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ON VIBRATION OF A BODY OF CYLINDRICAL FORM, WHICH IS IMMERSED IN A FLUID REGION—II

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ABSTRACT

We consider a case of cylindrical body, whose cross-section is composed of closed curve made up by two circular arcs intersecting each other. Small transverse vibration of this body, which is immersed in a region of ideal fluid, is treated, as an application of the author's general theory.

1. Potential Flow around the Cylindrical Body, whose Cross-Section is made up of two Circular Arcs.

Referring to Fig. 1, two circular arcs which intersect each other, is shown in z-plane. Points of intersection of two arcs, P, Q. are assumed to be situated on

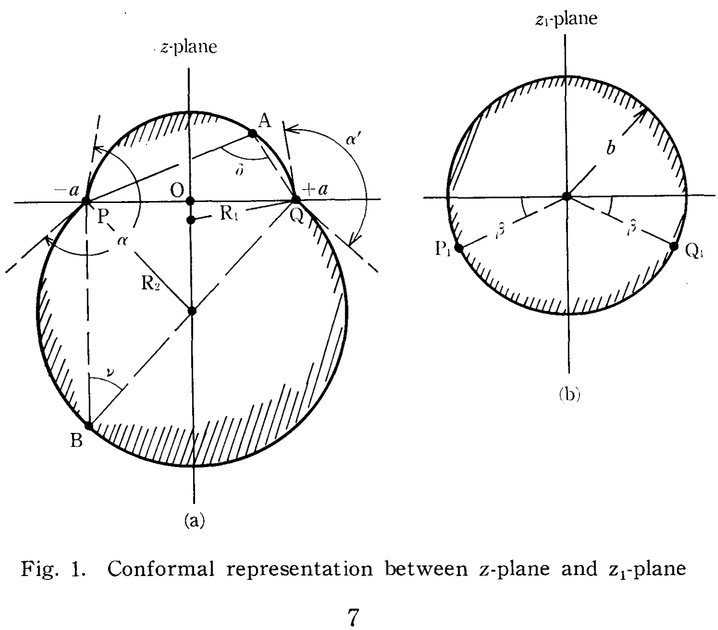


Fig. 1. Conformal representation between z-plane and z_1-plane

real axis, their ordinates being a and $-a$. Two arcs subtend angles $\angle PAQ=\delta$, $\angle PBQ=\nu$, respectively.

The conformal transformation, which transforms the inside (outside) of this closed curve in z-plane into inside (outside) of a circle (center at origin o_1, and radius b) in z_1-plane, will be given by relation of complex variables z and z_1-as follows;

$$\frac{z_1-Q_1}{z_1-P_1}=C\left[\frac{z-a}{z+a}\right]^n \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(1)$$

where C, n are constants. On any point on circle b, we have $z_1=be^{i\theta}$ and, especially,

$$P_1=be^{(\beta+\pi)i},\ Q=bc^{-i\beta}.$$

In order that the points at infinity $z_1\to\infty$ and $z\to\infty$ correspond each other, in such way that $z_1/z\to 1$, we must have $C=1$. Since we have

$$\frac{dz_1}{dz}=\frac{2an}{Q_1-P_1}\quad\frac{(z_1-Q_1)(z_1-P_1)}{(z-a)(z+a)}, \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots(2)$$

we have at

$$z\to\infty,\ z_1\to\infty;\ \frac{dz_1}{dz}\to\frac{2an}{Q_1-P_1}.$$

Therefore, we shall take

$$\frac{2an}{Q_1-P_1}=1, \quad \text{or} \quad an=b\cos\beta.$$

Geometrical relation concerning this figure (which is shown in Fig. 1 (a)), are seen to be as follows;

$$\left.\begin{aligned} &n\delta=\pi/2-\beta,\ n\nu=\frac{1}{2}(\pi+2\beta),\\ &\alpha=2\pi-(\nu+\delta),\\ &n=\frac{\pi}{\nu+\delta}=\frac{\pi}{2\pi-\alpha}=\frac{\pi}{\alpha'}, \end{aligned}\right\} \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(3)$$

α is the angle subtended (insidely) by two tangents to circular arcs at their meeting points P (or Q). Radii of two circular arcs are, respectively

$$R_1=\frac{a}{\sin\delta},\ R_2=\frac{-a}{\sin(\alpha+\delta)}.$$

Areas of two sectors are

$$A_1=\frac{a^2}{\sin\delta}\left[\frac{\theta_1}{\sin\delta}+\cos\delta\right],$$

$$A_2=\frac{a^2}{\sin(\alpha+\delta)}\left[\frac{\pi-\theta_2}{\sin(\alpha+\delta)}-\cos(\alpha+\delta)\right],$$

where

$$\theta_1=\pi-\delta,\quad \theta_2=2\pi-\alpha-\delta,$$

the total cross-sectional area being given by $A=A_1+A_2$.

