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ON A DESCRIPTIVE MODEL FOR INFORMATION SEEKING IN BAYESIAN DECISION TASK

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ABSTRACT

This paper presents a model of human information-purchasing behaviour in Bayesian decision task.

In this field, several models have been presented, particularly by EDWARDS. From the results of probability-estimation experiments, EDWARDS assumed that human underestimate the diagnostic value of information, and modified the optimal purchasing strategy of the decision task to describe the human behaviour.

WALLSTEN and others have reported data on this subject obtained through a series of experiments based on the suggestion by EDWARDS. Comparing the model and the experimental data, the author proposes a modification of the model in mathematical treatments and provides a new revised model.

Validity of this revised model is examined by comparing the model-behaviour and the experimental data reported by WALLSTEN and by others. It is also remarked by the author for human in the decision task to overestimate the diagnostic value of information. Finally, some implications of the revised model are mentioned.

§ 1. Introduction

The attractiveness of a particular course of action to a decision maker always depends on the states of nature. If the outcome resulting from an action depends on the prevailing states of nature, it is desirable and mostly possible to reduce the risk of the decision by having information about the states of nature prior to his terminal decision-making. However, in the most cases, such information is only available at the expense of time, effort, and/or money. In this situation, it is not reasonable to buy all the information but it is preferable to decide whether or not to seek the next one after each observation. Indeed, every real-life system has a function of sequential information seeking.

In mathematical theory of statistics, these situations are formulated in optional stopping and sequential decision model, or in short, Bayesian decision model. In these treatments, stopping rules are established so as to maximize expected values of the courses of actions. The expected values are calculated as functions of the diagnostic value of information and the observation cost, as well as the payoffs that may be incurred as a result of the terminal decision.

From the Bayesian point of view, EDWARDS (1965) formulated the situation of simple dichotomy form and specified the optimal stopping strategy which maximizes the expected value. On the other hand, several experimental studies on human performance in such information-seeking task were reported (e.g., WALLSTEN (1968)). In these experiments, such values were manipulated as (a) the diagnostic value of the information contained in an observation, (b) the prior probabilities of the states of nature, (c) the cost of observation and (d) the payoffs associated with the alternative decisions. The consequent effect upon the amount of information purchased were measured as the results of experiments.

However, it is observed that the maximizing expected value model gives some deviations from numerous experimental results. The subjects in experiment trend to purchase too large or too small amount of information depending upon the task condition (SLOVIC and LICHTENSTEIN (1971)). The model should be modified somehow in order to obtain more adequate values.

Among many possibilities to interpret the human behaviour and to modify the model, following three were attempted by several authors.

(1) *Utility*: WALLSTEN (1968) has assumed a power function for the relation between utility(u) and value(v). Under the optimal model, the utility function is assumed to be linear in v within the range used. Therefore the subjective probabilities for maximizing subjective expected utility are the same as the Bayesian probabilities for maximizing expected value. However, the expected value model could not predict human behaviour. Then he has assumed that the relation is a power function of the form $u(v)=kv^{\xi}$, $\xi<1$.

(2) *Strategies*: PITZ *et al.* (1969) have asserted that subjects adopte different stopping strategies during the course of the task. Then they have suggested several strategies that depend on another recognizable variables in the decision process. The critical-difference strategy specifies the critical value of difference between two different types of data for each decision. The fixed-sample-size strategy is assumed that subject purchases a fixed number of data following which he makes his decision. The "world series" strategy specifies the number of events of one kind that must occur before a decision is made.

(3) *Subjective Probability*: From the results of posterior probability-estimation experiments, EDWARDS (1965) has suggested that subjects misperceive the information contained in an observation. He has assumed that the subjects replace $E_\theta(z)$ with $E_\theta(z')=\omega E_\theta(z)$ in the optimal conditions of maximizing the expected value, where $E_\theta(z)$ is the expected value of log-likelihood ratio for an observation under the true state θ and ω is an inefficiency parameter. The value of ω is assumed to be between 0 and 1.

The modified models based on these interpretations were not necessarily given any degree of support because of the insufficient description of human behaviour (e.g., WALLSTEN (1968), PITZ *et al.* (1969)). However, EDWARDS' model does not satisfy the condition of the expected number of observations given the difference,

$E(n|d)$, in the data-generating process. Then the author presents a revised model in this paper.

As probability assessment may be an unfamiliar task in real life, subjects do not recognize the uncertainty relevant to the situation in terms of probabilities and do not become experienced in expressing their own subjective probabilities. A number of significant barriers on eliciting subjective probabilities have been reported (CHESLEY (1976)). Then we investigate the amount of information purchased by subjects instead of subjective probabilities eliciting from them. Validity of the revised model is examined by comparing the model-behaviour and the experimental data reported by WALLSTEN and by others. As a result, the author suggests that the condition of ω used in the revised model might be greater than 1 in order to perform better fit to the experimental results, while EDWARDS postulated that $\omega < 1$. Finally, some implications of the model are mentioned.

