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EFFECTS OF ELECTROSTATIC POTENTIAL AND MAGNETIC DRIFT ON DISSIPATIVE TRAPPED ELECTRON INSTABILITY

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ABSTRACT

Effects of equilibrium electrostatic potential and magnetic drift are investigated on the growth rate of the dissipative trapped electron instability. Formulation of OTT and MANHEIMER is extended to the lower collision frequency regime by including the magnetic drift. In the lower collision frequency regime, the electrostatic trapping has smaller destabilization effect than in the collisional limit considered by OTT and MANHEIMER.

§ 1. Introduction

The trapped electron instability has been extensively investigated in relation to the anomalous transport in tokamak-like tori[1-6,9-15]. In such devices, the instability is caused by the magnetically trapped electrons between local magnetic mirrors.

Depending on the collisionality, the instability is classified into two types[2,3]. The first is the dissipative trapped electron instability proposed by KADOMTSEV and POGUTSE[3]. This type grows when the electron temperature gradient exists and the collision frequency depends on the particle energy. The instability is understood as arising from the collisional relaxation of trapped electrons into circulating ones. The second is the collisionless trapped electron instability predicted by COPPI and REWOLDT[3], which is destabilized by the magnetic drift and the ion Larmor radius effect. This type purely grows (frequency $\omega_r \ll$ growth rate γ).

OTT and MANHEIMER[4] have investigated the effects of electrostatic potential on the dissipative trapped electron instability, in the collisional regime ($\omega_e^* \ll \nu_e^{eff}$, where ω_e^* is the electron diamagnetic frequency and ν_e^{eff} the effective collision frequency of the trapped electrons) without taking into account the magnetic drift. They have studied linear and nonlinear evolution of the instability by using equilibrium and oscillating electrostatic potentials. They have shown that the electrostatic trapping increases the growth rate owing to the broadening of the trapped region in velocity space and that the detrapping decreases the growth rate. WIMMEL[5] has recently investigated the effects of electrostatic trapping and detrapping by the wave potential for the dissipative trapped ion instability.

The purpose of this paper is to extend the work of OTT and MANHEIMER for the equilibrium electrostatic potential case[4] to the lower collision frequency regime. The equilibrium electrostatic potential is known to exist in the equilibrium state of tokamak[6,7,8]. OTT and MANHEIMER have considered the collisional limit ($\omega_e^* \ll \nu_e^{eff}$) and have neglected ω_e^* compared to ν_e^{eff} to calculate the growth rate. However, as the collision frequency decreases, ω_e^* can not be neglected. The effect of the magnetic drift on the dissipative trapped electron instability has usually been neglected. We will take account of the effect of the magnetic drift, in the investigation of the lower collision frequency regime.

We explain our model and notation in § 2. In § 3, we derive a dispersion relation including the effects of the magnetic drift and the equilibrium electrostatic potential. In § 4, the growth rate is numerically investigated with some discussions. Concluding remarks will be stated in § 5.

§ 2. Model and Notation

We consider an axisymmetric torus and employ the coordinate system (r, θ, ζ), where r denotes a minor radius of a magnetic surface, θ the poloidal angle and ζ the toroidal angle. The inverse aspect ratio is defined as $\varepsilon = r/R_0$, where R_0 is the major radius of the magnetic axis. We assumed that the magnetic surfaces have the concentric circular cross-sections. For the magnetic field, the step function model is employed for simplicity as

$$\vec{B} = B_0[1-\varepsilon\sigma(\theta)]\vec{e}_\zeta + B_\theta\vec{e}_\theta, \qquad B_0 \gg B_\theta, \tag{1}$$

where $\sigma(\theta) = \mathrm{sgn}(\cos\theta)$, the safety factor $q = rB_0/R_0B_\theta \simeq d\zeta/d\theta$ and $\theta = 0$ corresponds to the outside of the torus.

We assume an electrostatic perturbation $\vec{E}_1 = -\vec{\nabla}\tilde{\phi}_1$ with $\tilde{\phi}_1$ varying as

$$\tilde{\phi}_1(\theta, \zeta, t) = \phi_1(\theta)\exp(\mathrm{i}\, m_0\theta - \mathrm{i}\, l\zeta - \mathrm{i}\,\omega t), \tag{2}$$

where l is the toroidal mode number and $m_0 \simeq lq(r_0)$ the poloidal mode number at the mode rational surface $r = r_0$. The mode is localized near $r = r_0$. We assume that the ion Larmor radius is much smaller than the radial characteristic length of the wave, so we can confine ourselves to the local theory.

