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Study on the Forced Convective Heat Transfer of a Water Droplet

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Abstract

Evaporation rate from a droplet in the hot air stream was measured in the range of the Reynolds number from 70 to 600.

By using a specially designed droplet suspender, water was supplied to the droplet as much as the rate of evaporation so as to keep constant the droplet size during the measurement. As the result the following correlative equation was obtained:

$$Nu=2+0.192Re^{0.70}.$$

I. Introduction

The experimental research on the evaporation of a water droplet in the hot air stream is presented in this paper. By using a specially designed equipment a droplet was kept in a constant size during the measurement, namely the measurement was carried out in the steady state. This is a distinctive point of this research.

The problems of heat and mass transfer of liquid droplets are one of the important basic informations in order to analyze the operations of spray drying, spray cooling, humidification, combustion of liquid fuel spray and the heat transfer in the cooling tower and so forth. These subjects have been studied and published previously(1)~(15) and recently the many more researches have been published by Tsubouchi(16)~(17), Toei(18)~(19), Mori(20) and Kwan(21). They have used, however, a small solid sphere as the material. For example, Tsubouchi and Mori used the small thermistor probes. On the other hand, Ranz, Toei and Kwan used a water droplet but the ranges of Reynolds number of their experiments were not so large and only 200 or so. In the range of the high Reynolds number, Ranz correlated his experiments on a water droplet by connecting the other experiments on a solid sphere(3).

In our research the rate of evaporation of a water droplet suspended at the bottom

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of a glass capillary in the hot air stream was measured directly. The unique points in our research are as follows:

(1) The measurement was carried out in steady state. Water was supplied to the droplet as much as the rate of evaporation, so as to keep the droplet size constant during the measurement.

(2) We could hold a large droplet by this equipment. Therefore, the range of the Reynolds number was to be extended to larger value up to 600.

(3) The feed-water temperature was maintained approximately equal to the droplet temperature. Then, we assumed the inside temperature of the droplet to be uniform.

By these experimental devices the accurate results covering the extensive range of Reynolds number were obtained. These experimental results were a little different from those of others. That is to say at large Reynolds number the larger heat flux was found.

II. Experimental device

(1) General layout

The general layout of the experimental apparatus is shown in Fig. 1. Air introduced from a blower passes through a 2 KW electric tunnel heater and is heated up to 80~120°C. The hot air flows out from a nozzle whose diameter is 40 mm. The air velocity can be measured at a certain point of 1 mm over the nozzle by using a standard type Pitot-static tube whose outer diameter is 4 mm and a Chattock gage.

A water droplet is suspended 1 mm over the nozzle by a droplet suspender which is described in the next section.

(2) Device of the droplet suspender

The droplet suspender is made of a glass tube shown in Fig. 2. Its bottom end is drawn to the capillary whose outer diameter is about 0.2~0.4mm. Its outside is coated with thin lacquer film to prevent the adherence of water. The bottom end of the capillary is ground by the emery paper so as to make a flat surface, and here a water droplet can be kept stable.

(3) Measurement of the rate of evaporation

One end of the droplet suspender is connected with a microburette by a vinyl tube as shown in Fig. 2. This microburette can be operated up and down by a fine microscrew. In proportion to the difference of the levels of the droplet and the water surface in the burette, a suitable amount of water may be supplied to the droplet. If the rate of feed-water is controlled with the same rate of evaporation from the droplet surface, the size of the droplet can be a maintained constant during the whole period of the measurement. Flow rate of feed-water can be measured precisely with the scale of the microburette which is graduated by a pitch of 1/200cc. From the rate of feed water we can evaluate accurately the rate of heat supply to the droplet from the hot air.

The size of the droplet is observed by reading microscope and is controlled by the level of the burette so as to be maintained in a constant size. Moreover, photographs are taken in every 30 seconds and after the experiment the droplet diameter is read from the photographs with the accuracy of 1/100 mm by using the profile projector and its microscale.