§ 2. Bayesian Decision Task

When one is required to make a decision, he frequently has the option of deferring the decision until he has obtained relevant information at some additional cost. It makes the decision-making process complicated, since one must weigh the relative advantage of the information to be purchased against its cost. When the characteristics of the task are well specified, it is possible to obtain an optimal stopping strategy that will maximize expected value (EDWARDS (1965)).

Suppose there are two hypothetical states of nature, H_1 and H_2. If $P(H_1)$ is the prior probability that H_1 is true, and if $P(H_2)$ is the prior probability that H_2 is true, then the ratio $P(H_1)/P(H_2)$ is called prior odds in favour of H_1 and is denoted by Ω_0.

If $P(D|H_1)$ is the probability of getting information with the data, D, given the truth of H_1, and if $P(D|H_2)$ is the probability of the same data given H_2, then the ratio of $P(D|H_1)/P(D|H_2)$ is called the likelihood ratio and is represented by L. Bayes' theorem provides a simple rule for combining the prior probabilities and the likelihood ratios to obtain the posterior probabilities for each hypothesis, $P(H_1|D)$ and $P(H_2|D)$. The ratio of the two posterior probabilities is known as posterior odds and is denoted by Ω_1. The appropriate form of Bayes' theorem is

$$\Omega_1 = L\Omega_0 . \qquad (1)$$

In this paper, we will simplify the task conditions in order to characterize the difference between theoretical strategy and human behaviour. We assume that the hypotheses are binomial distributions and are distinct from the values of the binomial parameter θ. Then each datum comprising information is binary, say red event and blue event represented by r and b respectively. The payoffs, reward R and fine F, depend only upon whether the choice is correct or incorrect, and not upon the particular choice that is made, and the cost per observation, c, is constant through information-seeking process.

Furthermore, we assume the following three symmetry conditions:
Symmetry condition (1): The probability of observing each event under each hypo-

thesis is given by

$$P(r|H_1)=P(b|H_2)=p_0,$$

$$P(b|H_1)=P(r|H_2)=1-p_0,$$

where, without loss of generality, $p_0>0.5$.

Symmetry condition (2): The prior probability of each hypothesis is equal and then $\Omega_0=1$.

Symmetry condition (3): The absolute value of fine is equal to that of reward, but the sign is opposite, i.e., $F=-R$.

We use the probability of observing red event under each hypothesis as the binomial parameter of each one. When H_1 is true, then $\theta=\theta_1=P(r|H_1)=p_0$ and when H_2 is true, then $\theta=\theta_2=P(r|H_2)=1-p_0$.

Equation (1) is often used in terms of logarithms. Letting z_1 equal to log-likelihood ratio for the observation of red event and z_2 equal to log-likelihood ratio for the observation of blue event, the expected value of the log-likelihood ratio for an observation, $E_\theta(z)$, can easily be calculated:

$$E_\theta(z)=\theta z_1+(1-\theta)z_2,$$

$$=\theta \ln \frac{p_0}{1-p_0}+(1-\theta) \ln \frac{1-p_0}{p_0},$$

$$=(2\theta-1) \ln \frac{p_0}{1-p_0}.$$

When H_1 is true, $E_{\theta_1}(z)=(2p_0-1)\ln(p_0/(1-p_0))$, and when H_2 is true, $E_{\theta_2}(z)=-(2p_0-1)\ln(p_0/(1-p_0))$. Hence we obtain

$$E_{\theta_1}(z)=-E_{\theta_2}(z)=(2p_0-1)\ln \frac{p_0}{1-p_0}\equiv Z.$$

Z can be taken as a measure of the average diagnostic value of an observation. As the difference between Z and zero for a task increases, it can be expected that the posterior probabilities of the hypotheses will more quickly approach zero and one.

Briefly, the effect of Bayes' theorem is to change the log-odds in favour of either hypothesis by a constant amount following a red event observed and by an equal but opposite amount following a blue event observed. Hence, the difference, d, between the number of blue and red events observed should be the only relevant variable in making a decision. The observed sequential effects clearly rule out such a model as completely descriptive of behaviour.

Under these symmetry conditions, the optimal stopping probabilities are the same for both H_1 and H_2. To find the optimal stopping probability, it is appropriate to differentiate the expected value equation of the final decision with respect to the stopping probability, and set the resulting equation equal to zero. It depends only on the ratio $V=(R-F)/c$ and on Z, in terms of the equation

$$VZ-2\ln(p^*/(1-p^*))+(1/p^*)-(1/(1-p^*))=0, \qquad (2)$$

where p^* equals the optimal stopping probability (EDWARDS (1965), Eq. 18, p. 319).