Under the condition[11]

$$\omega_r < \omega_{be} = \frac{\sqrt{2\varepsilon}\, v_{the}}{qR_0}, \tag{3}$$

where ω_r is the wave frequency, ω_{be} the averaged trapped electron bounce frequency and v_{the} the electron thermal velocity, the trapped electron mode exists. We will take the equilibrium potential $\Phi(\theta)$ to be a constant value Φ_0 for $|\theta| < \pi/2$ and 0 for $|\theta| > \pi/2$.

When the equilibrium electrostatic potential exists, the total energy of a trapped electron is given by

$$E_{\mathrm{T}}=\frac{m_{\mathrm{e}}}{2}v^2-e\Phi, \tag{4}$$

where m_{e} and $-e$ are mass and charge of an electron. Using the conservation of electron total energy, we determine the region occupied by the trapped electron in velocity space as

$$\frac{\bar{v}_{\parallel}{}^2}{a^2}-\frac{\bar{v}_{\perp}{}^2}{b^2}<\frac{\hat{\Phi}}{|\hat{\Phi}|}, \tag{5}$$

where $a^2=\varepsilon|\hat{\Phi}|$, $b^2=(1-\varepsilon)|\hat{\Phi}|/2$ and $\hat{\Phi}=e\Phi_0/\varepsilon T_e$, and at $\theta=0$, $\bar{v}^2=m_e v^2/2T_e$ is evaluated. Thus, with introduction of the equilibrium potential, the region for the trapped electrons and the effective collision frequency are modified. In the case of $\Phi_0>0$, electrostatic trapping takes place. On the other hand, for $\Phi_0<0$, electrostatic detrapping occurs because the electrostatic anti-well pushes the magnetically trapped electrons out of the magnetic well.

§ 3. Dispersion Relation

We will derive a dispersion relation of the dissipative trapped electron mode which includes the effects of the electrostatic potential and the magnetic drift. There are three kinds of particles in this mode, that is, ions, circulating electrons and trapped electrons. The ions behave hydrodynamically under $k_{\parallel}v_{thi}\ll\omega$, where $k_{\parallel}$ is the parallel wave number. The perturbed density of ions is given by

$$\frac{n_{\mathrm{i}1}}{n_0}=\frac{\omega_e{}^*}{\omega}\frac{e\phi_1}{T_{\mathrm{e}}}, \tag{6}$$

where $\omega_e{}^*=(m_0/r)(cT_e/eB)(-\mathrm{d}\ln n_0/\mathrm{d}r)$ and the effects of ion temperature gradient are neglected.

The linearized drift kinetic equation for the electron velocity distribution function[10] is

$$\begin{aligned}\frac{\mathrm{d}f_{\mathrm{e}1}}{\mathrm{d}t}&=\left[\frac{\partial}{\partial t}+(\vec{v}_{\parallel}+\vec{v}_{\mathrm{B}})\cdot\vec{\nabla}\right]f_{\mathrm{e}1}\\&=-\vec{v}_{\mathrm{E}}\cdot\vec{\nabla}f_{\mathrm{e}0}-e(\vec{v}_{\parallel}+\vec{v}_{\mathrm{B}})\cdot\vec{\nabla}\tilde{\phi}_1\frac{\partial f_{\mathrm{e}0}}{\partial E_{\mathrm{k}}}-\nu_{\mathrm{T}}\left(f_{\mathrm{e}1}-\frac{e\tilde{\phi}_1}{T_{\mathrm{e}}}f_{\mathrm{e}}{}^{\mathrm{M}}\right),\end{aligned} \tag{7}$$

where $\vec{v}_{\mathrm{E}}$ is the electric drift velocity, $\vec{v}_{\mathrm{B}}$ the magnetic drift velocity, $E_{\mathrm{k}}=m_e v^2/2$ the electron kinetic energy and for the collision term the Krook model is employed. An effective collision frequency for the trapped electron is taken as[4]

$$\nu_{\mathrm{T}}=\frac{3\pi^{1/2}}{8}\frac{\nu_{\mathrm{ei}}}{2\varepsilon}\frac{1+K(E^{1/2})}{E^{3/2}}\frac{E}{E+(\hat{\Phi}/2)}, \tag{8}$$

which includes the electron-electron collision and

$$K(y)=\frac{2}{\pi^{1/2}}\left[\left(1-\frac{1}{2y^2}\right)\int_0^y\mathrm{d}z\exp(-z^2)+\frac{1}{2y}\exp(-y^2)\right], \tag{9}$$

