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THE BOUNDARY LAYER EQUATION $x'''+2xx''+2\lambda(1-x'^2)=0$ for $\lambda>-0.19880$

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ABSTRACT

The purpose of this paper is to show the existence of continuous solutions for the equation described in the title satisfying the conditions $x(0)=x'(0)=0, x'(\infty)=1$ and $0<x'(t)<1$ for $0<t<\infty$.

This paper concerns with the following boundary value problem which occurs in laminar boundary layer theory in hydrodynamics:

$$x'''+2xx''+2\lambda(1-x'^2)=0, \tag{1}$$

$$x(0)=x'(0)=0, \quad x'(\infty)=1, \tag{2}$$

$$0<x'(t)<1 \quad \text{for} \quad 0<t<\infty. \tag{3}$$

For $\lambda\geqq 0$ H. WEYL (1942) first proved that there exists a continuous solution of the problem.

For $\lambda<0$ ($|\lambda|$ small) S. P. HASTINGS (1971) first showed the existence of solutions as far as the authors know.

On the other hand, it is known (I. TANI (1957)) that the separation phenomenon of boundary layer occurs for $\lambda=-0.1988\cdots$, and M. IWANO (1974) tried to show the existence of solutions for negative λ as small as possible.

In this paper we shall extend the value of such λ as closely as possible to the value $-0.1988\cdots$.

Our method of proof, which is close to that of W. A. COPPEL (1960), owes to Kneser's property, which was shown by M. HUKUHARA (1967). Although we can

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solve this problem by using the continuity dependence property of solutions to initial data, because the equation (1) has the property that the solution for an initial value problem is unique, our proof was found by examining the paper of M. HUKUHARA (1967).

We shall state Kneser's property in Section 1 and apply it to our problem in Section 2.

It should be noticed that our method could be applied for not only negative λ but also every λ (>-0.19880, for example).

1. Kneser's property by M. HUKUHARA (1967)

Let $\mathcal{F}$ be a family of n-vector valued continuous functions (or curves in $R\times R^n$) satisfying the following conditions:

(A) Each curve x (or $\{(t, x(t)); t\in I_x\}$) of $\mathcal{F}$ is a graph of an n-vector valued continuous function defined on a compact definition interval I_x.

(B) If x belongs to $\mathcal{F}$, every partial arc ($x|_I$: the restriction of x to a compact sub-interval I of I_x) belongs to $\mathcal{F}$.

(C) $\mathcal{F}$ is compact in the metric space $\mathrm{D}=\mathrm{D}(R^{n+1})$ of all compact sets in R^{n+1}, where the distance $\mathrm{Dist}(A, B)$, $A, B\in\mathrm{D}$, is defined by

$$\mathrm{Dist}(A, B)=\inf\{\delta>0;\ O_\delta(A)\supset B, O_\delta(B)\supset A\},$$

$$O_\delta(A)=\{P\in R^{n+1};\ \mathrm{dist}(P, A)\leqq\delta\}.$$

(D) If x and y of $\mathcal{F}$ assume a same value for $t=\alpha$, the function which coincides with x for $t\leqq\alpha$ and with y for $t\geqq\alpha$ belongs to $\mathcal{F}$.

(E) The extreme points of maximal (with respect to the definition interval) curves (which can be shown to exist) belong to the boundary $\mathcal{B}=\partial\mathcal{D}$ of $\mathcal{D}$, where $\mathcal{D}$ is the compact set in R^{n+1} filled by the curves of $\mathcal{F}$.

The right (left) extreme point $(\beta_x, x(\beta_x))$ $(\alpha_x, x(\beta_x))$ of a maxial curve

$$x=\{(t, x(t));\ t\in I_x=[\alpha_x, \beta_x]\}$$

will be called a right (left) extreme point of $\mathcal{D}$. They form a set which we call the right (left) boundary denoted by $\mathcal{B}^r(\mathcal{B}^l)$. We define the right emission zone $\mathcal{Z}^+(\mathcal{E})$ of $\mathcal{E}(\subset\mathcal{D})$ by

$$\mathcal{Z}^+(\mathcal{E})=\{(t, x(t));\ x\in\mathcal{F} \text{ for which there exists } t_0\in I_x \text{ such that } (t_0, x(t_0))\in\mathcal{E} \text{ and } t\geqq t_0\}.$$

The set of points $P\in\mathcal{B}\setminus\mathcal{B}^r$ such that P is an isolated point in $\mathcal{B}\cap\mathcal{Z}^+(P)$ will be denoted by $\mathcal{B}^+$.