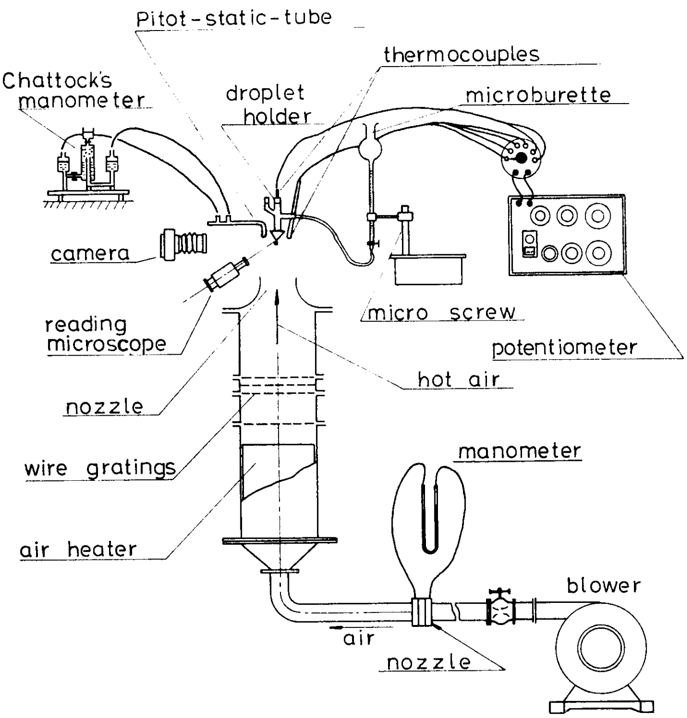


Fig. 1. General Layout of the Experimental Apparatus.

In order to keep a droplet size constant during the measurement, the water level in the burette must be controlled precisely. This was a delicate operation and was manipulated by hand, but once this has been mastered the droplet size was maintained constant. Fig. 3. (*a*) (*b*) show the examples of the progresses of feed-water rate and horizontal and vertical diameters of a droplet in a series of the experiment. Fig. 4 (*a*) (*b*) show the histograms of the distributions of the data shown in Fig. 3. Some of the photographs of the droplets are shown in Fig. 5. All data were plotted by these ways and their steadiness were verified. As we can see from these photographs and data, the profile of the droplet is not a perfect sphere but a ellipsoid of vertically longer, namely a prolate spheroid or a shape of pear.

