| Title | Overall optimization by coordinating center and local optimizations |
|---|---|
| Sub Title | |
| Author | 志水, 清孝(Shimizu, Kiyotaka) |
| Publisher | 慶応義塾大学藤原記念工学部 |
| Publication year | 1969 |
| Jtitle | Proceedings of the Fujihara Memorial Faculty of Engineering Keio University (慶応義塾大学藤原記念工学部研究報告). Vol.22, No.90 (1969. ) ,p.129(15)- 136(22) |
| JaLC DOI | |
| Abstract | An approach to optimization of a comlex industrial system is studied based on center-local optimizing structure. Final purpose to maximize total profit of an overall system is developed by considering coordination of center optimization of simplified overall system and local optimizations of subsystems. Our formulation justifies the local optimizations in relation to the center optimization so that the overall purpose is achieved. |
| Notes | |
| Genre | Departmental Bulletin Paper |
| URL | https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=KO50001004-00220090-0015 |

# Overall Optimization by Coordinating Center and Local Optimizations

(Received January 9, 1970)

Kiyotaka SHIMIZU*

**Abstract**

An approach to optimization of a comlex industrial system is studied based on center-local optimizing structure. Final purpose to maximize total profit of an overall system is developed by considering coordination of center optimization of simplified overall system and local optimizations of subsystems. Our formulation justifies the local optimizations in relation to the center optimization so that the overall purpose is achieved.

## I. Introduction

Ultimate purpose is optimization of an overall system or an entire factory. Control objective of the entire factory is to maximize gross profit gained by normal operation of the factory under given constraints. As a matter of fact, the factory consists of a lot of plants, each of which possesses constraints peculiar to that plant and a control objective such as maximizing efficiency. In this paper we consider optimizing control such that to the entire factory (or the overall system) an objective function is given based on economics and to each plant (or subsystem) a subobjective function peculiar to that plant is given based on technical or economic requirement. Furthermore all plants are mutually connected with certain constraints.

Posing such optimization problem is very practical. Actually the optimizing production control is executed at the factory by application of mathematical programming and the optimizing control for local objectives such as efficiency, quality, local profit etc. is carried out at each plant by application of proper mathematical programming. But at present situation the relation between overall objective and local ones is not examined closely. So our purpose is to study how we may connect the local objectives as plant efficiency etc. with the overall objective as the gross profit of the entire factory. From other point of view our purpose is to study how we may set the local objectives for the subsystems from a given overall economic objective.

This conception is essentially different from Decomposition Principle which divides

*志 水 清 孝 Assistant Professor, Faculty of Engineering, Keio University.

the overall goal and system into a set of subgoals and subsystems merely for problem solving. We are rather concerned with a point how to give a significance to the subobjectives peculiar to the subsystem in the overall objective of entire system, in other words how to unify local optimization problems which actually exist.

Taking up a factory manufacturing city-gas as an example, let us consider the above problem for its abstracted model. A gas-manufacturing factory consists of

(i) several plants which produce various kinds of gases respectively

(ii) blending process which mixes the gases being different qualities in order to obtain the specified quality

(iii) gas-supplying system to consumers.

Main purpose is to manufacture city-gas of specified quality as much as consumer demands with minimum cost. One plant may be of a continuous-type process and another a batch type. All of them are fairly complex processes for which we can assign local objectives such as efficiency, total calory, quality, production volume etc.

The local objective for each plant must be chosen among these or be specified by properly combining these performance functions. The local objective must be set up so that technical or economic condition for operating the plant is satisfied. In addition we need a set of process equations expressing input-output relations and constraints for each plant.

Now, it is inappropriate to set up a complex objective function and constraints for the entire factory by considering properties of each plant in detail. In fact it is impossible to solve a problem formulated by thinking of all conditions. An overall problem should be formulated by compromise of the objective, a scale of the problem and possiblity of computation. In that case, input-output relations of each plant may be simplified, mathematical models may be replaced by approximated equations such that center of a subsystem may be assumed fixed constants. These simplified equations may appear to be too simple to use for optimization of each plant, but such approximation is usually necessary for center optimization in reality.

