Title	On the effects of severity adjustment in multiple sampling plans of MIL-STD 105D type
Sub Title	
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Publisher	慶応義塾大学工学部
Publication year	1976
Jtitle	Keio engineering reports Vol.29, No.3 (1976.) ,p.23- 40
JaLC DOI	
Abstract	The multiple sampling plans of the MIL-STD-105D type are formulated as a MARKOV chain for analyzing the effects of severity adjustment.
Notes	
Genre	Departmental Bulletin Paper
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=KO50001004-00290003-0023

KEIO ENGINEERING REPORTS
VOL. 29, NO. 3, pp. 23-40, 1976

ON THE EFFECTS OF SEVERITY ADJUSTMENT IN MULTIPLE SAMPLING PLANS OF MIL-STD-105 D TYPE

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(*Received, Sep. 8, 1975*)

ABSTRACT

The multiple sampling plans of the MIL-STD-105 D type are formulated as a MARKOV chain for analyzing the effects of severity adjustment.

1. Introduction

Many papers have been published on the effects of severity adjustment in sampling plans of MIL-STD-105 D type. Omae, Yokoh and Takeuchi (1966) studied the switching probabilities of severity of inspection and the mean and variance of the number of lots prior to the switching by means of computer simulation under some restricted conditions. STEPHENES and LARSON (1967) investigated the stationary probabilities of normal, tightened and reduced inspections, the composite operating characteristic curve, and the average sample number (ASN), provided that the rule for "discontinuation of inspection" is not taken into account. KOYAMA (1974) applied the theory of MARKOV chain to analyze the dynamic characteristics of severity adjustment, and developed one- and two-directional switching theory. He also introduced the theory of signal flow graph into the severity adjustment. He calculated approximately the mean and variance of the number of lots prior to switching the severity of inspection.

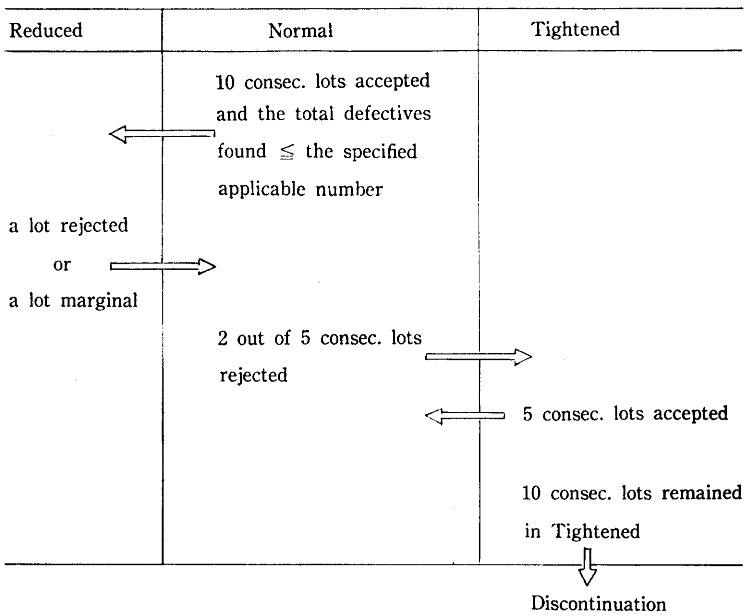
However, the double and multiple sampling plans have not been treated in the previous papers cited above. In section 3 the author will formulate the sampling plans of the MIL-STD-105 D type as a MARKOV chain with finite states, taking into consideration the rule for "discontinuation of inspection." His formulation includes all types of single, double and multiple sampling plans, where the concept of the set of states is introduced, so as to make clear the structure of transitions among the states.

The author will present the methods for computing the effects of severity adjustment. In section 4.1 we shall discuss the transition probabilities determining the probability law for the MARKOV chain. In section 4.2 we shall consider the number of lots inspected up to switching the severity of inspection. In section 4.3 we shall study the conditional expected proportion of lots inspected on Normal, Reduced and Tightened, the composite OC function and the average sample number.

2. Sampling System and MARKOV Chain

The rules of the severity adjustment in MIL-STD-105D may be summarized as follows:

Table 1. Switching Procedures in MIL-STD-105D.



A result of k-phase sampling inspection (with k samples of size n) for a series of m consec. lots may be represented as a sequence, consisting of the numbers of defective units found in km samples arranged in the order of inspection. Therefore, the practice of inspection may be regarded as a transition from one sequence to another sequence.

For example, we may show a result of sampling inspection (Normal, multiple, code letter N, AQL 0.10 in MIL-STD-105D) for a series of three lots as follows:

$$\overbrace{0100101}^{\text{1st lot}}\ \overbrace{000****}^{\text{2nd lot}}\ \overbrace{000****}^{\text{3rd lot}}$$

↑ ↗
Inspection is stopped here.

, where " * " means undetermined number. The result of sampling for the first lot is represented as below:

$$X_{11}=0,\quad X_{12}=1,\quad X_{13}=0,\quad X_{14}=0,\quad X_{15}=1,\quad X_{16}=0,\quad X_{17}=1$$

, where X_{mi} corresponds to the i-th sampling from the m-th lot. In order to formulate and to analyze a system of multiple sampling plans in MIL-STD-105 D as a stationary MARKOV chain, we suppose that every lot has the same lot percent deffective. X_{nj} and X_{mi} are random variables and independent of each other for $n \neq m$. We will treat a sequence mentioned above as a state of a MARKOV chain.

3. Formulation as a MARKOV Chain

3. 1. State Space

To relate the sampling system under consideration to MARKOV chain, the concepts of " state " and " set of states " are introduced, so as to completely describe the rules of switching. The transitions among these states have the MARKOV property in the sense that at any trial the probability of being in a particular state depend only upon the previous state.

We define the following notations:

$Ac(i, N)$: the i-th acceptance number in k-phase sampling plan when normal inspection is in effect, where $i=1, 2, \cdots, k$,

$Re(i, N)$: the i-th rejection number in k-phase sampling plan when normal inspection is in effect, where $i=1, 2, \cdots, k$.

Similarly, we introduce:

$Ac(i, R)$, $Re(i, R)$: when reduced inspection is in effect, where $i=1, 2, \cdots, k$,

$Ac(i, T)$, $Re(i, T)$: when tightened inspection is in effect, where $i=1, 2, \cdots, k$.

Then the realized value $\boldsymbol{x}$ of a random vector $\boldsymbol{X}=(X_1, X_2, \cdots, X_k)$ belongs to a k-dimentional product space $\boldsymbol{L}=L\times L\times\cdots\times L$, where L is a union of all nonnegative integers $\{0, 1, 2, \cdots\}$. Further we introduce k-dimensional product spaces $\boldsymbol{L}_1, \boldsymbol{L}_2, \cdots, \boldsymbol{L}_{10}$ corresponding to the random vectors $\boldsymbol{X}_1, \boldsymbol{X}_2, \cdots, \boldsymbol{X}_{10}$, respectively. Let us introduce a space $L_0=\{N, T, R, D\}$, where N, T, R and D correspond to the normal, tightened, reduced inspections and discontinuation of inspection, respectively.