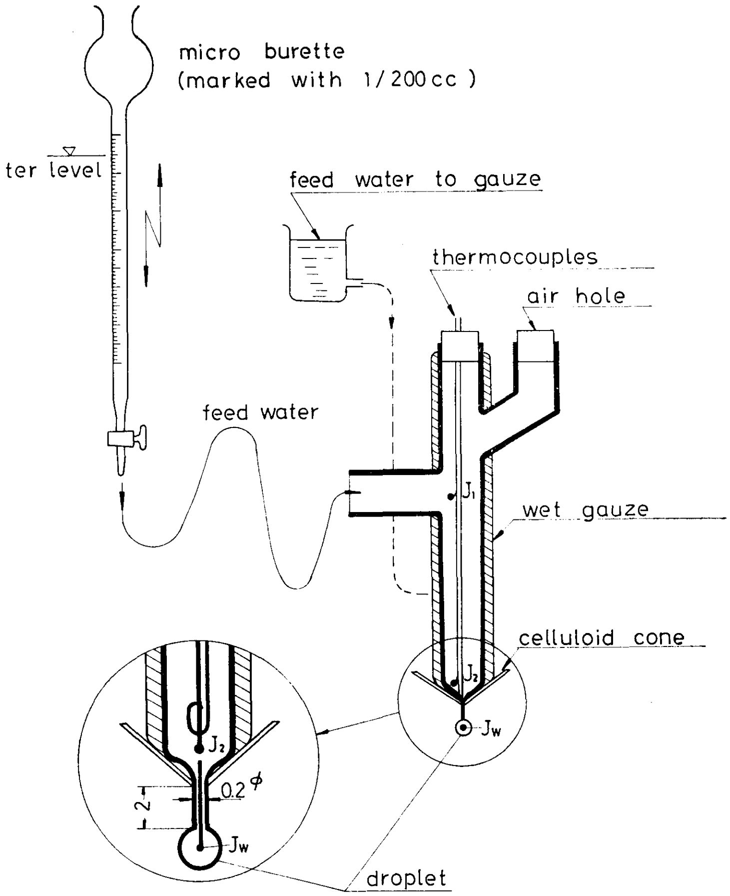
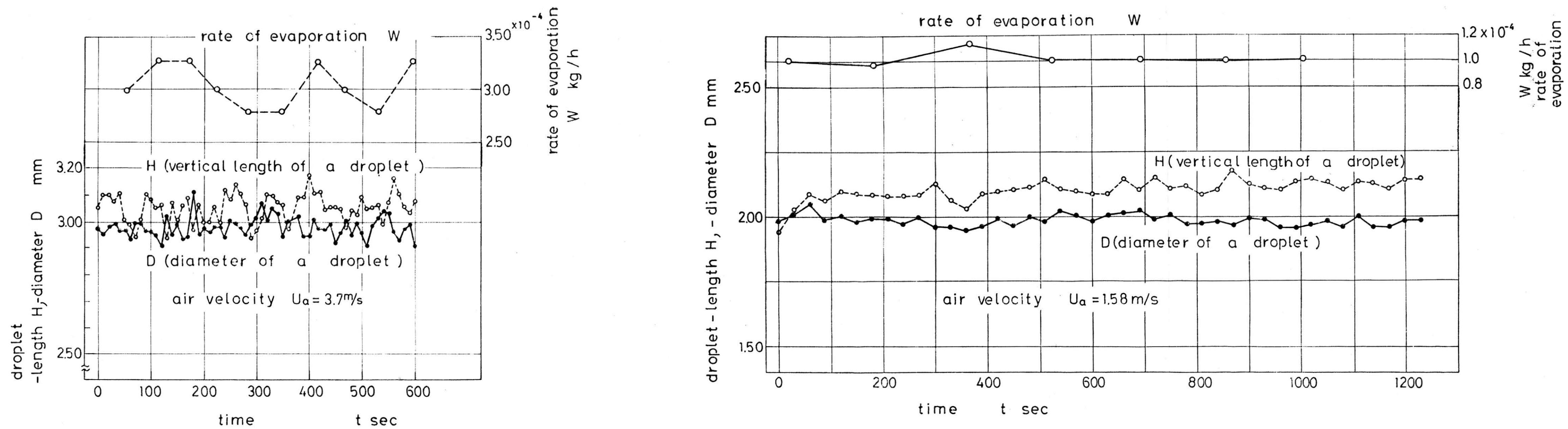


Fig. 2. Droplet Suspender.