The expected number of observations bought, n^*, depending on the stopping probability is given by following equation (EDWARDS (1965), Eq. 19, p. 320):

$$n^*Z=(2p^*-1)\ln(p^*/(1-p^*)) . \tag{3}$$

§ 3. EDWARDS' Modification and A Comment on his Descriptive Model

Numerous experiments have compared human estimations of Bayesian probabilities with the objective values (SLOVIC and LICHTENSTEIN (1971)). The results showed that over a wide range of conditions, subjects underestimated high probabilities and overestimated low probabilities. The term *conservative* is used to indicate that subjects revise their subjective probabilities in the face of evidence less than that prescribed by Bayes' theorem. PHILLIPS and EDWARDS (1966) have proposed that conservatism results from an inability to make full use of the information contained in an observation. They suggested that subjects' performance could be described by a modified form of Bayes' theorem (Eq. (1)) in which L was replaced by a subjective value L', where $L'=L^{\omega}$ and ω would usually be less than one.

EDWARDS (1965) made an assumption that $E_\theta(z)$ used by subjects in determining their optimal stopping probabilities are not the same as that used in the equations for maximizing the expected value, and that the subjects replace $E_\theta(z)$ with $E_\theta(z')=\omega E_\theta(z)$ in determining their optimal stopping probabilities, where ω is an inefficiency parameter and the value of ω is assumed to be between 0 and 1. Using this assumption, the modified forms of the equations for the subjectively optimal stopping probability, p_s', and for the expected number of observations, n_s', become as follows:

$$V\omega Z-2\ln(p_s'/(1-p_s'))+(1/p_s')-(1/(1-p_s'))=0 , \tag{4}$$

$$n_s'\omega Z=(2p_s'-1)\ln(p_s'/(1-p_s')) . \tag{5}$$

Based on the results of inference experiments, EDWARDS has suggested that each subject is characterized by a particular value of ω ranging from 0.05 to 0.50; 0.20 might be a convenient representative value.

However the EDWARDS' model does not satisfy the condition of the expected number of observations given the difference, d, between two different types of data. The condition should be objectively decided by the data-generating process. If m is the number of red events and $k(=n-m)$ the number of blue events in a particular order in n observations, the stopping rule is defined in terms of absolute difference, $d(=|m-k|)$.

From the symmetry conditions (1) and (2), we assume that $m\geqq k$ without loss of generality. Then the difference d is that $d=m-k=2m-n$.

The posterior odds Ω_n is

$$\Omega_n=L^d\Omega_0=L^d. \tag{6}$$

The relation among the critical difference, d^*, and the cutoff odds, Ω^*, for the optimal condition (Eq. (2)) is that

$$\Omega^* = p^*/(1-p^*) = (p_0/(1-p_0))^{d^*}, \tag{7}$$

then,

$$p^* = p_0^{d^*}/(p_0^{d^*} + (1-p_0)^{d^*}). \tag{8}$$

Scince we assumed $Z = (2p_0 - 1)\ln(p_0/(1-p_0))$, the substitution from Eq. (8) into Eq. (3) produces:

$$n^* = \frac{2d^*}{2p_0 - 1} \cdot \left[\frac{1-((1-p_0)/p_0)^{d^*}}{1-((1-p_0)/p_0)^{2d^*}}\right] - \frac{d^*}{2p_0 - 1}. \tag{9}$$

Eq. (9) is the equation for the expected number of observations given the difference d^*, $E(n|d^*)$, derived from the data-generating process, and the strategy specified by Eq. (9) is referred to as the critical-difference strategy in PITZ *et al.* (1969, Eq. (1), p. 3).

Eq. (9) may be rewritten

$$n^* = E(n|d^*) = \frac{d^*}{2p_0 - 1} \cdot \left[\frac{p_0^{d^*} - (1-p_0)^{d^*}}{p_0^{d^*} + (1-p_0)^{d^*}}\right]. \tag{10}$$

If subject inferes the log-likelihood ratios from Bayes' theorem inefficiently, the cutoff posterior odds Ω_s' for his subjectively optimal condition is

$$\Omega_s' = (L^{\omega})^{d_s'} = p_s'/(1-p_s'), \tag{11}$$

and

$$p_s' = p_0^{\omega d_s'}/(p_0^{\omega d_s'} + (1-p_0)^{\omega d_s'}). \tag{12}$$

Then we substitute the Eq. (12) into Eq. (5) and rewrite as follows:

$$\begin{aligned} \omega n_s' &= \frac{\omega d_s'}{2p_0 - 1} \cdot \left[\frac{p_0^{\omega d_s'} - (1-p_0)^{\omega d_s'}}{p_0^{\omega d_s'} + (1-p_0)^{\omega d_s'}}\right], \\ &= \frac{2\omega d_s'}{2p_0 - 1} \cdot \left[\frac{1-((1-p_0)/p_0)^{\omega d_s'}}{1-((1-p_0)/p_0)^{2\omega d_s'}}\right] - \frac{\omega d_s'}{2p_0 - 1}. \end{aligned} \tag{13}$$

While the right hand side of Eq. (13) is the expected number of observations given the differences $\omega d_s'$ then $\omega n_s' = E(n|\omega d_s')$. Without the case $\omega = 1$, the EDWARDS' model does not consist of the relationship between n_s' and d_s' derived from the data-generating process in the decision task. Namely the model does not satisfy the condition of the expected number of observations given d_s', $E(n|d_s')$, derived from the binomial distribution, that is $n_s' \neq E(n|d_s')$.