Now we define $2k$ sets A_N^m, R_N^m $(m=1, 2, \cdots, k)$ in a k-dimensional product space $\boldsymbol{L}$ as follows:

$$A_N^m=\{(x_1, x_2, \cdots, x_k)\,;\ Ac(i, N)<\sum_{j=1}^{i} x_j<Re(i, N) \text{ for } i=1, 2, \cdots, m-1,$$

$$\sum_{j=1}^{m} x_j \leqq Ac(m, N) \text{ and } x_j \leqq n(N) \text{ for } j=1, 2, \cdots, k\} \text{ for } m=1, 2, \cdots, k,$$

$$R_N^m=\{(x_1, x_2, \cdots, x_k)\,;\ Ac(i, N)<\sum_{j=1}^{i} x_j<Re(i, N) \text{ for } i=1, 2, \cdots, m-1,$$

$$\sum_{j=1}^{m} x_j \geqq Re(m, N) \text{ and } x_j \leqq n(N) \text{ for } j=1, 2, \cdots, k\} \text{ for } m=1, 2, \cdots, k-1$$

and

$$R_N^k=\{(x_1, x_2, \cdots, x_k)\,;\ Ac(i, N)<\sum_{j=1}^{i} x_j<Re(i, N) \text{ for } i=1, 2, \cdots, k-1,$$

$$\sum_{j=1}^{k} x_j \geqq Ac(k, N)+1 \text{ and } X_j \leqq n(N) \text{ for } j=1, 2, \cdots, k\},$$

where $n(N)$ corresponds to the sample size for each phase of normal inspection. Here we put $A_N=\bigcup_{m=1}^{k} A_N^m$, $R_N=\bigcup_{m=1}^{k} R_N^m$, corresponding to acceptance and rejection region of k-phase normal sampling inspection, respectively. Similarly, we introduce A_T^m, R_T^m $(m=1, 2, \cdots, k)$, A_T, R_T, A_R^m, R_R^m $(m=1, 2, \cdots, k)$, A_R and R_R, corresponding to the tightened and reduced inspection, where $n(T)$ and $n(R)$ denote the sample size, respectively.

Thus, we define subsets $N^{(n)}$ $(n=1, 2, \cdots, 10)$, $\bar{N}^{(1)}$, $\bar{N}^{(1,n)}$ $(n=1, 2, 3)$ and $\bar{N}^{(2)}$ of $\{N\}\times \boldsymbol{L}_1\times\cdots\times\boldsymbol{L}_{10}$ as follows:

$$N^{(n)}=\{N\}\times\overbrace{A_N\times\cdots\times A_N}^{n}\times\overbrace{\boldsymbol{L}_{n+1}\times\cdots\times\boldsymbol{L}_{10}}^{10-n},$$

$$\bar{N}^{(1)}=\{N\}\times R_N\times\boldsymbol{L}_2\times\boldsymbol{L}_3\times\cdots\times\boldsymbol{L}_{10},$$

$$\bar{N}^{(1,n)}=\{N\}\times R_N\times\overbrace{A_N\times\cdots\times A_N}^{n}\times\overbrace{\boldsymbol{L}_{n+2}\times\cdots\times\boldsymbol{L}_{10}}^{10-n-1},$$

$$\bar{N}^{(2)} = \bigcup_{n=1}^{3}[\{N\}\times R_N\times\overbrace{A_N\times\cdots\times A_N}^{n}\times R_N\times\overbrace{\boldsymbol{L}_{n+3}\times\cdots\times\boldsymbol{L}_{10}}^{10-n-2}]$$

$$\cup\ \{N\}\times R_N\times R_N\times\boldsymbol{L}_3\times\cdots\times\boldsymbol{L}_{10}.$$

For any $(x_0, \boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{10})\in\{N\}\times\overbrace{A_N\times\cdots\times A_N}^{10}$, we put

$$f(x_0, \boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{10})=\sum_{i=1}^{10}\sum_{j=1}^{10} x_{ij},$$

$$g(x_0, \boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{10})=n(N)\sum_{i=1}^{10} m(i),$$

where $m(i)$ denotes the inspected phase number, corresponding to $\boldsymbol{x}_i=(x_{i1}, x_{i2}, \cdots, x_{i10})$,

$$L_R(g(x_0, \boldsymbol{x}_1, \cdots, \boldsymbol{x}_{10})) = (\mathrm{AQL}/100)\, g(x_0, \boldsymbol{x}_1, \cdots, \boldsymbol{x}_{10})$$

$$-1.282\sqrt{(\mathrm{AQL}/100) g(x_0, \boldsymbol{x}_1, \cdots \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{10})},$$

where AQL means "acceptable quality level (percentage)."

We define $N_R^{(10)}$, $N_N^{(10)}$ as follows:

$$N_R^{(10)} = \{\boldsymbol{x}^{(10)} ;\ \boldsymbol{x}^{(10)} \in N^{(10)},\ f(\boldsymbol{x}^{(10)}) \leqq L_R(g(\boldsymbol{x}^{(10)}))\},$$

$$N_N^{(10)} = \{\boldsymbol{x}^{(10)} ;\ \boldsymbol{x}^{(10)} \in N^{(10)},\ f(\boldsymbol{x}^{(10)}) > L_R(g(\boldsymbol{x}^{(10)}))\}.$$

Similarly, we define R, $\bar{R}$, T, $T^{(1)}$, $T^{(n)}$ $(n=2, \cdots, 6)$, $T_0^{(D)}$, $T_n^{(D)}$ $(n=1, \cdots, 4)$, $S^{(0)}$ and $S^{(n)}$ $(n=1, \cdots, 5)$.

$$R = \{R\} \times \boldsymbol{L}_1 \times \cdots \times \boldsymbol{L}_{10},$$

$$\bar{R} = \{R\} \times R_R \times \boldsymbol{L}_2 \times \cdots \times \boldsymbol{L}_{10},$$

$$T = \{T\} \times \boldsymbol{L}_1 \times \cdots \times \boldsymbol{L}_{10},$$

$$T^{(1)} = \{T\} \times \overbrace{A_T \times \cdots \times A_T}^{5} \times \boldsymbol{L}_6 \times \cdots \times \boldsymbol{L}_{10},$$

$$T^{(n)} = \{T\} \times \overbrace{\boldsymbol{L}_1 \times \cdots \times \boldsymbol{L}_{n-2}}^{n-2} \times R_T \times \overbrace{A_T \times \cdots \times A_T}^{5} \times \overbrace{\boldsymbol{L}_{n+5} \times \cdots \times \boldsymbol{L}_{10}}^{10-n-4},$$

$$T^{(D)} = T - T^{(1)} \cup \cdots \cup T^{(6)},$$

$$T_0^{(D)} = T^{(D)} \cap \{T\} \times R_T \times \boldsymbol{L}_2 \times \cdots \times \boldsymbol{L}_{10},$$

$$T_n^{(D)} = T^{(D)} \cap \{T\} \times \overbrace{\boldsymbol{L}_1 \times \cdots \times \boldsymbol{L}_{10-n-1}}^{10-n-1} \times R_T \times \overbrace{A_T \times \cdots \times A_T}^{n},$$

$$S^{(0)} = \{D\} \times R_T \times L_2 \times \cdots \times L_{10},$$

$$S^{(n)} = \{D\} \times \overbrace{A_T \times \cdots \times A_T}^{n} \times \boldsymbol{L}_{n+1} \times \cdots \times \boldsymbol{L}_{10}.$$

In order to discuss the effects of severity adjustment in multiple sampling plans of MIL-STD-105 D type, we shall set up a MARKOV chain, the states of which consist of the following sample points,

$$(N, \boldsymbol{x}_1, \cdots\cdots\cdots, \boldsymbol{x}_n, \boldsymbol{x}_{n+1}^*, \cdots\cdots\cdots, \boldsymbol{x}_{10}^*) \in N^{(n)},$$

$$(N, \boldsymbol{x}_1, \boldsymbol{x}_2^*, \cdots\cdots\cdots\cdots\cdots\cdots, \boldsymbol{x}_{10}^*) \in \overline{N}^{(1)},$$

$$(N, \boldsymbol{x}_1, \cdots\cdots, \boldsymbol{x}_{n+1}, \boldsymbol{x}_{n+2}^*, \cdots\cdots\cdots\cdots, \boldsymbol{x}_{10}^*) \in \overline{N}^{(1,n)},$$

$$(N, \boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3^*, \cdots\cdots, \boldsymbol{x}_{10}^*),\quad (N, \boldsymbol{x}_1, \cdots\cdots, \boldsymbol{x}_{n+2}, \boldsymbol{x}_{n+3}^*, \cdots\cdots, \boldsymbol{x}_{10}^*) \in \overline{N}^{(2)},$$

$$(R, \boldsymbol{x}_1, \boldsymbol{x}_2^*, \cdots\cdots\cdots\cdots\cdots\cdots, \boldsymbol{x}_{10}^*) \in R,$$

$$(T, \boldsymbol{x}_1, \cdots\cdots\cdots, \boldsymbol{x}_{n+4}, \boldsymbol{x}_{n+5}^*, \cdots\cdots\cdots, \boldsymbol{x}_{10}^*) \in T^{(n)},$$

$$(T, \boldsymbol{x}_1, \boldsymbol{x}_2, \cdots\cdots\cdots\cdots\cdots\cdots, \boldsymbol{x}_{10}) \in T^{(D)}.$$

$$(D, \boldsymbol{x}_1, \boldsymbol{x}_2^*, \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots, \boldsymbol{x}_{10}^*) \in S^{(0)},$$

$$(D, \boldsymbol{x}_1, \cdots\cdots\cdots, \boldsymbol{x}_n, \boldsymbol{x}_{n+1}^*, \cdots\cdots\cdots, \boldsymbol{x}_{10}^*) \in S^{(n)},$$

where $\boldsymbol{x}_j^*$ means a dummy valued vector, corresponding to the possible results of sampling inspection which is not taken place in reality.