(4) Temperature of droplet, feed water and air

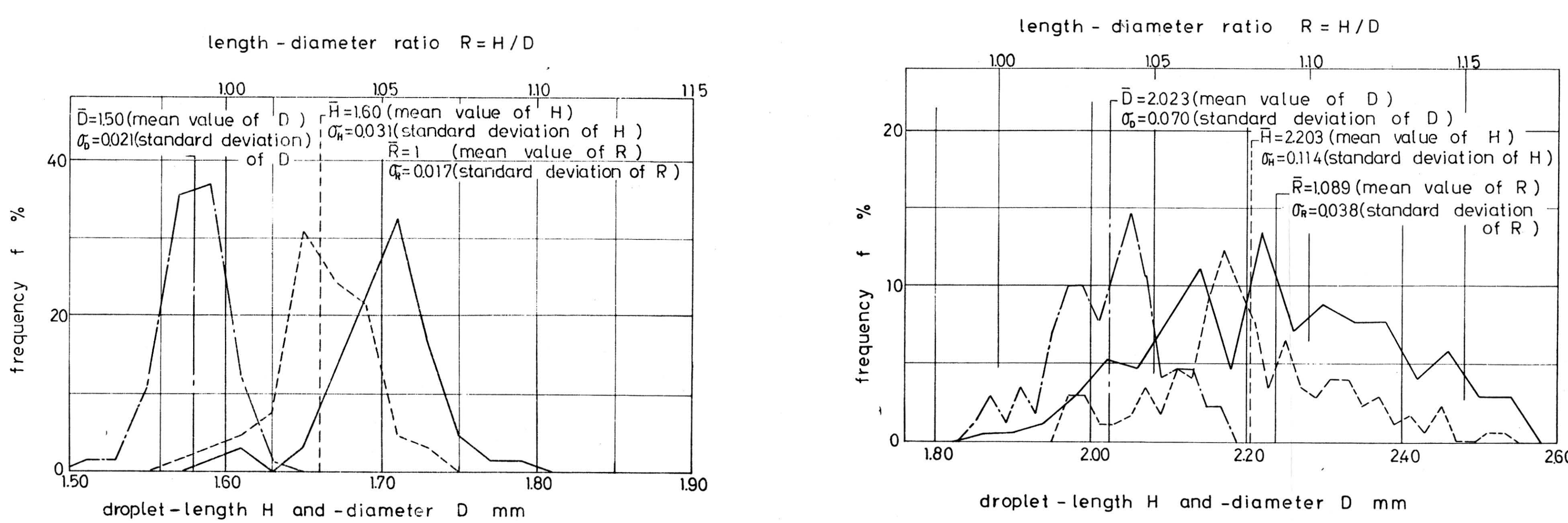
The droplet suspender is wrapped up by the wet gauze to which water is continuously supplied by siphon-effect. Owing to this device the temperature of the glass tube surface and also the water temperature in the tube can be expected to be close to the wet bulb temperature. As shown in Fig. 2 three thermo-junctions made of 0.1 mm Copper-Constantan wires are installed at three points in the glass tube marked J_1, J_2 and J_w and we mark the temperatures indicated by these junctions by letters T_1, T_2 and T_w, respectively. The junction J_w is in the center of the droplet and we can assume that the temperature T_w represents the droplet temperature. Hot air temperature T_a is also measured by another thermo-junction which is set beside the droplet. The relations between these temperatures are shown in Fig. 6.



(a) (b)

Fig. 3. Examples of the Progresses of Feed Water Rate and Horizontal and Vertical Diameter of the Droplet.

①



(a) (b)

Fig. 4. Distributions of the Data.

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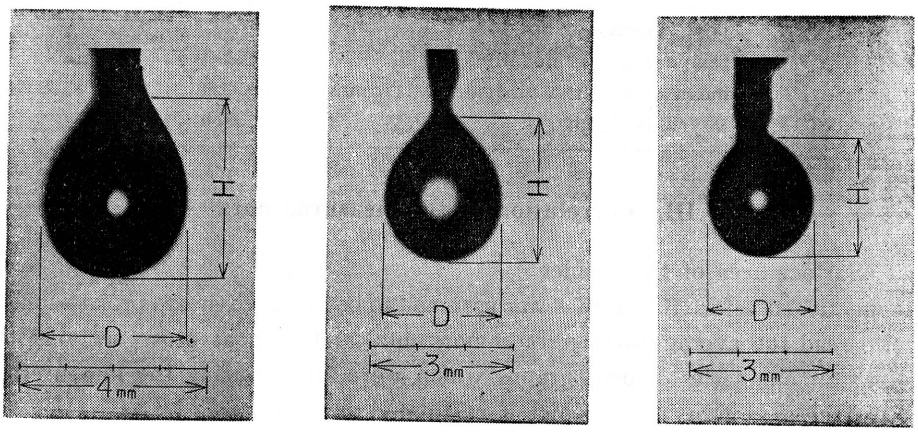

Re=185 Re=226 Re=134

Fig. 5. Photogrphs of Droplets.

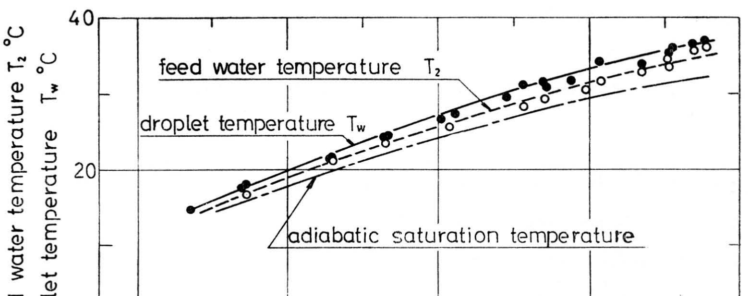

Fig. 6. Comparison of Temperatures.

