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A Contribution to Optimum Nonlinear Control

(on a system under an action of two disturbances)

(Received August 19, 1969)

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Abstract

In this paper, a controlled system under an action of two step disturbances, about which an interval of occurances and a ratio of magnitudes are known at the instant of the initiation of the first disturbance, is treated.

The optimum control in the sense of minimizing the maximum system error and the duration of the response is considered.

From this analysis, it is concluded that the response of the system can be improved by making use of the information about disturbances which can be known in advance. Further, it is also shown that the closed-loop control system can be constructed for a few cases.

I. Introduction

In the control theory, the closed-loop control has been paid large attention since it can eliminate undesirable effects of the disturbances. The open-loop control (in the sense of the scheduled control), has a good feature but the closed-loop control has not. The purpose of this paper is to point out such an aspect of the open-loop control.

When we wish to control some object, there may be some cases that the controlling action can be determined by not only the present state but also the future state of the system. This kind of problem was studied by Oldenburger.[1] In his paper, he discussed the responses of a controlled system to predetermined step and pulse disturbances. He showed that the response could be improved by making use of the parameters of the step and pulse disturbances known beforehand. The parameters were, for example, magnitude, instant of occurance, pulse duration and so on. Here arises a question—is there a case when one can know the instant of the occurrance of the disturbances or above mentioned quantities beforhand? If there were no case, such kind of study would be nonsense. We can, however show a following case as an example. In a prime mover, the load change, caused by its operation, often becomes disturbance. And in this case the occurance of the disturbance is

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determined by operating conditions. So the cotrolling action can start before the initiation of the disturbance if we want. Another example occurs in the case of a space vehicle. It is known in advance when the vehicle will enter or leave the earth's shadow. This information is used in the temperature control of the vehicle.

In this paper, the analysis by Oldenburger are extended to more general cases. We consider the optimum response for two step disturbances. We assume that the interval of occurances of two disturbances τ and the ratio of the magnitudes of two disturbances $\alpha=L_2/L_1$ become known at the instant when the system meets with the first disturbance. We consider the case when $0<\alpha\leqslant 1$.†

Here, "optimum" is used in the sense of minimizing the maximum system error and other criteria, as the transient duration.††

For systems where the rate of change of the controlling variable is bounded, the optimum transient in the above mentioned mean can be obtained by operating the system at the maximum rate of change of the controlling variable. In this paper, the rate of change of the controlling variable is called as "control".

The system under consideration is composed of two major parts; namely, the controlled system and the controller as shown in Fig. 1.

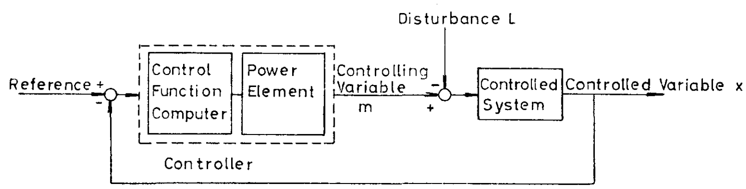

Fig. 1. Block diagram of a control system.

The controller has the following two elements:

(*a*) A power element with an output whose rate of change with respect to time is limited. Practically this limitation corresponds to the maximum power being bounded.

(*b*) A control-function computer used as a decision-making device which from the system error and its derivatives determines the instant at which the sign of the control is switched.

II. Nomenclature

The following nomenclature is used in the paper.

x : controlled variable

m : controlling variable

† When $\alpha=1$, these two step disturbances result in a pulse disturbance with magnitude $L=L_1=L_2$ and the duration τ.

†† In the case of a min-max performance criterion, the second criterion can be chosen. (See Johnson[3])

L : disturbance
t : time
t_1, t_2, t_e, t_f, t_g : switching times
Σ, Ω : switching functions
τ, α : parameters of disturbances

III. System equations

Consider the system showed by the differential equation

$$\dot{x}=m\,; \qquad |\dot{m}|\leq 1 \tag{1}$$

where $\dot{x}$ is the time rate of change of the controlled variable x, and m is the deviation in the controlling variable.

As an example we may have

$$x=\text{engine revolution,}$$
$$m=\text{throttle position}$$

for a prime mover, where these quantities are deviations from equilibrium values.

Equation (1) can be written equivalently in such form as

$$\begin{aligned}\dot{x}_1&=x_2\\ \dot{x}_2&=u\end{aligned} \tag{2}$$

where $x_1=x$, $x_2=\dot{x}$ and $u=\dot{m}$.

