γ_1 and γ_2 (with respect to new instantaneous x-axis) are related to original directions of motion β_1 and β_2 by the equation

$$\gamma_k=\beta_k-\mu\ (k=1,2) \tag{13}$$

When two circular cylinders are moving through the fluid in a prescribed manner, we must have $V_n=V_\nu$ on both wall surfaces $\xi=\xi_k$ $(k=1,2)$ of two cylinders. Thus, we could evaluate coefficients A_n, B_n, C_n $(n=1,2,\cdots)$ of eq. (5) by means of Fourier analysis, as mentioned above. We find it more convenient to take

$$\phi=\phi_1+\phi_2 \tag{14}$$

where ϕ_1 satisfies the boundary condition at wall of No. 1 cylinder $(\xi=\xi_1)$ while the No. 2 cylinder remain at rest. On the other hand, ϕ_2 satisfies the boundary condition at wall of No. 2 cylinder $(\xi=\xi_2)$ while the No. 1 cylinder is kept at stand-still. It is to be noted that the solution (14) gives us an instantaneous value of velocity potential ϕ, pertaining to the motion of an ideal fluid. Actual values of these coefficients obtained by this means are as given below. Thus, we put

$$\phi_k=\sum_n (A_{nk}\sin n\eta+B_{nk}\cos n\eta)\cdot$$
$$(\mathrm{sh}\,n\xi+C_{nk}\,\mathrm{ch}\,n\xi) \tag{15}$$

where we have (for $k=1,2$)

$$A_{nk}=-\frac{H_k}{nD_{nk}}[K^{(2)}_{n-1}(\lambda_k)-K^{(2)}_{n+2}(\lambda_k)]$$

$$B_{nk}=-\frac{G_k}{nD_{nk}}[2K^{(2)}_n(\lambda_k)]$$
$$+\frac{2c\dot{a}_k\cos\gamma_k\,\mathrm{ch}\,\xi_k}{nD_{nk}}[K^{(1)}_n(\lambda_k)]$$

$$H_k=\frac{\dot{a}_k c^2}{R_k}\sin\gamma_k$$

$$G_k=\frac{\dot{a}_k c^2}{R_k}\mathrm{sh}\,\xi_k\cos\gamma_k$$

$$D_{n1}=\frac{\operatorname{sh} n(\xi_2-\xi_1)}{\operatorname{sh} n\xi_2}$$

$$D_{n2}=\frac{\operatorname{sh} n(\xi_1-\xi_2)}{\operatorname{sh} n\xi_1}$$

It is to be noted that ξ_1 is positive, while ξ_2 is negative. In the above formula, following values of definite integrals were used, these being used already in author's previous report.

$$K_m^{(1)}(\lambda)=\frac{1}{2\pi}\int_0^{2\pi}\frac{\cos m\eta}{(\lambda+\cos\eta)}d\eta$$

$$=(-)^m\frac{2\varepsilon^{m+1}}{(1-\varepsilon^2)}$$

$$K_m^{(2)}(\lambda)=\frac{1}{2\pi}\int_0^{2\pi}\frac{\cos m\eta}{(\lambda+\cos\eta)^2}d\eta$$

$$=(-)^m\frac{4(m+1)\varepsilon^{m+2}}{(1-\varepsilon^2)^2}+(-)^m\frac{8\varepsilon^{m+4}}{(1-\varepsilon^2)^3}$$

wherein we write for shortness

$$\varepsilon=\frac{1}{\lambda+\sqrt{\lambda^2-1}}=\frac{1}{\operatorname{ch}\xi+|\operatorname{sh}\xi|}<1$$

$$\lambda=\operatorname{ch}\xi$$

m is positive integers, $0, 1, 2, \cdots$

Especially, for the values of ϕ_s $(s=1, 2)$ at two cylindrical walls (at which $\xi=\xi_p\,;p=1, 2)$ we have

$$\phi_s^{(p)}=\sum_n[A_{ns}^{(p)}\sin n\eta+B_{ns}^{(p)}\cos n\eta] \tag{16}$$

Actual values of coefficients $A_{ns}^{(p)}$ and $B_{ns}^{(p)}$ are as follows;