where $\nu_{ei}=(4\sqrt{2\pi}/3)n_0e^4\ln\Lambda/(m_e^{1/2}T_e^{3/2})$. In deriving eq. (8), as was done by OTT and MANHEIMER[4], we have neglected $e|\Phi|$ compared to $m_ev^2/2$ which is valid for $e|\Phi|/T_e\ll 1$. However, $e|\Phi|$ has been retained in comparison to $2\varepsilon(m_ev^2/2)$, since we have assumed $\varepsilon\ll 1$ and have expected possible significant effects for $e|\Phi|/T_e\simeq 2\varepsilon$. The equilibrium electron distribution function f_{e0} for the inhomogeneous plasma is

$$f_{e0}=f_e^M\left[1-\frac{v_\zeta}{\Omega_{\theta e}}\left\{\frac{1}{n_0}\frac{\mathrm{d}n_0}{\mathrm{d}r}-\frac{1}{T_e}\frac{\mathrm{d}T_e}{\mathrm{d}t}\left(\frac{3}{2}-\frac{E_K}{T_e}\right)\right\}\right], \tag{10}$$

where f_e^M is the Maxwellian distribution function and $\Omega_{\theta e}=eB_\theta/m_ec$. As the result of the usual ordering in ε, we have

$$\frac{\mathrm{d}f_{e1}}{\mathrm{d}t}\simeq\frac{ef_e^M}{T_e}\left(\frac{\mathrm{d}}{\mathrm{d}t}+i\omega+\nu_T\right)\tilde{\phi}_1-\frac{c}{R_0B_\theta}\frac{\partial f_e^M}{\partial r}\frac{\partial\tilde{\phi}_1}{\partial\zeta}-\nu_Tf_{e1}. \tag{11}$$

Using eq. (2), we can rewrite this

$$\frac{\mathrm{d}}{\mathrm{d}t}[f_{e1}\exp(\nu_Tt)]=\frac{ef_e^M}{T_e}\left[\frac{\mathrm{d}}{\mathrm{d}t}+i(\omega-\omega_E^*)\right]\tilde{\phi}_1\exp(\nu_Tt), \tag{12}$$

where $\omega_E^*=\omega_e^*[1+\eta_e(E-3/2)]$, $\eta_e=\mathrm{d}\ln T_e/\mathrm{d}\ln n_0$ and $E=m_ev^2/2T_e$. Integrating the above equation along the unperturbed guiding center orbit under the boundary conditions $\theta'(t'=t)=\theta$ and $r'(t'=t)=r$, we obtain

$$f_{e1}=\frac{ef_e^M}{T_e}\left\{\phi_1(\theta)+i(\omega-\omega_E^*)\int_{-\infty}^{t}\mathrm{d}t'\phi_1(\theta')\exp[-i(\omega+i\nu_T)(t'-t)]\right.$$
$$\left.\times\exp[i(m_0\theta-l\zeta')-i(m_0\theta-l\zeta)]\right\}. \tag{13}$$

In deriving eq. (13), we have again used the assumption $e|\Phi|/T_e\ll 1$. Defining $\xi=\zeta-q\theta$ in eq. (13), we have

$$(m_0\theta'-l\zeta')-(m_0\theta-l\zeta)=-\int_t^{t'}\mathrm{d}t''l\dot{\xi}.$$

Here we have confined ourselves to the local, shearless limit and have taken $q(r)\simeq q(r_0)\simeq m_0/l=$constant. Since we have employed the simple step model for $B(\theta)$ and $\Phi(\theta)$ as was stated in § 2, $l\dot{\xi}$ can be calculated easily and shown to be constant. From the equation of motion in the drift approximation, $l\dot{\xi}$ is given by $l\dot{\xi}=-\bar{\omega}_{De}E(1+v_\parallel^2/v^2)\sigma(\theta)$, $\bar{\omega}_{De}=\varepsilon\omega_e^*$ [1]. The trapped electrons exist only in the region $|\theta|<\pi/2$, so we can take $\sigma(\theta)=1$. The term $v_\parallel^2/v^2$ can be neglected, since it is higher order than 1 by ε for the trapped electrons. Thus $l\dot{\xi}\simeq-\bar{\omega}_{De}E$, which is constant under the assumption $e|\Phi|/T_e\ll 1$. Then we have

$$\int_t^{t'}\mathrm{d}t''l\dot{\xi}=-\bar{\omega}_{De}E(t'-t). \tag{14}$$

Since $\omega<\omega_{be}$, $\phi_1(\theta')$ in the integrand of eq. (13) can be replaced by a bounce average $<\phi_1>$. Then we obtain

$$f_{e1}=\frac{ef_e^M}{T_e}\left\{\phi_1(\theta)-\frac{\omega-\omega_E^*}{\omega-\bar{\omega}_{De}E+i\nu_T}<\phi_1>\right\}. \tag{15}$$