The set of points $P\in\mathcal{B}\setminus\mathcal{B}^r$ such that P is an accumulation point of $\mathcal{B}\cap\mathcal{Z}^+(P)$ will be denoted by $\mathcal{B}_+$. Then $\mathcal{B}$ is expressed as the disjoint sum of $\mathcal{B}^r$, $\mathcal{B}^+$ and $\mathcal{B}_+$, that is,

$$\mathcal{B}=\mathcal{B}^r\cup\mathcal{B}^+\cup\mathcal{B}_+.$$

A point $A(\alpha, a)\in\mathcal{D}\subset R\times R^n$ will be called a right Kneser point if it satisfies one of the following conditions:

(I) $A\in\mathcal{B}^r$.

(II) If $A\in\mathcal{B}^+\cup\mathcal{D}^o$ (the interior of $\mathcal{D}$), then the section of $\mathcal{Z}^+(A)$ by the hyperplane $t=\tau$ is a continuum for sufficiently small $\tau-\alpha>0$.

(III) If $A\in\mathcal{B}_+$, the union

$$S\cup(\mathcal{B}\cap\mathcal{Z}_\tau^+(A))$$

is a continuum for sufficiently small $\tau-\alpha>0$, where

$$\mathcal{Z}_\tau^+(A)=\{(t, x(t))\in\mathcal{Z}^+(A);\ t\leqq\tau\}$$

and

$$S=\{(t, x(t))\in\mathcal{Z}^+(A);\ t=\tau\}.$$

If the above family $\mathcal{F}$ further satisfies the following condition (F), then $\mathcal{F}$ will be called a right Kneser family:

(F) Each point of $\mathcal{D}$ is a right Kneser point and $\mathcal{B}^+$ is open in $\mathcal{B}$ and contained in $\mathcal{B}^l$.

M. Hukuhara (1967) proved the following fundamental

Theorem 0. *If $\mathcal{F}$ is a right Kneser family, then the intersection $\mathcal{Z}^+(\mathcal{E})\cap(\mathcal{B}^r\cup\mathcal{B}_+)$ is a continuum, when $\mathcal{E}$ is a continuum in $\mathcal{D}$.*

2. An Existence Theorem of Solutions

Theorem 1. *If $\lambda>-1/6$, then the equation (1) has a continuous solution satisfying (2), (3).*

Let $x(t)$ be a solution of (1) on some t-interval satisfying $x'(t)>0$ (so that $x(t)$ is increasing). We choose x as a new independent variable and $y=x'^2$ as a new dependent variable (D. Grohne and R. Iglisch (1945)). Then we have

$$\frac{d}{dt}=x'\frac{d}{dx}=y^{\frac{1}{2}}\frac{d}{dx},\quad x''=\frac{1}{2}\dot{y},\quad x'''=\frac{1}{2}y^{\frac{1}{2}}\ddot{y},$$

where a dot denotes differentiation with respect to x. The equation (1) is transformed into

$$(4)\qquad y^{\frac{1}{2}}\ddot{y}+2x\dot{y}+4\lambda(1-y)=0,\quad \dot{y}=\frac{dy}{dx},$$

the boundary condition (2) into

$$(5)\qquad y(0)=0,\quad y(\infty)=1$$

and the condition (3) into

$$0<y(x)<1 \quad \text{for} \quad 0<x<\infty. \tag{6}$$

The equation (4) is equivalent to the system

$$\dot{y}=z, \tag{7}$$

$$\dot{z}=-y^{-\frac{1}{2}}\{2xz+4\lambda(1-y)\}\equiv f(x, y, z).$$

Consequently, Theorem 1 is replaced by the following

Theorem 1′. *If* $\lambda>-\frac{1}{6}$, *then* (4) (*or* (7)) *has a continuous solution satisfying* (5), (6).

To prove this we will first prove the following

Lemma 1. *Let* $\lambda<0$, $0<\alpha\leqq 1$ *and* $0<\beta<1$. *If* $\gamma>0$ *is sufficiently large, the solution* $y(x)$ *of* (4) *determined by*

$$y(\alpha)=\beta, \quad \dot{y}(\alpha)=\gamma$$

satisfies that $\dot{y}(x)\geqq 2$ *as long as* $\alpha\leqq x\leqq\alpha+1$ *and* $\beta<y(x)<1$.