$$A_{n1}^{(1)}=\frac{-1}{n}\left[\frac{c^2\dot{a}_1}{R_1}\sin\gamma_1\right]\coth n(\xi_1-\xi_2)\cdot[K_{n-1}^{(2)}(\lambda_1)-K_{n+1}^{(2)}(\lambda_1)]$$

$$B_{n1}^{(1)}=\frac{-2}{n}[c\dot{a}_1\cos\gamma_1]\coth n(\xi_1-\xi_2)\cdot[\operatorname{sh}^2\xi_1K_n^{(2)}(\lambda_1)-\operatorname{ch}\xi_1K_n^{(1)}(\lambda_1)]$$

$$A_{n1}^{(2)}=\frac{-1}{n}\left[\frac{c^2\dot{a}_1}{R_1}\sin\gamma_1\right]\frac{1}{\operatorname{sh} n(\xi_1-\xi_2)}\cdot[K_{n-1}^{(2)}(\lambda_1)-K_{n+1}^{(2)}(\lambda_1)]$$

$$B_{n1}^{(2)}=\frac{-2}{n}[c\dot{a}_1\cos\gamma_1]\frac{1}{\operatorname{sh} n(\xi_1-\xi_2)}\cdot[\operatorname{sh}^2\xi_1K_n^{(2)}(\lambda_1)-\operatorname{ch}\xi_1K_n^{(1)}(\lambda_1)]$$

$$A_{n2}^{(1)}=\frac{-1}{n}\left[\frac{c^2\dot{a}_2}{R_2}\sin\gamma_2\right]\frac{1}{\operatorname{sh} n(\xi_1-\xi_2)}\cdot[K_{n-1}^{(2)}(\lambda_2)-K_{n+1}^{(2)}(\lambda_2)]$$

$$B_{n2}^{(1)}=\frac{-2}{n}[c\dot{a}_2\cos\gamma_2]\frac{1}{\operatorname{sh} n(\xi_1-\xi_2)}\cdot[-(\operatorname{sh}\xi_2)^2K_n^{(2)}(\lambda_2)+\operatorname{ch}\xi_2K_n^{(1)}(\lambda_2)]$$

$$A_{n2}^{(2)}=\frac{-1}{n}\left[\frac{c^2\dot{a}_2}{R_2}\sin\gamma_2\right]\coth n(\xi_1-\xi_2)\cdot[K_{n-1}^{(2)}(\lambda_2)-K_{n+1}^{(2)}(\lambda_2)]$$

$$B_{n2}^{(2)}=\frac{-2}{n}[c\dot{a}_2\cos\gamma_2]\coth n(\xi_1-\xi_2)\cdot[-(\operatorname{sh}\xi_2)^2K_n^{(2)}(\lambda_2)+\operatorname{ch}\xi_2K_n^{(1)}(\lambda_2)]$$

It will be observed that infinite series solutions (15) and (16) are absolutely convergent. After numerical estimation about some practical cases, we find that the convergence is fairly good, and we may stop at $n=1,2,\ldots$ to $\ldots,5$.

5. Hydrodynamical Forces acting on each Circular Cylinders

As mentioned above, we take as instantaneous location of x-axis, the straight line P_1P_2 of Fig. 2. When the time t elapses, the location of line P_1P_2 changes. Therefore, in order to evaluate fluid pressure p, we must have recourse to theory of hydrodynamics for the case of moving axes of reference. This has been done by the present author and is described in previous paper (Ref. (2)). In our case of two dimensional flow, it can be given in following form;

$$-\frac{1}{\rho}p+U=\frac{\partial\phi}{\partial t}+\frac{1}{2}\left\{\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2\right\}$$

$$-(c_x-\Omega y)\frac{\partial\phi}{\partial x}-(c_y+\Omega x)\frac{\partial\phi}{\partial y}+C \tag{17}$$

In this eq. (17), U is the potential of external force field, which we take here $U\equiv0$. C is an arbitrary constant independent of x, y but it may be a function of time t. Here also, we take $C\equiv0$. (c_x, c_y) is linear velocity of motion of the origin of our instantaneous frame of axes, Ω being its angular velocity of rotation about an axis perpendicular to x, y plane.