We can devide all the states in $L_0 \times \boldsymbol{L}_1 \times \cdots \times \boldsymbol{L}_{10}$ into groups that have the property that the transition probabilities from each state in a group S_k to each of the states in another group S_l when summed over all of the states in group S_l are the same for each state of group S_k. Formally, $\sum_{j \in S_l} p_{ij} = \sum_{j \in S_l} p_{i'j}$ for $i, i' \in S_k$. When this relation holds for any pair of groups k, l, we say that the MARKOV chain is mergeable with respect to the grouping of states. A mergeable process behaves as a MARKOV chain each of whose merged state k is the group S_k of the original MARKOV chain.

We can form the merged states, consisting of states which are contained in the following sets.

merged states

$$(N, \overbrace{\boldsymbol{x}_1, \cdots\cdots, \boldsymbol{x}_m}^{m}, \overbrace{\boldsymbol{L}_{m+1}, \cdots\cdots, \boldsymbol{L}_{10}}^{10-m}) \subset N^{(m)} \quad (m=1, 2, \cdots\cdots, 10),$$

$$(N, R_N, \boldsymbol{L}_2, \boldsymbol{L}_3, \cdots\cdots\cdots\cdots, \boldsymbol{L}_{10}) = \overline{N}^{(1)},$$

$$(N, R_N, \overbrace{\boldsymbol{x}_2, \cdots\cdots, \boldsymbol{x}_{m+1}}^{m}, \overbrace{\boldsymbol{L}_{m+2}, \cdots\cdots, \boldsymbol{L}_{10}}^{10-m-1}) \subset \overline{N}^{(1,m)} \quad (m=1, 2, 3),$$

$$(N, R_N, \overbrace{\boldsymbol{x}_2 \cdots\cdots, \boldsymbol{x}_{m+1}}^{m}, R_N, \overbrace{\boldsymbol{L}_{m+3}, \cdots\cdots, \boldsymbol{L}_{10}}^{10-m-2}) \quad (m=1, 2, 3),$$

$$(N, R_N, R_N, \boldsymbol{L}_3, \boldsymbol{L}_4, \cdots\cdots, \boldsymbol{L}_{10}) \subset \overline{N}^{(2)}$$

$$(R, I_R, \boldsymbol{L}_2, \boldsymbol{L}_3, \cdots\cdots\cdots\cdots, \boldsymbol{L}_{10}) \subset R,$$

where I_R equals to A_R or R_R,

$$(T, \overbrace{A_T, \cdots\cdots, A_T}^{5}, \overbrace{\boldsymbol{L}_6, \cdots\cdots, \boldsymbol{L}_{10}}^{5}) = T^{(1)},$$

$$(T, \overbrace{I_{1,T}, \cdots, I_{m-2,T}}^{m-2}, R_T, \overbrace{A_T, \cdots, A_T}^{5}, \overbrace{\boldsymbol{L}_{m+5}, \cdots, \boldsymbol{L}_{10}}^{10-m-4}) \subset T^{(m)} \quad (m=2, 3, \cdots, 6),$$

$$(T, I_{1,T}, I_{2,T}, \cdots\cdots\cdots, I_{10,T}) \subset T^{(D)},$$

$$(T, I_{1,T}, I_{2,T}, \cdots\cdots, I_{9,T}, R_T) \subset T_0^{(D)},$$

$$(T, \overbrace{I_{1,T}, \cdots\cdots, I_{10-m,T}}^{10-m}, \overbrace{A_T, \cdots\cdots A_T}^{m}) \subset T_m^{(D)},$$

where each of $I_{j,T}$ $(j=1, 2, \cdots\cdots, 10)$ equals to A_T or R_T.

Further, we consider another state D as an absorbing state, which corresponds to discontinuation of inspection.

We will introduce another sampling system as investigated by STEPHENES and LARSON (1967) in the case of single sampling. Instead of state D, we consider the following merged states $S^{(0)}$ and $S^{(m)}$ $(m=1, 2, \cdots, 5)$ defined as below:

$$(D,\ R_T,\ \boldsymbol{L}_2,\ \boldsymbol{L}_3,\ \cdots\cdots\cdots\cdots\cdots,\ \boldsymbol{L}_{10}) = S^{(0)}$$

$$(D,\ \overbrace{A_T,\ \cdots\cdots,\ A_T}^{m},\ \overbrace{\boldsymbol{L}_{m+1},\ \cdots\cdots,\ \boldsymbol{L}_{10}}^{10-m}) = S^{(m)}.$$

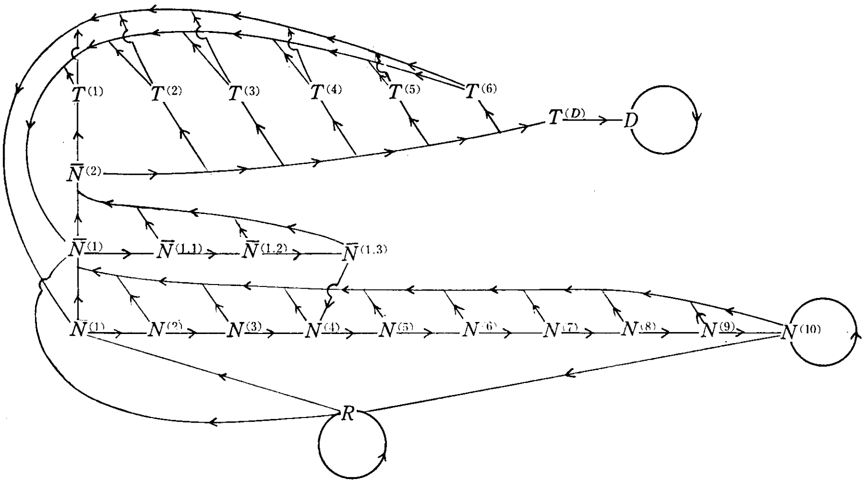


Fig. 1. Transition diagram of the operation of MIL-STD-105 D.

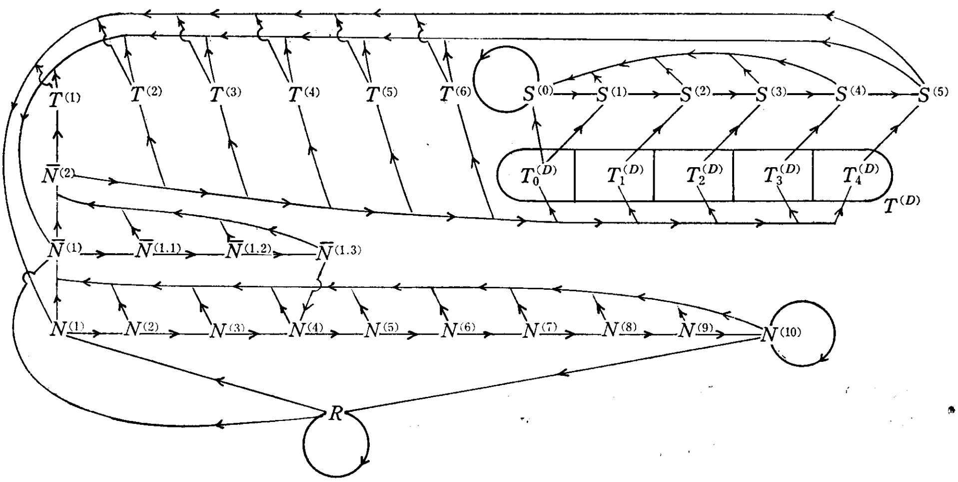


Fig. 2. Transition diagram of the modified operation of MIL-STD-105 D.

