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A PRACTICAL MODEL FOR MULTIPLE OBJECTIVE CORPORATE FINANCIAL PLANNING: A DEVIATION BALANCING GOAL-VECTOR APPROACH

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ABSTRACT

A practical model for multiobjective, multiperiod corporate financial planning, which is based on an actual business corporation in Japan, is presented. A deviation-balancing approach based on a goal-vector concept is applied to the model corporation.

After the characteristics of typical goals in a comprehensive financial plan are discussed, the essence of the goal-vector approach and the usefulness of an L-shaped utility function, a kind of deviation-balancing model, is argued. Then mathematical formulations necessary for the planning model, including the formulations of finanacial-statement variables, goal constraints and technical constraints, are developed. The results of computer solutions under the assumption of maintaining the existing production capacity and the assumption of expanding the capacity are presented and discussed. Detailed data and some technical notes of the goal-vector approach are provided in appendices.

INTRODUCTION

This paper presents a practical model for multiple objective corporate financial planning. A deviation-balancing goal-vector approach is applied to a model corporation, which is based upon an actual business corporation in Japan. Since the author's main concern is not to solve the real problem in a real company but to present a typical example of multiple objective financial planning, the events and numerical data have been greatly simplified so that the whole structure of the model can easily be grasped. The model corporation should therefore be considered as a fictitious corporation.

In *Section 1*, typical goals in a comprehensive financial planning are illustrated and the characteristics of the goal activities are pointed out. In *Section 2*, the essence of a goal-vector approach is explained and the usefulness of an L-shape model (a kind of deviation-balancing model) for the financial planning is argued. *Section 3* presents the physical, organizational and financial situations of the model corporation at the beginning of the planning period. In *Section 4*, mathematical formulations necessary for the model corporation are presented. Since most of the goals are expressed in financial terms, equations for preparing planned income statements and balance sheets, as well as goal constraints and technical constraints, are defined. In *Section 5*, the results of computer solutions under two major assumptions are presented and discussed; under the assumption of maintaining the existing production capacity and the assumption of expanding the capacity. Detailed data of the model corporation and technical aspects of the goal-vector approach are provided in *Appendices A and B.*

1. PROBLEMS IN FINANCIAL PLANNING GOALS

1.1 Background

The model corporation, a producer of main parts for electronic products, was incorporated nearly two decades ago. After a long history of business fluctuations, the company has established a good position in the electronics industry as one of the oligopolistic producers of basic parts for electronic equipment. The main production facilities have been completely modernized during these five years.

Now the top executives wish to have a somewhat comprehensive financial planning model through which they can *monitor* the company's financial situations for the next several years. The executives wish their financial planning model to cover four accounting periods beginning with this coming year.

Table 1. Initial Financial Condition
BALANCE SHEET
—as of the initial date of planning period—

ASSETS		LIABILITIES	
K_{C0} Cash and bank deposits	3,600,000	K_{P0} Accounts payable	1,645,800
K_{B0} Securities	1,800,000	K_{S0} Short-term borrowings	13,500,000
K_{R0} Accounts receivable	9,000,000	Total current liabilities	15,145,800
K_{G0} Inventories	4,000,000	K_{L0} Long-term borrowings	4,800,000
Total current assets	18,400,000	Total liabilities	19,945,800
K_{Q0} Direct factory equipment	3,375,800	**NET WORTH**	
K_{F0} Other factory equipment	3,800,000	K_{K0} Capital stock	5,000,000
K_{H0} Office equipment	930,000	K_{U0} Capital surplus	1,800,000
K_{I0} Land	800,000	K_{E0} Retained earnings	560,000
Total fixed assets	8,905,800	Total net worth	7,360,000
Total assets	27,305,800	Total liabilities & net worth	27,305,800

The balance sheet in Table 1 shows the initial financial condition of the company as of the end of period 0 (after paying income taxes, dividends to shareholders and bonuses to officers). The symbol K_{jt} on the head of each item in Table 1 denotes the dollar amount of item j at the end of period t(after deducting income distribution).

1.2 Development of Multiple Objectives

There has been much argument about the nature of the company's *ultimate objective*, but the top executives and their staff (decision analysts) could not come to an agreement. Moreover they know implicitly that even if they could reach a common understanding about the ultimate objective, it would only be represented in an abstract slogan such as "long-run profitability", "shareholder's wealth", "company's stable growth", and the like; the objective could hardly be formulated as a specific function of the decision variables that are controllable by the actual business activities.

On the other hand, the top executives know through experience that some important *subobjectives*, each of which can be described as a specific function of management processes, contribute to improvement of the company's financial situation. After listing and examining these subobjectives, the decision analysts rearranged them taking into consideration their applicability to mathematical formulations, and chose some of them as their practical targets. We shall call, for convenience, these operational objectives *goals*.

The following are the goals adopted by the model corporation; most of them may be regarded, in general, as typical financial planning goals:

i) To pay a proper amount of dividend in each fiscal period in order to maintain good relationships with current and future shareholders.

ii) To increase the net income after tax in each period, maintaining a favorable ratio thereof to the invested capital.

iii) To expand the sales of each class of products, taking into account a good balance between each product, for maintaining and improving the company's position in the oligopolistic market.

iv) To maintain a proper cash balance in each period in order to assure a good liquidity of funds.

v) To maintain good financial ratios among the assets, liabilities, equity, and income. This objective has been specified in the following four goals.

a) To maintain a proper rate of income on total assets.
b) To maintain a good ratio between current assets and current liabilities.
c) To maintain a good ratio of net worth to the total assets.
d) To maintain a proper balance between the total fixed assets and capital resources (long-term liabilities and net worth).

Questions and criticisms may be presented by theorists about the usefulness of traditionally-used financial ratios. It should be noted, however, that the corporate management cannot ignore the ratios when they are practically used by the outside financial analysts as the means of evaluating the "soundness" of the corporation.

Note: In actual corporate plans various "non-financial" objectives, e.g., the improvement of product quality, welfare of employees, and other organizational and environmental situations, may be listed up. In this model we regard these objectives as given conditions whose influences are reflected in planned fixed expenses and other financial parameters.

1.3 Characteristics of Goal Activities

The multiple goals of the model corporation suggest that we are dealing with the goal activities that have the following characteristics:

i) None of the goals has such a dominant priority to others that the *ordering method* of preemptive priority, which are widely used in traditional goal programming, can be applied. Under the ordering method, when a high priority goal cannot be sufficiently achieved the lower priority goals will be neglected; however, corporate executives would never think, for example, that if the goal of achieving a sufficient income could not be satisfied, the goals of improving market share and capital liquidity might be ignored. The more common occurrence is that management wishes to improve the level of achievement in most or all goals at the same time, while considering some relative differences of importance among the goals.

ii) It is seldom that the relationships between goals are substitutive enough to apply the *weighting method*, which is also widely used in traditional goal programming. It should also be noted that the minimization of the sum of the weighted unattainment of goals often leads to a largely " unbalanced " solution in the sense that the ratio of unattainment of each goal is quite different from the ratio of the weights. Assume a case where a top manager considers the importance of sales goals of products A and B to be equal and has given the same weight to each of them. In this case it would be natural to assume that he expects the ratio of unattainment of each goal should be close to 1:1. His expectation will scarcely be realized. The more probable result will be that one goal is largely attained while the other is partially or not at all attained as illustrated in Appendix B-5.

iii) It is quite usual that a comprehensive financial planning model is built for the purpose of monitoring future corporate situations over several periods, and the emphasis is placed upon a balance among the levels of performance in *multiple periods* for each goal item. Take the example of the goal of increasing net incomes over four periods. Corporate managers would prefer a steady growth of net income to the maximization of *total* income of these periods. None of the net incomes for these periods would have a dominant priority to, or would be substitutive for, the others. Such an independency of interperiodic goals amplifies the above-listed problems. It also suggests that in a comprehensive planning model there would be a number of goals for which a lot of variables, parameters and equations associated with multiperiod balance sheets, income statements and other statements should be prepared.

iv) It is very hard, as suggested above, to directly specify a tradeoff relationship among the multiple goals. On the other hand it is relatively easy for managers to specify a *minimum required level* for each goal, below which the managers do not want to allow the planned performance to go. For example, top managers would require: "the level of dividend must not go below the level of previous period", "the after-tax income must be at least above the level that will make the rate of return on equity capital $r\%$; "the sales of product A must be x units or more in order to maintain the present position in the market", "the year-end cash balance must be at least y dollars so that the probability of insolvency can be fairly little", and so forth. Corporate executives would also realize it impossible to *maximize* the level of performance in every goal, and would compromise to set a most desirable or *sufficient level* under the existing corporate situations, as a practical target.

v) The level of performance in each goal that is presented in financial terms can usually be assumed to be continuously divisible between minimum required and sufficient levels.

vi) The list of goals, the minimum required and sufficient levels in each goal are not completely stable, since uncertainty of the data is unavoidable. Therefore, it is desirable that the planning model is simple enough to allow many trial computations including the sensitivity analyses that deal with possible major changes of situations.

2. A GOAL-VECTOR APPROACH

The characteristics of goal activities mentioned above suggest that we need a new pragmatic approach for building an comprehensive financial planning model, instead of applying goal programming techniques in the traditional way.

A goal-vector approach, which the author has developed for extending the applicability of multigoal programming to practical planning problems, will be applied to our model corporation. We shall briefly explain the essence of the approach (FUSHIMI and YAMAGUCHI (1975)).

2.1 G-Vector and Related Utility Functions

We consider an m-dimensional *goal space* composed of $g_i \geq 0$, $i=1,2,\cdots,m$, which represents the level of i-th goal. The level of performance in each goal is assumed to be linear function $g_i(x) \equiv g_i(x_1, x_2, \cdots, x_n)$, $i=1,2,\cdots,m$, subject to linear constraints. When a *minimum required level* g_i^0 and a *sufficient level* g_i^s are determined by a decision maker for each goal, a vector $\overrightarrow{G^0G^s}$ will be obtained by connecting two points $G^0=(g_1^0, g_2^0, \cdots, g_m^0)$ and $G^s=(g_1^s, g_2^s, \cdots, g_m^s)$ in the goal space. The vector $\overrightarrow{G^0G^s}$, which we call *G-vector*, is a useful indicator to show the direction of desirability of goal attainment.

A variety of utility functions associated with the G-vector can be defined. Figure 1 illustrates typical examples of these utility functions assuming a two goal case.

The ideas corresponding to these figures are:

(a) To give the weight of $\lambda_i = g_i^s - g_i^0$, $i=1,2,\cdots,m$, to each goal, assuming a sufficient substitutivity between goals (—weighting model).

