Title	Trapped electron velocity distribution function in plasmas : I. Positiveness of the distribution function in electron plasma
Sub Title	
Author	小笠原, 正忠(Ogasawara, Masatada) 白井, 正司(Shirai, Shoji)
Publisher	慶応義塾大学工学部
Publication year	1971
Jtitle	Keio engineering reports Vol.24, No.7 (1971.) ,p.105- 108
JaLC DOI	
Abstract	Sufficient condition for the positiveness of the trapped electron velocity distribution function in electron plasma is analyzed in the case of one-dimensional stationary wave solution of Vlasov and Poisson equations. If the electrostatic potential φ has a value between φmin and some critical value φc, the sufficient condition for the positiveness is satisfied. The critical potential φc are evaluated in the limits of small and large φ.
Notes	
Genre	Departmental Bulletin Paper
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=KO50001004-00240007-0105

KEIO ENGINEERING REPORTS
VOL. 24 NO. 7

TRAPPED ELECTRON VELOCITY DISTRIBUTION FUNCTION IN PLASMAS I.

BY

MASATADA OGASAWARA and *SHOJI SHIRAI*

FACULTY OF ENGINEERING
KEIO UNIVERSITY
YOKOHAMA 1971

KEIO ENGINEERING REPORTS
VOL. 24, NO. 7, pp. 105-108, 1971

TRAPPED ELECTRON VELOCITY DISTRIBUTION FUNCTION IN PLASMAS

I. Positiveness of the Distribution Function in Electron Plasma

MASATADA OGASAWARA AND SHOJI SHIRAI

Dept. of Instrumentation Engineering, Keio University, Yokohama 223, Japan

(*Received Dec. 22, 1971*)

ABSTRACT

Sufficient condition for the positiveness of the trapped electron velocity distribution function in electron plasma is analyzed in the case of one-dimensional stationary wave solution of Vlasov and Poisson equations. If the electrostatic potential ϕ has a value between $\phi_{\min}$ and some critical value ϕ_c, the sufficient condition for the positiveness is satisfied. The critical potential ϕ_c are evaluated in the limits of small and large ϕ.

1. Introduction

The stationary one-dimensional nonlinear electrostatic wave has been studied by BERNSTEIN, GREENE and KRUSKAL (1957) with use of Vlasov and Poisson equations. The solutions of the basic equations were given as follows: If the potential and the distribution functions of ion and untrapped electron were given, Poisson equation allowed the distribution of the trapped electron to be calculated. The latter distribution function, however, is not always positive. An example was given by HATORI and SUGIHARA (1970). The trapped electron distribution was shown to be negative for some ranges of parameters of the untrapped electron velocity distribution.

SCHAMEL (1971) analyzed a necessary condition for the positiveness by assuming the trapped electron velocity distribution to be isotropic. Instead of the Poisson equation, TASSO (1969) used quasi-neutrality condition and showed that a sufficient condition for the positiveness was satisfied in an interval of potential. But his theory cannot take account of potential forms, i.e., sinusoidal, solitary and shock-like etc.. The aim of the present paper is to give a simple expression of a sufficient condition for the positiveness of the trapped electron velocity distribution function in an electron plasma.

2. Positiveness of $f_t(E)$

The basic equations for stationary electrostatic waves are

$$v\frac{\partial f(x,v)}{\partial x}+\frac{e}{m}\frac{d\phi(x)}{dx}\frac{\partial f(x,v)}{\partial v}=0\,, \tag{1}$$

$$\frac{d^2\phi(x)}{dx^2}=4\pi e\int_{-\infty}^{\infty}dv\,f(x,v)-4\pi e n_0\,, \tag{2}$$

where f and ϕ are the electron velocity distribution function in the wave frame and potential, respectively. Ion density is denoted by n_0. The general solution of equation (1) is

$$f=f(E)\,,\qquad E=\frac{1}{2}mv^2-e\phi\,. \tag{3}$$

Substituting (3) into (2) and denoting "trapped" and "untrapped" by subscripts t and u, we obtain

$$\int_{-e\phi}^{-e\phi_{\min}}dE\,f_t(E)[2m(E+e\phi)]^{-\frac{1}{2}}=g(e\phi), \tag{4}$$

where

$$g(e\phi)=-\frac{N(e\phi)}{4\pi e}+n_0-\int_{-e\phi_{\min}}^{\infty}dE f_u(E)[2m(E+e\phi)]^{-\frac{1}{2}} \tag{5}$$

is the density of trapped electron at x corresponding to $\phi(x)$. Because of (4), $g(e\phi_{\min})=0$. If the charge density $N(e\phi)=-d^2\phi/dx^2$ and n_0 and $f_u(E)$ are prescribed, $g(e\phi)$ is a known function and (4) is an integral equation of the convolution type for $f_t(E)$ and following BERNSTEIN, GREENE and KRUSKAL (1957), the solution is given by way of Laplace transform

$$f_t(E)=\frac{\sqrt{2m}}{\pi}\int_{-e\phi_{\min}}^{-E}dV\frac{dg(V)}{dV}\frac{1}{\sqrt{-E-V}}\,,\qquad E<-e\phi_{\min}\,. \tag{6}$$