(5) Conditions of the experiment

The experiments were carried out under the conditions of table 1.

Table 1. Items and Ranges of Experiments

item	symbol	range
flow velocity of hot air	u_a m/s	1.58～4.84
temperature of hot air	T_a °C	81～115
mean diameter of droplet	D mm	1.61～2.90
Reynolds number	R_e	70～600

III. Correlation of the measured data

(1) Surface area of the droplet

As mentioned before the droplet was assumed to be an approximate prolate spheroid and we read the average diameters of horizontal and vertical cross sections of a droplet from one series of photographs which were taken during the measurement. The surface area F m^2 of a droplet is calculated approximately by

$$F=\pi D\sqrt{\frac{H^2+D^2}{2}}-\frac{\pi}{4}d^2 \tag{1}$$

where, D : diameter of horizontal cross section of a droplet m

H : vertical length of a droplet m

d : diameter of the glass capillary m.

The second term of the right side of Eq. (1) is the decrease of the surface area which is caused by contact of a droplet to the capillary.

(2) Energy balance

Total heat flux q kcal/m^{2}h to the droplet surface from the hot air stream is

$$q_T=q_\alpha+q_r+q_\lambda+q_s \tag{2}$$

where, q_α : heat flux by convective heat transfer.

q_r : heat flux by radiation.

q_λ : heat flux by heat conduction.

q_s : heat flux by Ackermann's effect.

All of the energy transfered into the droplet is spent as the latent heat of evaporation and the temperature rise of the feed water. Then,

$$q_T=wL+cw(T_w-T_2) \tag{3}$$

where, w : rate of evaporation kg/h.

L : latent heat of evaporation kcal/kg.

c : specific heat of water kcal/kg °C.

a) Heat flux carried by feed water

This is shown in the last term in Eq. (3). Here $(T_w-T_2)\leqq 3$ °C and this was so small as compared with $L/c>500$. Then, this term could be neglected.

b) Heat flux by radiation q_r

q_r was calculated by assuming that the half side of the droplet viewed the hot air of temperature T_a and another half viewed the surroundings of temperature T_s. They were 2 % or less of total heat flux and could be neglected.

c) Heat transfer conducted through the glass capillary q_λ

As mentioned before the droplet and the glass tube temperatures were approximately equal to the wet bulb temperature. Then $q_\lambda \doteqdot 0$ may be considered. Moreover, in order to verify this assumption we tried some experiment changing the length of the capillary from 2 to 6 mm but there were no differences of evapolation rate in these experiments. It can be said from this consideration that there is no conductive heat transfer through the glass capillary.

d) Ackermann's effect

Ackermann's effect was taken into cosideration but it was so small that q_s was also neglected here.

e) Heat trasfer by convection q_α

Taking into consideration the above mentioned effects (*a*) (*b*) (*c*) and (*d*), heat flux by convection q_α can be simply written as,

$$q_\alpha = q_T \tag{4'}$$

$$\therefore \quad q_\alpha = wL \tag{5}$$

where the rate of evaporation w kg/h can be measured directly, then the heat flux by convection q_α can be calculated from Eq. (5).

IV. Experimental results

(1) Correlative equation

Following the precedent of Ranz we correlated our data as follows:

$$Nu = Nu_0 + K Re^m Pr^n \tag{6}$$

where Nu_0 is the Nusselt number when the flow velocity around the droplet is 0 and it is already obtained theoretically as $Nu_0=2$. In our experiments Prandtl number Pr was almost a constant, because only the air was used as the hot gas and the range of pressure and temperature was small.

Whole measured data were plotted in Fig. 7. We applied the least squares method to these points and took the following empirical equation:

$$Nu = 2 + 0.192\, Re^{0.70} \tag{7}$$

here, $70 < Re < 600.$

In Fig. 8 our result is compared with the other reference data[3][17][18][19]. Experimental conditions and formulae of such experiments are also shown in Table 2.

