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ON VIBRATION OF A BODY OF CYLINDRICAL FORM, WHICH IS IMMERSED IN A FLUID REGION—I

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ABSTRACT

We take up a body of cylindrical form, whose cross-section is made up of any smooth closed curve, and which is immersed in a region of ideal fluid. We suppose that this body is making transverse vibration of small amplitude. Regarding this case as a problem of two dimensional flow, and using complex velocity potential, the author has deduced general formula which gives us approximate value of resultant force of fluid pressure acting on surface of this body.

1. General Consideration

We consider two-dimensional motion of an ideal fluid. The fluid flow can be represented by complex velocity potential $w_1(z_1)=\phi_1+i\psi_1$, where $z_1=x_1+iy_1$ is complex variable. When a circular cylinder of radius b (its center being at origin of complex plane z_1) is placed in a uniform flow (of velocity $-U$, in direction of angle λ with x_1-axis) around it, the fluid flow thus set up may be given by

$$\begin{aligned} w_1(z_1)&=\phi_1+i\psi_1 \\ &=-U\left[z_1e^{-i\lambda}+\frac{b^2}{z_1e^{-i\lambda}}\right]. \qquad (1) \end{aligned}$$

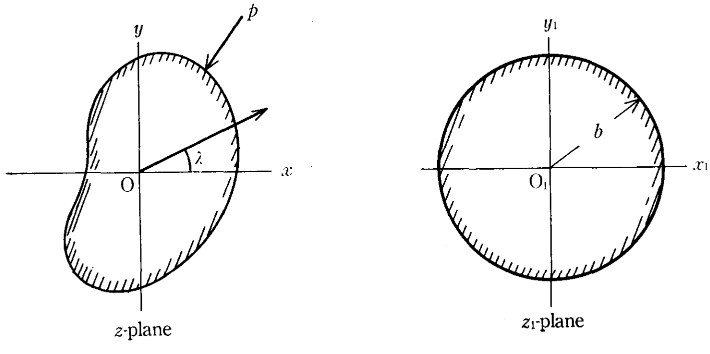

Fig. 1 Conformal Transformation of z-plane into z_1-plane.

Next, let us consider another cylinder of any form in the $z=x+iy$ plane. If we could connect this figure in z-plane to the above-mentioned circle of z_1-plane by means of conformal representation $z=F(z_1)$, the expression (1) will give us flow around the figure in z-plane, where we have $z_1=F^{-1}(z)$. Outer regions of two contours are represented conformally each other. The velocity at infinity, of the flow in z-plane will also be $-U$ (in direction λ) provided that we have $z\to\infty$, $(dz/dz_1)\to 1$ as $z_1\to\infty$.

Velocity of flow at any point in z-plane will be given by

$$v_x - iv_y = \frac{dw}{dx} = \frac{dw_1}{dz_1}\cdot\frac{dz_1}{dz},$$

where $w(z)$ is $w_1(z_1)$ regarded as function of z.

When the body in z-plane is moving with velocity U (in direction λ) through still water, the corresponding fluid motion will be given by

$$\begin{aligned} w_1(z_1) &= w(z) = \phi + i\psi \\ &= -U\left[z_1 e^{-i\lambda} + \frac{b^2}{z_1 e^{-i\lambda}}\right] + Ue^{-i\lambda}, \qquad (2) \end{aligned}$$

where we have $z_1=F^{-1}(z)$.

Let us now apply above fundamental formula to the case in which the cylinder in z-plane is making a vibratory motion. Assuming that the displacement of cylindrical body is given by

$$\xi \text{ (in direction } \lambda) = \varepsilon \sin \omega t,$$

where ε=amplitude, ω=angular frequency of vibration, and t=time, we have merely to put

$$U = d\xi/dt = \varepsilon\omega \cos \omega t \qquad (3)$$

into our formula (2). This is possible so far as we are considering the case of an incompressible ideal fluid.

Hydraulic pressure p caused by this fluid motion is given by

$$p = -\rho\frac{\partial\phi}{\partial t} = -\frac{\rho}{2}\{(v_x)^2 + (v_y)^2\},$$

where ρ is density of the fluid. Restricting ourselves to the case of a small motion, here we take approximately,

$$p = -\rho\frac{\partial\phi}{\partial t}. \qquad (4)$$

Velocity potential ϕ and stream function ψ can be put into following form, in case of vibration

$$\phi = \Phi\omega\varepsilon \cos \omega t, \qquad \psi = \Psi\omega\varepsilon \cos \omega t, \qquad (5)$$

where Φ, Ψ are functions of x and y. And we have

$$p=\rho\omega^2\varepsilon\Phi \sin \omega t. \quad \cdots\cdots (6)$$

Resulttant of this fluid pressure acting on surface of our cylindrical body in z-plane can be expressed as following forces:

$$X=-\oint p dy, \qquad Y=\oint p dx, \quad \cdots\cdots (7)$$

where the contour integral is to be taken around the boundary closed curve of the body. The above formula (7) may also be given in the following form

$$Y-iX=\oint p(dx+idy)=\oint p dz. \quad \cdots\cdots (8)$$

Now, let us consider the following expression

$$\Pi=w(z)dz=(\phi+i\psi)(dx+idy).$$

For the part

$$w_g(z)=-U\left[z_1e^{i\lambda}+\frac{b^2}{z_1e^{-i\lambda}}\right],$$

we have, along the boundary curve at which $z_1=be^{i\theta}$,

$$w_g(z)=\phi_g+i\psi_g=-2Ub\cos(\theta-\lambda)\,;\ \psi_g=0.$$

Therefore we have

$$\Pi=\phi_g(dx+idy). \quad \cdots\cdots (9)$$

Next, for the part

$$w_h(z)=U^{-i\lambda}z=\phi_h+i\psi_h,$$

we have

$$\phi_h=U(x\cos\lambda+y\sin\lambda),$$

$$\phi_h(dx+idy)=U\cos\lambda(xdx+ixdy)$$
$$+U\sin\lambda\,[ydx+iydy].$$

From which we obtain

$$\oint \phi_h(dx+idy)=iUe^{i\lambda}\oint xdy. \quad \cdots\cdots (10)$$

Combining these results, we find that the complex expression (8) of the force can be put into following form;

$$Y-iX=\rho\omega^2\varepsilon\sin\omega t\cdot$$

$$\left[-\oint\left\{z_1e^{-i\lambda}+\frac{b^2}{z_1e^{-i\lambda}}\right\}dz+ie^{i\lambda}\oint xdy\right]$$