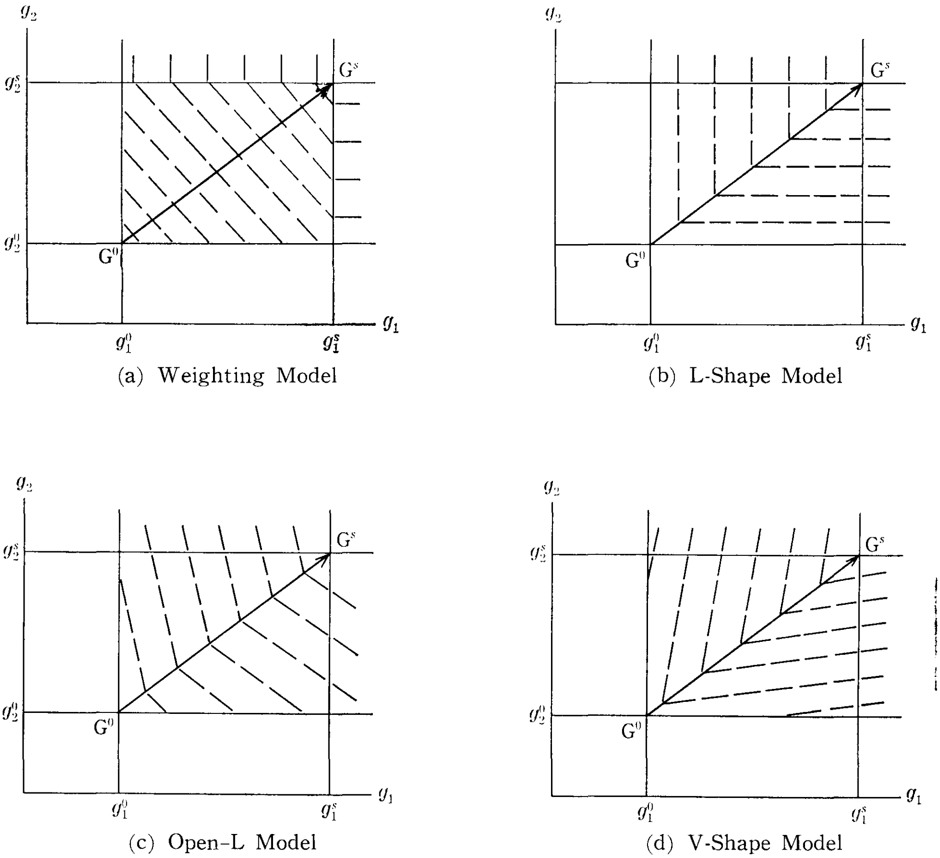

Figure 1 Examples of Utility Functions Associated with G-vector

(b) To minimize $\max_i \left\{ \frac{g_i^s - g_i(x)}{\lambda_i} \right\}$, or maximize $\min_i \left\{ \frac{g_i(x) - g_i^0}{\lambda_i} \right\}$ (—L-shape model).

(c) To assume that the goals are somewhat substitutive, but not substitutive enough to apply the weighting model (—Open-L model).

(d) To require a more rigid balance between goals than in the L-shape model (—V-shape model).

Theoretically, all of these utility functions are worth noticing, and perhaps there can be the best way of mixing these functions; from the pragmatic point of view, we adopt an *L-shape model* (the (b) above), which is a minimax utility function adjusted by the ratio of $\lambda_i = g_i^s - g_i^0$, $i = 1, 2, \cdots, m$. The reasons are:

i) As pointed out in i) and ii) of Section 1.3, corporate financial executives usually wishes to raise the level of planned performance in every goal, instead of assuming a preemptive priority or a sufficient substitutivity between goals, though considering a *relative* difference of importance among the goals. L-shaped utility function provides an easily understood projection of this goal-aiming process and usually brings about a better solution than the ordering and weighting models do.

(For further discussion, see Appendix B).

ii) The solution provides an improved ordering and continues to do this so long as possible, without overpowering anything else, in the same fashion as one achieves a Pareto optimum point. The V-shape model does not guarantee the Pareto optimality.

iii) Its mathematical formulation and computation are simple enough to apply to practical corporate planning models that have the requisites as mentioned in iii) and vi) of Section 1.3; The open-L and V-shape models do not meet the purposes of practical use in this sense.

2.2 Optimal Solution of L-shape Model

Generally, we may regard the L-shaped function as a *regret function* in which the value of regret equals the difference between the sufficient level and what is actually attained for each goal. Following L. J. Savage who first suggested the idea of using the regret function, we represent it as an L-shaped utility function, and then minimize the maximum regret value which is weighted by the ratio of

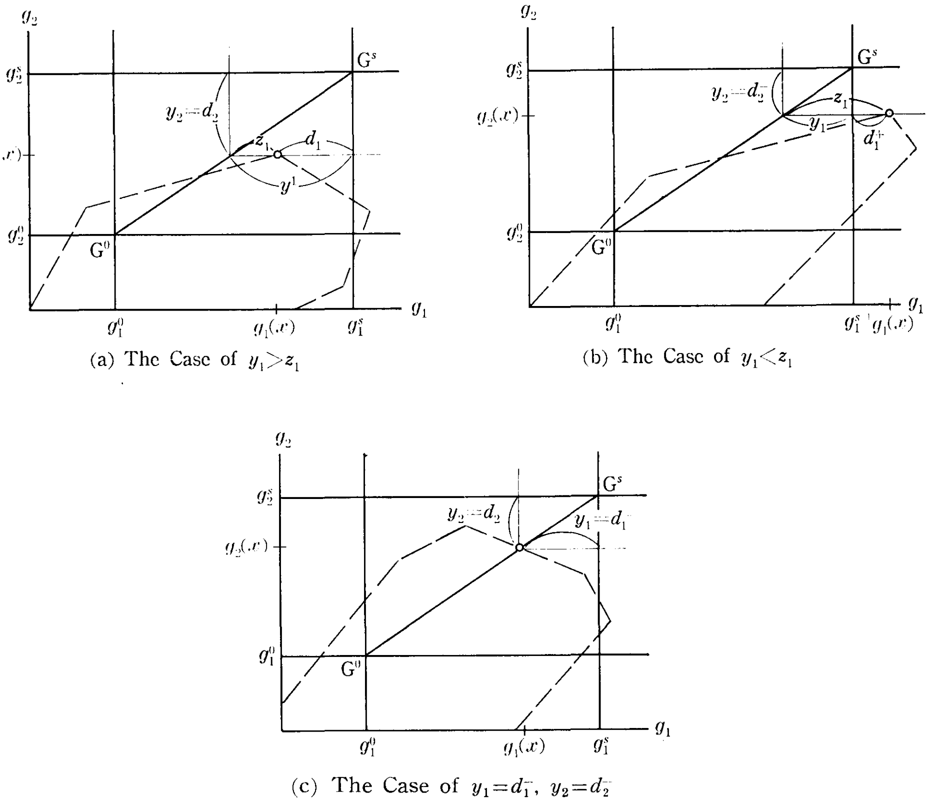

Figure 2 Illustration of L-shape Models (The broken lines show feasible zone)

$1/\lambda_i,\ i=1,2,\cdots,m.$

Thus adjusted regret R can be minimized by solving the following linear program:

(I) Minimize y_1

(II) Subject to

(a) $Ax+Iy-Iz=g^s$

(b) $y_i-\frac{\lambda_i}{\lambda_1}y_1=0,\ i=2,\cdots,m$

(c) $Ax\geq g^0$

(d) $Bx\leq L$

(e) $x,y,z\geq 0,$

where x is a column vector comprised of decision variables $x_j, j=1,2,\cdots,n$; y, z, g^s and g^0 are column vectors comprised of y_i, z_i, g_i^s and g_i^0, $i=1,2,\cdots,m$, respectively; A and B are $m\times n$ and $l\times n$ matrices respectively; I is an $m\times m$ identity matrix. We call, for convenience, (a), (b), (c) above *goal constraints*, and (d) *technical constraints*, though the latter may include various "policy constraints" established by managenent. The deviational variables $d_i^-=g_i^s-g_i(x)$ and $d_i^+=g_i(x)-g_i^s$, which are widely used in the usual goal programming, are obtained as:

(a) $y_i-z_i=d_i^-$ (when $y_i>z_i$)

(b) $z_i-y_i=d_i^+$ (when $y_i<z_i$).

These concepts are illustrated in Figure 2 assuming two-goal cases.

Usually, the technical constraints will prevent the attainment of $y_1=0$; if y_1 is anticipated to be zero, the executives will raise the "sufficient" level somewhat higher. On the contrary, if the regret under the "optimal" solution is very large, the executives will try to change some of the technical and/or goal constraints so that they can obtain a better plan.

3. PHYSICAL AND FINANCIAL CONDITIONS OF THE MODEL CORPORATION

Before applying the L-shaped function to the model corporation, we shall summarize the physical and financial conditions of the corporation.

3.1 Organization and Production Facilities

Major Department

The organization of the model corporation consists of three major departments; *production, sales,* and *general administrative* departments. In the production department there are three *direct manufacturing departments*, i.e., Department I (basic machining process of raw materials), Department II (secondary machining process), and Department III (assembling and finishing process), and several *indirect departments* (engineering, production control, material handling, R & D, and others).

Some activities in the indirect departments vary depending on the activities of the direct production departments. These effects are taken into consideration in estimating the *variable production costs*.

Fixed Assets

The fixed assets owned by the company are summarized below (the labels within quotation marks are the ones used in the balance sheet):

(i) A set of machines in Departments I and II, containing Q1 in Department I and Q2 in Department II—"Direct factory equipment"

(ii) Factory buildings and various production facilities other than Q1 and Q2—"Other factory equipment"

(iii) Buildings and other equipment for selling and administrative departments, and some intangible assets—"Office equipment"

(iv) Land and other non-depreciable fixed assets—"Land"

The decisions regarding how many pieces of Q1 and Q2 are to be held in period t will determine the production capacity in the period.

The existing direct factory equipment consists of machines with different useful lives, some of which will terminate within the planning period. So if the company wants to maintain or expand its production capacity, it will have to invest in some new equipment in the planning period. The equipment bought in a later period will require smaller number of workers than the equipment bought in earlier periods because of technological progress.

Labor Force

The employees necessary for each department are as follows: in Departments I and II the number of workers will vary with the numbers of Q1 and Q2; in Department III the number of workers will vary with the anticipated production volume in each period.

The indirect production departments, and selling and administrative departments will maintain given numbers of workers and managers in each period, regardless of the levels of production and sales.

3.2 Products and Materials

The Products

The company produces three classes of products that belong in a similar category. They are:

Product A: A high quality product that will be used for luxurious finished goods; the demand is anticipated to increase greatly in the future, however, it will be susceptible to influences from outside business fluctuations.

Product B: A middle-class product; the demand is anticipated to increase gradually year by year.

Product C: An economy-class product; the demand will be stable, and will hardly be influenced by outside business fluctuations.

Considering the restrictions of physical distribution, at least 20% of products that are to be sold in period t must be on hand at the end of period $t-1$.

The Materials

The company uses two kinds of direct raw materials: Type 1; raw materials that are bought from some specific subcontractors and foreign producers. Type 2; raw materials that can be bought any time they are needed. We call the former M1 and the latter M2. At least 20% of M1 consumed in period t must be on hand at the end of period $t-1$ taking into account the delay of order-filling by producers.

The costs of indirect materials and supplies are included in factory overhead costs. In our model it is assumed that there are no work-in-process inventories held at the end of each period.

Further details of production conditions will be given in Appendix A.