The transition diagrams of the operation of MIL-STD-105 D and the modified operation suggested by STEPHENES and LARSON are shown in Fig. 1 and in Fig. 2, respectively. Notice the transitions illustrated can be understood as the transitions from some (merged) states in a set, e. g. $N^{(3)}$, to some (merged) states in the corresponding sets, $\bar{N}^{(1)}$ and $N^{(4)}$.

We shall give the brief list of sets of states and the results of sampling inspection.

Set of States	Sampling Inspection
	Normal Inspection
1. $N^{(n)}$, $n=1, 2, \cdots\cdots, 10$	1. The consec. n lots have been accepted.
2. $\bar{N}^{(1)}$	2. A lot has been rejected.
3. $\bar{N}^{(1,n)}$, $n=1, 2, 3$	3. A lot has been rejected and followed by n consec. lots accepted.
4. $\bar{N}^{(2)}$	4. 2 out 5 consec. lots have been rejected.
	Reduced Inspection
5. R	5. A lot has been on reduced inspection.
	Tightened Inspection
6. $T^{(n)}$, $n=1, 2, \cdots\cdots, 6$	6. Tightened inspection has been instituted and 5 consec. lots from the n-th lot have been accepted.
7. $T^{(D)}$	7. 10 consec. lots have been on tightened inspection, and in those lots there is no 5 consec. lots accepted.
8. $S^{(n)}$, $n=0, 1, \cdots\cdots 5$	8. More than 10 lots have been on tightened inspection and the last n lots bave been accepted.

3. 2. Transition Probabilities

Now, we shall give the transition probabilities among the merged states. Let $X_{n,0}$ and $\boldsymbol{X}_n$ be a random variable which denotes the severity of sampling inspection for the n-th lot from the start, and a random vector which corresponds to the result of sampling inspection for the n-th lot, respectively. In order to construct a simple MARKOV chain from the series of random vecrtors $(X_{n,0}, \boldsymbol{X}_n)$, $n=1, 2, \cdots$, we must set up a sequence of random vectors $\boldsymbol{Z}_n=(X_{n,0}, \overbrace{\boldsymbol{X}_{n-m}, \boldsymbol{X}_{n-m+1},}^{10}$

$\cdots, \boldsymbol{X}_{n-m+9})$ $(m=0, 1, \cdots, 9)$, where m is determined uniquely for any n by the realized value of random vector $\boldsymbol{Z}_{n-1}$. It should be noted here that we must construct the above-mentioned simple MARKOV chain by adding dummy random vectors to the original series, because it does not form a multiple MARKOV chain of order 11 in itself.

Then, the transition probability from the merged state s_i to the merged state s_j is given by $P(\boldsymbol{Z}_{n+1} \in s_j/\boldsymbol{Z}_n \in s_i)=P(s_i, s_j)$.

Note that, if the transition probability is not specified, we shall put it equal to zero.

For any merged state $s_i=(x_0, D_1^{(i)}, D_2^{(i)}, \cdots, D_{10}^{(i)})$ of $L_0 \times \boldsymbol{L}_1 \times \cdots \times \boldsymbol{L}_{10}$, the function h_m is defined by the relation $h_m(s_i)=(x_0, D_m^{(i)})$, where $D_m^{(i)}$ is the $(m+1)$-th coordinate vector of s_i.

For $\forall s_i \subset N^{(m)}$, $\forall s_j \subset N^{(m+1)}$ $(m=1, 2, \cdots, 9)$

$$P(s_i, s_j)=P\{(X_{n,0}, \boldsymbol{X}_n)=h_{m+1}(s_j)\}, \quad \text{if } h_k(s_i)=h_k(s_j) \text{ for } k=1, \cdots, m,$$
$$=0, \quad \text{otherwise.} \tag{1}$$

For $\forall s_i \subset N^{(10)}$, $\forall s_j \subset N^{(10)}$

$$P(s_i, s_j)=P\{(X_{n,0}, \boldsymbol{X}_n)=h_{10}(s_j)\}, \quad \text{if } h_{k+1}(s_i)=h_k(s_j) \text{ for } k=1, \cdots, 9,$$
$$=0, \quad \text{otherwise.} \tag{2}$$

For $\forall s_i \subset N^{(m)}$, $\forall s_j \subset \bar{N}^{(1)}$ $(m=1, 2, \cdots, 9)$

$$P(s_i, s_j)=P\{(X_{n,0}, \boldsymbol{X}_n) \in h_1(s_j)\}=P\{(X_{n,0}, \boldsymbol{X}_n) \in (N, R_N)\}. \tag{3}$$

For $\forall s_i \subset N_N^{(10)}$, $\forall s_j \subset \bar{N}^{(1)}$

$$P(s_i, s_j)=P\{(X_{n,0}, \boldsymbol{X}_n) \in h_1(s_j)\}=P\{(X_{n,0}, \boldsymbol{X}_n) \in (N, R_N)\}. \tag{4}$$

For $\forall s_i \subset \bar{N}^{(1)}$, $\forall s_j \subset \bar{N}^{(2)}$

$$P(s_i, s_j)=P\{(X_{n,0}, \boldsymbol{X}_n) \in h_2(s_j)\}=P\{(X_{n,0}, \boldsymbol{X}_n) \in (N, R_N)\}. \tag{5}$$

For $\forall s_i \in \bar{N}^{(1)}$, $\forall s_j \in N^{(1,1)}$

$$P(s_i, s_j)=P\{(X_{n,0}, \boldsymbol{X}_n)=h_2(s_j)\}. \tag{6}$$

For $\forall s_i \subset \bar{N}^{(1,3)}$, $\forall s_j \subset N^{(4)}$

$$P(s_i, s_j)=P\{(X_{n,0}, \boldsymbol{X}_n) \in h_4(s_j)\}, \quad \text{if } h_{k+1}(s_i)=h_k(s_j) \text{ for } k=1, 2, 3,$$
$$=0, \quad \text{otherwise.} \tag{7}$$

For $\forall s_i \subset \bar{N}^{(1,m)}$, $\forall s_j \subset \bar{N}^{(1,m+1)}$ $(m=1, 2)$

$$P(s_i, s_j)=P\{(X_{n,0}, \boldsymbol{X}_n)=h_{m+2}(s_j)\}, \quad \text{if } h_k(s_i)=h_k(s_j) \text{ for } k=1, 2, \cdots, m+1,$$
$$=0, \quad \text{otherwise.} \tag{8}$$

For $\forall s_i \subset \bar{N}^{(1,m)}$, $\forall s_j \subset \bar{N}^{(2)}$ $(m=1, 2, 3)$

$$P(s_i, s_j) = P\{(X_{n,0}, \boldsymbol{X}_n) \in h_{m+2}(s_j)\} = P\{(X_{n,0}, \boldsymbol{X}_n) \in (N, R_N)\}. \quad (9)$$

For $\forall s_i \subset N_R^{(10)}$, $\forall s_j \subset R$

$$P(s_i, s_j) = P\{(\boldsymbol{X}_{n,0}, \boldsymbol{X}_n) \in h_1(s_j)\}. \quad (10)$$