From the expression (6), it follows that $\frac{dg}{dV}>0$ is a sufficient condition for the positiveness of $f_t(E)$. From (5) we have

$$\frac{dg(V)}{dV}=-\frac{1}{4\pi e}\frac{dN(V)}{dV}+\frac{1}{2\sqrt{2m}}\int_{-e\phi_{\min}}^{\infty}dE\frac{f_u(E)}{(E+V)^{\frac{3}{2}}}\,. \tag{7}$$

Then the sufficient condition for the positiveness is given by

$$\frac{dg}{dV} \geqq \frac{1}{2\sqrt{2m}} \int_{-e\phi_{\min}}^{\infty} dE \frac{f_u(E)}{(E+V)^{\frac{3}{2}}} - \frac{M}{4\pi e} > 0 , \tag{8}$$

where

$$M = Maximum\ of\ \frac{dN}{dV} . \tag{9}$$

For physically reasonable potential ϕ, M is positive. [c.f. VALEO, OBERMAN and KRUSKAL (1969).] If $f_u(E)$ is defined in the whole range of E, the first term of (8) goes to ∞ as $V \to e\phi_{\min}$ and decreases monotonically to zero as $V \to \infty$. Then V has a critical value $V_c(=e\phi_c)$ beyond which $\frac{dg(V)}{dV} < 0$. The result is similar to that of TASSO (1969).

In the limit of small and large ϕ, the critical potential ϕ_c can be derived analytically as follows: We have small ϕ expansion by making partial integration in (8),

$$\frac{1}{\sqrt{2m}} \left\{ \frac{f_u(-e\phi_{\min})}{\sqrt{e(\phi-\phi_{\min})}} + 2\sqrt{e(\phi-\phi_{\min})}\, f_u'(-e\phi_{\min}) + \cdots \right\} - \frac{M}{4\pi e} > 0 . \tag{10}$$

This expansion has meaning if $e(\phi-\phi_{\min}) \ll V_T$, V_T being the width of $f_u(V)$. With use of (10), the critical ϕ_c in the small ϕ limit is given by

$$\phi_c - \phi_{\min} = \frac{1}{e} \left[\frac{4\pi e f_u(-e\phi_{\min})}{\sqrt{2m}\, M} \right]^2 . \tag{11}$$

When V is much larger than $V_0 + V_T$, V_0 being defined by $f_u(V_0)$=maximum, it follows that

$$\frac{1}{\sqrt{2m}} \int_{-e\phi_{\min}}^{\infty} \frac{f_u(E)dE}{(E+V)^{\frac{3}{2}}} \approx \frac{n_u}{V} , \tag{12}$$

where n_u, the untrapped electron number density, is defined by

$$n_u = \int_{-e\phi_{\min}}^{\infty} \frac{f_u(E)dE}{\sqrt{2m(E+V)}} . \tag{13}$$

According to MONTGOMERY and JOYCE (1969), n_u are connected with n_0, unperturbed number density, by

$$n_u = \frac{n_0}{\sqrt{1+\frac{V}{V_T}}} \approx n_0 \sqrt{\frac{V_T}{V}} . \tag{14}$$

Then in the limit of large ϕ, it follows from (8), (12) and (14) that

$$\phi_c = \frac{1}{e} \left[\frac{2\pi e n_0 \sqrt{V_T}}{M} \right]^{\frac{2}{3}} . \tag{15}$$

For the intermediate value of ϕ, the following expression for the positiveness criterion will be convenient

$$\frac{1}{\sqrt{2me(\phi-\phi_{\min})}} \frac{f_u(-e\phi_{\min}) + 4f_u(3e\phi - 4e\phi_{\min})}{6} - \frac{M}{4\pi e} > 0 , \tag{16}$$

where Simpson's formula has been used for evaluating the integral.

For examples we take a potential given by

$$\phi(x)=\frac{\phi_0}{\cosh^l(kx)}. \tag{17}$$

This expresses a solitary wave if k is real and l is a positive integer. It follows from (17) that

$$M=\frac{k^2}{e}(3l+2), \qquad \phi_{\min}=0. \tag{18}$$

When k is pure imaginary and $l=-1$, the potential $\phi(x)$ given by (17) reduces to a sinusoidal one, and it follows that

$$M=\frac{k^2}{e}, \qquad \phi_{\min}=-\phi_0. \tag{19}$$

It is to be noted that in both cases M are positive and the critical values ϕ_c exist.

REFERENCES

BERNSTEIN I. B., GREENE J. M. and KRUSKAL M. D. (1957): Exact Nonlinear Plasma Oscillations, Phys. Rev. **108**, 546.

HATORI T. and SUGIHARA R. (1970): Private Communication.

MONTGOMERY D. and JOYCE G. (1970): Shock-Like Solutions of Electrostatic Vlasov Equation, J. Plasma Physics **3**, 1.

SCHAMEL H. (1971): Stationary Solutions of the Electrostatic Vlasov Equation, Plasma Physics **13**, 491.

TASSO H. (1969): Nonlinear Quasi-Neutral Electrostatic Plasma Waves and Shock Waves, Plasma Physics **11**, 663.

VALEO E. J., OBERMAN C. and KRUSKAL M. D. (1969): Property of Large-Amplitude Steady Electrostatic Waves, Phys. Fluids **12**, 1246.