3.3 Revenue and Cost Estimation

For simplifying and making effective the planning process, all expenses of the model corporation are classified into two types, *variable* and *fixed* expenses, in relation to the production volumes. The revenue, expenses and contribution margin per unit of each product are summarized in Table 2. The anticipated price increases in the revenue and expense items are shown in the last column of the table.

All of the direct material costs are variable in proportion to the production level in each period.

Most of the labor costs vary depending on the number of workers, not on the production volume. In Departments I and II, the numbers of direct workers will vary according to the changes of the numbers of equipment held in each period. In Department III, since the number of workers can be adjusted to the production

Table 2. Revenue, variable expenses and production hours for each product

Revenue and variable expenses per unit of product	Product A	Product B	Product C	Price increase anticipated
Selling price	$1,200	$800	$600	8% a year
Variable expenses				
Material M1	400	250	150	5% 〃
Material M2	100	50	30	8% 〃
Labor*	120	80	80	See Table 14 and 15.
Indirect & Selling**	80	70	60	See Table 16.
Total variable expenses	$ 700	$450	$320	
Contribution margin	$ 500	$350	$280	

* The labor cost in Dept. III is assumed to be variable.

** This item includes the variable portions of factory overhead and selling expenses.

level of each period, the direct labor costs of the department are regarded as variable in proportion to the production volume in each period.

There are some variable portions in the indirect manufacturing costs and selling expenses. The variable indirect costs are statistically determined through the input-output analysis of the interdepartmental relationships.

Fixed costs in the direct and indirect departments, and fixed selling and general administrative expenses will be given as parameters for the mathematical formulations in Section 4.

The end-of-period inventories of products are evaluated by the *direct costing* method on the assumption of first-in first-out (FIFO) for simplifying the computations.

Further details on costs, revenues and other financial transactions will be given with the mathematical formulations in Section 4.

Note: The net income calculated by *direct costing* will be different from the one calculated by *absorption costing.* This difference or "error" will be negligible as compared with the errors involved in the estimation of future costs, revenues and other conditions.

4. MATHEMATICAL FORMULATION

4.1 Decision Variables Defined

The decision variables used for our model corporation are presented in Table 3. The subscript t affixed to each decision variable represents the fiscal period t $(t=1, 2, 3, 4)$; accordingly there are sixty-four decision variables.

The last five variables in Table 3 are expressed in symbols implying *journal entries* for corresponding balance-sheet accounts as employed by IJIRI (1965). For example, X_{CSt} implies that the amount is debited to "Cash and deposits" account and credited to "Short-term borrowings" account in period t (refer to the balance sheet in table 1). We could express *all* of the decision variables in terms of "journal-entry variables", but this is not convenient for our model, since too many variables for describing physical activities would be required to do so.

The financial-statement variables, goal constraints and technical constraints will be defined by using the above-mentioned variables and the parameters presented in Table 4.

4.2 Financial-Statement Variables Defined

4.2.1 Variables for Income Statement

The variables necessary to prepare the income statement for each fiscal period are as follows. These variables, which are indicated with a symbol V, are linearly dependent.

(i) *Total net sales in period t* is the sum of the net sales of each product, which is obtained by multiplying the unit price by the quantity sold;

Table 3. Decision variables Defined

	Symbol	Description
1	X_{Q1t}	The number of equipment Q1 to be invested at the beginning of period t.
2	X_{Q2t}	The number of equipment Q2 to be invested at the beginning of period t.
3	X_{PAt}	The quantity of product A to be produced in period t.
4	X_{PBt}	The quantity of product B to be produced in period t.
5	X_{PCt}	The quantity of product C to be produced in periud t.
6	X_{SAt}	The quantity of product A to be sold in period t.
7	X_{SBt}	The quantity of product B to be sold in period t.
8	X_{SCt}	The quantity of product C to be sold in period t.
9	X_{MAt}	The volume of material M1 for product A to be purchased in period t.*
10	X_{MBt}	The volume of material M1 for product B to be purchased in period t.
11	X_{MCt}	The volume of material M1 for product C to be purchased in period t.
12	X_{CSt}	The dollar amount of short-term borrowings to be made at the beginning of period t.
13	X_{CLt}	The dollar amount of long-term borrowings to be made at the beginning of period t.
14	X_{BCt}	The dollar amount of securities to be purchased in period t.
15	X_{CBt}	The dollar amount of securities to be sold in period t.
16	X_{ECt}	The dollar amount of dividends to be paid at the end of period t.

* The unit of the material corresponds to the unit of each product.

(1) $$V_{1t}=a_{1t}X_{SAt}+a_{2t}X_{SBt}+a_{3t}X_{SCt}$$

(ii) *The quantity of finished product j ($j=A, B, C$) on hand at the end of period t* is: the quantity on hand at the end of period $t-1$ plus the quantity produced in period t minus the quantity sold in period t;

(2) $$V_{Gjt}=V_{Gj,t-1}+X_{Pjt}-X_{Sjt}, \qquad j=A, B, C$$

(iii) *The quantity of material M1 for product j ($j=A, B, C$) on hand at the end of period t* is determined in the same way as above;

(3) $$V_{Mjt}=V_{Mj,t-1}+X_{Mjt}-X_{Pjt}, \qquad j=A, B, C$$

Note: V_{Gjt} and V_{Mjt} do not appear in the income statement; they are used for determining the dollar amounts of net income and inventories. The amounts of V_{Gjo} and V_{Mjo} are given as the initial conditions.

(iv) *The total amount of fixed direct labor costs in period t* is obtained by multiplying the average payroll by the number of workers in departments I and II and adding them together;

(4) $$V_{2t}=a_{4t}\left(b_{1t}+\sum_{k=1}^{t} b_{2k}X_{Q1k}+\sum_{k=1}^{t} b_{3k}X_{Q2k}\right),$$

Table 4. Parameters Defined

Symbol	Definition	$t=1$	$t=2$	$t=3$	$t=4$
a_{1t}	Unit selling price of product A	$1,200	$1,296	$1,400	$1,512
a_{2t}	Unit selling price of product B	800	864	933	1,008
a_{3t}	Unit selling price of product C	600	648	700	756
a_{4t}	Average labor cost per worker	20,000	24,000	28,300	33,100
a_{5t}	Predetermined depreciation charges for existing Q1 and Q2	1,156,600	689,200	354,500	70,400
a_{6t}	Acquisition cost per unit of equipment Q1 in period t	700,000	750,000	800,000	850,000
a_{7t}	The same for Q2 in period t	180,000	190,000	200,000	210,000
a_{8t}	Variable production costs per unit of product A	640	689	750	811
a_{9t}	$=a_{8t}-a_{8,t-1}$	47	49	61	61
a_{10t}	Variable production costs per unit of product B	410	449	487	528
a_{11t}	$=a_{10t}-a_{10,t-1}$	31	39	38	41
a_{12t}	Variable production costs per unit of product C	290	322	354	388
a_{13t}	$=a_{12t}-a_{12,t-1}$	24	32	32	34
a_{14t}	Cost of material M1 per unit of product A	400	420	441	463
a_{15t}	$=a_{14t}-a_{14,t-1}$	19	20	21	22
a_{16t}	Cost of material M1 per unit of product B	250	263	276	289
a_{17t}	$=a_{16t}-a_{16,t-1}$	62	13	13	13
a_{18t}	Cost of material M1 per unit of product C	150	158	165	174
a_{19t}	$=a_{18t}-a_{18,t-1}$	7	8	7	9
a_{20t}	Cost of material M2 per unit of product A	100	108	117	126
a_{21t}	The same for product B	50	54	58	63
a_{22t}	The same for product C	30	32	35	38
a_{23t}	Variable labor costs and variable indirect costs per unit of product A	140	161	192	222
a_{24t}	The same for product B	110	132	153	176
a_{25t}	The same for product C	110	132	154	176
a_{26t}	Fixed overhead costs	1,700,000	1,975,000	2,272,000	2,613,000
a_{27t}	Variable selling expenses per unit of product A	60	65	70	76
a_{28t}	The same for product B	40	43	47	50
a_{29t}	The same for product C	30	32	35	38
a_{30t}	Fixed expenses in selling and administrative Depts	1,800,000	2,056,000	2,336,000	2,640,000
a_{31t}	Depreciation charge on office equipment	69,000	69,000	69,000	69,000
a_{32t}	The existing long-term borrowings paid back in period t	1,600,000	1,300,000	1,000,000	500,000

Symbol	Definition	$t=1$	$t=2$	$t=3$	$t=4$
a_{33t}	Remaining book value of the existing Q1 and Q2 scrapped at the end of period	177,600	290,600	456,000	55,100
a_{34t}	Remaining book value of Q2 invested at the beginning of period 1 and scrapped at the end of period 4	—	—	—	28,600
b_{1t}	The total number of workers for the existing Q1 and Q2	150	118	70	14
b_{2t}	The number of workers for a unit of Q1 invested in period t	7	7	6	6
b_{3t}	The same for Q2	3	3	3	2
b_{4t}	The maximum production hours of the existing Q1	10,000	10,000	6,000	4,000
b_{5t}	The same of the existing Q2	64,000	48,000	32,000	0
c_1	Depreciation rate for Q1	28%			
c_2	Depreciation rate for Q2	36.9%			
c_3	Effective rate of income tax	52%			
c_4	Machine hours in Dept I necessary for a unit of product A	0.4 Hrs			
c_5	The same for product B	0.4 〃			
c_6	The same for product C	0.32 〃			
c_7	Machine hours in Dept II necessary for a unit of product A	2.4 〃			
c_8	The same for product B	1.6 〃			
c_9	The same for product C	1.6 〃			

where k is the period when new direct equipment is to be bought.

(v) *Factory depreciation charge for Q1 and Q2 in period t* is the sum of the depreciation charge on each of Q1 and Q2 in period t, and each depreciation charge is obtained by multiplying the beginning-of-period book value by the depreciation rate, which reflects the declining balance depreciation.

$$V_{3t}=a_{5t}+\sum_{k=1}^{t} c_1(1-c_1)^{t-k}a_{6k}X_{Q1k}+\sum_{k=1}^{n} c_2(1-c_2)^{t-k}a_{7k}X_{Q2k} \tag{5}$$

(vi) *The total amount of costs of products sold in period t* is the sum of the following: total variable costs, fixed direct labor costs, depreciation charges for equipment Q_1 and Q_2, and other factory overhead;

$$\begin{aligned} V_{4t}=&(a_{8t}X_{SAt}+a_{10t}X_{SBt}+a_{12t}X_{SCt})\\ &-(a_{9t}V_{GA,t-1}+a_{11t}V_{GB,t-1}+a_{13t}V_{GC,t-1})\\ &-(a_{15t}V_{MA,t-1}+a_{17t}V_{MB,t-1}+a_{19t}V_{MC,t-1})\\ &+V_{2t}+V_{3t}+a_{26t} \end{aligned} \tag{6}$$

(vii) *Gross income in period t* is obtained by subtracting the costs of products sold from the total net sales in period t;

(7) $$V_{5t}=V_{1t}-V_{4t}$$

(viii) *The total amount of selling and administrative expenses in period t* is the sum of the variable selling expenses, fixed labor and other expenses for selling and administrative activities, and depreciation charge on office equipment;

(8) $$V_{6t}=a_{27t}X_{SAt}+a_{28t}X_{SBt}+a_{29t}X_{SCt}+a_{30t}+a_{31t}$$

(ix) *Operating income in period t* is obtained by subtracting selling and administrative expenses from the gross income;

(9) $$V_{7t}=V_{5t}-V_{6t}$$

(x) *Interest received at the end of period t* is the sum of the following: 6 per cent of half the average balance of "Cash and bank deposits", and 8 per cent of average balance of "Securities" in period t;

(10) $$\begin{aligned}V_{8t}&=0.06\times0.5\times0.5\times(K_{C,t-1}+K_{Ct})+0.08\times0.5\times(K_{B,t-1}+K_{Bt})\\&=0.015(K_{C,t-1}+K_{Ct})-0.04(K_{B,t-1}+K_{Bt})\end{aligned}$$

(xi) *Interest paid at the end of perid t* is the sum of the following: 10 per cent of the average short-term borrowings, and 11 per cent of the beginning-of-period balance of long-term borrowings;

(11) $$V_{9t}=0.05(K_{S,t-1}+K_{St})+0.11K_{Lt}$$

Note: It is assumed that matured long-term borrowings are repaid at the beginning of the next period.

