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Author	藤田, 広一(Fujita, Hiroichi) 柳瀬, 智夫(Yanase, Tomoo)
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Reflection and Transmission of a Plane Electromagnetic Wave at a Moving Boundary

(Received April 9, 1969)

Hiroichi FUJITA*
Tomoo YANASE**

Abstract

The present paper is concerned with the problem of a moving boundary in the questions of relative electromagnetic theory, particularly the reflection and transmission of a plane electromagnetic waves at the moving boundary. Here the two media are stationary, but the boundary moves. It is found that Snell's law on reflection and transmission in the stationary state can be extended to the dynamic state. The frequency shift of reflected wave and transmitted wave occurs not because of moving media but of moving boundary. In case of TE wave, it is found that the absolute value of the amplitude of the reflected wave is equal to that of the incident wave and is independent of the angle of incidence, when the speed of the boundary is light speed in the second medium. Sum of the power reflection coefficient and the power transmission coefficient is not equal to unity. In case of TM wave, the expression of Brewster angle is extended; when the speed of the boundary is equal to the light speed in medium 2 the Brewster angle is 90°, and when it exceeds the light speed in medium 2 the Brewster angle vanishes. In this paper the above phenomena are described in detail that have never reported.

I. Introduction

Problems in the relative electromagnetic theory are much discussed recently, especially of the reflection and transmission of a plane wave at a interface, where two media move relatively. In these problems. there are two different cases; one is on moving media, and another is on moving boundary. For the first case, problems where two media move parallel with interface were studied in detail by C. Yeh,[1] T. Shiozawa,[2][3] T. Hosono[4][5] and others. S. N. Stolyarov,[6] C. S. Tsai and B. A. Auld[7] and others studied the problems where the media moved vertically to the interface.

This paper concerns with the second case: not moving media, but moving boundary. Here the media are stationary but the boundary moves. This means that

*藤 田 広 一 Professor, Faculty of Engineering, Keio University.
**柳 瀬 智 夫 Graduate Student, Faculty of Engineering, Keio University.

medium 1 changes to medium 2 or vice versa.

Such a physical phenomenon may possibly occur when atmospheric gas is ionized by the sunlight. For instance in Fig. 1, if the light travels in the y direction and ionizes the atmosphere, then the permittivity ε_2 becomes ε_1 and the bonudary moves at a constant speed u in the y direction. This model is suitable to the problem of relative electromognetic theory. When the surface of sun explodes violently, the height of ionosphere on the earth changes rapidly. In such changes, the media don't move, but the boundary does.

It is assumed that both media are homogeneous, isotropic, and lossless. Two cases are investigated. In the first case incident wave is TE wave to the surface, and in the second case, TM wave. In conclusion we obtain the following features by the moving boundary that has never been reported. (1) The angle of reflection is not equal to the angle of incidence, and the angle of transmission doesn't satisfy the Snell's law at a stationary. The Snell's law is extended in this paper. (2) For $\varepsilon_1 > \varepsilon_2$, the expression of total reflection is extended. (3) The frequencies of reflected wave and transmitted wave are not equal to one of incident wave. The cause that makes this phenomenon is not moving media but moving boundary. (4) In case of TE wave, when the speed of the boundary is light speed in medium 2, the absolute value of the amplitude of the reflected wave is equal to that of the incident wave and is independent of the angle of incidence. (5) It is found that $R+T\neq 1$, where R is power reflection coefficient and T is power transmission coefficient. (6) In case of TM wave, the expression of Brewster angle is extended. When the speed of the boundary exceeds the light speed in medium 2 the Brewster angle vanishes. (7) Between the frequencies and amplituds it is found that the following equation holds,

$$\frac{E_i}{\omega_i}+\frac{E_r}{\omega_r}=\frac{E_t}{\omega_t} \qquad \text{(TE wave)} \tag{1}$$

$$\frac{H_i}{\omega_i}+\frac{H_r}{\omega_r}=\frac{H_t}{\omega_t} \qquad \text{(TM wave)} \tag{2}$$

where i, r, and t represent incidence, reflection, and transmission, respectively.