§ 4. Revised Model

Now we suppose that a subject is characterized by a ω. He cutoff his seeking behaviour as follows:

$$\Omega_s' = A_s \Omega_0,$$

and

$$\Omega_s'' = B_s \Omega_0,$$

where A_s is the likelihood ratio just large enough to transform his prior odds into the upper posterior odds-cutoff, and B_s is the likelihood ratio just small enough to transform his prior odds into the lower posterior odds-cutoff. The conditional probability that he will accept H_2 when H_1 is correct will be called β_s. Similarly, the conditional probability that he will accept H_1 when H_2 is correct will be called α_s. We assume that he lets

$$A_s=(1-\beta_s)/\alpha_s\,,$$

$$B_s=\beta_s/(1-\alpha_s)\,.$$

However, his strategy seems objectively as follows:

$$A'=((1-\beta_s)/\alpha_s)^{1/\omega}=A_s^{1/\omega}\,,$$

$$B'=(\beta_s/(1-\alpha_s))^{1/\omega}=B_s^{1/\omega}\,.$$

The operating characteristic function to the procedure, $L'(\theta)$, is the probability that the sequential process will terminate with the acceptance of H_2 when θ is the true parameter. We obtain the approximation formula

$$L'(\theta_1)\sim(1-A')B'/(B'-A')\,,$$

when H_1 is true, and

$$L'(\theta_2)\sim(A'-1)/(A'-B')\,,$$

when H_2 is true (WALD (1947), Eq. (3: 43), p. 50).

Hence we obtain the approximation formula of the expected number of observations $E_\theta(n)$. The equation can be written as

$$E_\theta(n)\sim[L'(\theta)\ln(\beta_s/(1-\alpha_s)) \\ +(1-L'(\theta))\ln((1-\beta_s)/\alpha_s)]/E_\theta(z')\,,$$

if $E_\theta(z')\neq 0$ (WALD (1947), Eq. (3: 57), p. 53).

Now we define that v_{ij} is the payoff (reward or fine) received with subject's choice, H_j, when H_i is correct. In the same manner as EDWARDS, we treat observations as though they were continuous, so that stopping points can be reached precisely.

Now the expected gain is as follows:

$$E(v)=p(H_1)[(1-\beta_s)v_{11}+\beta_s v_{21}-cE_{\theta_1}(n)] \\ +p(H_2)[\alpha_s v_{12}+(1-\alpha_s)v_{22}-cE_{\theta_2}(n)]\,. \tag{14}$$

To find the optimal values of α_s and β_s, it is appropriate to differentiate Eq. (14) with respect to α_s and β_s, set the resulting equations equal to zero, and slove them as a system of simultaneous equations in two unknowns.

The operations are straight forward and conventional. It is convenient to define

$$X_0=(\alpha_s\beta_s)^{1/\omega}-((1-\alpha_s)(1-\beta_s))^{1/\omega}\,,$$

$$X_1=((\alpha_s\beta_s)^{1/\omega}-(\beta_s(1-\beta_s))^{1/\omega})/X_0\,,$$

$$X_2=((\beta_s(1-\beta_s))^{1/\omega}-((1-\alpha_s)(1-\beta_s))^{1/\omega})/X_0\ ,$$

$$X_3=((\alpha_s(1-\alpha_s))^{1/\omega}-((1-\alpha_s)(1-\beta_s))^{1/\omega})/X_0\ ,$$

$$X_4=((\alpha_s\beta_s)^{1/\omega}-(\alpha_s(1-\alpha_s))^{1/\omega})/X_0\ ,$$

and

$$Y=\ln\alpha_s-\ln(1-\alpha_s)-\ln(1-\beta_s)+\ln\beta_s\ .$$

Then, the final forms of the equations become

$$\frac{\partial E(v)}{\partial \alpha_s}=\frac{-P(H_1)c}{E_{\theta_1}(z')}\cdot\Bigg[\frac{-1}{X_0^2\omega}[\beta_s(1-\beta_s)]^{1/\omega}\{[\alpha_s(1-\alpha_s)]^{(1-\omega)/\omega}-\alpha_s^{(1-\omega)/\omega}\beta_s^{1/\omega}$$
$$-(1-\alpha_s)^{(1-\omega)/\omega}(1-\beta_s)^{1/\omega}\}Y+\frac{X_1}{1-\alpha_s}-\frac{X_2}{\alpha_s}\Bigg]$$
$$+\frac{-P(H_2)c}{E_{\theta_2}(z')}\cdot\Bigg[\frac{1}{X_0^2\omega}[\alpha_s(1-\alpha_s)]^{(1-\omega)/\omega}\{[\beta_s(1-\beta_s)]^{1/\omega}-\alpha_s^{(1-\omega)/\omega}\beta_s^{1/\omega}$$
$$-(1-\alpha_s)^{(1-\omega)/\omega}(1-\beta_s)^{1/\omega}\}Y+\frac{X_3}{1-\alpha_s}-\frac{X_4}{\alpha_s}\Bigg]+P(H_2)(v_{12}-v_{22})=0\ ,$$