For $\forall s_i \subset R - \bar{R}$, $\forall s_j \subset R$

$$P(s_i, s_j) = P\{X_{n,0}, \boldsymbol{X}_n) \in h_1(s_j)\}. \quad (11)$$

For $\forall s_i \subset \bar{R}$, $\forall s_j \subset N^{(1)}$

$$P(s_i, s_j) = P\{(X_{n,0}, \boldsymbol{X}_n) \in h_1(s_j)\}. \quad (12)$$

For $\forall s_i \subset \bar{R}$, $\forall s_j \subset \bar{N}^{(1)}$

$$P(s_i, s_j) = P\{(X_{n,0}, \boldsymbol{X}_n) \in h_1(s_j)\} = P\{(X_{n,0}, \boldsymbol{X}_n) \in (N, R_N)\}. \quad (13)$$

For $\forall s_i \subset \bar{N}^{(2)}$, $\forall s_j \subset T^{(m)}$ $(m=1, 2, \cdots, 6)$

$$P(s_i, s_j) = \prod_{k=1}^{m+4} P\{X_{n+k,0}, \boldsymbol{X}_{n+k}) \in h_k(s_j)\}. \quad (14)$$

For $\forall s_i \subset \bar{N}^{(2)}$, $\forall s_j \subset T^{(D)}$

$$P(s_i, s_j) = \prod_{k=1}^{10} P\{(X_{n+k,0}, \boldsymbol{X}_{n+k}) \in h_k(s_j)\}. \quad (15)$$

For $\forall s_i \subset T^{(m)}$, $\forall s_j \subset N^{(1)}$ $(m=1, 2, \cdots, 6)$

$$P(s_i, s_j) = P\{X_{n,0}, \boldsymbol{X}_n) = h_1(s_j)\}. \quad (16)$$

For $\forall s_i \subset T^{(m)}$, $\forall s_j \subset \bar{N}^{(1)}$ $(m=1, 2, \cdots, 6)$

$$P(s_i, s_j) = P\{(X_{n,0}, \boldsymbol{X}_n) \in h_1(s_j)\} = P\{(\boldsymbol{X}_{n,0}, \boldsymbol{X}_n) \in (N, R_N)\}. \quad (17)$$

For $\forall s_i \subset T^{(D)}, D$

$$P(s_i, D) = 1 \quad (18)$$

$$P(D, D) = 1 \quad (19)$$

In the modified sampling system, we must add the following transition probabilities.

For $\forall s_i \subset T^{(m)}$, $\forall s_j \subset S^{(0)}$ $(m=0, 1, \cdots, 4)$

$$P(s_i, s_j) = P\{(X_{n,0}, \boldsymbol{X}_n) \in h_1(s_j)\} = P\{X_{n,0}, \boldsymbol{X}_n) \in (T, R_T). \quad (20)$$

For $\forall s_i \subset T^{(m)}$, $\forall s_j \subset S^{(m+1)}$ $(m=0, 1, \cdots, 4)$

$$P(s_i, s_j) = P\{(X_{n,0}, \boldsymbol{X}_n) \in h_{m+1}(s_j)\} = P\{(X_{n,0}, \boldsymbol{X}_n) \in (T, A_T)\}. \quad (21)$$

For $\forall s_i \subset S^{(m)},\ \forall s_j \subset S^{(0)}\ (m=0, 1, \cdots, 4)$

$$P(s_i, s_j)=P\{(X_{n,0}, \boldsymbol{X}_n) \in h_1(s_j)\}=P\{X_{n,0}, \boldsymbol{X}_n) \in (T, R_T)\}. \tag{22}$$

For $\forall s_i \subset S^{(5)},\ \forall s_j \subset N^{(1)}$

$$P(s_i, s_j)=P\{(X_{n,0}, \boldsymbol{X}_n)=h_1(s_j)\}. \tag{23}$$

For $\forall s_i \subset S^{(5)},\ \forall s_j \subset \overline{N}^{(1)}$

$$P(s_i, s_j)=P\{(X_{n,0}, \boldsymbol{X}_n) \in h_1(s_j)\}=P\{(X_{n,0}, \boldsymbol{X}_n) \in (N, R_n). \tag{24}$$

If the fraction defective of each lot is p (fixed), we put

$$P\{(X_{n,0}, \boldsymbol{X}_n)=(x_0, x_1, \cdots, x_k)\}$$
$$= \prod_{j=1}^{k} \binom{n(x_0)}{x_j} p^{x_j}(1-p)^{n(x_0)-x_j}. \tag{25}$$

Table 2. State Descriptions and Transition Probability Matrix.
MARKOV chain for MIL-STD-105 D System.

$$P = \begin{array}{c|c|c|c|c|}
\text{merged state} \backslash \text{merged state} & R & \overline{N}^{(1)}\ \overline{N}^{(2)}\ \overline{N}^{(1.1)} \cdots \overline{N}^{(1.3)}\ N^{(1)} \cdots N^{(10)} & T^{(1)} \cdots T^{(6)}\ T^{(D)} & D \\
\hline
R & \boldsymbol{R}_R & \boldsymbol{R}_N & \boldsymbol{O} & \boldsymbol{O} \\
\hline
\overline{N}^{(1)}, \overline{N}^{(2)}, \overline{N}^{(1.1)}, \cdots, \overline{N}^{(1.3)}, N^{(1)}, \cdots, N^{(10)} & \boldsymbol{N}_R & \boldsymbol{N}_N & \boldsymbol{N}_T & \boldsymbol{O} \\
\hline
T^{(1)}, \cdots, T^{(6)}, T^{(D)} & \boldsymbol{O} & \boldsymbol{T}_N & \boldsymbol{O} & \boldsymbol{T}_D \\
\hline
D & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & 1 \\
\hline
\end{array}
= \left\| \begin{array}{c|c} \boldsymbol{O}_Q & \boldsymbol{O}_D \\ \hline \boldsymbol{O} & 1 \end{array} \right\|$$

We can calculate the numerical values for the formulas (1)–(24) from the relation (25) over the effective range of percent defective p. Thus we can get the transition matrix $\boldsymbol{P}=(p_{ij})$. $\boldsymbol{P}$ is partitioned as in Table 2.

4. Analyses on the Effects of Severity Adjustment

4.1 Transition Probabilities determining the probability laws for the sequence $\boldsymbol{Z}_1, \boldsymbol{Z}_2, \cdots, \boldsymbol{Z}_n, \cdots$.

The purpose of this section is to give the probabilities $P_n(\mathfrak{N})$, $P_n(\mathfrak{R})$, $P_n(\mathfrak{T})$ and $P_n(D)$ on Normal, Reduced, Tightened and Discontinuation, where $\mathfrak{N}$, $\mathfrak{R}$ and $\mathfrak{T}$ are the unions of the merged states which are contained in $\{N\}\times\boldsymbol{L}_1\times\boldsymbol{L}_2\times\cdots\times\boldsymbol{L}_{10}$, $\{R\}\times\boldsymbol{L}_1\times\boldsymbol{L}_2\times\cdots\times\boldsymbol{L}_{10}$ and $\{T\}\times\boldsymbol{L}_1\times\boldsymbol{L}_2\times\cdots\times\boldsymbol{L}_{10}$, respectively.

Since normal inspection is generally used at the start in MIL-STD-105 D, we assign the initial probabilities $P\{(X_{0,1}, \boldsymbol{X}_1)=(N, \boldsymbol{x}_k)\}=p_{N,k}$ to $(N, \boldsymbol{x}_k, \boldsymbol{L}_2, \boldsymbol{L}_3, \cdots, \boldsymbol{L}_{10})$ for $\forall \boldsymbol{x}_k \in A_N$, and $P\{X_{0,1}, \boldsymbol{X}_1) \in (N, R_N)\}=p_{N,R}$ to $(N, R_N, \boldsymbol{L}_2, \boldsymbol{L}_3, \cdots, \boldsymbol{L}_{10})$, respectively. Thus, we have an initial probability vector

$$\text{merged state} \longrightarrow \quad R \;\; \bar{N}^{(1)}\, \bar{N}^{(2)}\, \bar{N}^{(1,1)} \cdots \bar{N}^{(1,3)} \quad N^{(1)} \quad N^{(2)} \cdots\cdots N^{(10)} \quad T^{(1)} \cdots\cdots T^{(6)} \quad T^{(D)}\, D$$

$$\boldsymbol{p}_0 = (0\,0\,p_{N,R}\,0 \cdots\cdots\cdots\cdots 0\,p_{N,1}\,p_{N,2} \cdots\cdots\cdots\cdots p_{N,M}\,0 \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots 0)$$

, where M means the number of all merged states in $N^{(1)}$.