IV. Optimum control function

We discuss the optimum control in the sense of minimizing the maximum deviation and the response time. We introduce the following equation as the control (or switching) function,

$$\sum = x+\frac{1}{2}\{\dot{x}\}=x+\frac{1}{2}\dot{x}|\dot{x}| \tag{3}$$

Theorem 1[2]. For a second-order system (Equation (1) or (2)) in any given initial state ($x=x(0)$, $\dot{x}=\dot{x}(0)$) there is an optimum two-or-less-phase transient (the phase of an optimum transient is defined as that part of a transient for which the control u is a constant value) to the equilibrium state ($x=\dot{x}=0$) obtained by Equations

$$u=-\text{sgn}\ \Sigma \qquad \text{when}\ \Sigma\neq 0 \tag{4}$$

$$u=\ \ \text{sgn}\ x \qquad \text{when}\ \Sigma=0 \tag{5}$$

where

$$\text{sgn}\ A\begin{cases}=+1 & \\ =\ \ 0 & \text{when} \\ =-1 & \end{cases}\quad\begin{matrix}A>0\\ A=0\\ A<0.\end{matrix}$$

Equations (4) and (5) indicate that the optimum control is a type of "bang-bang". In this case the control function $\sum$ is determined by x and $\dot{x}$, therefore it is possible to construct the closed-loop control system.

V. Optimum control for a step disturbance

Consider an optimum response for a step disturbance as exprssed in Fig. 2. We assume that this disturbance acts upon the system at the same place where the controlling variable acts. (See Fig. 1.) So Equation (1) becomes

$$\dot{x}=m-L \tag{6}$$

Therefore, by the step disturbance the state of the system which was at equilibrium for $t<0$ becomes to $(x, \dot{x})=(0, -L)$. The optimum control can be obtained by Theorem 1. Fig. 3 shows an optimum response for the step disturbance.

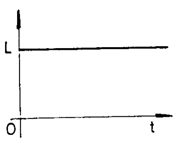

Fig. 2. A step disturbance.

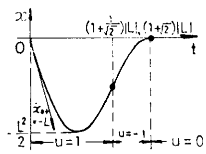

Fig. 3. An optimum response for a step disturbance.

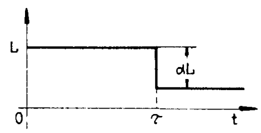

Fig. 4. Two step disturbances.

VI. Optimum control for two step disturbances

Consider an optimum control for two step disturbances shown in Fig. 4. In Fig. 4 τ is an interval of two disturbances and α is a ratio of the magnitudes of two disturbances. We consider the case when $0<\alpha\leq 1$.

From a view of the feedback control, the control detemined by Eqs. (4) and (5) is in a sense the optimum control for this system. We call the optimum control of this kind as the conventional optimum control.

Getting an information about τ and α beforehand, it is possible to improve the response by making use of them. In analysis it is assumed that this information is available from the instant of the initiation of the first step disturbance.

Case 1.

When τ is in the range $0<\tau\leq\frac{2-\alpha+\sqrt{2-\alpha^2}}{2}|L|$, it is impossible to obtain the better response than the conventional optimum response carried out by Eq. (4) and (5). If there are no disturbances except these two, we obtain the optimum response shown in Fig. 5. The trajectory on a phase plane is shown in Fig. 6.

The switching time of the control t_1 and t_2 in Fig. 4 are as follows, respectively

$$\begin{aligned} t_1 &= (1-\alpha)|L| + (\alpha|L|\tau + (1-\alpha)L^2/2)^{1/2} \\ t_2 &= (1-\alpha)|L| + 2(\alpha|L|\tau + (1-\alpha)L^2/2)^{1/2} \end{aligned} \tag{7}$$

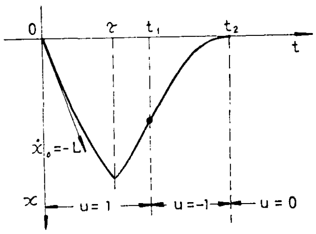

Fig. 5. Optimum response for fwo step disturbances$\left(0<\tau\leqslant\frac{2-\alpha+\sqrt{2-\alpha^2}}{2}|L|\right)$.

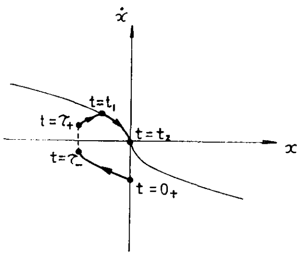

Fig. 6. Phase-plane trajectory of the optimum response.