From various kinds of gases produced by each plant is made city-gas through the blending process at which mixed gas satisfies:

(i) specified calory

(ii) volume flow-rate

(iii) combustibility condition

(iv) density, CO percentage etc.

These conditions become constraints to center system optimization. Desired volume of production changes depending on consumer's demand and it becomes a disturbance to the optimizing system.

Center optimization determines main decision variables defining operating conditions which mean mainly load distribution (main raw material) of each plant. Then local optimization determines within remained freedom manipulated variables of each plant based on sub-objective function plant by plant.

But there arises a question whether local optimizations are consistent with overall optimization (maximization of profit). Namely there is a worry whether an effort obtaining maximum calory or best quality of each plant gas results in decreasing total profit of the entire factory. The problem is how we can arrange local optimizations for various subobjective functions to center optimization aiming maximum profit without incosistency.

## II. Formulation of problem

We consider our problem as follows. Although an overall goal is to maximize an objecctive function $P$ defined for the entire system, each subsystem empirically possesses subgoal which maximizes or minimizes a subobjective function $P_n$ peculier to each subsystem ($n=1, 2, \cdots, N$). For example an overall objective function $P$ denotes profit as mentioned before, subobjective functions $P_n$ are efficiency, production volume, cost quality, respectively. Since a minimum problem can be converted to a maximum one easily, we suppose that unifying all subproblems as maximum problems, each subsystem has own subgoal to maximize its subobjective function.

Our factory consists of $N$ plants, each of which has input vectors $\underline{x}_n$, $\underline{m}_n$, an output vector $\underline{y}_n$. Both $\underline{x}_n$ and $\underline{m}_n$ are decision variables, where $\underline{x}_n$ is the raw material feed and $\underline{m}_n$ is the manipulated variable for operation of the plant.

Let us formulate our problem as follows, simplifying description to understand only essence of the problem:

<u>*Center Optimization Problem*</u>

$$\max_{\underline{x}} \quad P(\underline{y}^*, \underline{x}, \underline{m}) \tag{1}$$

$$\text{subj. to} \quad \underline{f}(\underline{y}^*, \underline{x}, \underline{m})=\underline{0} \tag{2}$$

$$\underline{g}(\underline{y}^*, \underline{x}, \underline{m})\geqslant\underline{0} \tag{3}$$

$$\underline{y}_n^*=\underline{f}_n^*(\underline{x}_n, \underline{m}_n), \quad n=1, 2,\cdots, N \qquad (4)^*$$

*(i) In Eq. (4) (6) we consider the case when $\underline{y}_n$ is expressed explicity as a function of $\underline{x}_n$, $\underline{m}_n$. In general cases, however, process equations are described as $\underline{F}_n(\underline{y}_n, \underline{x}_n, \underline{m}_n)=\underline{0}$.

(ii) Process optimization problem is often formulated as determining the optimal temperature $T$ of the furnace for a certain objective. In order to regulate the furnace temperature $T$ to such a state, we must control an actual manipulated variable, feed rate of fuel $f$. Thus we should formulate fuel $f$ to be a decision variable $m_n$. It is for convinience of simplification of optimization problem to take $T$ as an independent decision variable. If we desire to consider $\underline{y}_n$ as something which seems to be really output, we should consider $T$ as one element of internal state $\underline{z}_n$ of the process. By thinking of the internal state $\underline{z}_n$, process equations are described as follows.

where $\underline{x}$, $\underline{m}$ and $\underline{y}^*$ are composite vectors such as $\underline{x}=(\underline{x}_1, \underline{x}_2, \cdots, \underline{x}_N)$, $\underline{m}=(\underline{m}_1, \underline{m}_2, \cdots, \underline{m}_N)$ and $\underline{y}^*=(\underline{y}_1^*, \underline{y}_2^*, \cdots, \underline{y}_N^*)$ respectively.