Proof. Write the equation (4) as

$$\ddot{y}=-4\lambda y^{-\frac{1}{2}}(1-y)+4y^{\frac{1}{2}}-4\frac{d}{dx}\left(xy^{\frac{1}{2}}\right).$$

Along the arc $y(x)$ with $\beta\leqq y(x)\leqq 1$ we have

$$\ddot{y}\geqq -4\frac{d}{dx}\left(xy^{\frac{1}{2}}\right)$$

and hence a quadrature gives that

$$\begin{aligned}\dot{y}(x)&\geqq\gamma-4xy^{\frac{1}{2}}+4\alpha\beta^{\frac{1}{2}}\\ &\geqq\gamma-4(\alpha+1)\geqq\gamma-8\end{aligned}$$

for $\alpha\leqq x\leqq\alpha+1$. Hence, if $\gamma\geqq 10$, the assertion of Lemma 1 is true.

Remark. The property similar to that of Lemma 1 holds for not only negative λ but also for every λ.

Proof of Theorem 1′. Let us define three continuous functions $\underline{\Omega}(x, y)$, $\bar{\Omega}(x, y)$ and $\omega(x)$ as follows:

$$\underline{\Omega}(x, y)=2xy^{-\frac{1}{2}}(1-y) \qquad (x>0,\ 0<y\leqq 1),$$

$$\bar{\Omega}(x, y)=2x^{-1}y \qquad (x>0,\ 0<y\leqq 1),$$

$\omega(x)$ is defined implicitly by

$$x^2\omega^{-\frac{3}{2}}(1-\omega)=N \quad (x>0,\ 0<\omega<1), \tag{8}$$

where N is a constant with $\frac{2}{3}<N<1+2\lambda$. Differentiation of (8) gives

$$(9)\qquad \dot{\omega}(x)=4N^{-\frac{1}{2}}\omega^{\frac{1}{4}}\frac{(1-\omega)^{\frac{3}{2}}}{3-\omega}.$$

We shall show that if $0<x<\infty$ and $\omega(x)\leqq y<1$, then

$$(10)\qquad \underline{\Omega}(x,y)<\bar{\Omega}(x,y),\quad \underline{\Omega}(x,y)<2,$$

$$(11)\qquad \underline{\Omega}_x(x,y)+\underline{\Omega}_y(x,y)\underline{\Omega}(x,y)-f(x,y,\underline{\Omega}(x,y))>0,$$

$$(12)\qquad \bar{\Omega}_x(x,y)+\bar{\Omega}_y(x,y)\bar{\Omega}(x,y)-f(x,y,\bar{\Omega}(x,y))>0.$$

In fact,

$$\underline{\Omega}(x,y)\leqq\underline{\Omega}(x,\omega)=2N^{\frac{1}{2}}\omega^{\frac{1}{4}}(1-\omega)^{\frac{1}{2}}<2\omega^{\frac{1}{4}}(1-\omega)<2,$$

$$\bar{\Omega}(x,y)\geqq\bar{\Omega}(x,\omega)=2N^{-\frac{1}{2}}\omega^{\frac{1}{4}}(1-\omega)^{\frac{1}{2}}>2\omega^{\frac{1}{4}}(1-\omega),$$

hence (10) is valid;

$$\begin{aligned}\Omega_x+\Omega_y\Omega-f&=2y^{-\frac{1}{2}}(1-y)\{1+2\lambda-x^2y^{-\frac{3}{2}}(1-y)\}\\&\geqq 2y^{-\frac{1}{2}}(1-y)\{1+2\lambda-x^2\omega^{-\frac{3}{2}}(1-\omega)\}\\&=2y^{-\frac{1}{2}}(1-y)(1+2\lambda-N)>0,\end{aligned}$$

$$\begin{aligned}\bar{\Omega}_x+\bar{\Omega}_y\bar{\Omega}-f&=2x^{-2}y+4y^{\frac{1}{2}}+4\lambda y^{-\frac{1}{2}}(1-y)\\&>2x^{-2}\omega+4\lambda\omega^{-\frac{1}{2}}(1-\omega)\\&=2x^{-2}\omega+4\lambda Nx^{-2}\omega\\&=2(1+2\lambda N)x^{-2}\omega>0,\end{aligned}$$

hence (11), (12) are valid.