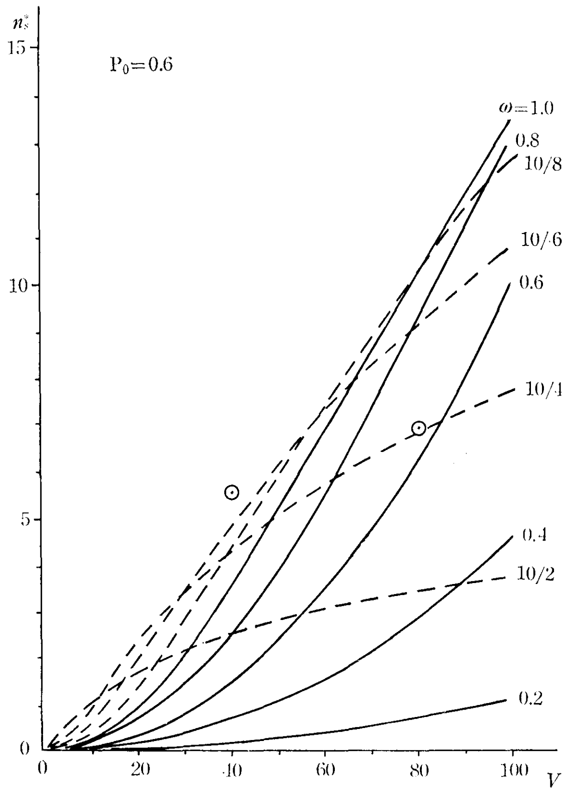

Fig. 1. n_s^* as a function of V for sample values of ω when $p_0=0.6$ The mark ⊙ indicates the average number of observations in FUKUKAWA (1974).

$$\frac{\partial E(v)}{\partial \beta_s}=\frac{-P(H_1)c}{E_{\theta_1}(z')}\cdot\Big[\frac{-1}{X_0{}^2\omega}[\beta_s(1-\beta_s)]^{(1-\omega)/\omega}\{[\alpha_s(1-\alpha_s)]^{1/\omega}-\alpha_s{}^{1/\omega}\beta_s{}^{(1+\omega)/\omega}$$
$$-(1-\alpha_s)^{1/\omega}(1-\beta_s)^{(1+\omega)/\omega}\}Y+\frac{X_1}{\beta_s}-\frac{X_2}{1-\beta_s}\Big]$$
$$+\frac{-P(H_2)c}{E_{\theta_2}(z')}\cdot\Big[\frac{1}{X_0{}^2\omega}[\alpha_s(1-\alpha_s)]^{1/\omega}\{[\beta_s(1-\beta_s)]^{(1-\omega)/\omega}-\alpha_s{}^{1/\omega}\beta_s{}^{(1-\omega)/\omega}$$
$$-(1-\alpha_s)^{1/\omega}(1-\beta_s)^{(1-\omega)/\omega}\}Y+\frac{X_3}{\beta_s}-\frac{X_4}{1-\beta_s}\Big]+P(H_1)(v_{21}-v_{11})=0\,.$$

Explicit solutions of these equations are not to be anticipated.

If the symmetry conditions (1), (2) and (3) apply, it follows that $E_{\theta_1}(z')=-E_{\theta_2}(z')=\omega Z$, $\Omega_s'=1/\Omega_s''$, $A'=1/B'$, $\alpha_s=\beta_s=1-p_s{}^*$ and the optimal condition is

$$V\omega Z-\frac{2}{\omega}\cdot\Big[\frac{p_s{}^{*(1-\omega)/\omega}(1-p_s{}^*)^{(1-\omega)/\omega}}{(p_s{}^{*1/\omega}+(1-p_s{}^*)^{1/\omega})^2}\Big]\ln\frac{p_s{}^*}{1-p_s{}^*}$$
$$-\Big[\frac{p_s{}^{*1/\omega}-(1-p_s{}^*)^{1/\omega}}{p_s{}^{*1/\omega}+(1-p_s{}^*)^{1/\omega}}\Big]\cdot\Big[\frac{1}{p_s{}^*}+\frac{1}{1-p_s{}^*}\Big]=0\,, \qquad (15)$$