*Local Optimization Problems*

$$\max_{\underline{m}_n} \quad P_n(\underline{y}_n, \underline{x}_n, \underline{m}_n) \tag{5}$$

$$\text{subj. to} \quad \underline{y}_n=\underline{f}_n(\underline{x}_n, \underline{m}_n) \tag{6)*}$$

$$\underline{g}_n(\underline{y}_n, \underline{x}, \underline{m}_n)\geq 0 \tag{7}$$

$$n=1, 2, \cdots, N$$

In particular, the gas-making factory specifies Eq. (2) as $\underline{z}_d-\underline{F}(\underline{y}^*)=\underline{0}$ ($\underline{z}_d$ is a given constant) which means the blended gas must satisfy a specific quality and quantity. Since $\underline{z}=\underline{F}(\underline{y}^*)$, the performance Eq. (1) is expressed as $P'(\underline{z}, \underline{x}, \underline{m})=P(\underline{y}^*, \underline{x}. \underline{m})$, Eq. (3) means other inequality constraints. Eq. (4) represents suitably simplified input-output relations (in other words, the approximated ones) and $\underline{y}_n^*$ is an approximation to an output vector when the simplified system equations are used.

The overall objective function is generally represented as follows:

Profit

$$P=P(\underline{y}^*, \underline{x}, \underline{m})=\sum_{n=1}^{N} \underline{c}_{1n}{}^T\underline{y}_n{}^*-\underline{c}_{2n}{}^T\underline{x}_n-\underline{c}_{3n}{}^T\underline{m}_n \tag{8)**}$$

here $\underline{c}_{jn}$ represents price per unit and there is a case when $\underline{c}_{jn}$ is a function of $\underline{y}_n^*$, $\underline{x}_n$, $\underline{m}_n$ (for example price of production is a function of quality and/or price of raw material is one of purchasing quality). However, we consider them as constants here.

---

$$\underline{y}_n=\underline{f}'_n(\underline{x}_n, \underline{m}_n, \underline{z}_n) \tag{R・1}$$

$$\underline{z}_n=\underline{h}_n(\underline{x}_n, \underline{m}_n) \tag{R・2}$$

$$\underline{g}_n(\underline{y}_n, \underline{x}_n, \underline{m}_n, \underline{z}_n)\geq \underline{0} \tag{R・3}$$

From a view-point of decision function, $\underline{z}_n$ will be regarded as a part of $\underline{m}_n$ in Eq. (6). Thus it is a decision vector of the subsystem. In that case, of course, there should be a freedom of realizing $\underline{z}_n$ in (R・2) with respect to $\underline{m}_n$ in (R・1) (R・2).

Meanwhile, from a view-point of process property, $\underline{z}_n$ can be regarded as a part of the out put vector $\underline{y}_n$. But in that case the process equation is not solved in a form of $\underline{y}_n=\underline{f}_n(\underline{x}_n, \underline{m}_n)$ explicitly and (R・1) and (R・2) are regarded as equivalent to process equation expressed in implicit form $\underline{F}_n(\underline{y}_n, \underline{x}_n, \underline{m}_n)=\underline{0}$.

** There exists a factor $\alpha$ in a profit function which cannot be described merely with economic income and outcome; $P(\underline{y}, \underline{x}, \underline{m}; \alpha)$. Thus, the overall optimization problem must be formulated such that real profit should be improved by consideration those factors (quality, efficiency, safety, etc., most of which are dependent on technical factors of each plant) as an objective function or constraints of plant level.

Meanwhile $P_n$'s are subobjective functions which must be defined with an appropriate manner based on economics or techniques.

A feature of this multi-goal system is that among decision variables coming into the objective function $P$, a vector $\underline{x}=(\underline{x}_1, \underline{x}_2, \cdots, \underline{x}_N)$ is decided by the second level (center manager) and $\underline{m}=(\underline{m}_1, \underline{m}_2, \cdots, \underline{m}_N)$ is desided by the first level (local managers of the subsystems) (see Fig. 1). The reason may be understood from the fact mentioned in previous section.