Taking the velocity potential ϕ to be given by eqs. (14), (15), and putting it into eq. (17), we can estimate the amount of hydrodynamic forces exerted by the fluid pressure p, upon two wall surfaces of circular cylinders, thus;

(1) Hydrodynamical Force acting upon Wall of No. 1 Circular Cylinder.

$$F_x=\int_0^{2\pi}[-p]\left[\left(\frac{c}{R_1}\right)^2\frac{1}{\operatorname{ch}\xi_1+\cos\eta}-\operatorname{ch}\xi_1\right]\cdot\left[\frac{c}{\operatorname{ch}\xi_1+\cos\eta}\right]d\eta \tag{18}$$

$$F_y=\int_0^{2\pi}[-p]\left[\frac{c^2}{R_1}\frac{\sin\eta}{(\operatorname{ch}\xi_1+\cos\eta)^2}\right]d\eta \tag{19}$$

(2) Hydrodynamic Force acting upon Wall of No. 2 Circular Cylinder.

$$F_x=\int_0^{2\pi}[+p]\left[\left(\frac{c}{R_2}\right)^2\frac{1}{\operatorname{ch}\xi_2+\cos\eta}-\operatorname{ch}\xi_2\right]\cdot\left[\frac{c}{\operatorname{ch}\xi_2+\cos\eta}\right]d\eta \tag{20}$$

$$F_y=\int_0^{2\pi}[-p]\left[\frac{c^2}{R_2}\frac{\sin\eta}{(\mathrm{ch}\,\xi_2+\cos\eta)^2}\right]d\eta \tag{21}$$

In what follows, we shall show actual expressions of hydrodynamic forces F_x, F_y for the case in which $a_1\neq 0$, $a_2\equiv 0$, this being done for presenting simpler expressions. Task of evaluation of F_x, F_y may be done, conveniently, in following four steps.

(A) Effect of the Term $\partial\phi/\partial t$

As we see from eq. (17), we must first evaluate the effect of term $\partial\phi/\partial t$ upon F_x and F_y. This is done, for No. 1 cylinder, by evaluating following definite integrals:

$$F_x=-\rho\int_0^{2\pi}\frac{\partial\phi}{\partial t}\left[\frac{c}{R_1}\frac{\mathrm{sh}\,\xi_1}{\mathrm{ch}\,\xi_1+\cos\eta}-\frac{E_1}{R_1}\right]\cdot\left[\frac{c}{\mathrm{ch}\,\xi_1+\cos\eta}\right]d\eta \tag{22}$$

$$F_y=-\rho\int_0^{2\pi}\frac{\partial\phi}{\partial t}\left[\frac{c^2}{R_1}\frac{\sin\eta}{\mathrm{ch}\,\xi_1+\cos\eta}\right]d\eta \tag{23}$$

Value of velocity potential ϕ, contained in these definite integrals, being the value at $\xi=\xi_1$, we take it in form of eq. (16), in which we have $s=1$, $p=1$. Moreover, differentiation $\partial/\partial t$, with respect to time t, must be carried out in two ways. The first one is with regard to $\dot{a}_1$, and write

$$\frac{d}{dt}(\dot{a}_1)=\ddot{a}_1$$

The second is with respect to a_1, which is contained implicitly in coefficients $A_{ns}^{(p)}$ and $B_{ns}^{(p)}$ (through c, E_1, etc.) and this term may conveniently be expressed formally as

$$\frac{\partial\phi}{\partial a_1}\frac{\partial a_1}{dt}=\frac{\partial\phi}{\partial a_1}\dot{a}_1$$

We may express this fact by writing

$$\frac{d\phi}{dt}=\frac{\partial\phi}{\partial\dot{a}_1}\ddot{a}_1+\frac{\partial\phi}{\partial a_1}\dot{a}_1$$

The author finds it more convenient to put

$$A_{n1}^{(1)}=[D\dot{a}_1]a_{n1}^{(1)}$$

$$B_{n1}^{(1)}=[D\dot{a}_1]b_{n1}^{(1)}$$

etc., where $a_{n1}^{(1)}, b_{n1}^{(1)}$ are numerical (non-dimensional) coefficients, which depend implicitly on a_1.