Using the transition matrix $\boldsymbol{P}=(p_{ij})$ and the initial probability vector $\boldsymbol{p}_0$, we get the n-step probability vector $\boldsymbol{p}_n=\boldsymbol{p}_0\boldsymbol{P}^n=(p^{(n)}(i))$, $i \in S$, where S is the union of the merged states which are contained in $\{R, N, T\}\times\boldsymbol{L}_1\times\boldsymbol{L}_2\times\cdots\times\boldsymbol{L}_{10}$ and D. We put $P_n(\mathfrak{N})=\sum_{i\in\mathfrak{N}} p^{(n)}(i)$, $P_n(\mathfrak{R})=\sum_{i\in\mathfrak{R}} p^{(n)}(i)$, $P_n(\mathfrak{T})=\sum_{i\in\mathfrak{T}} p^{(n)}(i)$ and $P_n(D)$.

4.2 The First Entrance Time Distribution

The author presents here the methods to give the conditional probability of the first entrance to the merged state k in the n-step, given that the MARKOV chain was initially at the merged state i.

The conditional first entrance time distribution from $i \in \mathfrak{N}$ to $\mathfrak{R}$ is given by

$$\begin{aligned} {}_{\mathfrak{T}}f_{i\mathfrak{R}}^{(1)} &= p_{i\mathfrak{R}}/(1-p_{i\mathfrak{T}}), \\ {}_{\mathfrak{T}}f_{i\mathfrak{R}}^{(n)} &= \sum_{k\in\mathfrak{N}} \{1/(1-p_{i\mathfrak{T}})\} p_{ik},\, {}_{\mathfrak{T}}f_{k\mathfrak{R}}^{(n-1)}, \end{aligned} \tag{26}$$

where ${}_{\mathfrak{T}}f_{i\mathfrak{R}}^{(n)}$ is the first entrance probability from the merged state $i \in \mathfrak{N}$ to any merged state of $\mathfrak{R}$ at the n-step with the taboo set $\mathfrak{T}$.

Matrix formulation of (26) is as follows:

$$ {}_{\mathfrak{T}}\boldsymbol{F}(1)=({}_{\mathfrak{T}}f^{(1)}_{i\mathfrak{R}})=\boldsymbol{D}_{\mathfrak{T}}\,\boldsymbol{N}_{\mathfrak{R}}\,\mathbf{1}, $$

$$ {}_{\mathfrak{T}}\boldsymbol{F}(n)=({}_{\mathfrak{T}}f_{i\mathfrak{R}})=\boldsymbol{D}_{\mathfrak{T}}\,\boldsymbol{N}_{N}\,{}_{\mathfrak{T}}\boldsymbol{F}(n-1)\mathbf{1}=(\boldsymbol{D}_{\mathfrak{T}}\,\boldsymbol{N}_{N})^{n-1}\cdot(\boldsymbol{D}_{\mathfrak{T}}\,\boldsymbol{N}_{\mathfrak{R}})\mathbf{1}, \tag{27} $$

where $\mathbf{1}$ is the vector of the form $(1\ 1\cdots1)'$ and $\boldsymbol{D}_{\mathfrak{T}}$ is the diagonal matrix with the diagonal elements equal to $1/(1-p_{i\mathfrak{T}})$, $i\in\mathfrak{N}$.

The moment of order r of the conditional first entrance time distribution from $i\in\mathfrak{N}$ to $\mathfrak{R}$ is given by

$$ {}_{\mathfrak{T}}m^{(r)}_{i\mathfrak{R}}=\sum_{n=1}^{\infty}n^{r}\cdot{}_{\mathfrak{T}}f^{(n)}_{i\mathfrak{R}}=\{\sum_{j\in\mathfrak{R}}p_{ij}+\sum_{k\in\mathfrak{N}}p_{ik}\cdot\sum_{n=1}^{\infty}(n+1)^{r}\,{}_{\mathfrak{T}}f^{(n)}_{k\mathfrak{R}}\}/(1-p_{i\mathfrak{T}}). \tag{28} $$

Matrix formulation of (28) is as follows:

$$ {}_{\mathfrak{T}}\boldsymbol{M}^{(r)}=(\boldsymbol{E}-\boldsymbol{D}_{\mathfrak{T}}\,\boldsymbol{N}_{N})^{-1}\cdot\boldsymbol{D}_{\mathfrak{T}}\{\boldsymbol{N}_{R}\mathbf{1}+\boldsymbol{N}_{N}({}_{r}C_{0}\cdot{}_{\mathfrak{T}}\boldsymbol{M}^{(0)}+{}_{r}C_{1}\,{}_{\mathfrak{T}}\boldsymbol{M}^{(1)} + \cdots\cdots+{}_{r}C_{r-1}\,{}_{\mathfrak{T}}\boldsymbol{M}^{(r-1)})\}, \tag{29} $$

where $\boldsymbol{E}$ is the identity matrix.

The conditional first entrance time distribution from $i\in\mathfrak{N}$ to $\mathfrak{T}$ is given by

$$ {}_{\mathfrak{R}}\boldsymbol{F}(n)=({}_{\mathfrak{R}}f^{(n)}_{i\mathfrak{T}})=(\boldsymbol{D}_{\mathfrak{R}}\,\boldsymbol{N}_{N})^{n-1}\cdot(\boldsymbol{D}_{\mathfrak{R}}\,\boldsymbol{N}_{T})\cdot\mathbf{1}, \tag{30} $$

where $\boldsymbol{D}_{\mathfrak{R}}$ is the diagonal matrix with the diagonal elements equal to $1/(1-p_{i\mathfrak{R}})$, $i\in\mathfrak{N}$.

The moment of order r of the conditional first entrance time distribution from $i\in\mathfrak{N}$ to $\mathfrak{T}$ is given by

$$ {}_{\mathfrak{R}}m^{(r)}_{i\mathfrak{T}}=\{\sum_{k\in\mathfrak{T}}p_{ik}+\sum_{j\in\mathfrak{N}}p_{ij}\cdot\sum_{n=1}^{\infty}(n+1)^{r}\cdot{}_{\mathfrak{R}}f^{(n)}_{j\mathfrak{T}}\}/(1-p_{i\mathfrak{R}}). \tag{31} $$

Matrix formulation of (31) is as follows:

$$ {}_{\mathfrak{R}}\boldsymbol{M}^{(r)}=(\boldsymbol{E}-\boldsymbol{D}_{\mathfrak{R}}\,\boldsymbol{N}_{N})^{-1}\boldsymbol{D}_{\mathfrak{R}}\{\boldsymbol{N}_{T}\mathbf{1}+\boldsymbol{N}_{N}({}_{r}C_{0}\,{}_{\mathfrak{R}}\boldsymbol{M}^{(0)}+{}_{r}C_{1}\,{}_{\mathfrak{R}}\boldsymbol{M}^{(1)} + \cdots\cdots+{}_{r}C_{r-1}\,{}_{\mathfrak{R}}\boldsymbol{M}^{(r-1)})\}. \tag{32} $$

The conditional first entrance time distribution from $i\in\mathfrak{T}$ to $\mathfrak{R}$ is given by

$$ {}_{D}\boldsymbol{F}(1)=({}_{D}f^{(1)}_{i\mathfrak{R}})=\boldsymbol{T}_{N}\mathbf{1}. \tag{33} $$