(xii) *Before-tax net income in period t* is obtained by: operating income plus interest received minus interest paid minus loss on equipment disposals;

(12) $$V_{10t}=V_{7t}+V_{8t}-V_{9t}-a_{33t}-a_{34,4}X_{Q2,1}$$

Note: In this model it is assumed that salvage values of equipment Q1 and Q2 are zero after deducting the expenses for disposal. The Q2 invested at the beginning of period 1 will be scrapped at the end of period 4. The last term in Eq. (12) represents it.

(xiii) *After-tax net income in period t* is obtained by subtracting income tax from the before-tax net income;

(13) $$V_{11t}=(1-c_3)V_{10t}$$

Note: Here the tax rate is assumed to be constant for simplicity. In practice the payment of income taxes and dividends may be done a few months after the end of each period, but in this model it is assumed that both of them are paid at the end of period t. The possible "errors" caused by this simplification will be lessened by discounting the rates of dividend and income tax taking into account the average delay of payments.

4.2.2 Variables for Balance Sheet

The equations (14) through (17) presented below define the supplementary variables that are used for preparing balance sheets. A symbol W is used for these supplementary variables. The equations (18) through (26) define the amount of each of the balance sheet items at the end of period t.

(i) *Repaying the accounts payable*: All of the accounts payable at the end of period $t-1$ and 80 per cent of the costs of raw materials purchased in period t are repaid in period t;

$$W_{1t}=K_{P,t-1}+0.8(a_{14t}X_{MAt}+a_{16t}X_{MBt}+a_{18t}X_{MCt}+a_{20t}X_{PAt}+a_{21t}X_{PBt}+a_{22t}X_{PCt}) \tag{14}$$

(ii) *Collecting the accounts receivable*: All of the accounts receivable at the end of period $t-1$ and 70 per cent of net sales in period t are collected in period t;

$$W_{2t}=K_{R,t-1}+0.7V_{1t} \tag{15}$$

(iii) *Repaying the short-term borrowings*: All of the short-term borrowings kept at the end of period $t-1$ are repaid in period t;

$$W_{3t}=K_{S,t-1} \tag{16}$$

(iv) *Repaying the long-term borrowings*: The amount of repaying the long-term borrowings at the beginning of period t is the sum of the following: the existing long-term borrowings repaid in cash according to the predetermined contracts, and 1/5 of new borrowings which are paid back at the beginning of each of the following 5 periods after borrowings;

$$W_{4t}=a_{32t}+\sum_{k=0}^{t-1}0.2X_{CL,k} \tag{17}$$

(v) *Cash and bank deposits*: The balance of cash and bank deposits at the end of period t is: the balance at the end of period $t-1$ plus total receipts in period t minus total disbursements in period t;

$$\begin{aligned}K_{Ct}=K_{C,t-1}&+(V_{8t}+W_{2t}+X_{CBt}+X_{CSt}+X_{CLt})\\&-(W_{1t}+a_{23t}X_{PAt}+a_{24t}X_{PBt}+a_{25t}X_{PCt}+V_{2t}+a_{26t}+a_{27t}X_{SAt}\\&+a_{28t}X_{SBt}+a_{29}X_{SCt}+a_{30t}+V_{9t}+X_{BCt}+W_{3t}+W_{4t}\\&+c_3V_{10t}+a_{6t}X_{Q1t}+a_{7t}X_{Q2t}+X_{ECt})\end{aligned} \tag{18}$$

(vi) *Securities*: The balance of securities at the end of period t is: the beginning-of-period balance plus purchased securities minus sold securities;

$$K_{Bt}=K_{B,t-1}+X_{BCt}-X_{CBt} \tag{19}$$

(vii) *Accounts receivable*: The accounts receivable at the end of period t is 30 per cent of net sales in period t;

$$K_{Rt}=0.3V_{1t} \tag{20}$$

(viii) *Inventories*: Inventories on hand at the end of period t is: the beginning-of-period balance+(costs of finished goods produced−costs of finished goods sold)+(material M1 purchased−M1 consumed) in period t;

(21) $$K_{Gt}=a_{8t}V_{GAt}+a_{10t}V_{GBt}+a_{12t}V_{GCt}+a_{14t}V_{MAt}+a_{16t}V_{MBt}+a_{18t}V_{MCt}$$

Note: The end-of-period inventories are evaluated by means of direct costing.

(ix) *Direct factory equipment*: The book value at the end of period t is: the book value (after depreciation) at the end of period $t-1$ plus new investment at the beginning of period t minus declining balance depreciation at the end of period t minus residual book value of the equipment scrapped at the end of period t;

(22) $$K_{Qt}=K_{Q,t-1}+\sum_{k=1}^{t}a_{6k}X_{Q1k}(1-c_1)^{t-(k-1)}+\sum_{k=1}^{t}a_{7k}X_{Q2k}(1-c_2)^{t-(k-1)}$$
$$-a_{5t}-a_{33t}-a_{34,4}X_{Q2,1},\qquad k \text{ is the investment period}$$

(x) *Accounts payable*: Accounts payable at the end of period t is 20 per cent of raw materials purchased in period t;

(23) $$K_{Pt}=0.2(a_{14t}X_{MAt}+a_{16t}X_{MBt}+a_{18t}X_{MCt}+a_{20t}X_{PAt}+a_{21t}X_{PBt}+a_{22t}X_{PCt})$$

(xi) *Short-term borrowings*: The end-of-period balance of the short-term borrowings is equal to the new borrowings at the beginning of the period;

(24) $$K_{St}=X_{CSt}$$

Note: The end-of-period balance will be paid back in the following period, then new borrowings will be made conforming to the fund requirements of the period.

(xii) *Long-term borrowings*: The end-of-period balance of the long-term borrowings is: the beginning-of-period balance−current pay back+current new borrowings;

(25) $$K_{LT}=K_{L,t-1}-W_{4t}+X_{CLt}$$

(xiii) *Retained earnings*: The end-of-period balance of retained earnings after making the income distribution is: the beginning-of-period balance+after-tax net income−dividends paid;

(26) $$K_{Et}=K_{E,t-1}+V_{11t}-X_{ECt}$$

(xiv) *Other balance sheet items*: In this model K_{Ft}, K_{Ht}, K_{It}, K_{Kt} and K_{Ut} are determined independently of the decisions mentioned above.

4.3 Goal Constraints

4.3.1 Minimum Required and Sufficient Levels

The minimum required level (abbreviated to MRL) g_{it}^{0} and sufficient level (SFL) g_{it}^{s}, ($i=1,2,\cdots,10$; $t=1,2,3,4$) for each goal have been determined as follows:

Goal 1 (goal of dividend): The MRL g^0_{1t} is 8% of the capital stock, i.e., $400 thousand, and the SFL g^s_{1t} is 20%, i.e., $1,000 thousand for each period.

Note: The amount of bonus to officers, which is regarded as a part of income distribution under the Japanese accounting laws, is very small as compared with dividends to shareholders. So we assume the amount to be included in the dividend goal.

Goal 2 (goal of net income): Since the tax rate is constant in our model, the goal of after-tax net income can be converted to the goal of *before-tax* net income, which we use here. The MRL g^0_{2t} and SFL g^s_{2t} have been determined as percentages of the initial net worth. They are:

	g^0_{2t}	g^s_{2t}
$t=1$:	12% ($ 880 thousand)	36% ($2,650 thousand)
$t=2$:	14% ($1,030 〃)	38% ($2,800 〃)
$t=3$:	16% ($1,180 〃)	40% ($2,950 〃)
$t=4$:	18% ($1,320 〃)	42% ($3,090 〃)

The amounts within parentheses above are rounded.

Note: Much attention may be paid to the return on equity capital and to the retained earnings. In this model, since the dividend goal has been given and the capital stock is constant, the goal of net income can be substituted for them.

Goals 3, 4, 5 (goals of sales volume for products A, B, C): The MRLs g^0_{3t}, g^0_{4t}, g^0_{5t} and the SFLs g^s_{3t}, g^s_{4t}, g^s_{5t} are as follows:

Product A

	g^0_{3t}	g^s_{3t}
$t=1$:	5,000 units	10,000 units
$t=2$:	8,000 〃	15,000 〃
$t=3$:	11,000 〃	20,000 〃
$t=4$:	14,000 〃	25,000 〃

Product B

	g^0_{4t}	g^s_{4t}
$t=1$:	5,000 units	10,000 units
$t=2$:	6,000 〃	12,000 〃
$t=3$:	7,000 〃	14,000 〃
$t=4$:	8,000 〃	16,000 〃

Product C

	g^0_{5t}	g^s_{5t}
$t=1$:	5,000 units	10,000 units
$t=2$:	5,000 〃	10,000 〃
$t=3$:	5,000 〃	10,000 〃
$t=4$:	5,000 〃	10,000 〃

Goal 6 (goal of cash balance): The MRL g_{6t}^{0} and the SFL g_{6t}^{s} have been determined as:

	g_{6t}^{0}	g_{6t}^{s}
$t=1$:	$ 2,200 thousand	$ 4,200 thousand
$t=2$:	$ 2,200 〃	$ 5,200 〃
$t=3$:	$ 2,200 〃	$ 6,200 〃
$t=4$:	$ 2,200 〃	$ 7,200 〃

As for the following four goals, MRLs and SFLs are so close that we may assume that $g_{it}^{0}=g_{it}^{s}$ $(i=7, 8, 9, 10)$.

Goal 7: This goal is expressed in the ratio of "before-tax net income/total assets". The MRL and SFL of the goal are 5 per cent for each period.

Goal 8: This goal is expressed in the ratio of "current assets/current liabilities". The MRL and SFL of the goal are 100 per cent for each period.

Goal 9: The ratio of "net worth/total assets" is used for this goal. The MRL and SFL of the goal are 27 per cent for each period.