Along the arc $\omega(x)$ we have

$$\begin{aligned}\underline{\Omega}(x,\omega(x))-\dot{\omega}(x)&=2N^{\frac{1}{2}}\omega^{\frac{1}{4}}(1-\omega)^{\frac{1}{2}}-4N^{-\frac{1}{2}}\omega^{\frac{1}{4}}\frac{(1-\omega)^{\frac{3}{2}}}{(3-\omega)}\\&=2N^{-\frac{1}{2}}\omega^{\frac{1}{4}}(1-\omega)^{\frac{1}{2}}\left(N-\frac{2(1-\omega)}{3-\omega}\right)\\&\geqq 2N^{-\frac{1}{2}}\omega^{\frac{1}{4}}(1-\omega)^{\frac{1}{2}}\left(N-\frac{2}{3}\right)>0\end{aligned}$$

and hence

$$(13)\qquad \underline{\Omega}(x,\omega(x))>\dot{\omega}(x)\quad(0<x<\infty).$$

Let $\{\alpha_n\}$ $\{\beta_n\}$ be any sequence such that

$$0<\alpha_n<\frac{1}{5}, \qquad \beta_n=5\alpha_n,$$

$$\lim_{n\to\infty}\alpha_n=0.$$

For the moment we will fix n and define a continuous fuction $\omega_n(x)$ by

$$\omega_n(x)=\begin{cases}\beta_n & (\alpha_n\leqq x\leqq c_n),\\ \omega(x) & (c_n\leqq x<\infty),\end{cases}$$

where $c_n(>\alpha_n)$ is uniquely determined by the condition $\omega(c_n)=\beta_n$.

For an arbitrary $u(>\alpha_n+1)$ we will define subsets in the (x, y, z)-space as follows:

$$S_0=\{(x, y, z);\ \alpha_n\leqq x<u,\ y=1,\ z=0\},$$

$$S_1=\{(x, y, z);\ x=\alpha_n,\ \omega_n(x)<y<1,\ \underline{\Omega}(x, y)<z<\bar{\Omega}(x, y)\},$$

$$S_2=\{(x, y, z);\ \alpha_n\leqq x<u,\ \omega_n(x)\leqq y<1,\ z=\bar{\Omega}(x, y)\},$$

$$S_3=\{(x, y, z);\ \alpha_n\leqq x<u,\ y=\omega_n(x),\ \underline{\Omega}(x, y)<z<\bar{\Omega}(x, y)\},$$

$$S_4=\{(x, y, z);\ \alpha_n\leqq x<u,\ \omega_n(x)\leqq y<1,\ z=\underline{\Omega}(x, y)\},$$

$$S_5=\{(x, y, z);\ \alpha_n\leqq x<u,\ y=1,\ \underline{\Omega}(x, y)<z\leqq\bar{\Omega}(x, y)\},$$

$$S_6=\{(x, y, z);\ x=u,\ \omega_n(x)\leqq y\leqq 1,\ \underline{\Omega}(x, y)\leqq z\leqq\bar{\Omega}(x, y)\}.$$

$\mathcal{D}$ is the compact domain in the (x, y, z)-space surrounded by $S_0, S_1,\cdots, S_6$. Hence, the boundary $\mathcal{B}=\partial\mathcal{D}$ of $\mathcal{D}$ is $\bigcup_{i=0}^{6}S_i$, which is a disjoint sum. Let $\mathcal{F}$ be the set of all continuous functions satisfying (4), which are defined on compact intervals and the graphs of which are contained in $\mathcal{D}$.

We shall show that the set $\mathcal{F}$ forms a right Kneser family. To see this we have only to confirm the condition (F). Since the equation (4) has the uniqueness property for an initial value problem, we have that

$$S_0\subset\mathcal{B}_+.$$

The inequality (11) shows that

$$S_4\subset\mathcal{B}^r$$

and (12), (13) show that

$$S_2\cup S_3\subset\mathcal{B}^+\cap\mathcal{B}^l.$$

It is clear that

$$S_1\subset\mathcal{B}^+\cap\mathcal{B}^l, \qquad S_5\cup S_6\subset\mathcal{B}^r.$$

Hence, we have that

$$\mathcal{B}^+=S_1\cup S_2\cup S_3, \qquad \mathcal{B}_+=S_0,$$

$$\mathcal{B}^r=S_4\cup S_5\cup S_6, \qquad \mathcal{B}^+\subset\mathcal{B}^l$$

and $\mathcal{B}^+$ is an open subset of $\mathcal{B}$. Consequently, it was shown that $\mathcal{F}$ is a right Kneser family.