The moment of order r of the conditional first entrance time distribution from $i\in\mathfrak{T}$ to $\mathfrak{R}$ is given by

$$ {}_{D}m^{(r)}_{i\mathfrak{R}}=\sum_{j\in\mathfrak{R}}p_{ij}. \tag{34} $$

Matrix formulation of (34) is as follows:

$$ {}_{D}\boldsymbol{M}^{(r)}=\boldsymbol{T}_{N}\mathbf{1}. \tag{35} $$

The conditional first entrance time distribution from $i\in\mathfrak{T}$ to D is given by

$$ {}_{\mathfrak{R}}\boldsymbol{F}(1)=({}_{\mathfrak{R}}f_{iD})=\boldsymbol{T}_{D}. \tag{36} $$

The moment of order r of the conditional first entrance time distribution from $i \in \mathfrak{T}$ to D is given by

$$ {}_{\mathfrak{R}}m_{iD}^{(r)} = p_{iD}. \tag{37}$$

Matrix formulation of (37) is as follows:

$$ {}_{\mathfrak{R}}\boldsymbol{M}^{(r)} = \boldsymbol{T}_D \mathbf{1}. \tag{38}$$

The first entrance time distribution from $i \in \mathfrak{R}$ to $\mathfrak{R}$ is given by

$$\boldsymbol{F}(n) = (f_{i\mathfrak{R}}^{(n)}) = \boldsymbol{R}_R^{n-1} \boldsymbol{R}_N \mathbf{1}. \tag{39}$$

The moment of order r of the first entrance time distribution from $i \in \mathfrak{R}$ to $\mathfrak{R}$ is given by

$$m_{i\mathfrak{R}}^{(r)} = \sum_{j \in \mathfrak{R}} p_{ij} + \sum_{k \in \mathfrak{R}} p_{ik} \sum_{n=1}^{\infty} ({}_rC_0 n^r + {}_rC_1 n^{r-1} + \cdots\cdots + {}_rC_r n^0) f_{k\mathfrak{R}}^{(n)}.$$

$$m_{i\mathfrak{R}}^{(r)} = 1 + \sum_{k \in \mathfrak{R}} p_{ik} ({}_rC_0 m_{k\mathfrak{R}}^{(r)} + {}_rC_1 m_{k\mathfrak{R}}^{(r-1)} + \cdots\cdots + {}_rC_{r-1} m_{k\mathfrak{R}}^{(1)}). \tag{40}$$

Matrix formulation of (40) is as follows:

$$\boldsymbol{M}^{(r)} = (\boldsymbol{E} - \boldsymbol{R}_R)^{-1} \{ \mathbf{1} + \boldsymbol{R}_R ({}_rC_1 \boldsymbol{M}^{(r-1)} + {}_rC_2 \boldsymbol{M}^{(r-2)} + \cdots\cdots + {}_rC_{r-1} \boldsymbol{M}^{(1)}) \}. \tag{41}$$

The first entrance time distribution frome $i \in S-D$ to D is given by

$$\boldsymbol{F}(n) = (f_{iD}^{(n)}) = \boldsymbol{Q}_Q^{n-1} \boldsymbol{Q}_D. \tag{42}$$

The moment of order r of the first entrance time distribution from $i \in S-D$ to D is given by

$$m_{iD}^{(r)} = 1 + \sum_{k \in S-D} p_{ik} ({}_rC_0 m_{kD}^{(r)} + {}_rC_1 m_{kD}^{(r-1)} + \cdots\cdots + {}_rC_{r-1} m_{kD}^{(1)}). \tag{43}$$

Matrix formulation of (43) is as follows:

$$\boldsymbol{M}^{(r)} = (\boldsymbol{E} - \boldsymbol{Q}_Q)^{-1} \{ \mathbf{1} + \boldsymbol{Q}_Q ({}_rC_1 \boldsymbol{M}^{(r-1)} + {}_rC_2 \boldsymbol{M}^{(r-2)} + \cdots\cdots + {}_rC_{r-1} \boldsymbol{M}^{(1)}) \}. \tag{44}$$

The probability that the first severity adjustment of inspection occurs at the n-th lot from a specified start is considered.

1. Normal $\rightarrow$ Reduced

We assign the initial probabilities $P\{(X_{0,1}, \boldsymbol{X}_1) = (N, \boldsymbol{x}_k)\} = p_{N,k}$ to $(N, \boldsymbol{x}_k, \boldsymbol{L}_2, \boldsymbol{L}_3, \cdots, \boldsymbol{L}_{10})$ for $\forall \boldsymbol{x}_k \in A_N$, and $P\{X_{0,1}, \boldsymbol{X}_1) \in (N, R_N)\} = p_{N,R}$ to $(N, R_N, \boldsymbol{L}_2, \boldsymbol{L}_3, \cdots, \boldsymbol{L}_{10})$, respectively. Thus, we have an initial probability vetor merged state $\bar{N}^{(1)}\ \bar{N}^{(2)}\ \bar{N}^{(1,1)} \cdots \bar{N}^{(1,3)}\ N^{(1)} \cdots\cdots\cdots\cdots N^{(2)} \cdots\cdots N^{(10)}$

$$\boldsymbol{p}_1=(p_{N,R}\,0\cdots\cdots\cdots\cdots 0\,p_{N,1}\,p_{N,2}\cdots\cdots\cdots\cdots p_{N,M}\,0\cdots\cdots\cdots 0).$$

$$_Tf_{N,R}^{(n)}=\sum_{i\in\mathfrak{N}} P(\boldsymbol{Z}_1=i)\,_{\mathfrak{T}}f_{i\mathfrak{N}}^{(n-1)}=\boldsymbol{p}_1'\,_{\mathfrak{T}}\boldsymbol{F}(n-1),$$

$$_Tm_{N,R}^{(r)}=\sum_{n=1}^{\infty} n^r\,_Tf_{N,R}^{(n)}=\sum_{n=1}^{\infty}\sum_{i\in\mathfrak{N}} n^r P(\boldsymbol{Z}_1=i)\,_{\mathfrak{T}}f_{i\mathfrak{N}}^{(n-1)}$$

$$=\sum_{i\in\mathfrak{N}} P(\boldsymbol{Z}_1=i)\sum_{k=1}^{r}{}_rC_k\,{}_{\mathfrak{T}}m_{i\mathfrak{N}}^{(k)}$$

$$=\sum_{k=1}^{r}{}_rC_k\,\boldsymbol{p}_1'\,_{\mathfrak{T}}\boldsymbol{M}^{(k)}$$

, where $_Tf_{N,R}^{(n)}$ means the first entrance probability from Normal to Reduced at the n-th sampling inspection with taboo inspection, Tightened.

2. Normal → Tightened

$$_Rf_{N,T}^{(n)}=\boldsymbol{p}_1'\,_{\mathfrak{R}}\boldsymbol{F}(n-1),$$

$$_Rm_{N,T}^{(r)}=\sum_{k=1}^{r}{}_rC_k\,\boldsymbol{p}_1'\,_{\mathfrak{R}}\boldsymbol{M}^{(k)}.$$

3. Tightened → Normal

The merged state function g is defined by the relation

$$\begin{aligned} g(s)&=m+4, && s\in T^{(m)}\cap\mathfrak{T},\\ &=10\quad, && s\in T^{(D)}\cap\mathfrak{T},\\ &=1\quad, && \text{otherwise.}\end{aligned}$$

We assign the initial probabilities $\prod_{k=1}^{m+4} P\{(X_{n,0},\boldsymbol{X}_n)\in h_k(s_i)\}$ to $s_i\in T^{(D)}\cap\mathfrak{T}$, and $P\{(X_{n,0},\boldsymbol{X}_n)\in h_k(s_j)\}$ to $s_j\in T^{(m)}\cap\mathfrak{T}$ $(m=1,2,\cdots,6)$, respectively.

$$_Df_{T,N}^{(g(S_j)+1)}=\sum_{s_j\in T^{(m)}\cap\mathfrak{T}} P(\boldsymbol{Z}_1\in s_j)/\{1-P(\boldsymbol{Z}_1\in T^{(D)}\cap\mathfrak{T})\},$$

$$_Dm_{T,N}^{(r)}=\sum_{m=1}^{6} g(s_j\in T^{(m)}\cap\mathfrak{T})\cdot P(\boldsymbol{Z}_1\in T^{(m)}\cap\mathfrak{T})/\{1-P(\boldsymbol{Z}_1\in T^{(D)}\cap\mathfrak{T})\}.$$