Goal 10: The ratio of "fixed assets/(long-term borrowings+net worth)" is used for this goal. The MRL and SFL of the goal are 100 per cent for each period.

4.3.2 Mathematical Formulations of the Goal Constraints

For convenience, we will use the goal constraint shown below, instead of using two equations of (II)-(a) and (II)-(b):

(III) $$g_{it}(x)+\frac{\lambda_{it}}{\lambda_{1,1}}y_{1,1}-z_{it}=g_{it}^{s}, \qquad i=1, \cdots, 6;\quad t=1, \cdots, 4.$$

Also we will use the symbol y for $y_{1,1}$ and then write:

Goal 1 (dividend)

(27) $$X_{ECt}+y-z_{1t}=1{,}000 \text{ (thousand dollars)}, \qquad t=1, 2, 3, 4$$

Goal 2 (net income before tax)

(28) (a) $V_{10,1}+2.95y-z_{2,1}=2{,}650$ (thousand dollars)

(b) $V_{10,2}+2.95y-z_{2,2}=2{,}800$

(c) $V_{10,3}+2.95y-z_{2,3}=2{,}950$

(d) $V_{10,4}+2.95y-z_{2,4}=3{,}090$

Goal 3 (sales volume of product A):

(29) (a) $X_{PA1}+8.3y-z_{3,1}=10{,}000$ (units)

(b) $X_{PA2}+11.6y-z_{3,2}=15{,}000$

(c) $X_{PA3}+15.0y-z_{3,3}=20,000$

(d) $X_{PA4}+18.3y-z_{3,4}=25,000$

Goal 4 (sales volume of product B):

(30) (a) $X_{PB1}+8.3y-z_{4,1}=10,000$ (units)

(b) $X_{PB2}+10.0y-z_{4,2}=12,000$

(c) $X_{PB3}+11.6y-z_{4,3}=14,000$

(d) $X_{PB4}+13.0y-z_{4,4}=16,000$

Goal 5 (sales volume of product C):

(31) $X_{PCt}+8.3y-z_{5t}=10,000$ (units), $t=1,2,3,4$

Goal 6 (cash balance):

(32) (a) $K_{C1}+3.3y-z_{6,1}=4,200$ (thousand dollars)

(b) $K_{C2}+5.0y-z_{6,2}=5,200$

(c) $K_{C3}+6.6y-z_{6,3}=6,200$

(d) $K_{C4}+8.3y-z_{6,4}=7,200$

As to the goals 7 through 10, since $\lambda_{it}=0$, we formulate the applicable constraints on these goals as:

(IV) $$g_{it}(x)\geq g_{it}^{0}, \quad i=7,8,9,10;\ t=1,2,3,4$$

Goal 7:

(33) $$V_{10t}-0.05(K_{Pt}+K_{St}+K_{Lt}+K_{Kt}+K_{Ut}+K_{Et})\geq 0, \quad t=1,2,3,4$$

Goal 8:

(34) $$K_{Ct}+K_{Bt}+K_{Rt}+K_{Gt}-K_{Pt}-K_{St}\geq 0, \quad t=1,2,3,4$$

Goal 9:

(35) $$K_{Kt}+K_{Ut}+K_{Et}-0.27(K_{Pt}+K_{St}+K_{Lt}+K_{Kt}+K_{Ut}+K_{Et})\geq 0, \quad t=1,2,3,4$$

Goal 10:

(36) $$K_{Qt}+K_{Ft}+K_{Ht}+K_{It}-(K_{Lt}+K_{Kt}+K_{Ut}+K_{Et})\leqq 0, \quad t=1,2,3,4$$

4.4 Technical Constraints

(i) The production hours in Department I are limited to 2,000 hours per piece of Q1;

(37) $$c_4X_{PAt}+c_5X_{PBt}+c_6X_{PCt}\leq b_{4t}+\sum_{k=1}^{t}2000X_{Q1k}, \quad t=1,2,3,4.$$

where k is the period in which new Q1 is to be purchased.

(ii) The constraint of production hours in Department II is defined in the same way as above;

(38) $$c_7X_{PAt}+c_8X_{PBt}+c_9X_{PCt}\leq b_{5t}+\sum_{k=1}^{t}2{,}000X_{Q2k},\qquad t=1,2,3,4$$

wher k is the period in which new Q2 is to be purchased.

(iii) New short-term borrowings available in period t cannot exceed the net amount of current assets minus accounts payable at the end of period t-1;

(39) $$X_{CSt}+K_{P,t-1}-(K_{C,t-1}+K_{B,t-1}+K_{R,t-1}+K_{G,t-1})\leq 0,\qquad t=1,2,3,4$$

(iv) New long-term borrowings available in period t cannot exceed 70 per cent of newly purchased equipment in the period;

(40) $$X_{CLt}-0.7(a_{6t}X_{Q1t}+a_{7t}X_{Q2t})\leq 0,\qquad t=1,2,3,4$$

(v) The quantity of the end-of-period inventories of product j $(j=A,B,C)$ must be at least 20 per cent of the quantity to be sold during the next period;

(41) $$V_{Gj,t-1}-0.2X_{Sjt}\geq 0,\qquad j=A,B,C;\ t=1,2,3,4$$

Table 5. The Optimal Solution in the Case of Constant Production Capacity

Decision Variable	Optimal Value			
	$t=1$	$t=2$	$t=3$	$t=4$
X_{Q1t}*	0	0	2	1
X_{Q2t}*	8	8	8	16
X_{PAt}	14,000	17,621	18,081	18,447
X_{PBt}	3,365	4,038	4,711	5,385
X_{PCt}	8,538	3,365	3,365	3,365
X_{SAt}	15,524	17,713	18,154	18,447
X_{SBt}	2,073	4,173	4,846	5,385
X_{SCt}	9,254	3,893	2,499	1,461
X_{MAt}	17,173	17,801	18,213	18,447
X_{MBt}	1,061	4,308	4,954	5,385
X_{MCt}	8,132	3,614	2,292	1,461
X_{CSt}	$12,773**	$13,856	$14,057	$12,006
X_{CLt}	0	$ 1,064	$ 2,240	$ 2,947
X_{BCt}	0	0	0	0
X_{CBt}	0	0	0	0
X_{ECt}	$ 204	$ 204	$ 204	$ 204

* In this run, the values of X_{Q1t} and X_{Q2t} are predetermined so that the total numbers of Q_{1t} and Q_{2t} are kept constant, i.e., 5 and 40 respectively.

** The dollar amounts are shown in thousands.

(vi) The quantity of the end-of-period inventories of M1 for product j ($j=A, B, C$) must be at least 20 per cent of M1 to be consumed in the next period;

(42) $$V_{Mj,t-1}-0.2X_{Pjt}\geq 0, \qquad j=A, B, C; \quad t=1, 2, 3, 4$$

(vii) The amount of securities on hand must not go below the level of the initial condition, i.e., \$1,800 thousand;

(43) $$K_{Bt}\geq 1,800 \text{ (thousand)}, \qquad t=1,2,3,4$$

and this completes our formulation.

5. RESULTS AND DISCUSSION

The decision analysts of the model corporation can try various computations using the mathematical formula presented above. Here we shall present the results of two kinds of computations, which would provide the top executives with mean-

Table 6. Pro Forma Income Statement in the Case of Constant Production Capacity (in thousands of dollars)

	$t=1$		$t=2$		$t=3$		$t=4$	
Net sales		24,615		28,506		32,066		35,863
Cost of products sold								
Variable costs	12,532		14,652		16,588		18,654	
Fixed direct labor	3,480		3,984		4,358		4,502	
Overhead	1,700		1,975		2,272		2,613	
Depreciation	1,898	19,610	1,795	22,406	2,168	25,386	2,811	28,580
Gross income		5,005		6,100		6,680		7,283
Selling & Gen. Adm. expenses								
Variable selling expense	1,231		1,427		1,605		1,799	
Fixed expenses	1,800		2,056		2,336		2,640	
Depreciation	69	3,100	69	3,552	69	4,010	69	4,508
Operating income		1,905		2,548		2,670		2,775
Other income or loss								
Interest received	240		228		222		189	
Interest paid	1,666		1,657		1,835		1,939	
Loss on equipment disposal	178	−)1,604	291	−)1,720	456	−)2,069	284	−)2,034
Net income before tax		301		828		601		741
Corporate incom tax		157		431		313		385
Net income after tax		144		397		288		356
Retained earnings from prior years		560		501		695		780
Earnings available for dividends		704		898		983		1,136
Dividends		204		204		204		204
Retained earnings		500		694		779		932

ingful suggestions.

5.1 A Test Run under the Assumption of Keeping the Initial Production Capacity

In the first run of the computer solution, we assumed a case where the top executives of the model corporation would maintain the policy of holding the same production capacity as the initial condition through the planning period, making the necessary replacement of equipment.

In this test run, we intentionally omitted the constraints corresponding to Eq. (II)-(c) so that we could arrive at a solution even when the degrees of goal-attainmeints were very low.

The optimal solution, which was obtained by minimizing y under the planned constraints of equations (1) through (43), is summarized in Table 5. These are programs in pertinent units. The resulting income statement and balance sheet for these solutions are given in Table 6 and 7. The degrees of attainment of goals 1 through 6 are represented in Figure 3. The figure shows that if the

Table 7. Pro Forma Balance Sheet in the Case of Constant Production Capacity (in thousands of dollars)

	$t=1$	$t=2$	$t=3$	$t=4$
ASSETS				
K_{Ct} Cash and bank deposits	2,271	2,811	2,401	566
K_{Bt} Securities	1,800	1,800	1,800	1,800
K_{Rt} Accounts receivable	7,384	8,552	9,620	10,759
K_{Gt} Inventories	5,675	5,882	6,176	5,891
Total current assets	17,130	19,045	19,997	19,016
K_{Qt} Direct factory equipment	2,951	2,595	3,381	4,707
K_{Ft} Other factory equipment	3,590	3,380	3,170	2,960
K_{Ht} Office equipment	861	792	723	654
K_{It} Land	800	800	800	800
Total fixed assets	8,202	7,567	8,074	9,121
Total assets	25,332	26,612	28,071	28,137
LIABILITIES				
K_{Pt} Accounts payable	2,058	2,298	2,444	2,622
K_{St} Short-term borrowings	12,773	13,856	14,057	12,006
Total current liabilities	14,831	16,154	16,501	14,628
K_{Lt} Long-term borrowings	3,200	2,964	3,991	5,778
Total liabilities	18,031	19,118	20,492	20,406
NET WORTH				
K_{Kt} Capital stock	5,000	5,000	5,000	5,000
K_{Ut} Capital surplus	1,800	1,800	1,800	1,800
K_{Et} Retained earnings	501	694	779	931
Total net worth	7,301	7,494	7,579	7,731
Total liabilities & net worth	25,332	26,612	28,071	28,137