The set

$$\mathcal{E}=\{(\alpha_n, \beta_n, z);\ \underline{Q}(\alpha_n, \beta_n)\leqq z\leqq\bar{Q}(\alpha_n, \beta_n)=10\}$$

is a continuum in $\mathcal{D}$, so that, by Theorem 0

$$C(u)\equiv\mathcal{Z}^+(\mathcal{E})\cap(S_0\cup S_4\cup S_5\cup S_6)$$

is a continuum in $\mathcal{D}$ (hence, in $\mathcal{B}$). We have by Lemma 1 that

$$\mathcal{Z}^+(\mathcal{E})\cap S_5\neq\phi.$$

On the other hand, we have by the uniqueness property of (4) for an initial value problem that

$$\mathcal{Z}^+(\mathcal{E})\cap S_0=\phi.$$

The point $(\alpha_n, \beta_n, \underline{Q}(\alpha_n, \beta_n))$ belongs to $\mathcal{Z}^+(\mathcal{E})\cap S_4$. Therefore, we have by the connectedness of $C(u)$ that $\mathcal{Z}^+(\mathcal{E})\cap S_6$ is a nonempty compact set and therefore the set

$$H(u)\equiv\{P\in\mathcal{E};\ \mathcal{Z}^+(P)\cap S_6\neq\phi\}$$

is also a nonempty compact set. Furthermore, we have by the property of $\mathcal{F}$ that

$$H(u_1)\subset H(u_2) \quad \text{if} \quad u_1>u_2.$$

Hence, the family

$$\{H(u);\ u>\alpha_n+1\}$$

has a finite intersection property and hence

$$H_n=\cap\{H(u);\ u>\alpha_n+1\}$$

is nonvoid. Consequently, if $(\alpha_n, \beta_n, \gamma_n)\in H_n$, then the solution $y_n(x)$ of (4) determined by

$$y(\alpha_n)=\beta_n, \qquad \dot{y}(\alpha_n)=\gamma_n$$

exists on $\alpha_n\leqq x<\infty$ and satisfies $y_n(\infty)=1$.

For each $\alpha>0$ the sequences $\{y_n(x)\}$ and $\{\dot{y}_n(x)\}$ are equi-bounded on $[\alpha, \infty)$. By using Ascoli-Arzela's theorem and an usual diagonalization arguement we can choose a subsequence $\{y_{n_k}(x)\}$ which converges uniformly to a continuous function $y(x)$ on $(0, \infty)$. This function $y(x)$ can be shown to satisfy (4), (5) and (6).

3. More precise estimates for λ

In order to have a more precise estimate for λ we shall construct $\underline{\Omega}(x, y)$ by the following form

$$\underline{\Omega}(x, y)=2xy^{-\frac{1}{2}}(1-y)u(y),$$

where $u(y)$ is a continuous function on $0\leqq y\leqq 1$ such that the following conditions are satisfied:

(i) C^1 and piecewise C^2 on $0<y<1$,

(ii) $1\leqq u\leqq 2$, $u'\leqq 0$ on $0<y<1$,

$u(1)=1$, $\lim_{y\to 1}(1-y)u'(y)=0$,

(iii) $g(y)>0$ on $0<y<1$,

(iv) there exists a constant N such that $N>h(y)$ on $0\leqq y\leqq 1$ and $-\frac{1}{2}<N<0$.

Here, v, g, h are defined as follows:

$$v(y)=1+\frac{1+y}{1-y}(u-1)-2yu' \quad (\geqq 1 \text{ from (ii)}),$$

$$g(y)=\frac{3-y}{1-y}-2y\left(\frac{u'}{u}+\frac{v'}{v}\right),$$

$$h(y)=\frac{1}{g(y)}\left\{v+u\left(y\frac{v'}{v}-\frac{3-y}{2(1-y)}\right)\right\}.$$