Case 2.

When τ is in the range $\frac{2-\alpha+\sqrt{2-\alpha^2}}{2}|L|<\tau\leq(1+\alpha+\sqrt{2}\sqrt{1+\alpha^2})|L|$, the response is improved in the sense of minimizing the maximum system error and the response time. The optimum response is shown in Fig. 7. We shall call the control corresponding to this response as "the improved optimum control I" in order to distinguish from the conventional optimum control determined by Eqs. (4) and (5).

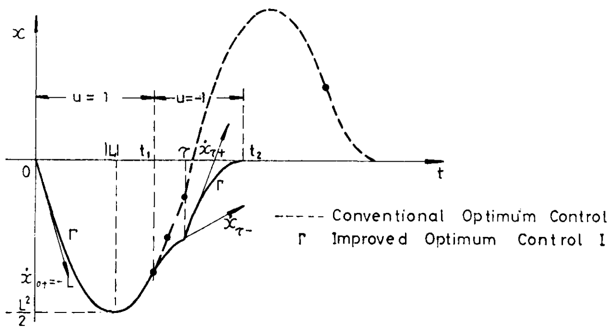

Fig. 7. Optimum responce for two step disturbances $\left(\frac{2-\alpha+\sqrt{2-\alpha^2}}{2}|L|<\tau\leq[1+\alpha+\sqrt{2}\sqrt{1+\alpha^2}]|L|\right)$.

The control schedule for this response is given as follows;

$$\begin{aligned} u &= 1 \quad &\text{for}\quad 0<t\leq t_1 \\ u &= -1 \quad &\text{for}\quad t_1<t\leq t_2 \end{aligned} \tag{8}$$

$$u = 0 \qquad \text{for} \qquad t > t_2$$

where

$$\begin{aligned} t_1 &= (1-\alpha)|L| + (\tfrac{1}{2}(1-\alpha)^2L^2 + \alpha|L|\tau)^{1/2} \\ t_2 &= (1-\alpha)|L| + 2(\tfrac{1}{2}(1-\alpha)^2L^2 + \alpha|L|\tau)^{1/2} \end{aligned} \tag{9}$$

In Fig. 7 the dashed curve shows the response obtained by the conventional optimum control.

Fig. 8 shows the phase plane trajectories of the optimum response obtained by the conventional and the improved method.

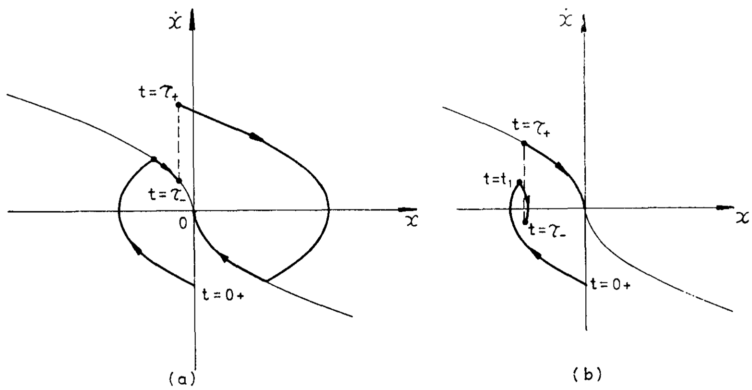

Fig. 8. Phase-plane trajectory of the optmum response.
(*a*) by the conventional method, (*b*) by the improved method.

This result shows that substantial improvement is done by our method.

Case 3.

When τ is in the range $(1+\alpha+\sqrt{2(1+\alpha^2)})|L| < \tau \leq (1+\alpha)(1+\sqrt{2})|L|$, it is possible to improve the response time by making use of the advanced information. Namely the state of the system can be brought to the equilibrium state at the instant of the initiation of the second disturbance. Fig. 9 shows the response of the system. Fig. 10 shows the phase plane trajectories obtained by the conventional and improved method. We shall call this type of control considered here as "the improved optimum control II". The switching time of the control t_e, t_f and t_g in Fig. 9 are, respectively

$$\begin{aligned} t_e &= \tau - \alpha|L| - (\tfrac{1}{2}(\tau + (1+\alpha)|L|)^2 - (1+\alpha^2)L^2)^{1/2} \\ t_f &= \tfrac{1}{2}(\tau + (1-\alpha)|L|) \\ t_g &= |L| + (\tfrac{1}{2}(\tau - (1+\alpha)|L|)^2 - (1+\alpha^2)L^2)^{1/2} \end{aligned} \tag{10}$$

Case 4

When τ is in range $\tau > (1+\alpha)(1+\sqrt{2})|L|$, the same improvement as in Case 3 is possible. The response of the system is shown in Fig. 11.