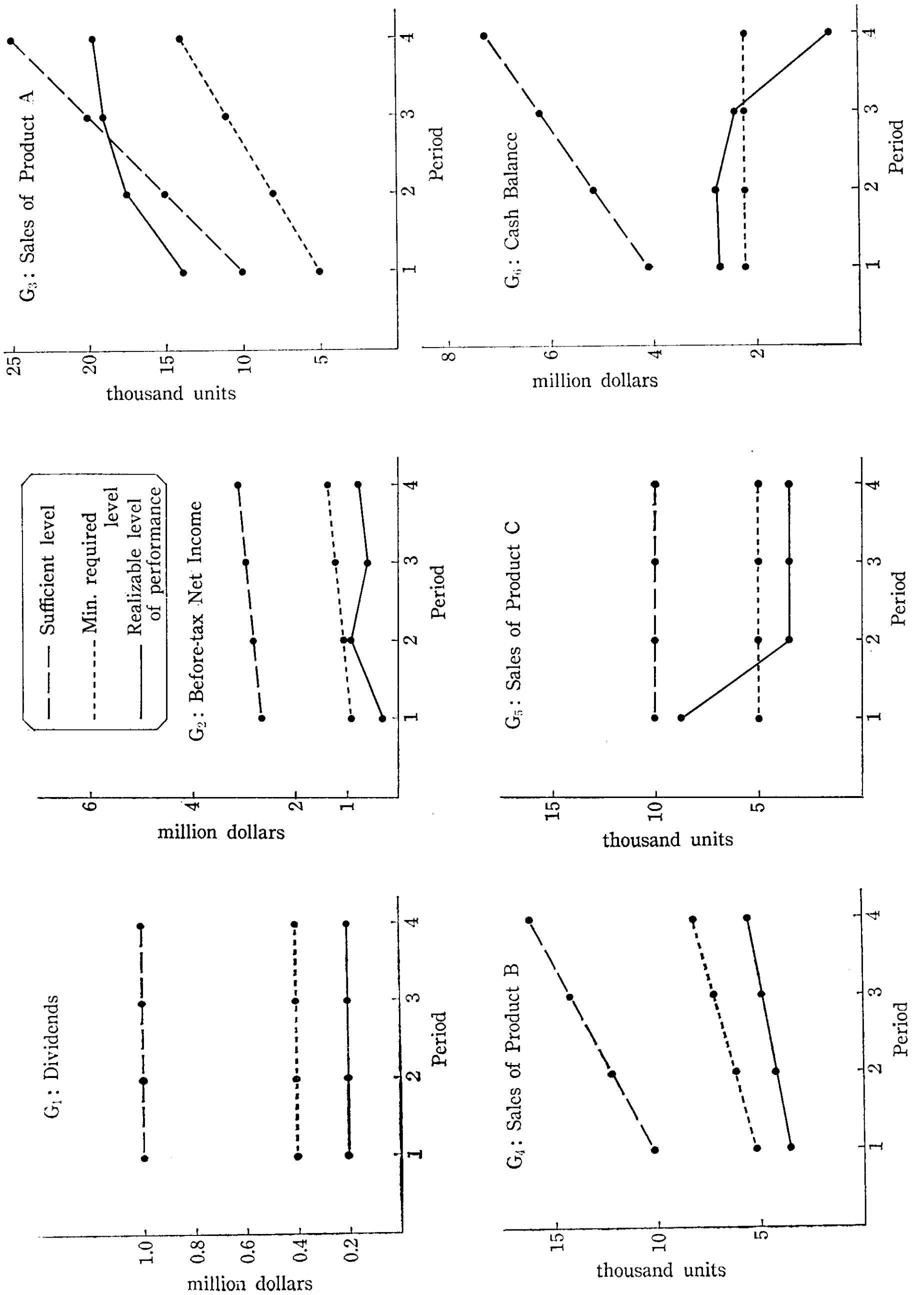

Figure 3 The Degree of Goal Attainment in the Case of Constant Production Capacity

top executives persisted the plan to keep the existing production capacity, the degrees of goal-attainment would be seriously low; in most of the goals even the minimum required levels could *not* be attained because of the cost pressure, especially of the rapid increase in labor costs. Such a result would be undesirable for the management. We therefore provide another test run under the assumption that the production capacity may be expanded. The test is to see whether this is desirable to improve the planned attainment of goals, and to see what financial position can be expected. This is shown in the following section.

5.2 A Test Run under the Assumption of Expanding the Production Capacity

The numbers of Q1 and Q2 purchased in period t, i.e., X_{Q1t} and X_{Q2t} ($t=1, 2, 3, 4$), must be integers. Although our model corporation is greatly simplified as compared with the real corporation, we found it almost impossible to reach the "optimal" solution of the mixed-integer program within an economic computation time. So we applied a practical alternative way of computation to obtain the second best solution.

First, we performed a computation assuming the condition that any real numbers of Q1 and Q2 were allowed. Under this assumption, the optimal values of

Table 8. The Final Solution in the Case of Flexible Production Capacity

Decision Variable	Optimal Value			
	$t=1$	$t=2$	$t=3$	$t=4$
X_{Q1t}	2(7)*	1(8)	2(8)	2(9)
X_{Q2t}	4(36)*	10(38)	11(41)	21(46)
X_{PAt}	14,000	17,824	18,135	22,700
X_{PBt}	10,516	10,745	13,978	14,327
X_{PCt}	11,497	8,954	8,954	8,954
X_{SAt}	15,565	17,886	19,048	22,700
X_{SBt}	10,565	11,392	14,048	14,327
X_{SCt}	11,088	9,279	8,630	8,954
X_{MAt}	17,248	18,119	19,778	22,700
X_{MBt}	10,996	11,923	14,104	14,327
X_{MCt}	11,044	9,149	8,695	8,954
X_{CSt}	$15,905**	$18,431	$21,885	$25,627
X_{CLt}	0	$ 1,240	$ 807	$ 1,059
X_{BCt}	0	$ 28	0	0
X_{CBt}	0	0	$ 28	0
X_{ECt}	$ 875	$ 875	$ 875	$ 875

* The numbers in parentheses are total pieces in period t.
** The dollar amounts are shown in thousands.

X_{Q1t} $(t=1, 2, 3, 4)$ and X_{Q2t} $(t=1, 2, 3, 4)$ were 2.32, 0.06, 2.27, 2.39, and 5.22, 8.72, 9.58, 23.38 respectively. Next, we rounded the decimal numbers of Q1 and Q2, and under these tentative numbers of equipment —2, 0, 2, 2 for X_{Q1t}, and 5, 9, 10, 23 for X_{Q2t} respectively—we performed another computer run.

Then examining the values of slack variables and shadow prices for the Q1 and Q2 constraints, we subtracted 1 from $X_{Q2,4}$ and added 1 to $X_{Q2,3}$. Under these revised tentative numbers of Q1 and Q2, we made another run and obtained a smaller value of y.

Reiterating the same way of improving the solution systematically, we obtained a final result as shown in Table 8, where y was 125.5 thousands. Expressing the rate of regret r as percentage of the length of the G-vector, $r=y/\lambda_{1,1}=20.9\%$. The income statement, the balance sheet, and the degrees of goal attainments in the final result are presented in Table 9, 10, and Figure 4 respectively. These results suggest the expected goal attainments will be considerablly improved if the production capacity is carefully expanded.

It should be noted that the results also suggest that the goal vector approach, assuming the L-shaped utility (regret) function, is very useful for practically achieving a well-balanced goal-attainment as compared with the traditional goal programming approach such as the ordering and weighting methods. Using the

Table 9. Pro Forma Income Statement in the Case of Flexible Production Capacity (in thousands of dollars)

	$t=1$		$t=2$		$t=3$		$t=4$	
Net sales		32,111		38,186		44,699		55,534
Cost of products sold								
Variable costs	16,322		19,557		23,055		28,840	
Fixed direct labor	3,520		4,344		5,037		5,826	
Overhead	1,700		1,975		2,272		2,613	
Depreciation	2,024	23,566	2,260	28,136	2,726	33,090	3,820	41,099
Gross income		8,545		10,050		11,609		14,435
Selling & Gen. Adm. expenses								
Variable selling expense	1,606		1,907		2,240		2,782	
Fixed expenses	1,800		2,056		2,336		2,640	
Depreciation	69	3,475	69	4,032	69	4,645	69	5,491
Operating income		5,070		6,018		6,964		8,944
Other income or loss								
Interest received	255		270		294		317	
Interest paid	1,822		2,062		2,314		2,689	
Loss on equipment disposal	178	−)1,745	291	−)2,083	456	−)2,476	170	−)2,542
Net income before tax		3,325		3,935		4,488		6,402
Corporate income tax		1,729		2,046		2,334		3,329
Net income after tax		1,596		1,889		2,154		3,073
Retained earnings from prior years		560		1,281		2,296		3,576
Earnings available for dividends		2,156		3,170		4,450		6,649
Dividends		874		874		874		874
Retained earnings		1,281		2,296		3,576		5,775

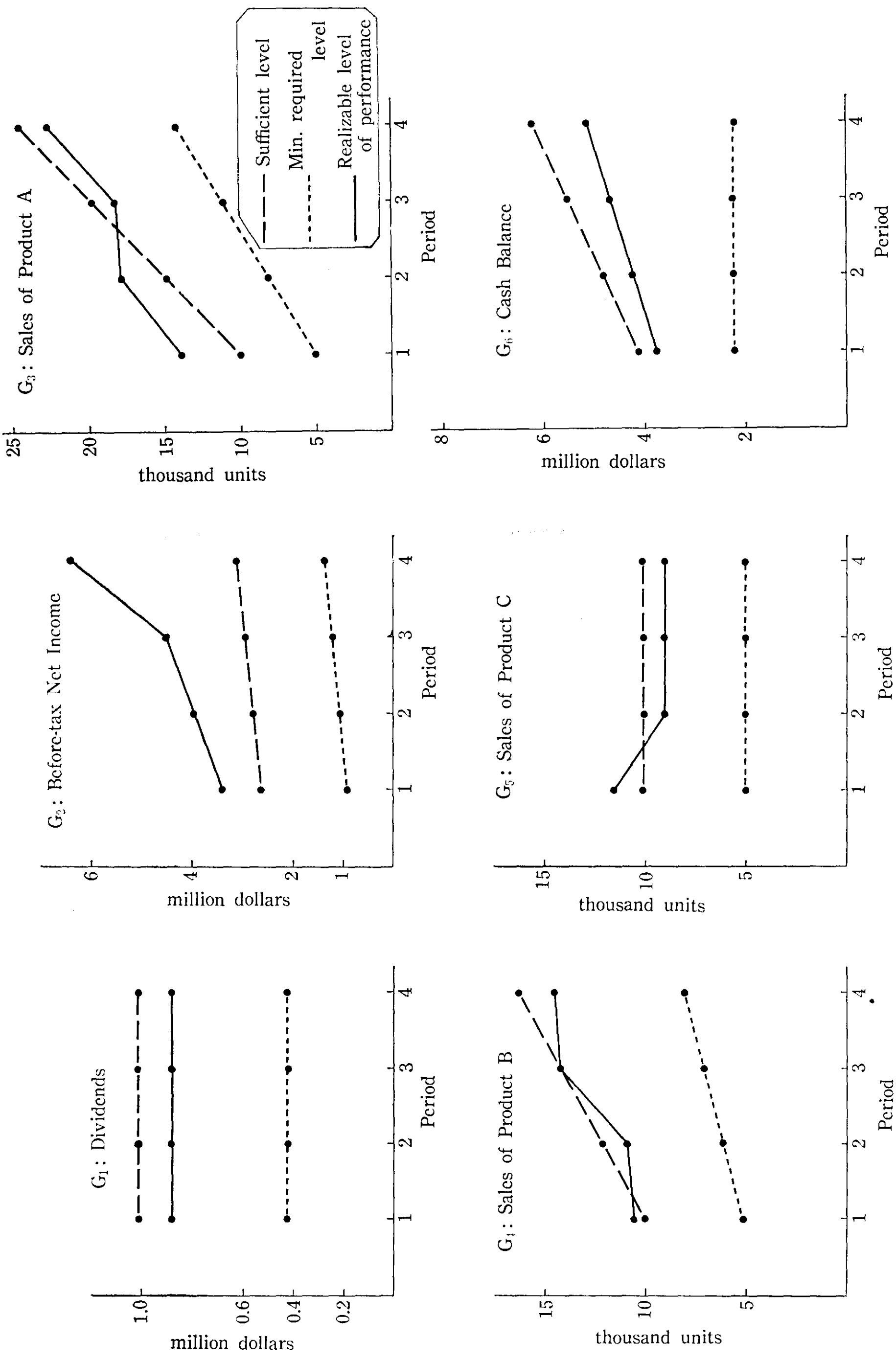

Figure 4 The Degree of Goal Attainments in the Case of Flexible Production Capacity