This has a usual form[1-4, 9-15] with the magnetic drift except for the equilibrium electrostatic potential included in ν_T. In order to obtain the electron density perturbation, we integrate eq. (15) over the velocity space, by changing the variables

$$d\vec{v}=\pi\left(\frac{2T_e}{m_e}\right)^{3/2}d\tilde{E}\tilde{E}^{1/2}d\lambda B\left[1-\lambda B+\frac{e\Phi}{T_e\tilde{E}}\right]^{-1/2},$$

where $\tilde{E}=E_T/T_e$, $\lambda=\mu/\tilde{E}T_e$ and μ is the magnetic moment. Using the assumption $e|\Phi|/T_e\simeq 2\varepsilon\ll 1$ again, the electron density perturbation is given by

$$\frac{n_{e1}}{n_0}=\frac{e}{T_e}\left\{\phi_1(\theta)-\frac{1}{\pi^{1/2}}\int_s^\infty dEE^{1/2}\exp(-E)\frac{\omega-\omega_E{}^*}{\omega-\bar{\omega}_{De}E+i\nu_T}\right.$$
$$\left.\times\int_\alpha^\beta d\lambda B(\theta)\left[1-\lambda B(\theta)+\frac{e\Phi(\theta)}{T_eE}\right]^{-1/2}<\phi_1>\right\}, \tag{16}$$

where $\alpha=1/B(\theta=\pi/2)$, $\beta=[1+(e\Phi(0)/ET_e)]/B(0)$, $s=0$ for $\Phi_0>0$ and $s=e|\Phi_0|/2\varepsilon T_e$ for $\Phi_0<0$. The trapped electrons exist only in the region $|\theta|<\pi/2$. Hence, it is reasonable to take $\phi_1(\theta)=\phi_1=$constant for $|\theta|<\pi/2$ and $\phi_1(\theta)=0$ for $|\theta|>\pi/2$. Thus the bounce average of perturbed potential becomes $<\phi_1>=\phi_1$. After integrating eq. (16) with respect to λ, we have

$$\frac{n_{e1}}{n_0}=\frac{e\phi_1}{T_e}\left[1-\frac{2}{\pi^{1/2}}\int_s^\infty dEE^{1/2}\exp(-E)\right.$$
$$\left.\times\frac{\omega-\omega_E{}^*}{\omega-\bar{\omega}_{De}E+i\nu_T}\left(2\varepsilon+\frac{e\Phi}{T_eE}\right)^{1/2}\right], \tag{17}$$

where we have used the relation $1-[B(0)/B(\pi/2)]\simeq 2\varepsilon$.

By use of the quasi-neutrality condition $n_{e1}=n_{i1}$, and the assumptions $\omega_r>\gamma$ in $\omega=\omega_r+i\gamma$ and $2\varepsilon\simeq e|\Phi|/ET_e\ll 1$, we obtain $\omega_r=\omega_e{}^*$ and

$$\gamma\left(\hat{\Phi},\frac{\omega_e{}^*}{\nu_e{}^{eff}}\right)=(4\varepsilon/\pi)^{1/2}\eta_e\omega_e{}^*I\left(\hat{\Phi},\frac{\omega_e{}^*}{\nu_e{}^{eff}}\right), \tag{18}$$

where $\nu_e{}^{eff}\equiv\nu_{ei}/2\varepsilon$ and

$$I\left(\hat{\Phi},\frac{\omega_e{}^*}{\nu_e{}^{eff}}\right)=\int_s^\infty dE\frac{(\omega_e{}^*/\nu_T)}{[1+(\omega_e{}^*/\nu_T)^2(1-\varepsilon E)^2]}\left(2+\frac{\hat{\Phi}}{E}\right)^{1/2}E^{1/2}\left(E-\frac{3}{2}\right)e^{-E}. \tag{19}$$

The collisional limit $(\omega_e{}^*/\nu_e{}^{eff}\ll 1)$ of this expression reduces to the corresponding expression of Ott and Manheimer. The expression (18) is valid as long as the condition $\omega_r>\gamma$ holds.