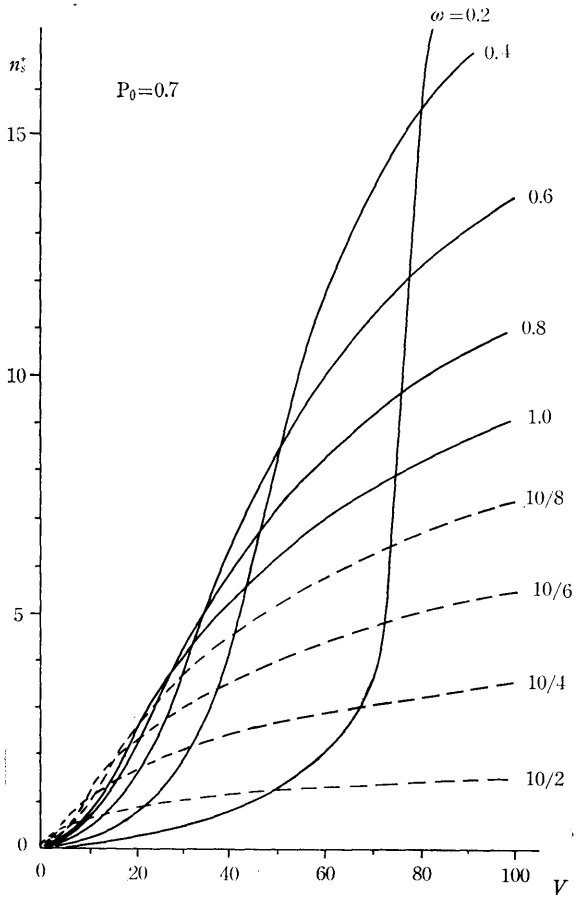

Fig. 2. $n_s{}^*$ as a function of V for sample values of ω when $p_0=0.7$.

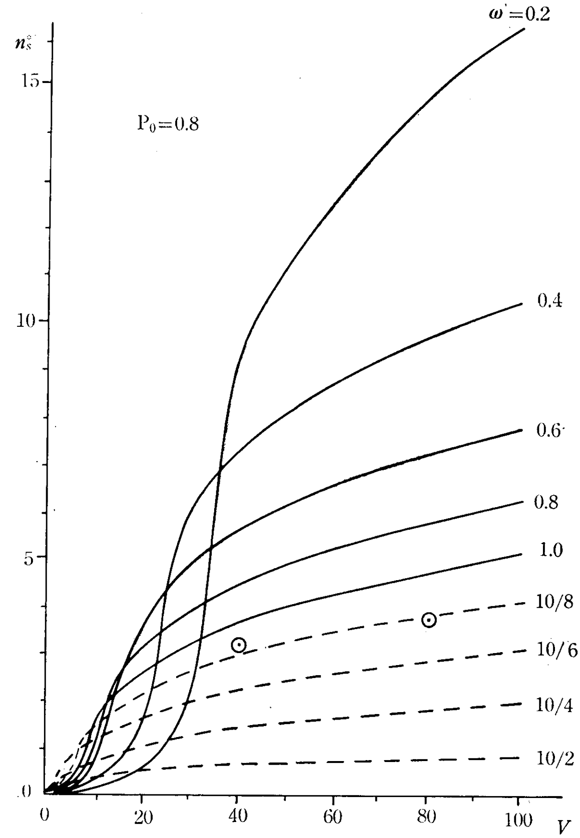

Fig. 3. n_s^* as a function of V for sample values of ω when $p_0=0.8$. The mark ⊙ indicates the average number of observations in FUKUKAWA (1974).

and the expected number of observations is

$$E_{\theta_1}(n)=E_{\theta_2}(n)=n_s^*$$

$$=\frac{1}{\omega Z}\cdot\left[\frac{p_s^{*1/\omega}-(1-p_s^*)^{1/\omega}}{p_s^{*1/\omega}+(1-p_s^*)^{1/\omega}}\right]\ln\frac{p_s^*}{1-p_s^*}. \qquad (16)$$

The optimal p_s^* and n_s^* are calculated from Eqs. (15) and (16), given ω, p_0 and V. To find the optimal p_s^*, we utilize an approximation method (NEWTON's method) to Eq. (15). Figs. 1, 2 and 3 show a plot of n_s^* as a function of V and ω for the fixed p_0, and in Figs. 1 and 3 the average number of purchases made by the subjects in the experiment (FUKUKAWA (1974)) are also plotted.