Fig. 12 shows that control of above three (conventional, improved I and improved II) should be used for particular values of τ, L and α.

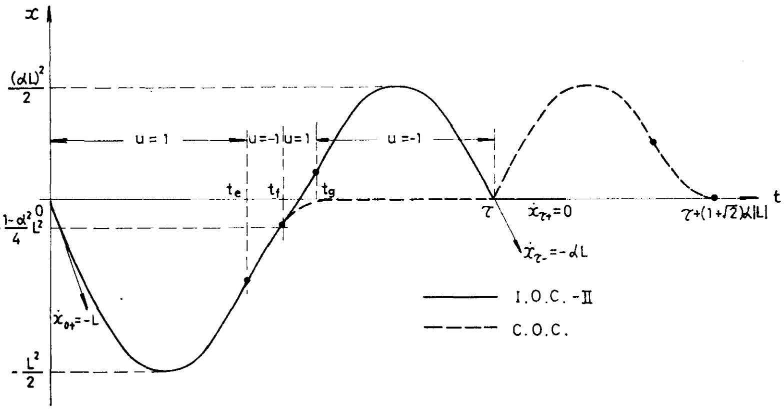

Fig. 9. Optimum response for two step disturbances
$([1+\alpha+\sqrt{2}\sqrt{1+\alpha^2}]|L|<\tau\leq(1+\alpha)(1+\sqrt{2})|L|)$.

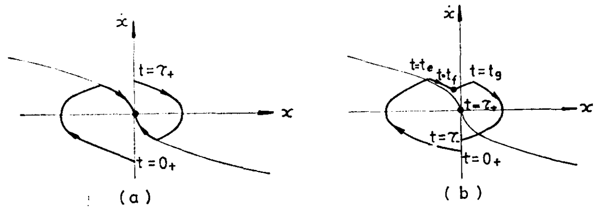

Fig. 10. Phase plane trajectory of the optimum response.
(*a*) by the conventional method, (*b*) by the improved method.

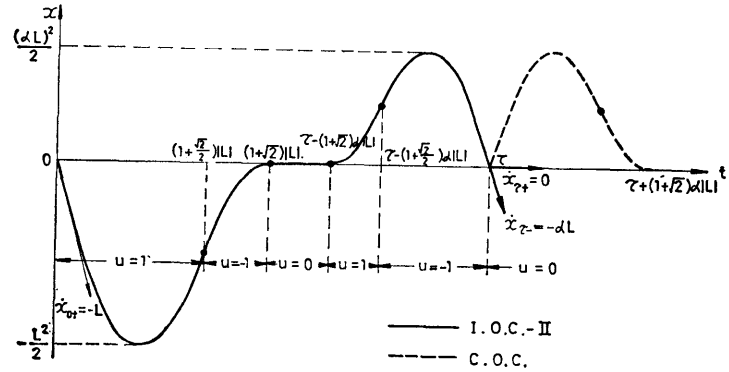

Fig. 11. Optimum response for two step disturbances
$(\tau>(1+\alpha)(1+\sqrt{2})|L|)$.

α	0.1	0.2	0.5	0.8	1.0
$\frac{(2-\alpha)+\sqrt{2-\alpha^2}}{2}$	1.65	1.60	1.41	1.19	1.00
$[1+\alpha+\sqrt{2}\sqrt{1+\alpha^2}]$	2.51	2.64	3.08	3.61	4.00

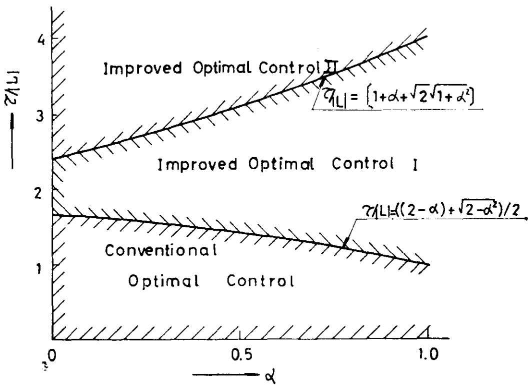

Fig. 12.