In this chapter $\bar{\Omega}(x, y)$, $\omega(x)$ are chosen as follows:

$$\bar{\Omega}(x, y)=4x^{-1}y,$$

and $y=\omega(x)$ is defined implicitly by

$$x^2=\frac{y^{\frac{3}{2}}\left(1+\frac{2N}{u}\right)}{(1-y)v(y)}.$$

Differentiation of this relation with respect to y gives

$$2x\frac{dx}{dy}=\frac{y^{\frac{1}{2}}}{(1-y)u}\left\{\frac{g(y)}{v}(N-h(y))+1\right\}.$$

Hence, we have

$$\frac{dx}{dy}>0$$

and from (ii)

$$(1-y)v(y) \to 0 \quad \text{as} \quad y \to 1.$$

Consequently, $y=\omega(x)$ is surely a continuous function defined for $0<x<\infty$ and

$$y \to 1 \quad \text{as} \quad x \to \infty.$$

Then, for $\lambda>N$, $\underline{\Omega}$, $\bar{\Omega}$, ω shall be shown to satisfy (10′), (11), (12), (13), where

(10′) $$\underline{\Omega}(x,y)<\bar{\Omega}(x,y), \quad \bar{\Omega}(x,y)<4$$

(In Lemma 1, if $\gamma\geqq 12$, then $\dot{y}(x)\geqq 4$).

(10′)
$$\underline{\Omega}(x,y)\leqq\underline{\Omega}(x,\omega)=2\frac{\omega^{\frac{3}{4}}\sqrt{1+\frac{2N}{u}}}{\sqrt{1-\omega}\sqrt{v}}\omega^{-\frac{1}{2}}(1-\omega)u$$

$$<4\omega^{\frac{1}{4}}(1-\omega)^{\frac{1}{2}}<4,$$

$$\bar{\Omega}(x,y)\geqq\bar{\Omega}(x,\omega)=4\frac{\sqrt{1-\omega}\sqrt{v}}{\omega^{\frac{3}{4}}\sqrt{1+\frac{2N}{u}}}\omega\geqq 4\omega^{\frac{1}{4}}(1-\omega)^{\frac{1}{2}}$$

$$>\underline{\Omega}(x,y),$$

(11)
$$\underline{\Omega}_x+\underline{\Omega}_y\underline{\Omega}-f=2y^{-\frac{1}{2}}(1-y)u\left\{1+\frac{2\lambda}{u}-x^2y^{-\frac{3}{2}}(1-y)v\right\}$$

$$>2y^{-\frac{1}{2}}(1-y)u\left\{1+\frac{2N}{u}-(\omega^{-1}(y))^2y^{-\frac{3}{2}}(1-y)v\right\}$$

$$=0,$$

(12)
$$\bar{\Omega}_x+\bar{\Omega}_y\bar{\Omega}-f=12x^{-2}y+8y^{\frac{1}{2}}+4\lambda y^{-\frac{1}{2}}(1-y)$$

$$>12\frac{(1-\omega)v}{\omega^{\frac{3}{2}}\left(1+\frac{2N}{u}\right)}\omega+4\lambda\omega^{-\frac{1}{2}}(1-\omega)$$

$$=4\omega^{-\frac{1}{2}}(1-\omega)\left(\frac{3v}{1+\frac{2N}{u}}+\lambda\right)>0,$$

(13)
$$\underline{\Omega}(x,\omega(x))>\dot{\omega}(x)$$

$$\Leftrightarrow \quad 2xy^{-\frac{1}{2}}(1-y)u>\frac{1}{\frac{dx}{dy}} \qquad (y=\omega(x))$$

$$\Leftrightarrow \quad 2x\frac{dx}{dy}>\frac{y^{\frac{1}{2}}}{(1-y)u}$$

$$\Leftrightarrow \quad \frac{g(y)}{v}(N-h(y))+1>1$$

$$\Leftarrow \quad N>h(y),\ g(y)>0 \text{ and } v\geqq 1 \text{ (by (ii), (iii) and (iv))}.$$

If we can construct such u, we have by the proof quitely similar to that of Theorem 1′ that for $\lambda>N$ (1) has a continuous solution satisfying (2), (3).