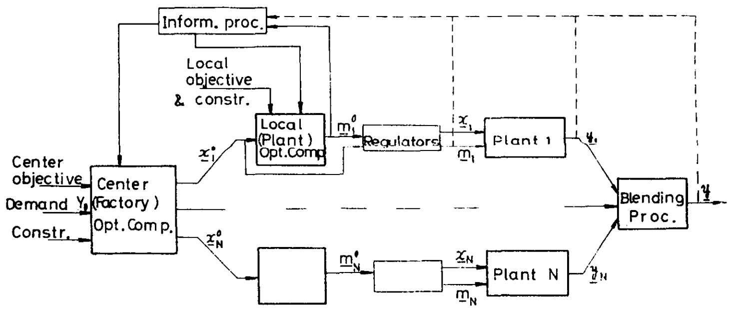


**Fig. 1.**

Now we should consider what is our true goal. Of course our true goal is optimization of the overall system, that is, maximization of total profit. Therefore, if all factors related with cost were formulated in the cost target $P$ and system constraints (system equations and inequality constraints) representing system characteristics with high accuracy, the problem would be summarized to determine $\underline{x}$ and $\underline{m}$ such that $P$ is maximized under all system constraints $(\underline{y}_n=\underline{f}_n(\underline{x}_n, \underline{m}_n)$, $\underline{f}(\underline{y}, \underline{x}, \underline{m})=\underline{0}$, $\underline{g}(\underline{y}, \underline{x}, \underline{m})\geq\underline{0})$. In reality, however, such formulation for a large-scale complex system restricts obtaining an optimal solution by numerical calculation. Usually an optimization problem needs to simplify the mathematical model as simple as the problem is solvable and the obtained solution is still effective.

Then for our system, what does it mean for us to maximize $P$ with respect to $\underline{x}$, maximizing efficiency etc., $P_n$, with respect to $\underline{m}_n$, $n=1, 2, \cdots, N$?. From a viewpoint of maximum $P$, maximum $P_n$ might result in reverse effect.

As a matter of fact, theoretically speaking, only maximum $P$ must be our goal. There does not exist an essentially multi-goal system except in a competitive situation.

In industry, however, they put in force optimization of the subsystem independent to overall optimization without unified view. What makes it appropriate to maximize local objective functions when an overall objective function exists ? The simplest case is that maxima $P_n$, $n=1, 2, \cdots, N$ are regarded to almost equivalent to maximum $P$ or to be proportional each other, since there is little interaction among subsystems. It seems to be no problem when $P$ is completely decomposed into $\sum P_n$.

We have another reason justifying the multigoal system formulated above. The purpose of this paper is to study the structual feature of such *center-local optimizing system.* Namely, the system is too complex to formulate all cost factors into $P$. Although we know that to keep efficiency maximum is profitable to economics, we cannot formulat eeffect of efficiency into overall profit function $P$ because of complexity of mathematical model of subsystems. Therefore, from the empirical fact that maximum $P_n$ contributes effectively for maximization of true profit we justify separate subgoals and local optimizations. In addition, although the integrated problem is too complex to solve, subproblems containing less variables are comparatively easy. The fact that such formulation makes numerical calculation possible justifies further appropriateness.

On the other hand, we can say from other view-point effects obliged to be lost by simplification (or approximation) of the center problem when overall problem is formulated as one integrated problem. We expect by this that true overall performance would be improved.

But from a mathematical point of view local optimizations become a kind of constraints for the overall optimization process.

In the two-level decision process local optimizations in the first level play a role of constraints to center optimization in the second level. Therefore, as $\underline{m}_n{}^0(\underline{x}_n{}^0)$ is a solution for local optimization of the subproblem $n$ given $\underline{x}_n{}^0$, there can exist $\{\underline{x}_n{}^0, \underline{m}_n{}^*\}$ being different to $\{\underline{x}_n{}^0, \underline{m}_n{}^0(\underline{x}_n{}^0)\}$, which maximizes $P$.

If we consider the local optimization problems as constraints to the center one, the following formulation in the center-local decision structure will be given and it will be significant in practice.