VII. Numerical example

Table 1 shows an example of comparison between the conventional optimum control and the improved optimum control I. These are calculated for $\tau/L=2$. Fig. 13 is helpful to understand the improvement carried out by the imroved optimum control I.

VIII. Synthesis of optimum control function

For value of τ in the range $0<\tau\leq(2-\alpha+\sqrt{2-\alpha^2})|L|/2$, the optimum response of the preceding section can be obtained by use of Eqs. (4) and (5), which enable us to construct the closed-loop control system as shown in Fig. 1.

For value of τ in the range $(2-\alpha+\sqrt{2-\alpha^2})|L|/2<\tau\leq(1+\alpha+\sqrt{2}\sqrt{1+\alpha^2})|L|$, we can find the control function

$$\Omega=x+\tfrac{1}{2}(\dot{x}+\alpha L)\cdot|\dot{x}+\alpha L|-\alpha L(\tau-t) \tag{11}$$

And the improved optimum control I is obtained by

$$u=-\operatorname{sgn}\Omega \qquad \text{when} \qquad \Omega\neq 0 \tag{12}$$

$$u=\operatorname{sgn} x \qquad \text{when} \qquad \Omega=0 \tag{13}$$

Using Eqs. (12) and (13), it is possible to construct the closed-loop system.

It is impossible to obtain the improved optimum control II as closed one.

Table. 1. ($\tau/L=2$)

	α	0.1	0.2	0.5	1.0
Conventional Optimum Control	response time (t/L)	2.94	3.26	4.06	5.32
	maximum system error (x/L^2)	0.5	0.5	0.5	0.914
Improved Optimum Control—I	response time (t/L)	2.45	2.50	2.62	2.83
	maximum system error (x/L^2)	0.5	0.5	0.5	0.5

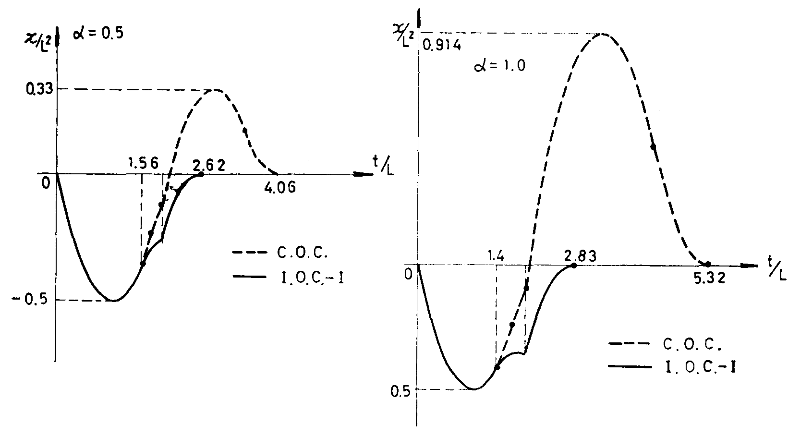

Fig. 13. Optimum response for two step disturbances ($\tau/L=2$).

IX. Conclusions

Optimum response was obtained for a second-order system with the bounded input. The term "optimum" was used in the sense of minimizing the maximum system error and the duration of the response.

For the response corresponding to two step disturbances, we showed the several improvement, making use of such informations as the duration of the occurances of two disturbances τ and the ratio of the magnitudes of two disturbances α. According to the values of τ, α and L, three cases were given.

1) Improvement was impossible (Case 1).
2) The response was able to be improved in the sense of minimizing the maximum system error and the duration of the response (Case 2).
3) The system was able to be brought to the equilibrium state at the instant of the initiation of the second disturbance (Cases 3 and 4).

The switching times of control were given and the optimum control was discussed as the open-loop control. So in the case when the effect of other noises is large, this type of control should not be used. For 1) and 2), however, it was possible to construct the closed-loop control system.

X. Acknowledgement

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References

(1) R. Oldenburger, R. C. C. Chang, "Optimum Nonlinear Control for Step and Pulse Disturbances," Trans. ASME, vol. 87, series D, No. 1, 1965.
(2) R. Oldenburger, "Optimal Control," Holt Rinehalt and Winston, 1966.
(3) C. D. Johnson, "Optimal Control with Chebyshev Minimax Performance Index," Trans. ASME, vol. 89 series D, No. 89, 1967.