In z_1-plane, suppose that the circular cylinder is immersed in a region of ideal fluid, which is flowing uniformly with velocity of $-U_\infty e^{i\lambda}$, the flow caused around the circular cylinder will be given by the following complex velocity potential

$$w_1(z_1)=U_\infty[\phi_1+i\psi_1]$$
$$=-U_\infty\left[z_1e^{-i\lambda}+\frac{b^2}{z_1e^{-i\lambda}}\right]. \qquad \cdots\cdots(4)$$

On the other hand, if the body represented by the figure of z-plane in Fig. 1 (a), is placed in the same incoming stream of $-U_\infty e^{-i\lambda}$, the flow around it will be given by function $w(z)=w_1(z_1)$, where we replace z_1 in expression (4) by $F(z)$, which is connected to z_1 by the relation (1), and which we may conveniently write $z_1=F(z)$.

Lastly, if the body in z-plane is moving with velocity $U_\infty e^{i\lambda}$, while the surrounding fluid is at rest at infinity, the flow around the body will be given by

$$w(z)=U_\infty[\phi_s+i\psi_s]$$
$$=-U_\infty\left[z_1e^{-i\lambda}+\frac{b^2}{z_1e^{-i\lambda}}\right]+U_\infty e^{-i\lambda}z \qquad \cdots\cdots(5)$$

wherein, we replace z_1 by $F(z)$.

2. Vibration of Cylindrical Body of Fig. 1 (a), which is Immersed in Fluid Region.

Let us consider the case in which body of Fig. 1 (a) is making a vibratory motion, with displacement as given by

$$\xi=\varepsilon\sin\omega t. \qquad \cdots\cdots(6)$$

ε is the amplitude (which is assumed to be very small), ω is the circular frequency, of vibration. The direction of this vibration is to take place in direction making an angle λ with the real axis. The fluid motion set up in this circumstance will be given by the expression (5), provided that we replace U_∞ by $d\xi/dt=\omega\varepsilon\cos\omega t$ assuming that the fluid motion is one of potential flow. The fluid pressure will be given by

$$p_s=\rho\omega^2\varepsilon\sin\omega t\cdot\phi_s,$$

provided that the amplitude of vibration ε is very small, and that the fluid is incompressible. The resultant force $(X_s,\ Y_s)$ acting on the body may be found by

$$Y_s - iX_s = \oint p_s dx, \quad \cdots\cdots (7)$$

where the contour integral extend to contour-line of the body (Fig. 1 (a)).

By the argument given in the author's previous paper, the expression (7) can be reduced into the following form;

$$Y_s - {}_iX_s = \rho\omega^2\varepsilon \sin \omega t.$$

$$\left[-\oint \left\{ z_1 e^{-i\lambda} + \frac{b^2}{z_1 e^{-i\lambda}} \right\} dz \right.$$

$$\left. + ie^{i\lambda} \oint xdy \right]. \quad \cdots\cdots (8)$$

The acceleration of motion of the body being

$$d^2\xi/dt^2 = -\omega^2\varepsilon \sin \omega t,$$

we put

$$\left. \begin{array}{l} X_s = M_x(A\rho\omega^2\varepsilon \sin \omega t), \\ Y_s = M_y(A\rho\omega^2\varepsilon \sin \omega t) \end{array} \right\}, \quad \cdots\cdots (9)$$

where A is cross-sectional area. Then we have

$$M_y - iM_x$$

$$= -\oint \left\{ z_1 e^{-i\lambda} + \frac{b^2}{z_1 e^{-i\lambda}} \right\} dz + ie^{i\lambda} \oint xdy, \quad \cdots\cdots (10)$$

If we know the relation

$$z_1 = z + c_0 + \frac{c_1}{z} + \frac{c_2}{z^2} \quad \cdots\cdots (11)$$

for very large value of $|z|$, $|z_1|$, we can evaluate contour integral in r. h. s. of eq. (10) and we obtain

$$\oint \left\{ z_1 e^{-i\lambda} + \frac{b^2}{z_1{}^{-i\lambda}} \right\} dz = 2\pi i\, [e^{-i\lambda} c_1 + b^2 e^{i\lambda}].$$

Also, we have

$$\oint xdy = A.$$

Thus, finally we obtain

$$M_x + iM_y = 2\pi \frac{c_1}{A} e^{-i\lambda} + \left(2\pi \frac{b^2}{A} - 1 \right) e^{i\lambda} \quad \cdots\cdots (12)$$

It will be seen that the vector (M_x, M_y) is not in direction of angle λ (which is direction of vibratory motion). The value of coefficient c_1 in eq. (11) can be found in the following manner; We have for a large value of $|z|$,

$$S=\left[\frac{z-a}{z+a}\right]^n=\left[\frac{1-(a/z)}{1+(a/z)}\right]^n$$

$$=1-2n\left(\frac{a}{z}\right)+2n^2\left(\frac{a}{z}\right)^2-\left\{n^3+\frac{1}{3}n(n^2+2)\right\}\left(\frac{a}{z}\right)^3+\cdots\cdots$$

and, putting this expression into eq. (1), we obtain

$$z_1=\frac{Q_1-SP_1}{1-S}=z\left[1+\left(n+\frac{P_1}{a}\right)\left(\frac{a}{z}\right)+\frac{1}{3}\left(n^2-1\right)\left(\frac{a}{z}\right)^2+\cdots\cdots\right],$$

whence we have

$$c_0=(na+P_1)=-ib\sin\beta,$$

$$c_1=\frac{1}{3}(n^2-1)a^2.$$

Numerical Examples

In illustration of our formula thus attained, we take three special cases as shown in Fig. 2 (a), (b), and (c).