The subjective posterior odds Ω_s^* is

$$\Omega_s^*=p_s^*/(1-p_s^*)=(p_0/(1-p_0))^{\omega d_s^*},$$

but the objective posterior odds Ω' in data-generating process is

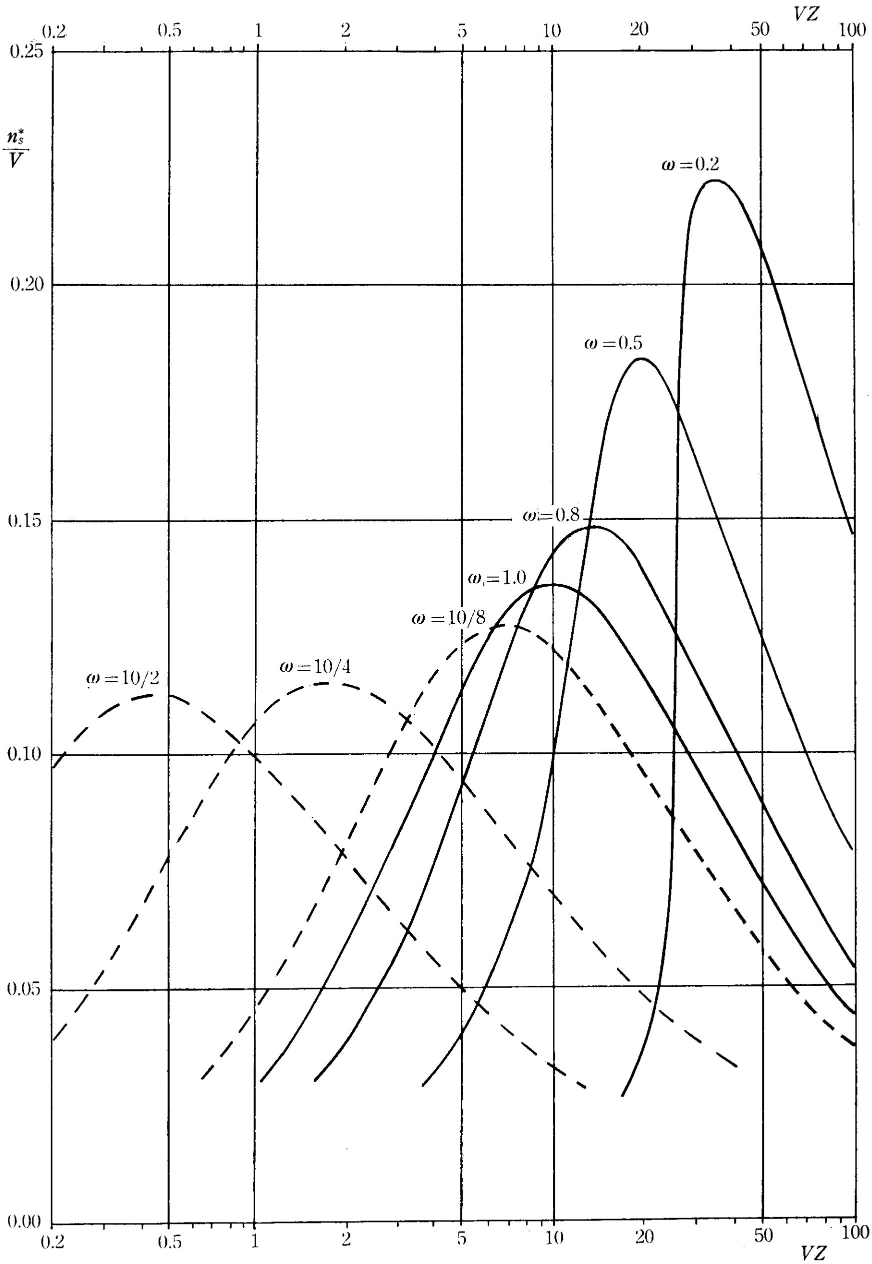

Fig. 4. n_s^*/V as a function of VZ for sample values of ω.

$$\Omega' = (p_0/(1-p_0))^{d_s^*} = (p_s^*/(1-p_s^*))^{1/\omega} \,. \tag{17}$$

Then, substitution from Eq. (17) into Eq. (16) produces:

$$n_s^* = \frac{d_s^*}{2p_0 - 1} \cdot \left[\frac{p_0^{d_s^*} - (1-p_0)^{d_s^*}}{p_0^{d_s^*} + (1-p_0)^{d_s^*}} \right]. \tag{18}$$

This Eq. (18) satisfy the condition of the expected number of observations given the difference, that is $n_s^* = E(n|d_s^*)$.

In the same way as EDWARDS (1965, Eq. (20), p. 320), divide Eq. (16) by Eq. (15):

$$\frac{n_s^*}{V} = \frac{n_s^*\omega Z}{V\omega Z} = \left\{ \left[\frac{p_s^{*1/\omega} - (1-p_s^*)^{1/\omega}}{p_s^{*1/\omega} + (1-p_s^*)^{1/\omega}} \right] \ln \frac{p_s^*}{1-p_s^*} \right\} \Big/$$
$$\left\{ \frac{2}{\omega} \cdot \left[\frac{p_s^{*(1-\omega)/\omega}(1-p_s^*)^{(1-\omega)/\omega}}{p_s^{*1/\omega} + (1-p_s^*)^{1/\omega}} \right] \ln \frac{p_s^*}{1-p_s^*} - \left[\frac{p_s^{*1/\omega} - (1-p_s^*)^{1/\omega}}{p_s^{*1/\omega} + (1-p_s^*)^{1/\omega}} \right] \right.$$
$$\left. \cdot \left[\frac{1}{p_s^*} + \frac{1}{1-p_s^*} \right] \right\}.$$

Since V is a constant in this equation, the maximum value of n_s^* will occur where n_s^*/V is a maximum. But the maximum value depends on the parameter ω. Fig. 4 presents plots of n_s^*/V as a function of VZ for sample values of ω.