Table 10. Pro Forma Balance Sheet in the Case of Flexible Production Capacity (in thousands of dollars)

	$t=1$	$t=2$	$t=3$	$t=4$
ASSETS				
K_{Ct} Cash and bank deposits	3,782	4,573	5,363	6,155
K_{Bt} Securities	1,800	1,828	1,800	1,800
K_{Rt} Accounts receivable	9,633	11,456	13,410	16,660
K_{Gt} Inventories	5,961	7,047	8,523	9,130
Total current assets	21,176	24,904	29,096	33,745
K_{Qt} Direct factory equipment	3,504	3,813	4,641	6,972
K_{Ft} Other factory equipment	3,590	3,380	3,170	2,960
K_{Ht} Office equipment	861	792	723	654
K_{It} Land	800	800	800	800
Total fixed assets	8,755	8,785	11,386	11,386
Total assets	29,931	33,689	45,131	45,131
LIABILITIES				
K_{Pt} Accounts payable	2,745	3,021	3,470	4,081
K_{St} Short-term borrowings	15,905	18,432	21,885	25,627
Total current liabilities	18,650	21,453	25,355	29,708
K_{Lt} Long-term borrowings	3,200	3,140	2,699	2,848
Total liabilities	21,850	24,593	28,054	32,556
NET WORTH				
K_{Kt} Capital stock	5,000	5,000	5,000	5,000
K_{Ut} Capital surplus	1,800	1,800	1,800	1,800
K_{Et} Retained earnings	1,281	2,296	3,576	5,775
Total net worth	8,081	9,096	10,376	12,575
Total liabilities & net worth	29,931	33,689	38,430	45,131

formula above, the decision analysts will be able to perform many other test runs assuming a variety of conditions of goals, expected changes in market conditions, expected changes in revenue and expense factors, and so forth.

APPENDIX A

DETAILED DATA FOR THE MODEL CORPORATION

A-1 Equipment and Depreciation

A-1.1 Direct Factory Equipment

Machinery Q1: Both the economic and legal lives are 7 years. The legal salvage value for taxation purposes is 10% of acquisition cost, but the actual salvage value is always zero. The declining-balance method of depreciation is applied (de-

Table 11. Book Values and Depreciations of the Existing Q1 (in thousands of dollars)

time of acquisition	acquisition cost			end-of-period book value					workers per machine
	unit cost	number of machines	amount	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	
−5	$500	2	$1,000	$193.5	$139.3	0			8 pers.
−4	500	1	500	134.4	96.7	$69.6	0		8
−3	550	1	550	205.3	147.8	106.4	$76.6	0	7
−2	600	0	0	0	0	0	0	0	—
−1	650	1	650	468.0	337.0	242.6	174.7	$125.8	7
Total book value				$1,001.2	720.8	418.6	251.3	125.8	
Depreciation charge				280.4	201.8	117.2	70.4	35.2	
Loss on disposal				0	100.4	50.1	55.1	0	

Table 12. Book Values and Depreciations of the Existing Q2 (in thousands of dollars)

time of acquisition	acquisition cost			end-of-period book value					workers per machine
	unit cost	number of machines	amount	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	
−5	$125	16	$2,000	0					—
−4	125	8	1,000	0					—
−3	140	8	1,120	$280.4	0				4 pers.
−2	150	8	1,200	477.8	$301.5	0			4
−1	160	16	2,560	1,615.4	1,019.3	$643.2	0		3
Total book value				2,374.6	1,320.8	643.2	0		
Depreciation charge				876.2	487.4	237.3	0		
Loss on disposal				177.6	190.2	405.9	0		

preciation rate is 28%).

Machinery Q2: The economic and legal lives are 4 and 5 years respectively. Declining-balance depreciation is applied (depreciation rate is 36.9%).

Production capacities per piece of Q1 and Q2 are 2,000 hours. The amounts of depreciation and book value of the existing direct factory equipment are summarized in Table 11 and 12.

The expected acquisition costs and the numbers of workers per piece of Q1 and Q2 are shown in Table 13. The number of workers necessary for each of Q1 and Q2 has been and will be gradually decreasing because of technological progress.

A-1.2 Other Factory Equipment

(i) Buildings and Structures:

Acquisition cost (10 years ago): $1,000,000

Depreciable life: 30 years

Table 13. Acquisition Costs of Direct Factory Equipment and Workers Needed

time of acquisition (end-of-period)	Q1 (per unit)		Q2 (per unit)	
	acquisition cost	the number of workers	acquisition cost	the number of workers
0	$700,000	7	$180,000	3
1	750,000	7	190,000	3
2	800,000	6	200,000	3
3	850,000	6	210,000	2

Depreciation: straight-line method; depreciation charge is $30,000/year
Book value at the end of period 0: $700,000

(ii) Other equipment (group depreciation):
Acquisition cost (5 pears ago): $4,000,000
Depreciable life: 20 years average
Depreciation: straight-line method; depreciation charge is $180,000/year
Book value at the end of period 0: $3,100,000

(iii) Total:
Depreciation charge: $210,000/year
Book value at the end of period 0: 3,800,000

A-1.3 Office Equipment

(i) Buildings and basic facilities:
Acquisition cost (10 years ago): $400,000
Depreciable life: 40 years
Depreciation: straight-line method; depreciation charge is $9,000/year
Book value at the end of period 0: $310,000

(ii) Other facilities (group depreciation):
Acquisition cost (3 years ago): $800,000
Depreciable life: 12 years average
Depreciation: straight-line method; depreciation charge is $60,000/year
Book value at the end of period 0: $620,000

(iii) Total:
Depreciation charge: $69,000/year.
Book value at the end of period 0: $930,000

A-1.4 Land

Acquisition cost $800,000 (estimated replacement cost is $10,000,000)
Depreciation: non-depreciable

A-2 Labor Costs

A-2.1 Direct Production Departments

The number of workers necessary for the existing Q1 and Q2, and expected

Table 14. Direct Labors for the Existing Q1 and Q2

period	Q1		Q2		total number of workers	average cost per head	rate of increase
	the No. of machines	the No. of workers	the No. of machines	the No. of workers			
1	5	38	32	122	150	$20,000	——
2	5	38	24	80	118	24,000	20%
3	3	22	16	48	70	28,300	18%
4	2	14	0	0	14	33,100	17%

Table 15. Variable Labor Costs in Dept. III

period	product A	product B	product C
1	$120	$ 80	$ 80
2	140	100	100
3	170	120	120
4	200	140	140

"fixed direct labor costs" are shown in Table 14.

The number of workers in Department III is variable depending on the production volume. Average production volumes per head are:

Product A: 167 units
Product B: 250 units
Product C: 250 units

The average direct labor cost per head is the same as in Departments I and II; the variable labor cost per unit of each product is summarized in Table 15.

A-2. 2 Indirect Departments

The total amount of indirect labor costs in each period is:

period	*amount*
1	$ 1,200,000
2	1,440,000
3	1,700,000
4	2,000,000

A-3 Revenues and Expenses

A-3. 1 Anticipated Contribution Margin

Anticipated revenue, variable expenses, and contribution margin for each product in each period are summarized in Table 16.

Table 16. Anticipated Revenue, Variable Expenses, and Contribution Margin per Unit of Products

	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$
Selling price					
Product A	$1,111	$1,200	$1,296	$1,400	$1,512
〃 B	741	800	864	933	1,008
〃 C	556	600	648	700	756
M1 cost					
Product A	$381	$400	$420	$441	$463
〃 B	238	250	263	276	289
〃 C	143	150	158	165	174
M2 cost					
Product A	$ 93	$100	$108	$117	$126
〃 B	46	50	54	58	63
〃 C	28	30	32	35	38
Variable labor cost					
Product A	$100	$120	$140	$170	$200
〃 B	67	80	100	120	140
〃 C	67	80	100	120	140
Variable indirect cost and selling expense	*				
Product A	$75 (56)	$80 (60)	$86 (65)	$92 (70)	$98 (76)
〃 B	65 (37)	70 (40)	75 (43)	80 (47)	86 (50)
〃 C	56 (28)	60 (30)	64 (32)	69 (35)	74 (38)
Total variable expenses	**				
Product A	$649 (593)	$700 (640)	$754 (689)	$820 (750)	$887 (811)
〃 B	416 (379)	450 (410)	492 (449)	534 (487)	578 (528)
〃 C	294 (266)	320 (290)	354 (322)	389 (354)	426 (388)
Contribution margin					
Product A	$462	$500	$542	$580	$625
〃 B	325	350	372	399	430
〃 C	262	280	294	311	330

* The amounts in parentheses are variable selling expenses.
** The amounts in parentheses are variable production costs.

A-3.2 Fixed Expenses

(i) Fixed factory overhead costs are as follows:

period	*amount*
1	$ 500,000
2	535,000
3	572,000
4	613,000

Table 17. Fixed Expenses in Selling and General Adm. Dept.

period	variable selling expense	labor expense	other expense	depreciation charge
1	5% of sales	$1,000,000	$800,000	$69,000
2	〃	1,200,000	856,000	69,000
3	〃	1,420,000	916,000	69,000
4	〃	1,660,000	980,000	69,000

Table 18. Payback Schedule for the Existing Long-term Borrowings

end of period	−5	−4	−3	−2	−1	0	1	2	3	4
The balance of type-1 borrowings		1,500	1,200	900	600	300	0			
			1,000	800	600	400	200	0		
				1,000	800	600	400	200	0	
					500	400	300	200	100	0
						2,000	1,600	1,200	800	400
The balance of type-2 borrowings	700	600	500	400	300	200	100	0		
		2,100	1,800	1,500	1,200	900	600	300	0	
Total balance						4,800	3,200	1,900	900	400
The amount paid back at the beginning of the following period						1,600	1,300	1,000	500	400
						1,600	1,300	1,000	500	400
Interest paid							352	209	99	44

* Since the repayment is made at the beginning of each period, the interest of period t is calculated as: the balance at the end of period $t \times 11\%$.