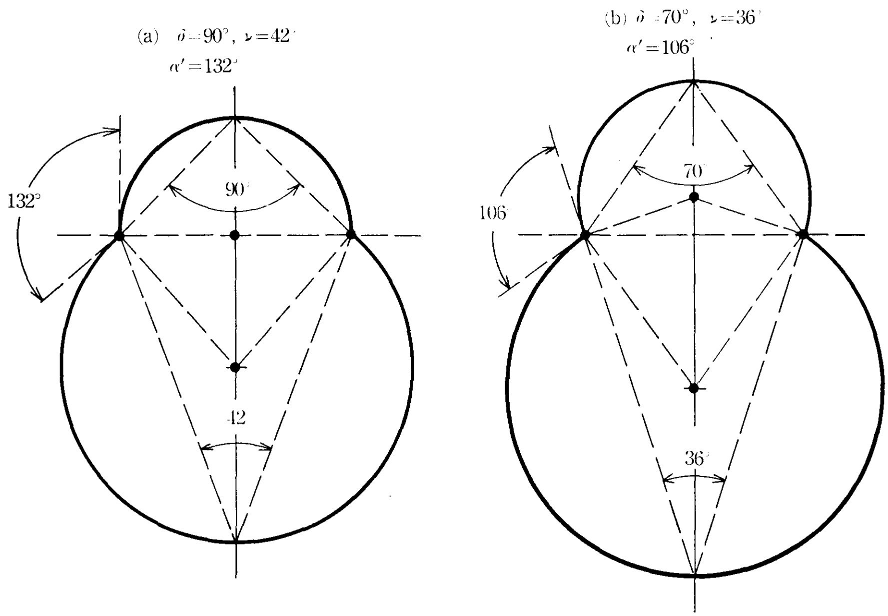


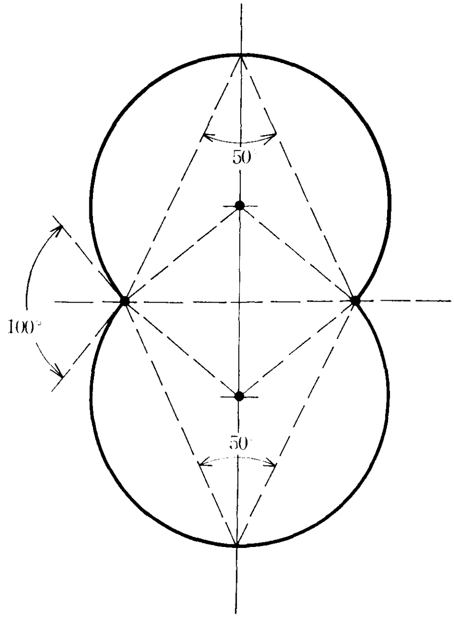
Fig. 2. Three examples of application of author's formula (12)

Respective values of (M_x, M_y) which we may call coefficients of virtual mass, are as follows;

(a) $\delta=90°$, $\nu=42°$, $\alpha'=132°$,

$\alpha=228°$ $n=1.365$

(c) $\delta=50°$, $\nu=50°$
$\alpha'=100°$



$(c_1=0.288a^2, \qquad A=8.07a^2)$

$M_x+iM_y=0.228e^{-i\lambda}+1.340e^{i\lambda}.$

(b) $\delta=70°, \qquad \nu=36°, \qquad \alpha'=106°$

$\alpha=254°, \qquad n=1.698$

$(c_1=0.630a^2, \qquad A=11.21a^2)$

$M_x+iM_y=0.353e^{-i\lambda}+1.12e^{i\lambda}.$

(c) $\delta=50°, \qquad \nu=50°, \qquad \alpha'=100°,$

$\alpha=260°, \qquad n=1.800$

$(c_1=0.746a^2, \qquad A=9.40a^2)$

$M_x+iM_y=0.498e^{-i\lambda}+1.16e^{i\lambda}.$

3. Concluding Remark.

In two successive papers thus developed, the author has given the general formula for coefficients of virtual mass for cylindrical bodies which are vibrating in a region of ideal fluid. Cross-section of cylindrical body is composed of closed contour which may be either a smooth closed curve or two-circular-arc figure. It is to be noted that, in general, direction of vector (M_x, M_y) of virtual mass do not coincide with the direction of vibration of the body. Only in case of the body in symmetrical sectional form, the coincidence may take place, if it is vibrating in direction of axis of symmetry of the figure.

Additional Note: In the above treatment, we have confined ourselves to case of small vibrations. But, it can be extended to case of vibrations of finite amplitude, as we shall see later on.

References

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