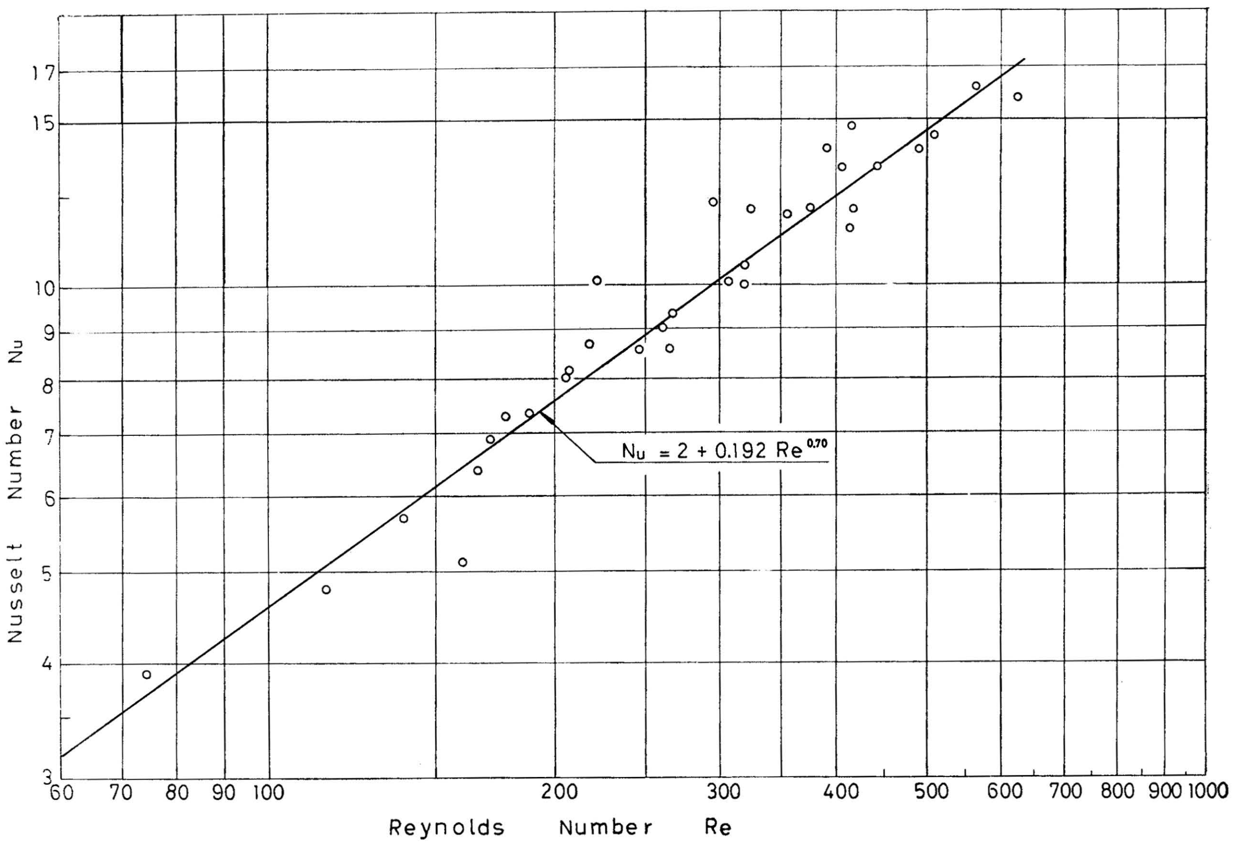

Fig. 7. Experimental Results and Correlative Equation.

