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A NOTE ON KADOMTSEV'S ANGULAR SPECTRUM OF ION SOUND WAVE

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ABSTRACT

Arguments are made of the use of delta function angular spectrum for ion sound wave in KADOMTSEV's theory. Saturation level obtained by numerical solution of wave kinetic equation is reasonably explained by assuming that KADOMTSEV's delta function angular spectrum corresponds to what is obtained by collecting all the modes in a broad angular spectrum into the most effective mode.

KADOMTSEV[1] derived the expression of stationary spectrum of ion sound instability. He assumed the angular part of the spectrum to be a sum of two delta functions which do not vanish only at some nonzero angles $\pm\theta_0$ from the direction of the electron drift velocity v_d. Since the linear growth rate has a maximum along the direction of drift velocity, it will be natural to assume a broad angular spectrum that has a maximum at $\theta=0$. By computer simulation BISKAMP and CHODURA[2] showed two symmetric off-center maxima in angular spectrum. But their spectrum is far from delta function. The use of the delta function in the KADOMTSEV's theory in reference [1] might be just to simplify the analysis. Then what follows from the simplification? This is the point which we will report in this brief note.

For the purpose of comparison, we have numerically solved wave kinetic equation in two dimensional wave number space by taking account of the nonlinear Landau damping without making delta function assumption[3]. Initially we gave a flat spectrum in θ and in wave number K. In Fig. 1, the angular distribution of the normalized spectrum $I(x, \theta, \tau)$ for the electrostatic potential of the wave is given as a function of θ, where $x=k\lambda_{De}$ and $\tau=\omega_{pi}t$ are normalized wave number and time, λ_{De} and ω_{pi} being electron Debye length and ion plasma frequency

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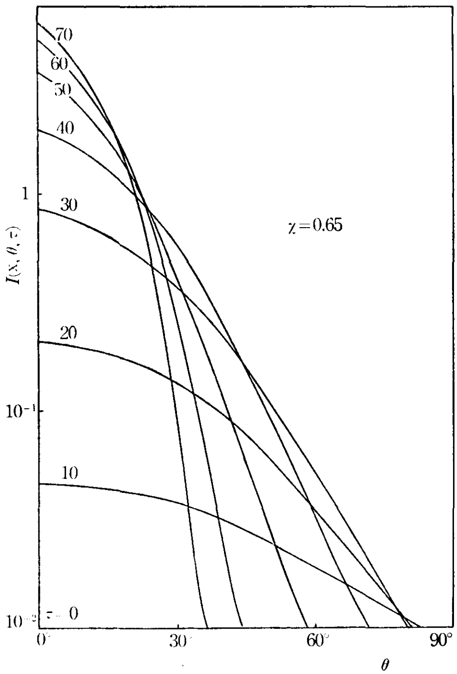

Fig. 1.

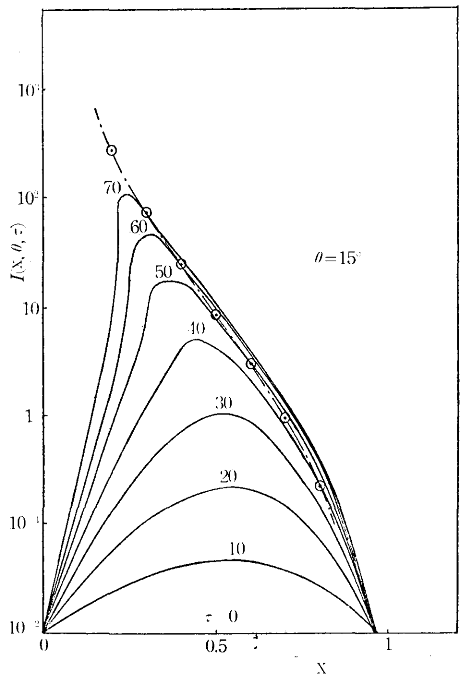

Fig. 2.