In order to evaluate first contour integral in r. h. s. of this formula, we observe that the contour may be replaced by a circle of very large radius R, whose center is at origin of z-plane. We have

$$z_1 z+\frac{c_1}{z}+\frac{c_2}{z^2}+\cdots\cdots, \qquad \cdots\cdots(11)$$

c_1, c_2, $\cdots\cdots$ being complex constants. Thus we see that

$$\oint\left\{z_1 e^{-i\lambda}+\frac{b^2}{z_1 e^{-i\lambda}}\right\}dz\to 2\pi i(c_1 e^{-i\lambda}+b^2 e^{i\lambda})$$

as R tends to infinity. Hence we have

$$Y-iX=\rho\omega^2\varepsilon\sin\omega t\cdot[-2\pi i(c_1 e^{-i\lambda}+b^2 e^{i\lambda})+ie^{i\lambda}A], \qquad \cdots\cdots(12)$$

where $A=\oint zdy$ is area of the cross section. Further, if we put

$$\left.\begin{aligned}X&=-(d^2\xi/dt^2)\rho AM_x\\ Y&=-(d^2\xi/dt^2)\rho AM_y\end{aligned}\right\}, \qquad \cdots\cdots(13)$$

we have

$$M_x+iM_y=-\frac{2\pi}{A}(c_1 e^{-i\lambda}+b^2 e^{i\lambda})+e^{i\lambda} \qquad \cdots\cdots(14)$$

2 Application to Cylinder of Given Form of Cross Section

One way to give form of cross-section of cylindrical body in fairly general form is to put (convergence being assured).

$$z=z_1+\frac{A_1}{z_1}+\frac{A_2}{z_1^2}+\cdots\cdots+\frac{A_N}{z_1^N}+\cdots\cdots, \qquad \cdots\cdots(15)$$

where A_N are complex constants which we write

$$A_N=b^{N+1}B_N exp(i\alpha_N),$$

B_N and α_N being real constants. Putting $z_1=be^{i\theta}$, coordinates of contour line are given by

$$\left.\begin{aligned}x&=b\left[\cos\theta+\sum_{N=1}^{\infty}B_N\cos(N\theta-\alpha_N)\right]\\ y&=b\left[\sin\theta-\sum_{N=1}^{\infty}B_N\sin(N\theta-\alpha_N)\right]\end{aligned}\right\}. \qquad \cdots\cdots(16)$$

Cross-sectional area is given by

$$A=\oint xdy=\pi b^2\left[1-\sum_{N=1}^{\infty} NB_N^2\right].$$

Also we have, from our formula (12),

$$c_1=-A_1=-b^2B_1exp\,(i\alpha_1)$$

Therefore, by the general formula we have

$$M_y-iM_x=i\left[e^{i\lambda}-\frac{2\pi b^2}{A}\left\{-B_1exp(i\alpha_1-i\lambda)+e^{i\lambda}\right\}\right]$$

$$=i\,[(1-k)\,e^{i\lambda}+kB_1exp\,(i\alpha_1-i\lambda)] \quad \cdots\cdots(17)$$

where we have put, for shortness,

$$k=\frac{2\pi b^2}{A}=2\Big/\left[1-\sum_{N=1}^{\infty} NB_N^2\right]. \quad \cdots\cdots(18)$$

For instance we may quote the following simple cases.

(a) Circle Here we put $B_1=0$, $B_2=0$, $\cdots\cdots$ $k=2$,

and we have

$$M_x=\cos\lambda, \qquad M_y=\sin\lambda.$$

(b) Ellipse. Here we have $B_N=0$ $(2\leq N)$, $k=2/(1-B_1^2)$,

We take also $\alpha_1=0$, B_1 a positive number. Then

$$M_x+iM_y=(k-1)e^{i\lambda}-kB_1e^{-i\lambda}.$$

The equation of ellipse being

$$z=b\,[e^{i\theta}+B_1e^{-i\theta}],$$

we have also

Major axis $=2b(1+B_1)$

Minor axis $=2b(1-B_1)$

Area of cross-section $=\pi b^2(1-B_1^2)$.

3. Concluding Remark

The author has given, in this short note, a general formula which gives us coefficient of virtual mass for cylindrical bodies which is vibrating in a region of ideal fluid. Cross-section of cylindrical body is composed of closed contour which is represented by the equation (15) or (16). Mathematically speaking, the equation (15) or (16) may represent any closed curve. But, from the stand-point of technical application, its practical use must be confined to closed smooth curve whose tangent do not make discontinuous change.

From the author's formula (14), we observe that, in general, the direction of coeffcient vector (M_x, M_y) of virtual mass do not coincide with the direction of vibration of the body. Only in case of the body in symmetrical sectional form, the coincidence may take place, if it is vibrating in direction of axis of symmetry.

Additional Note: In the above treatment, we have confined ourselves to case of small vibrations. But, it can be extended to case of vibrations of finite amplitude, as we shall see later on.

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