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Plastic Deformation of a Wire with Circular Section in Coiling

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Abstract

The deformed shape of the section of circular wire after plastic pure bending is obtained as a epitrocoid, and the degree of the deformation of the contour line of the cross-section of the wire is expressed by $1/4c$, where c is spring index.

I. Displacement of beam under pure bending

Now, consider a beam under pure bending, and set the origin 0 at any point on the neutral axis and take the coordinate axes as shown in Fig. 1. (*a*), (*b*).

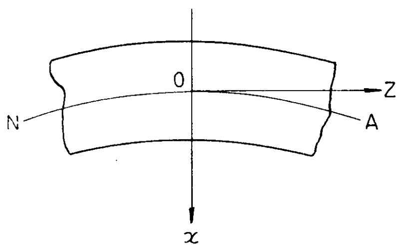

Fig. 1. (*a*)

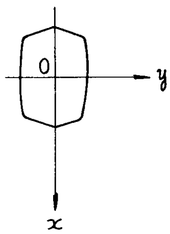

Fig. 1. (*b*)

The strain components in the beam are given as follows:

$$\varepsilon_x=\varepsilon_y=\nu x/R, \quad \varepsilon_z=-x/R, \quad \gamma_{yz}=\gamma_{zx}=\gamma_{xy}=0, \tag{1}$$

by the Theory of Simple Bending, where R is radius of curvature of the neutral axis and ν is the Poisson's ratio.

By the definition of strain and eq. (1) we obtain

$$\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}=\frac{\nu x}{R}, \quad \frac{\partial w}{\partial z}=-\frac{x}{R}. \quad \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}=\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}=\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}=0 \tag{2}$$

where u, v, w, are components of displacement.

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By integrating (2), displacements are obtained as follows:

$$u=\frac{\nu(x^2-y^2)}{2R}+\frac{z^2}{2R},\quad v=\frac{\nu xy}{R},\quad w=\frac{-xz}{R}. \tag{3}$$

From (3), it can be seen that the plane z=const. remains as a plane after deformation (Bernoulli's Assumption for the Theory of Simple Bending).

When deformation of the sectional form is in question, it is sufficient to consider only the components of displacement

$$u=\nu(x^2-y^2)/2R,\quad v=\nu xy/R. \tag{4}$$

If the elastic deformation is neglected, the plastic deformation of beam under pure bending are obtained by eq. (4), putting $\nu=1/2$, which means that the material is incompressible,

$$u=(x^2-y^2)/4R,\quad v=xy/2R. \tag{5}$$

II. Shape of the section

Now, assuming that the contour of the section of wire is circle of r_0 radius, we have the eq. of the contour line as follows:

$$x_0{}^2+y_0{}^2=r_0{}^2,\quad x_0=r_0\cos\theta,\quad y_0=r_0\sin\theta,$$

as shown in Fig. 2.

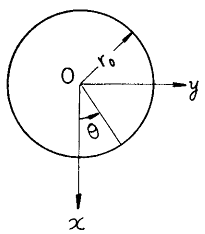

Fig. 2.

Then the plastic displacement at the contour is

$$\begin{aligned} u&=(x^2-y^2)/4R=(r_0{}^2/4R)\cos 2\theta,\\ v&=x_0y_0/2R=(r_0{}^2/4R)\sin 2\theta. \end{aligned} \tag{6}$$

The shape after coiling may be obtained from the next equations

$$\begin{aligned} x(=x_0+u)&=r_0\cos\theta+(r_0{}^2/4R)\cos 2\theta,\\ y(=y_0+v)&=r_0\sin\theta+(r_0{}^2/4R)\sin 2\theta. \end{aligned} \tag{7}$$

These are the equations for Epitrocoid as shown in Fig. 3, and it may be said that the degree of the deformation of the contour line of the wire is expressed by the next value

$$r_0/4R=1/4c,$$

where c is spring index, and R is mean coil radius.

III. Percentage of increase of wire diameter by coiling

From eq. (7), y is max. when θ satisfies the following conditions:

$$\frac{dy}{d\theta}=r_0\cos\theta+\frac{r_0{}^2}{4R}2\cos 2\theta=r_0\left(\cos\theta+\frac{1}{2c}\cos 2\theta\right)=0,$$

$$2\cos^2\theta + 2c\cdot\cos\theta - 1 = 0,$$

$$\cos\theta = -\frac{c}{2}\left[1 \mp \sqrt{1+\frac{2}{c^2}}\right].$$

If $c>5$; $\frac{2}{c^2} \leqq \frac{1}{12.5} = 0.08 \ll 1$;

$$\cos\theta \cong -\frac{c}{2}\left[1-1-\frac{1}{c^2}\right] = \frac{1}{2c}, \quad \sin\theta = \sqrt{1-\cos^2\theta} \cong 1 - \frac{1}{8c^2}.$$

Therfore, $\Delta = \frac{y_{max}}{r_0} - 1 = \sin\theta + \frac{1}{2c}\cos\theta\sin\theta - 1 \cong \frac{1}{8c^2}$. (8)

Table 1. Percentage of increase Δ and c.

c	4	5	7	10	12	15
$\Delta=1/8c^2$	1/128	1/200	1/392	1/800	1/1,152	1/1,800
Δ%	0.78	0.5	0.255	0.125	0.087	0.056

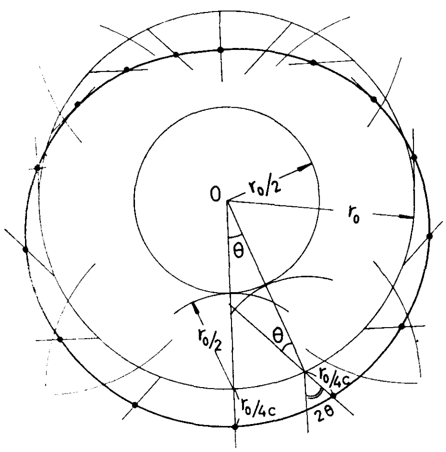

Fig. 3.

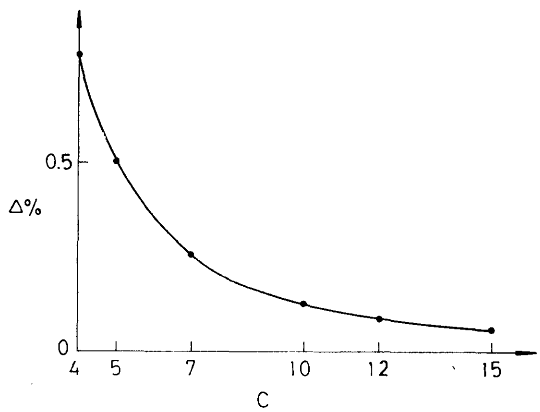

Fig. 4.

Generally, this value of Δ is smaller than the tolerances of wire diameter.