Furthermore, we have

$$\frac{\partial}{\partial t}[a_{n1}^{(1)}]=\frac{\partial}{\partial a_1}[a_{n1}^{(1)}]\dot{a}_1=\alpha_{n1}^{(1)}\dot{a}_1$$

$$\frac{\partial}{\partial t}[b_{n1}^{(1)}]=\frac{\partial}{\partial a_1}[b_{n1}^{(1)}]\dot{a}_1=\beta_{n1}^{(1)}\dot{a}_1$$

Strictly speaking, values of coefficients $\alpha_{n1}^{(1)}$, $\beta_{n1}^{(1)}$, etc., are to be obtained by actuating process of differentiation, which is very complicated. However, the author has found a practical method of estimating them approximately, which will be reported subsequently. Under these preparatory notices, we obtain

$$F_x = \sum_n \rho n\pi\left(\frac{c}{D}\right)\left(\frac{D}{R_1}\right)\left(\frac{D}{\cos\gamma_1}\right)\cdot\frac{\tanh n(\xi_1-\xi_2)}{\operatorname{sh}\xi_1}\cdot[b_{n1}^{(1)}]\left[\frac{dB_{n1}^{(1)}}{dt}\right] \tag{24}$$

$$F_y = \sum_n \frac{\rho n\pi D}{\sin\gamma_1}\tanh n(\xi_1-\xi_2)\cdot[a_{n1}^{(1)}]\left[\frac{dA_{n1}^{(1)}}{dt}\right] \tag{25}$$

Therein, we have

$$\frac{dB_{n1}^{(1)}}{dt} = D\ddot{a}_1 b_{n1}^{(1)} + D\frac{\partial b_{n1}^{(1)}}{\partial a_1}(\dot{a}_1)^2$$

$$\frac{dA_{n1}^{(1)}}{dt} = D\ddot{a}_1 a_{n1}^{(1)} + D\frac{\partial a_{n1}^{(1)}}{\partial a_1}(\dot{a}_1)^2$$

Summing up these results, we see that hydrodynamic forces F_x, F_y may be expressed in following form;

$$\left.\begin{aligned} F_x &= K_x\ddot{a}_1 + S_x(\dot{a}_1)^2 \\ F_y &= K_y\ddot{a}_1 + S_y(\dot{a}_1)^2 \end{aligned}\right\} \tag{26}$$

We observe that, coefficients K_x, K_y of acceleration term $\ddot{a}_1$ just coincide with those obtained (in Report I) for the case of small vibration. Terms $S_x(\dot{a}_1)^2$, $S_y(\dot{a}_1)^2$ are newly added terms representing non-linearity for the case of finite vibrations.

(B) Contribution by the term $[(\partial\phi/\partial x)^2+(\partial\phi/\partial y)^2]$. Here we have

$$\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2 = \frac{1}{h^2}\left[\left(\frac{\partial\phi}{\partial\xi}\right)^2+\left(\frac{\partial\phi}{\partial\eta}\right)^2\right]$$

Also, we have, on the wall surface

$$-\frac{1}{h}\frac{\partial\phi}{\partial\xi} = V_\nu$$

and total effect of the term $(1/h^2)[(\partial\phi/\partial\xi)]^2$ on F_x, F_y is null, due to consideration of symmetry. So that we need only to put

$$\frac{1}{2}\rho\frac{1}{h^2}\left[\left(\frac{\partial\phi}{\partial\eta}\right)^2\right]$$

into formulae (18) and (19), instead of p there; We thus obtain the following values;

$$F_x = -\frac{\rho}{2c}\int_0^{2\pi}\left[\sum_n\{nA_n\cos n\eta - nB_n\sin n\eta\}\cdot\{\operatorname{sh} n\xi_k + C_n\operatorname{ch} n\xi_k\}\right]^2$$

$$\cdot\left[\frac{c}{R_k}\operatorname{sh}\xi_k - \frac{E_k}{R_k}(\operatorname{ch}\xi_k+\cos\eta)\right]d\eta \tag{27}$$

$$F_y=-\frac{\rho}{2}\int_0^{2\pi}[\sum_n\{nA_n\cos n\eta-nB_n\sin n\eta\}\cdot\{\operatorname{sh} n\xi_k+C_n\operatorname{ch} n\xi_k\}]^2\frac{\sin\eta}{R_k}d\eta \tag{28}$$

Actual values of F_x, F_y can be obtained in form of infinite series, consisting of each term in form of squares and products of $A_{n1}^{(1)}$ and $B_{n1}^{(1)}$. We observe that these component parts of forces F_x, F_y are proportional to $(\dot{a}_1)^2$.