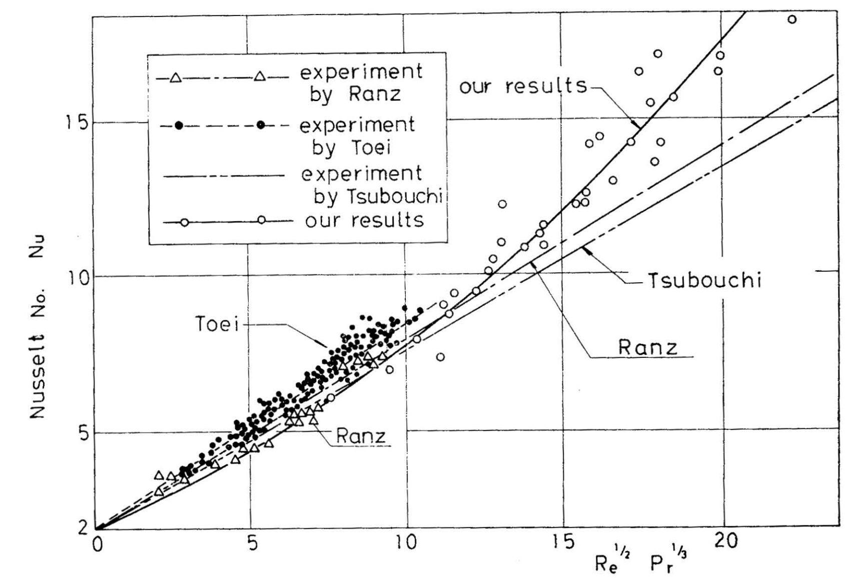

Fig. 8. Comparison with Reference Data.

Table 2. Correlative Equations of Forced Convective Heat Transfer of a Sphere.

Introducers of Formula	Year	Formula	Range of Re	No. of Eq.
Johnstone, Pigford & Chapin	1941	$Nu=0.714Re^{1/2}Pr^{1/2}$		(8)
Williams	1942	$Nu=0.37Re^{0.6}$	$17\sim7\times10^4$	(9)
Kramer	1946	$Nu=(0.97+0.68Re^{0.5})Pr^{0.3}$	$1\sim1000$	(10)
Yuge & Umehara	1950	$Nu=0.86Re^{1/2}$	$2\times10^3\sim5\times10^4$	(11)
Drake & Backer	1950	$Nu=2+0.40Re^{0.55}$	$0.1\sim2\times10^5$	(12)
Tang, Duncan & Schweger	1958	$Nu=3.10Pr+0.55PrRe^{1/2}$	$30\sim3000$	(13)
Yuge	1958	$Nu=2+0.49Re^{0.5}$ $Nu=2+0.3Re^{0.566}$	$10\sim1800$ $1800\sim1.5\times10^5$	(14)
Tsubouchi & Masuda	1963	$Nu=2+0.52Re^{1/3}Pr^{1/3}$ $Nu=2+0.57Re^{1/2}Pr^{1/3}$	$1.8\times10^{-4}\sim0.6$ $0.6\sim2400$	(15)
Ranz & Marshall	1952	$Nu=2(1+0.3Pr^{1/3}Re^{1/2})$	$1\sim9\times10^4$	(16)
Toei	1966	$Nu=2+0.65Re^{1/2}Pr^{1/3}$	~200	(17)

(2) Discussions on the experimental results

Our results coincide with the other data approximately. Especially at the low Reynolds number, we can see a very good agreement. However, at the high Reynolds number our results show rather larger values of the Nusselt number. In other words as we can see from Table 2 and Fig. 8 the exponent to the Reynolds number m is equal to 0.70 in our equation, but $m=0.5$ in most equations.

Eq. (16) correlated by Ranz is known as the most useful equation to the forced convective heat transfer. His experiments on the water droplet were carried out only in the range of Reynolds number of ~200. He extrapolated, however, five times beyond his experimental range in conformity to the experiments of solid sphere, and Eq. (16) was introduced. The exponent to the Reynolds number $m=0.5$ was verified closely by an experiment of Tsubouchi which was carried out by using the thermistor probe.

However, if we observe in detail whole measured values on liquid droplets introduced by Ranz and Toei in Fig. 8, it may be seen that they are not in a straight line of $m=0.5$ but in a concave curve. Moreover, it may be seen that the measured values by Rantz and our data are smoothly connected in only one simple curve of $m=0.70$. In other words, it may be unreasonable to apply the equation for solid sphere to the liquid droplet in large Reynolds number.