For example, if

$$u(y)=1+0.18(1-y)^{\frac{1}{3}},$$

then we have $N=-0.1962$. Furthermore, we have $N=-0.19880$ if $u(y)$ is defined as follows:

$$u(y)=u_i(y) \quad \text{for} \quad t\in I_i \ (i=1,2,\cdots,10),$$

where

$$u_1(y)=1.192803-0.043576y-0.018938y^2-0.010436y^3-0.011921y^4,$$

$$u_2(y)=u_1(y)+0.000128(y-0.2)^3-0.012599(y-0.2)^4,$$

$$u_3(y)=u_2(y)+0.0010388(y-0.32)^3-0.0222989(y-0.32)^4,$$

$$u_4(y)=u_3(y)-0.0000747(y-0.44)^3-0.0409591(y-0.44)^4,$$

$$u_5(y)=u_4(y)+0.0019263(y-0.52)^3-0.080957(y-0.52)^4,$$

$$u_6(y)=u_5(y)+0.0064752(y-0.6)^3-0.193242(y-0.6)^4,$$

$$u_7(y)=u_6(y)+0.0054272(y-0.68)^3-0.431317(y-0.68)^4,$$

$$u_8(y)=u_7(y)+0.0015922(y-0.74)^3-0.825141(y-0.74)^4,$$

$$u_9(y)=u_{10}(y)+a(y-0.78)^2+b(y-0.78)+c,$$

$$u_{10}(y)=1+0.192(1-y^{1.087})^{0.2535-0.008y},$$

$$c=u_8(0.78)-u_{10}(0.78), \quad b=u_8'(0.78)-u_{10}'(0.78), \quad a=b^2/4c,$$

$$\alpha=-2c/b=0.866\cdots$$

and

$$I_1=[0,0.2), \quad I_2=[0.2,0.32), \quad I_3=[0.32,0.44), \quad I_4=[0.44,0.52),$$

$$I_5=[0.52,0.6), \quad I_6=[0.6,0.68), \quad I_7=[0.68,0.74), \quad I_8=[0.74,0.78),$$

$$I_9=[0.78,\alpha), \quad I_{10}=[\alpha,1].$$

Remark. Professor M. IWANO (1975) pointed out that the solution thus constructed in our paper is not of algebraic type (which means that $x'(t)-1$ tends to zero with the order of a certain negative power of t as $t\to\infty$). Hence, from a result of P. HARTMAN (1964) the solution is of exponential type (which means

that $x'(t)-1$ tends to zero exponentialy as $t\to\infty$). Consequently, from results of P. HARTMAN (1964) and R. IGLISCH and F. KEMNITZ (1955) we seem to have all solutions of (1) satisfying (2), (3) for $\lambda>-0.19880$.

Our consideration to this problem is still unsatisfactory. Further precise analysis is our forthcoming problem.

REFERENCES

COPPEL, W.A. (1960): On a differential equation of boundary-layer theory, Philos. Trans. Roy. Soc. London Ser. A 253, 101～136.

GROHNE, D. and IGLISCH, R. (1945): Die laminare Grenzschicht an der längsangeströmten ebenen Platte mit schrängen Absaugen und Ausblasen, Veröffentlichung des mathematischen Instituts der technischen Hochschule, Braunschweig, Bericht 1/45.

HARTMAN, P. (1964): On the asymptotic behavior of solutions of a differential eqution in boundary layer theory, Z. Angew. Math. Mech. 44, 123～128.

HARTMAN, P. (1964): Ordidary Differential Equation, John Wiley and Sons, Inc., 519～534.

HASTINGS, S.P. (1971): An existence theorem for a class of nonlinear boundary value problems including that of Falkner and Skan, Journal of Diff. Eqns. 9, 580～590.

HUKUHARA, M. (1967): Familles knesériennes et le problème aux limites pour l'èquation differentielle ordinaire du second ordre, I, Publ. RIMS, Kyoto Univ. Ser. A, 3, 243～270.

IGLISCH, R. and KEMNITZ, F. (1955): Über die in der Grenzschichttheorie auftretende Differentialgleichung $f'''+ff''+\beta(1-f'^2)=0$ für $\beta<0$ bei gewissen Absauge und Ausblasegesetzen, in '50 Jahre Grenzschichtforschungs' (H. GÖRTLER and W. TOLLMIEN, Eds.), Vieweg, Braunschweig.

IWANO, M. (1974): The boundary layer equation $f'''+2ff''+2\lambda(1-f'^2)=0$, $\lambda<0$, Boll. Un. Mat. Ital. 4(10), 1～15.

TANI, I. (1957): The theory of viscous fluids, Iwanami (in Japanese).

WEYL, H. (1942): On the differential equations of the simplest boundary-layer problems, Ann. of Math. 43, 381～407.