4. Tightened → Discontinuation

$$_Nf_{T,D}^{(11)}=\sum_{s_i\in T^{(D)}\cap\mathfrak{T}} P(\boldsymbol{Z}_1\in s_i)/\{1-\sum_{m=1}^{6} P(\boldsymbol{Z}_1\in T^{(m)}\cap\mathfrak{T})\}=1.$$

5. Reduced → Normal

We assign the initial probabilities $P\{(X_{0,1},\boldsymbol{X}_1)\in(R,A_R)\}=p_{R,A}$ to $(R, A_R, \boldsymbol{L}_2, \boldsymbol{L}_3,$

$\cdots, \boldsymbol{L}_{10})$, and $P\{(X_{0,1}, \boldsymbol{X}_1) \in (R, R_R)\} = p_{R,R}$ to $(R, R_R, \boldsymbol{L}_2, \boldsymbol{L}_3, \cdots, \boldsymbol{L}_{10})$. Thus, we have an initial probability vector $\boldsymbol{p}_2 = (p_{R,A}\ p_{R,R})$.

$$f_{R,N}^{(n)} = \sum_{i \in \mathfrak{R}} p(\boldsymbol{Z}_1 = i) f_{i\mathfrak{R}}^{(n-1)} = \boldsymbol{p}_2' \boldsymbol{F}(n-1),$$

$$m_{R,N}^{(r)} = \sum_{n=1}^{\infty} n^r \cdot f_{R,N}^{(n)} = \sum_{i \in \mathfrak{R}} P(\boldsymbol{Z}_1 = i) \cdot \sum_{k=1}^{r} {}_rC_k \cdot m_{i\mathfrak{R}}^{(k)} = \sum_{k=1}^{r} {}_rC_k\, \boldsymbol{p}_2' \boldsymbol{M}^{(k)}.$$

6. Normal → Discontinuation

The moment generating function $G_{N,D}$ of the distribution of the number of lots inspected from Normal to Discontinuation is given as follows:

$$G_{N,D} = \sum_{n=0}^{\infty} e^{\theta n} f_{N,D}^{(n)}$$

$$= \sum_{k=0}^{\infty} \left\{ \sum_{j=0}^{\infty} \frac{\theta^j}{j!} \sum_{n=0}^{\infty} n^j f_{N,T}^{(n)} \cdot \sum_{i=0}^{\infty} \frac{\theta^i}{i!} \sum_{n=0}^{\infty} n^i f_{T,N}^{(n)} \right\}^k$$

$$\cdot p^k \sum_{j=0}^{\infty} \frac{\theta^j}{j!} \sum_{n=0}^{\infty} n^j f_{N,T}^{(n)} \cdot \sum_{i=0}^{\infty} \frac{\theta^i}{i!} \sum_{n=0}^{\infty} n^i f_{T,D}^{(n)} \cdot q$$

$$= \sum_{k=0}^{\infty} \left\{ \sum_{j=0}^{\infty} \frac{\theta^j}{j!} m_{N,T}^{(j)} \sum_{i=0}^{\infty} \frac{\theta^i}{i!} m_{T,N}^{(i)} p \right\}^k \sum_{j=0}^{\infty} \frac{\theta^j}{j!} m_{N,T}^{(j)} \sum_{i=0}^{\infty} \frac{\theta^i}{i!} m_{T,D}^{(i)} q,$$

where $q = P(\boldsymbol{Z}_n \in T^{(D)} \cap \mathfrak{T} / \boldsymbol{Z}_{n-1} = \bar{N}^{(2)})$ and $p = 1 - q$.

4.3 Evaluation of the modified system of sampling plans

We can evaluate the system of modified sampling plans suggested by STEPHENES and LARSON from the following three aspects:

1. the expected proportion of lots inspected
2. the composite OC curve
3. the average sample number

Assuming that the fraction defective p remains constant, it is meaningful to study the limiting distribution. Under this assumption, it is not difficult to see that the MARKOV chain in question is irreducible and possesses a non-zero stationary distribution.

Let S^*, $\mathfrak{T}^*$ denote the union of all merged states which are contained in $L_0 \times \boldsymbol{L}_1 \times \cdots \times \boldsymbol{L}_{10}$, $\{T, D\} \times \boldsymbol{L}_1 \times \cdots \times \boldsymbol{L}_{10}$, respectively. By using the stationary probability $P(\boldsymbol{Z} = i)$ for $\forall i \in S^*$ and the merged state function g, we can derive the expected proportion of time which Tightened, Normal and Reduced are in effect. Namely,

$$P^*(\mathfrak{T}^*) = \sum_{i \in \mathfrak{T}^*} P(\boldsymbol{Z} = i) \cdot g(i) / \sum_{j \in S^*} P(\boldsymbol{Z} = j) \cdot g(j), \tag{45}$$

$$P^*(\mathfrak{N}) = \sum_{i \in \mathfrak{N}} P(\boldsymbol{Z} = i) \cdot g(i) / \sum_{j \in S^*} P(\boldsymbol{Z} = j) \cdot g(j). \tag{46}$$

$$P^*(\mathfrak{R}) = \sum_{i \in \mathfrak{R}} P(\boldsymbol{Z}=i) \cdot g(i) / \sum_{j \in S^*} P(\boldsymbol{Z}=j) \cdot g(j). \tag{47}$$

Let $n_{\mathfrak{T}}$, $n_{\mathfrak{N}}$, $n_{\mathfrak{R}}$ be the average sample number per lot for the tightened, normal and reduced sampling plans, respectively. Then we have

$$n_{\mathfrak{T}} = \sum_{m=1}^{k} m \cdot n(T) \cdot P\{(X_{n,0}, \boldsymbol{X}_n) \in (T, A_T^m \cup R_T^m)\}, \tag{48}$$

$$n_{\mathfrak{N}} = \sum_{m=1}^{k} m \cdot n(N) \cdot P\{(X_{n,0}, \boldsymbol{X}_n) \in (N, A_N^m \cup R_N^m)\}, \tag{49}$$

$$n_{\mathfrak{R}} = \sum_{m=1}^{k} m \cdot n(R) \cdot P\{(X_{n,0}, \boldsymbol{X}_n) \in (R, A_R^m \cup R_R^m)\}. \tag{50}$$

Let $E(n_c)$ denote the amount of inspection required per lot, then we have

$$E(n_c) = p^*(\mathfrak{T}^*) \cdot n_{\mathfrak{T}} + P^*(\mathfrak{N}) \cdot n_{\mathfrak{N}} + P^*(\mathfrak{R}) \cdot n_{\mathfrak{R}}. \tag{51}$$

Let $P_{\mathfrak{T}}$, $P_{\mathfrak{N}}$, $P_{\mathfrak{R}}$ be the proportions of lots expected to be accepted when using Tightened, Normal and Reduced, respectively. Let P_c be the composite proportion of lots expected to be accepted under the weighting of the proportionate use of the different sampling plans. Then we have

$$P_c = P^*(\mathfrak{T}^*) \cdot P_{\mathfrak{T}} + P^*(\mathfrak{N}) \cdot P_{\mathfrak{N}} + P^*(\mathfrak{R}) P_{\mathfrak{R}}. \tag{52}$$

These measures $P_{\mathfrak{T}}$, $P_{\mathfrak{N}}$, $P_{\mathfrak{R}}$ as functions of p denote usual OC functions for the respective sampling plans. The composite measure P_c as a function of p represents the composite OC function of the set of this sampling plans used as a system.

Acknowledgements

I would like to thank Prof. Heihachi Sakamoto, Keio University, for his many helpful suggestions and encouragements. Thanks are also extended to Prof. Kinji Takagi, Musashi Insitute of Technology, and Dr. Makoto Maejima, Keio University, for their many useful comments. It is pleasure to acknowledge my gratitude to Dr. Takeshi Koyama, Mitsubishi Electric Company Inc., for his kind advices and stimulating papers.

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