This result means that in the large Reynolds number heat transfer of liquid droplet is more active than that of solid sphere. In the large Reynolds number, the following facts were observed:

a) Ripple on the droplet surface.

b) Slow circulation of water in the droplet.

It can be seen that these two effects may change the condition of gas-liquid interface which influences directly to the heat transfer and heat flux may increase in comparison with those of the solid sphere.

The circulation of water in the droplet may be due to the momentum transport from the outer gas flow and from the feed water which flows into the top of the droplet. As illustrated in Fig. 9, the velocity of the feed water is much smaller than the induced velocity by the outer gas flow. Therefore, the circulation of water can be understood as a characteristic of large droplet itself.

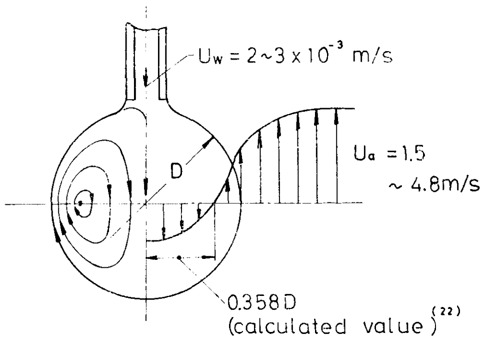

Fig. 9. Illustration of Flow Pattern in the Droplet.

V. Conclusions

We have carried out an experiment on the convective heat transfer between a water droplet and the hot air. The original points of this experiment were as follows:

(1) The experiment could be carried out in the steady state, namely the droplet size was maintained constant during the measurement by using a special device.

(2) The rate of evaporation was measured directly by a microburette.

(3) The range of Reynolds number in our experiment was 70～600, and larger than in the other previous researches.

From this experimental study we obtained a correlative equation on the forced convecive heat transfer of a water droplet

$$Nu=2+0.193\ Re^{0.70}$$

$$\text{at } Re=70\sim600.$$

VI. Acknowledgements

The authors wish to express their thanks to Mr. K. Miyake, Mr. K. Kuboki and Mr. M. Kusaka for their earnest assistance in this experimental work.

Discussions

This paper was presented at the 3rd Symposium on Heat Transfer in Japan (1966) and questions and comments were raised at that time. Here we compile these questions and our replies.

Questions and comments

(*D*—1) By this experimental method how rigorously could the steady state be maintained ?

(*D*—2) In Fig. 7 there is some scatter of the measured values. What is the reason ? Doesn't it mean the unsteadiness of the droplet size ?

(*D*—3) The results are correlated to a function of the Reynolds number. In this experiment, however, the droplet size is maintained almost a constant and only the gas velocity is varied. Therefore, these results should be considered as no effect of the Reynolds number but the effect of flow velocity only.

(*D*—4) On the considerably large droplet: the oscillation and deformation of the droplet profile are supposed. Were they observed ?

(*D*—5) Was the humidity of the hot air controlled or measured ?

Authors' replies

(*A*—1) The steadiness of the droplet size, the flow velocity and the air temperature can be seen from Fig. 3 and 4. We tried to calculate the standard deviation σ from these results and the most of them are 0.05 or less. Therefore it is concluded that the steadiness of this experiment is not perfect but enough.

(*A*—2) As the questioner pointed out, some scatter of the measured values was unavoidable in our experiment. As it was discussed in *D*—1 the scatter of the droplet size and the rate of feed water may not be its cause. We consider that the cause is mainly on the water temperature.

The temperatures of the droplet and feed water showed some differences. From these data it was considered that the heat balance was disturbed.

(*A*—3) The ranges of the experimental conditions are shown in Table 1. The droplet size is also taken in the wide range as well as the flow velocity. Then it is considered that our result does not depend on the flow velocity but closely on the Reynolds number.

(*A*—4) Oscillation of the droplet profile has been observed and this must be one of the causes of large heat flux in the large Reynolds number.

(*A*—5) The humidity of the hot air was not controlled. But relative humidity of the hot air (60～110 °C) was very small and there might be no effect on evaporation.

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