Figs. 1. and 2. Time variation of normalized angular spectrum $I(x, \theta, \tau)$ for $x=0.65$ (Fig. 1) and for $\theta=15°$ (Fig. 2), starting from $I(x, \theta, 0)=10^{-2}$ under $m/M=10^{-3}$, $T_e/T_i=20$ and $v_d/v_e=0.4$. Modified Kadomtsev spectrum is plotted with dotted circle.

respectively. In Fig. 1 the angular spectrum is seen to be narrow but different from delta function. In Fig. 2, larger k part saturates in shorter time due to ion nonlinear Landau damping which transfers energy from large k toward small k. The saturated part of the spectrum is slightly larger than the modified KADOMTSEV spectrum[3] shown with dotted circle. We mean the modified KADOMTSEV spectrum by what is obtained by introducing an upper cut off due to the linear ion Landau damping into the usual KADOMTSEV spectrum. The linear ion Landau damping has also been taken into account in our calculation. These result mean that Kadomtsev's delta function angular spectrum has stronger effect of nonlinear Landau damping than the broad one. In other word KADOMTSEV's spectrum overestimates the effect of nonlinear Landau damping on the ion sound wave.

In the broad spectrum, there are many modes with different angles ranging from $-\theta_0$ to θ_0. The minimum velocity of beat wave is obtained by the combination of θ_0 with $-\theta_0$ modes. If θ_0 is decreased in KADOMTSEV's saturated spectrum[1] $I(k)\sim\theta_0^{-2}$, $I(k)$ is increased. This corresponds to an increase of the beat wave velocity and hence to less energy loss. In the case of the broad spectrum, beat waves with larger velocities also take part in the wave-particle interaction, i.e., in the process of saturation. As a result the relative importance of KADOMTSEV's beat wave becomes small. This leads to weaker saturation and hence to higher saturation level than KADOMTSEV's case as shown in Fig. 2.

The above can actually be shown as follows. Equation for normalized spectrum $I(k)$ is given by

$$\frac{\partial I(k,\tau)}{\partial \tau} = 2\gamma_k I(k,\tau) - \text{NLD},$$

where γ_k is the growth rate of the ion sound instability. Assuming the expression for the spectrum $I(\boldsymbol{k})$ to be separable in k and θ_k as $I(\boldsymbol{k}) = J(k)A(\theta_k)$, we obtain the angular integral in the nonlinear Landau damping term NLD as[1],[3],[4]

$$B(\theta_k) = \int_{-\pi}^{\pi} d\theta\, A(\theta) \sin^2 2(\theta_k - \theta).$$

Kadomtsev's and broad angular spectra

$$A_{\text{Kad}}(\theta) = [\delta(\theta-\theta_0) + \delta(\theta+\theta_0)]/2,$$

$$A_{\text{Broad}}(\theta) = \begin{cases} 1/2\theta_0, & \text{for} \quad -\theta_0 \leqq \theta \leqq \theta_0 \\ 0, & \text{otherwise}, \end{cases}$$

yield $B_{\text{Kad}}(\theta_k) = (1 - \cos 4\theta_k \cos 4\theta_0)/2$ and $B_{\text{Broad}}(\theta_k) = (1 - \cos 4\theta_k \times \sin 4\theta_0)/2$. Then it follows $B_{\text{Kad}} > B_{\text{Broad}}$ for small θ_0. Since NLD term is proportional to the angular integral $B(\theta_k)$, we can conclude that KADOMTSEV's angular spectrum has larger effects of nonlinear Landau damping. Here we have used a flat spectrum for A_{Broad} for simplicity. If we employ smooth curve such as parabola, the inequality would be satisfied more sufficiently. Since $\int A_{\text{Kad}}\, d\theta = \int A_{\text{Broad}}\, d\theta$, all the distributed modes in the broad spectrum concentrate into two modes $\pm\theta_0$ in KADOMTSEV's case.

As a conclusion KADOMTSEV's delta function angular spectrum picks up the most effective contribution out of many angular modes and as a natural result the effect of nonlinear Landau damping is overestimated by collecting all the distributed angular modes into two dominantly effective modes θ_0 and $-\theta_0$.

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