(C) Effect of Term which contains the Factor Ω in eq. (17)

In this case we have to put

$$\rho\Omega\left[y\frac{\partial\phi}{\partial x}-x\frac{\partial\phi}{\partial y}\right]$$

in place of p in eqs. (18), (19). But we have, by definition of eq. (1),

$$\frac{\partial\xi}{\partial x}=\frac{2c[c^2-(x^2-y^2)]}{S}$$

$$\frac{\partial\xi}{\partial y}=-\frac{4cxy}{S}$$

$$\frac{\partial\eta}{\partial x}=\frac{4cxy}{S}$$

$$\frac{\partial\eta}{\partial y}=\frac{2c[c^2-(x^2-y^2)]}{S}$$

where we put, for shortness

$$S=[c^2-(x^2-y^2)]^2+[2xy]^2$$

So that we have, by inter-change of independent variables from (x, y) to (ξ, η);

$$y\frac{\partial\phi}{\partial x}-x\frac{\partial\phi}{\partial y}=\frac{\partial\phi}{\partial\xi}\operatorname{ch}\xi\sin\eta-\frac{\partial\phi}{\partial\eta}\operatorname{sh}\xi\cos\eta \tag{29}$$

Thus, we have to replace p in eqs. (20) and (21) by the expression

$$\rho\Omega\left[\frac{\partial\phi}{\partial\xi}\operatorname{ch}\xi_k\sin\eta-\frac{\partial\phi}{\partial\eta}\operatorname{sh}\xi_k\cos\eta\right]$$

The result will be given in a form of infinite series about factors $A_{n1}^{(1)}, B_{n1}^{(1)}$. Moreover, if we confine our-selves to the case in which $\dot{a}_1\neq 0$, $a_2\equiv 0$, we readily see that the angular velocity Ω of rotation of moving axes is proportional to $\dot{a}_1$. Thus finally, we infer that we may write

$$F_x=Q_x(\dot{a}_1)^2,\ F_y=Q_y(\dot{a}_1)^2$$

where factors Q_x, Q_y are easily determined by infinite series (14), (15) for velocity potential ϕ.

(D) Effect of Term containing c_x and c_y.

Her we have, also by interchange of independent variables from (x, y) to (ξ, η),

$$\rho\left[c_x\frac{\partial\phi}{\partial x}+c_y\frac{\partial\phi}{\partial y}\right]$$

$$=\rho\frac{\partial\phi}{\partial\xi}\left[\frac{c_x}{c}(1+\mathrm{ch}\,\xi\cos\eta)-\frac{c_y}{c}\mathrm{sh}\,\xi\sin\eta\right]$$

$$+\rho\frac{\partial\phi}{\partial\eta}\left[\frac{c_x}{c}\mathrm{sh}\,\xi\sin\eta-\frac{c_y}{c}(1+\mathrm{ch}\,\xi\cos\eta)\right] \tag{30}$$

So that we replace p in eqs. (18), (19) by this amount (30) (where we put $\xi=\xi_k$). Thus, the same inference as we made for above item (C) can also be made about the present item (D).

6. Concluding Remark.

Thus far, the author has reported results of his analytical study about vibration of two circular cylinders which are immersed in a region of ideal fluid, which extend to infinity. The magnitude of displacements a_1 and a_2 are taken to be of finite amounts (not necessarily of small amounts), this leading us to non-linear theory. In the above presentation, only main line of analysis is given. The author has made more detailed treatment, together with some examination of numerical values. It is hoped that these matters will be reported in subsequent papers.

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[CORRECTION to Fig. 2: $r_1\to\beta_1$; $r_2\to\beta_2$]