§ 4. Numerical Results

In this section, we will numerically integrate eq. (18) to obtain the growth rate γ, taking $\omega_e{}^*/\nu_e{}^{eff}$ as a parameter. Numerical parameters $m_i/m_e=2000$, $T_e/T_i=2$, $\varepsilon=1/4$ and $q=2.5$ are used in the following calculations. The condition $\omega_{bi}<\omega<\omega_{be}$ becomes $\varepsilon^{3/2}/q<\sqrt{b}<63.2\varepsilon^{3/2}/q$, where $b=(k_\theta\rho_i)^2/2$ with k_θ the poloidal wave number,

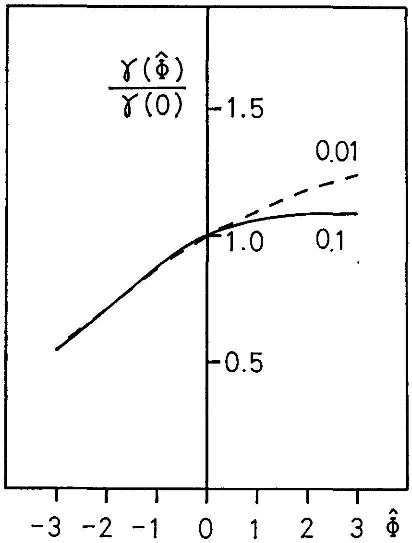

Fig. 1. The growth rates normalized by the usual liner growth rate as a function of the electrostatic potential for two typical collision frequencies $\omega_e^*/\nu_e^{eff}=0.01$ (collisional regime: OTT and MANHEIMER) and 0.1 (lower collision frequency regime: Ours).

ρ_i the ion gyro-radius and $\omega_{be}/\omega_{bi}=63.2$ for the above parameters. We have also the inequality $e|\Phi|/T_e \ll 1$. From this condition, we have $0<|\hat{\Phi}|\ll 1/\varepsilon$.

The growth rates $\gamma(\hat{\Phi},\omega_e^*/\nu_e^{eff})$ with $\hat{\Phi}=0$ and $\eta_e=1$ are $\gamma(0,0.01)=0.024\omega_e^*$ and $\gamma(0,1.0)=0.492\omega_e^*$. Hence the expression of the growth rate derived by use of the assumption $\omega_r>\gamma$ is valid for $\omega_e^*/\nu_e^{eff}<1$.

Numerically calculated growth rates are given in Fig. 1 as a function of the equilibrium potential with the parameters $\omega_e^*/\nu_e^{eff}=0.01$ and $\omega_e^*/\nu_e^{eff}=0.1$. Electrostatic trapping ($\Phi>0$) and detapping ($\Phi<0$) have respectively the destabilizing and the stabilizing effects. For $\omega_e^*/\nu_e^{eff}=0.01$, the growth rate agrees with that of OTT and MANHEIMER. In the lower collision frequency regime (*e. g.*, $\omega_e^*/\nu_e^{eff}=0.1$), the growth rate becomes smaller than that of OTT and MANHEIMER for $\Phi>0$. This is because OTT and MANHEIMER have considered the collisional limit ($\omega_e^*/\nu_e^{eff}\ll 1$) and have neglected the term $(\omega_e^*/\nu_e^{eff})^2(1-\varepsilon E)^2$ in the expression (18). On the other hand, the effect of this term in the case of electrostatic detrapping ($\Phi<0$) is negligibly small. In this case the difference between the growth rates for $\omega_e^*/\nu_e^{eff}=0.1$ and 0.01 is less than 1%. This is because the collision frequency ν_T, which is proportional to $\nu_e^{eff}/[E+(\hat{\Phi}/2)]$, increases effectively for $\hat{\Phi}<0$. Therefore a change of the parameter ω_e^*/ν_e^{eff} from 0.01 to 0.1 gives a negligibly small change of the growth rate.

§ 5. Conclusion

We have extended OTT and MANHEIMER's theory valid in the collisional limit

($\omega_e^* \ll \nu_e^{eff}$) to the lower collision frequency regime by taking account of the term $(\omega_e^*/\nu_e^{eff})^2(1-\varepsilon E)^2$ in the dispersion relation. The growth rate is calculated numerically under the assumption $\omega_r > \gamma$. In the lower collision frequency regime the electrostatic trapping has smaller destabilization effect than in the collisional regime. As the collision frequency decreases further, the magnetic drift resonance enhances the instability remarkably, but the assumption $\omega_r > \gamma$ breaks down. Thus it is important to study the effects of the electrostatic trapping and detrapping in such a collisionless regime, not using the assumption $\omega_r > \gamma$.

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