(ii) Fixed expenses in the selling and general administrative departments are summarized in Table 17.

A-4 Pay-back Schedule for Long-term Borrowings

The existing long-term borrowings payable in the future consist of two types; the borrowings made for financing the funds for Q1 and Q2, and the borrowings for other (more basic) facilities. The former borrowings are paid back with a uniform amount through five years, and the latter through seven years respectively. Since the model corporation has finished modernizing the basic facilities, the latter type of new borrowings will not be made in the coming planning period. The pay-back schedule of the existing long-term borrowings is summarized in Table 18.

APPENDIX B

FURTHER TECHNICAL DISCUSSION

B-1 Theoretical Basis for the L-shape Model

We shall briefly explain a theoretical basis why we prefer L-shaped function to others for our financial planning model (see 2.1).

Logical tracing of a typical goal-pursuing process that we are dealing with is as follows. When corporate executives find all of the minimum required levels definitely achievable under the given constraints, they will raise the desired level of each goal a little higher, e.g., to g_i^1, $i=1,2,\cdots,m$. And if all of these levels can also be assured, they will raise the desired levels again to g_i^2, $i=1,2,\cdots,m$. Thus the executives will repeat this process until reaching a sufficient level g_i^s, $i=1,2,\cdots,m$, beyond which the increase of utility will supposedly saturate.

By connecting the points $G^r=(g_1^r, g_2^r, \cdots, g_m^r)$, $r=0,1,2,\cdots,s$, goal vectors $\overrightarrow{G^{r-1}G^r}$, $r=1,2,\cdots,s$, are obtained in the goal space.

In each stage of these processes, the zone of $g_i \geq g_i^r$ for every i can be regarded as a "satisfactory" zone in the stage. Synthesizing these concepts into a complete picture, we arrive at the image as shown in Figure 5, where the L-shaped isoutility lines that have the zenith at the points $G^r, r=1,2,\cdots,m$, run. Therefore if we assume, for practical approximation, that all the G^r are on the G-vector, we will naturally arrive at the L-shape model.

The L-shape model usually provides us with a balanced attainment of multiple goals in the sense that z_i/λ_i, $i=1,2,\cdots,m$, which is the deviation of each $g_i(x)/\lambda_i$ from the one with the minimum achievement, are fairly small. The reason is that most of the goals in our model are mutually competitive; if the level of a goal which

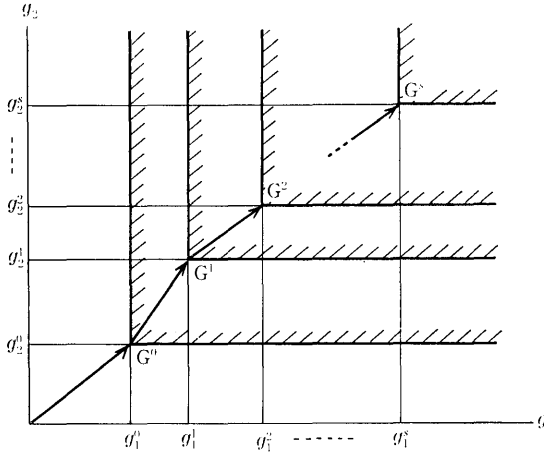

Figure 5 Satisfactory Zones in Each Stage of Goal-pursuing Process

has the minimum achievement is raised at a stage of computations, the level of achievement of other competitive goals will be lower at the next stage because of the technical constraints and/or the incompartibility between goals.

B-2 Optimal Solution of the L-shape Model

We measure a relative degree of utility in the L-shape model by the length of G-vector. Utility U and regret R at any point $G^k=(g_1^k, g_2^k, \cdots, g_m^k)$ on the G-vector can be measured by the length of $\overrightarrow{G^0G^k}$ and $-|\overrightarrow{G^sG^k}|$ respectively. Hence, the utility and regret at any point of $G(x)=(g_1(x), g_2(x), \cdots, g_m(x))$ in the space of $g_i \geq g_i^0$, excluding the sufficient zone that $g_i \geq g_i^s$ for every i (in which the regret $R=0$), can be measured by:

(B.1) (a) $$U=\min_i \left\{ \frac{g_i(x)-g_i^0}{u_i} \right\}$$

(b) $$R=\max_i \left\{ \frac{g_i^s-g_i(x)}{u_i} \right\}$$

where u_i is the i-th element of the vector for which $\sqrt{\sum_{i=1}^{m} u_i^2}=1$. Introducing the supplementary variables y_i, $i=1, 2, \cdots, m$, as in Eq. (II), R can also be measured by:

(B.2) $$R=\sqrt{\sum_{i=1}^{m} (y_i)^2} = y_1\sqrt{\sum_{i=1}^{m} (\lambda_i/\lambda_1)^2}.$$

Since $\sqrt{\sum_{i=1}^{m} (\lambda_i/\lambda_1)^2}>0$, the minimization of y_1 necessarily minimizes R.

The optimal point of the L-shape model is always found in the region of *Pareto optimum*; the tangent of every point in the region of Pareto optimum to the g_i axis must be between $-\infty$ and 0, and the tangent of every L-shaped isoutility line to the g_i axis is $-\infty$ or 0.

B-3 The Variation of Sufficient Zone

There might be a goal k whose sufficient level is lower than its minimum required level. A goal associated with cost reduction is a typical example. We can deal with this kind of goals in the same procedure as mentioned previously by assuming that $g_k^0 > g_k^s$ (therefore $\lambda_k < 0$).

Similarly, if there is a case where a decision maker wishes to decrease both of the excess and deficit deviations from a sufficient level g_k^s, i.e., to decrease the absolute value $|g_k(x)-g_k^s|$, we will divide the goal into two, goals $k1$ and $k2$, assuming that $g_{k1}^0 > g_{k1}^s$ (therefore $\lambda_{k1}<0$), $g_{k2}^0 < g_{k2}^s$ (therefore $\lambda_{k2}>0$), and $g_{k1}^s = g_{k2}^s$. The goal of maintaining a certain level of cash balance is a typical example.

Take, for instance, a two-goal case where $g_1^0=300$, $g_1^s=100$, $g_2^0=80$, $g_2^s=200$. In this case, the goal constraints for the L-shape model will be:

(B.3) (a) $g_1(x) - y_1 + z_1 = 100$

(b) $g_2(x) + y_2 - z_2 = 200$

(c) $120y_1 - 200y_2 = 0$ (or $3y_1 - 5y_2 = 0$)

(d) $g_1(x) \leq 300$

(e) $g_2(x) \geq 80$,

where x, y_i, z_i are non-negative.

B-4 Open-L Utility Function

As the test runs of our model corporation suggest, L-shape model gives us very useful solutions from the practical point of view. There may be the argument that the underlining assumption of L-shape model, where the degree of utility is dependent upon the minimum level of performance, is a little too rigid. It may be argued that in Figure 6, the relative utilities of the points K, A and B—U(K), U(A), U(B)—are not the same, but $U(\mathrm{K}) < U(\mathrm{A}) < U(\mathrm{B})$; in the same way, the relative utilities of the points K, P, Q are probably $U(\mathrm{K}) < U(\mathrm{P}) < U(\mathrm{Q})$.

If a decision analyst wants to reflect this idea to his financial planning model, an *open-L utility function* may be more theoretical. The broken lines in Figure 6 show the open-L isoutility lines. By means of an adequate conversion of variables, we can apply the similar technique to this problem as applied to the L-shape model (for detail, see FUSHIMI and YAMAGUCHI, 1975).

Theoretically, we can take the way to apply L-shaped utility functions to one group of goals, and open-L functions to others. A practical difficulty of the open-L model is that it is hard to determine a definite tradeoff between each pair of goals, and that the model requires a more complicated calculation.

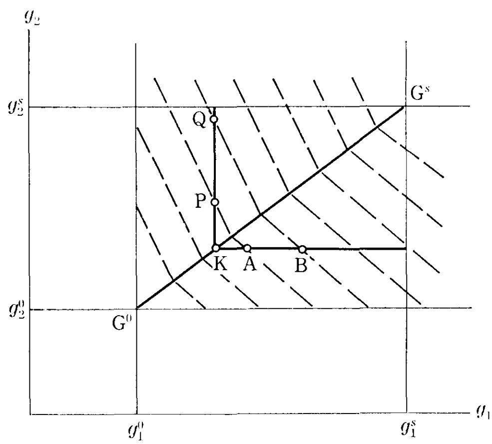

Figure 6 Open-L Utility Function

B-5. The Weighting Type of Utility Functions

Suppose the relationship between any two goals is fully substitutive, then the angle of open-L isoutility lines in Figure 6 will become wide enough to assume the isoutility lines crossing the G-vector vertically in the space of $g_i^0 \leq g_i \leq g_i^s$ (see Figure 1-(a)). The weighting type of utility functions corresponds to this extreme case of open-L utility functions.

The computation for weighting model is easier than open-L model; the goal-vector approach easily suggests the weights by means of $g_i^s - g_i^0 = \lambda_i$, $i=1, 2, \cdots, m$. It should be noted, however, that its solution can lead a very unbalanced goal-attainment as suggested in ii) of section 1.3. Figure 7 illustrates the reason assuming two goal cases; the optimal solution in which $d_1^- : d_2^- = \lambda_1 : \lambda_2$ will be realized only in a special case such as (a) of the figure. The more probable case is that one goal is fully or largely attained while the other goal is slightly or not attained as shown in (b) and (c) of the figure. We can see a typical example of this shortcoming in Chapter 9 of LEE (1972), where the levels of performance for the goals of paying extra dividends over five periods are very unbalanced even though an equal weight was given to each goal. Chapter 10 of the same book shows a similar problem in a sales effort allocation model.

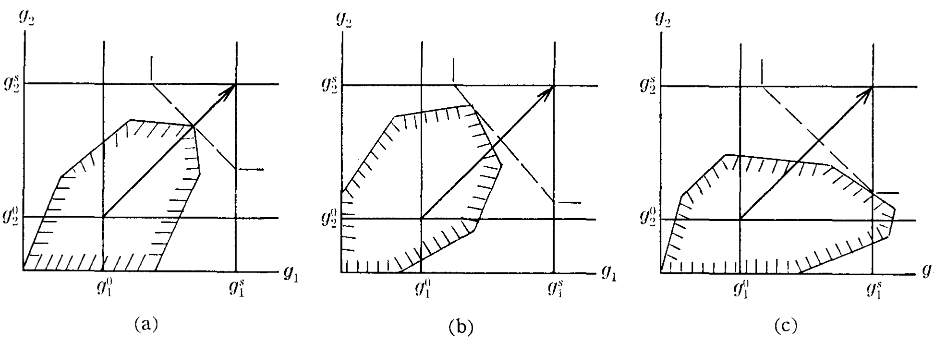

Figure 7 Solutions of Weighting-Type Problems (The shaded are feasible regions)

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