§ 5. Concluding Remarks

The features of human performance which were shown in the results of experiments on optional stopping, were as follows:

(a) Subjects were only partially sensitive to variables, that is, the amount of variation in the number of observations was more stable with V than the theoretical one in both cases of $p_0=0.6$ and 0.8.

(b) Subjects purchased too few observations in the task, where the diagnostic impact of an observation was great.

(c) Subjects tended to purchase too many or too few observations depending on V in the task where each observation was less diagnostic.

WALLESTEN (1968), PITZ *et al.* (1969), PITZ (1969), FUKUKAWA (1974) and others found these features in their studies.

In these decision problems where the average diagnostic impact of an observation is small, it is predicted by EDWARDS that the subjects would buy less observations under the all values of V. The results of experiments showed that they bought less observations under the higher values of V, but excess under the lower values. EDWARDS' prediction is that the subjects would buy considerably more than the optimal number of observations where the average diagnostic impact of an observation is great. However the observations they bought were less but not significantly different from the optimal.

In contrast with the EDWARDS' prediction, the revised model conforms to the features in case $\omega > 1$. That is, subjects have a tendency to overestimate a probabilistic information in optional stopping and sequential decision-making, on the contrary to underestimate in simple probability-estimation task. These results may be able to interpret as follows. Subjects in the information-seeking decision task treat each data sampled from a probabilistic state as it were a certain fact, and depend in excess on the observations.

There exists two commonly means of the subjective probabilities. One is subjective estimate that a particular individual infers on the objective probability and the other is subject's degree of belief as a measure of confidence that a particular individual has in the truth of a particular proposition. It could be asserted that the estimates of subjective probabilities reported by subjects do not accurately reflect their uncertainty under the decision-situation, and therefore the predictions are invalid (WALLSETN (1968)). Subjects' feeling about uncertainty would differ in simple estimation task and in information-purchasing task.

The results showed that human behaviour might be explained with the aid of the revised model. However, it is not in full but, we can describe the feature(c) in partially. That is, in the case $p_0=0.6$ and $V=40$ (see Fig. 1), the average number of the subjects' observations showed above the appropriate value of the revised model. The experimental results reported previously showed that subjects have sometimes had the prior opportunities to stop at the terminal difference d, or a larger one, within a series of observations (WALLSTEN (1968), PITZ *et al.* (1969), FUKUKAWA (1974)). The implication is that subjects would make a decision at a small value of d when the number of observations, n, was large, even though they had deferred to commit themselves to a decision when d was as large or larger, but n was small. In this research, it is assumed that the expected value model is adequate, but the influence about probability is misperceived. However, these results suggest the necessity of further investigations on (i) whether the decision is closely related to the likelihood ratio of the observation or not, and (ii) if it were the case, the exact relation.

Supposing that human-behaviour is based on the degree of belief, the confidence might be affected by "hope" to achieve or "fear" to fail, and by other utilities related to the task situation. Subjects might expect not only to maximize their rewards, but they might also have pleasure (utility) of "success" or displeasure (negative utility) of "failure" in their decision outcomes.

Other proposals by previous investigations are two hypotheses: (i) the constraining effect of success probability, and (ii) the risk-preference tendency. In the hypothesis (i), it is assumed that the success probability associated with the goal object perform as a constraint on choice behaviour in some decision situation (FEATHER (1959)). In (ii), it is assumed that the risks are maintained at constant levels particular to the decision makers (O'CONNOR *et al.* (1972)). While they are weakened by the difficulty of identification of these utilities and risks, the present findings may be partially explained by these hypotheses.

It is of interest to look at the individual differences in stopping behaviour. Within each task used in the experiments, the subjects were put in rank order according to their average number of observations bought. The large and consistent individual differences existed in the subjects' information-seeking behaviour (WALLSTEIN (1968), HERSHMAN *et al.* (1970), FUKUKAWA (1974)). These effects depend on subject's propensity for taking risk and purchasing an insurance policy in order to avoid a small chance of a large loss. While FEATHER (1959) hypothesized and found that attainment attractiveness of a goal object varies inversely with the associated success probability and that the assumed covariation tends to be more apparent in (i) achievement oriented than in relaxed situations, and (ii) ego-related (skill) than in chance-related situations. Subjects might perceive higher achievement values in correct decision with less observations than in correct decision with many observations. The effect of task involvement might constitute a deviation from the expected value model (RONEN (1974)). There will be an issue regarding the extent to which the subject's personality traits affect on their behaviour in these tasks.

Finally, it must be pointed out that the optimal stopping difference is decimal but actual differences are integers. Then subjects can not stop buying observations at the optimal difference precisely. These models cannot be applied exactly. Still, preliminary results suggest that the revised model is a good first approximation for some subjects. As a further research, it may consider to apply some strategies of information-seeking, like PITZ *et al.* (1969), but with misperceived observations.

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