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Space-Charge-Waves in Electoron Beam with Velocity Distribution

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Under a single-velocity assumption there exist two space charge waves (fast and slow waves) in an electron beam and they can be analysed easily. Effects of volocity spread of an electron beam, however, cannot be neglected in the region where the beam is not accelerated sufficiently or it is disturbed by strong electric and magnetic lenses.

The another first studied this problem by hydro-dynamic approximation method. But it was shown that this method is not useful for the problem, esspecially for explanation of the damping mechanism of the wave. Next method we tried is analysis by Boltzmann-Vlasov equation.

When a normalized d.c. velocity distribution of an electron beam is described by $f_0(v)$, the dispersion equation of the space charge waves is given as

$$D(\omega,\ k)=1-\omega_p^2\int_{-\infty}^{\infty}\frac{f(v)}{(\omega-kv)^2}dv\ , \tag{1}$$

where a small a.c. perturbation is expressed in the form exp $j(\omega t-kz)$, and ω_p is the plasma angular frequency of the electron beam.

The path of the integration in Eq. (1) must be carefully chosen. Because ω and k are brought out by Laplace- and Fourier-transformations, and then the position of the pole ω/k in the complex-v-plane should be decided from the original definitions of such transformations. Landau, in his classical paper, considered this problem for the case of real k. However, for the case of complex k, which is required for investigating a spacial propagation characteristics of a parturbation in an electron beam, it has not been studied yet. On the bases of analytic continuation it can be shown that the generalized Landau contour is as follows: If $Re\ k<0$, the pole ω/k is assumed to be in the lower-half of the complex-v-plane, or in other word the contour is deformed to pass upward of the pole when $Im(\omega/k)>0$. If $Re\ k<0$, the pole ω/k is assumed to be in the upper-half of the v-plane.

The velocity distribution of an electron beam is expressed by a rational function as:

$$f_0(v)=\frac{\prod_n (v/v_T-\beta_n)}{\prod_m (v/v_T-\alpha_m)}\quad , \tag{2}$$

where all coefficients shoud be chosen to satisfy $f_0(v)\geqq 0$ and $f_0(v=\pm\infty)=0$. Substituting Eq. (2) into Eq. (1) and utilizing residue theorem one obtains the following expression:

$$D(\omega,\ k)=1-2\pi j\cdot cv_T\epsilon\left(\frac{\omega_p}{kv_T}\right)^2\sum_l\frac{\prod_n(\alpha_l-\beta_n)}{(\omega/kv_T-\alpha_l)^2\prod_{m(\neq l)}(\alpha_l-\alpha_m)}=0\ , \quad (3)$$

where $\epsilon=1$ if $Re\ k>0$ and $\epsilon=-1$, if $Re\ k<0$, attributing to generalized Landau contour. It should be noted that the solution of Eq. (3) can be obtained analytically. If we solve Eq. (3) about k, k's are complex. This means the space-charge-waves grow or decrease in space. If Eq. (2) is single humped, they exhibit spacial Landau damping.

Next, a case of the velocity distribution has a step-function-discontinuity is considered. It is described as:

$$f_0(v)=c\ s(v-v_d)\frac{\prod_n(v/v_T-\beta_n)}{\prod_m(v/v_T-\alpha_m)}\ , \quad (4)$$

Eq. (4) is not changed in value, if we multiply it by $\lim_{\delta\to 0}(v-v_d)^\delta$. After some manipulations Eq. (1) can be written as

$$D(\omega,\ k)=1-cv_T\left(\frac{\omega_p}{kv_T}\right)^2\cdot\left[\sum_l\frac{\log(v_d/v_T-d_l)\prod_n(\alpha_l-\beta_n)}{(\omega/kv_T-\alpha_l)^2\prod_{m(\neq l)}(\alpha_l-\alpha_m)}\right.$$

$$\left.+\frac{\prod_n(\omega/kv_T-\beta_n)}{\prod_m(\omega/kv_T-\alpha_m)}\left\{\log(v_d/v_T-\omega/kv_T)\sum_m\frac{1}{\omega/kv_T-\alpha_m}-\frac{1}{\omega/kv_T-v_d/v_T}\right\}\right]=0, \quad (5)$$

where principle value should be taken the value of logarithm, but when the Landau contour is deformed we must take care of taking its "principle value".

We calculated $k(\omega)$ chracteristics with following distribution.

$$f_0(v)=c\ s(v-v_d)\frac{1}{1+\dfrac{v^2-v_d^2}{v_T^2}+\dfrac{1}{2}\left(\dfrac{v^2-v_d^2}{v_T^2}\right)^2} \quad (6)$$

This distribution approximates the half-maxwellian one. The results show that, although slow space charge wave is a simple propagating wave, fast wave exhibits spatial Landau damping.