I. *Center Optimization*

$$\max_{\underline{x}} \; P(\underline{y}^*, \underline{x}, \underline{m}) \tag{9}$$

subj. to

$$\underline{f}(\underline{y}^*, \underline{x}, \underline{m})=\underline{0} \tag{10}$$

$$\underline{g}(\underline{y}^*, \underline{x}, \underline{m})\geq\underline{0} \tag{11}$$

$$\underline{y}_n{}^*=\underline{f}_n{}^*(\underline{x}_n, \underline{m}), \quad n=1, 2,\cdots, N \tag{12}$$

*Local Decision-Making w. r. t.* $\underline{m}_n$ — constraints

$$P_n(\underline{y}_n, \underline{x}_n, \underline{m}_n)\geq\beta_n \tag{13}$$

$$\underline{y}_n=\underline{f}_n(\underline{x}_n, \underline{m}_n) \tag{14}$$

$$\underline{g}_n(\underline{y}_n, \underline{x}_n, \underline{m}_n)\geq\underline{0}, \quad n=1, 2,\cdots, N \tag{15}$$

II. *Center Optimization*

Eq. (9)

Eq. (10)$\sim$Eq. (12)

*Local Decision-Making w. r. t.* $\underline{m}_n$

$$\left.\begin{array}{l} P_n(\underline{y}_n, \underline{x}_n, \underline{m}_n)=\beta_n \\ \text{Eq. (14)} \\ \text{Eq. (15)} \end{array}\right\} \text{constraints} \qquad (16)$$

III. *Center Optimization*

Eq. (9)

$$\left.\begin{array}{l} \text{Eq. (10)} \sim \text{Eq. (12)} \\ \textit{Local Decision-Making w. r. t. } \underline{m}_n \\ \max_{\underline{m}_n} \ P_n(\underline{y}_n, \underline{x}_n, \underline{m}_n) \\ \text{subj. to Eq. (14)} \\ \qquad\quad \text{Eq. (15)} \end{array}\right\} \text{constraints} \qquad (17)$$

In case I and II subsystem performance indexes $P_n$ are treated as constraints completely. In particular $P_n \geq \beta_n$ gives a kind of *satisfaction condition* such that if $P_n$ is greater than $\beta_n$ it is satisfactory. Therefore if it is within permissible freedom we can take plural number of constraints $P_{ni} \geq \beta_{ni}$, $i=1, 2, \cdots, I_n$, instead of Eq. (13). Requirement $P_n=\beta_n$ arises when that plant is designed to operate with efficiency $\beta_n$. These restriction on efficiency etc. contributes to improving overall profit. In fact, although loss by catalist deterioration is not calculated in profit function $P$, it is known that by keeping efficiency $P_n \geq \beta_n$ catalist life is prolonged and it causes profit larger.

In case III maximizing profit $P$ and maximizing efficiency $P_n$ are primarily two different problems. The reason why we maximize the subobjective functions $P_n$ is explained before.

When we consider the local problems as constraints the equations obtained by solving the local problem

$$\underline{m}_n{}^0=\underline{m}_n{}^0(\underline{x}_n)=\underline{f}_{cn}(\underline{x}_n)$$

is an equation for plant operation which gives value of manipulated variables $\underline{m}_n$ when raw material $\underline{x}_n$ is given.

We will move a weight onto center optimization when reliability of center optimization for true profit increases.

Let us consider an iterative method to obtain a numerical solution. The iteration will repeat alternately a process such that a center manager determines $\underline{x}^0$ for given $\{\underline{m}_n\}$ and then local managers determine $\underline{m}_n{}^0$ for $\underline{x}_n$ given from the center respectively. Since the center determines $\underline{x}$, $P$ must be sensitive to $\underline{x}$ for significance of center optimization. If $P$ changes much when $\underline{m}_n$ changes little, it is inappropriate for $\underline{m}_n$ to be determined at local. But it is bad for convergence of two-level iteration scheme that when $\underline{m}_n^0(\underline{x}_n)$ changes a little, $\underline{x}^0$ maximizing $P$ given $\underline{m}_n^0$ changes much. There remains a lot of discussion on convergence and sensitivity for this center-local decision-making.

**References**

1. Arrow, K. J., "Control in Large Organizations", Management Science, Vol. 10 No. 3, Apr., 1964
2. Lefkowitz, I., "Multilevel Approach Applied to Control System Design" Proc. JACC ,1965, ASME
3. Lasdon, L. S., Schoeffler, J. D., "Decentralized Plant Control", No. 3.3-3-64, 19th Annual ISA Conference, Oct. 1964, New York