II. Fundamental theory

In this paper it is assumed that the boundary moves but the two media are stationary. homogeneous, isotropic and lossless. Then we need moving boundary conditions. The boundary conditions at a moving boundary are[9]

$$\boldsymbol{n}\times(\boldsymbol{E}_1-\boldsymbol{E}_2)-u(\boldsymbol{B}_1-\boldsymbol{B}_2)=0 \tag{3}$$

$$\boldsymbol{n}\times(\boldsymbol{H}_1-\boldsymbol{H}_2)+u(\boldsymbol{D}_1-\boldsymbol{D}_2)=0 \tag{4}$$

Here $\boldsymbol{E}$ and $\boldsymbol{H}$ are the intensities respectively of the electric and magnetic field, $\boldsymbol{D}$ is the electric displacement and $\boldsymbol{B}$, the magnetic induction. The subscripts 1 and 2 represent medium 1 and 2, respectively. $\boldsymbol{n}$ is the unit normal vector on the boundary and u is speed of the boundary from medium 1 to medium 2.

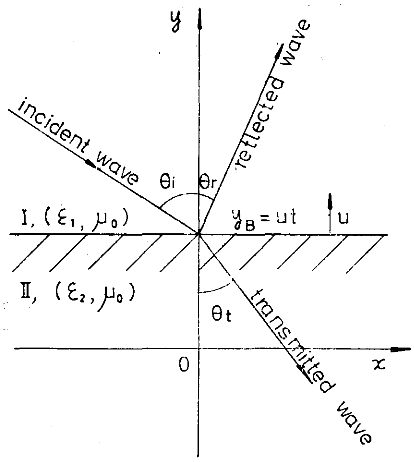

Fig. 1. Reflection and transmission of a plane wave.

III. Reflection and transmission of plane wave

We take a system of coordinates as shown in Fig. 1, and the boundary moves to the y direction. The permittivities and permeabilities of medium 1 and 2 are (ε_1, μ_0) and (ε_2, μ_0), respectively. The subscripts i, r and t represent incidence, reflection and transmission. For example the angle of incidence is θ_i, the electric field of reflected wave is E_r: and the freqnency of transmitted wave is ω_t. The coordinate of the boundary (y_B) is

$$y_B = ut \tag{5}$$

(*a*) TE Wave

The incident wave, the reflected wave and the transmitted wave are represented by

$$\begin{aligned} &E_{iz} = E_i \exp j(k_{ix}x + k_{iy}y - \omega_i t) \\ &H_{iz} = 0 \end{aligned} \tag{6}$$

$$\begin{aligned} &E_{rz} = E_r \exp j(k_{rx}x + k_{ry}y - \omega_r t) \\ &H_{rz} = 0 \end{aligned}$$

$$\begin{aligned} &E_{tz} = E_t \exp j(k_{tz}x + k_{ty}y - \omega_t t) \\ &H_{tz} = 0 \end{aligned} \tag{8}$$

The phases are invariant at the boundary ($y_B = ut$).

$$k_{ix}x - (\omega_i - uk_{iy})t = k_{rx}x - (\omega_r - uk_{ry})t = k_{tx}x - (\omega_t - uk_{ty})t \tag{9}$$

Then the following equations hold.

$$k_{ix} = k_{rx} = k_{tx} \tag{10}$$

$$\omega_i - uk_{iy} = \omega_r - uk_{ry} = \omega_t - uk_{ty} \tag{11}$$

From equations (10) and (11) the angle of reflection is

$$\cos\theta_r=\frac{2\beta n_1+\cos\theta_i(1+\beta^2 n_1^2)}{1+2\beta n_1\cos\theta_i+\beta^2 n_1^2} \tag{12}$$

The angle of transmission is

$$\cos\theta_t=\frac{\beta n_2(1+\beta n_1\cos\theta_i)-Q}{1+\beta n_1\cos\theta_i-\beta n_2 Q} \tag{13}$$

where $\beta=u/c$, and c is the speed of light in vacuum, n_1 and n_2 are refractive indexes.

$$Q=\left[\left(\beta n_1+\cos\theta_i\right)^2+\sin\theta_i\times\left(1-\frac{n_1^2}{n_2^2}\right)\right]^{\frac{1}{2}} \tag{14}$$

The frequency of reflected wave is

$$\frac{\omega_r}{\omega_i}=\frac{1+2\beta n_1\cos\theta_i+\beta^2 n_1^2}{1-\beta^2 n_1^2} \tag{15}$$

The frequency of transmitted wave is

$$\frac{\omega_t}{\omega_i}=\frac{1+\beta n_1\cos\theta_i-\beta n_2 Q}{1-\beta^2 n_2^2} \tag{16}$$

And from the moving bondary conditions expressed by equations (3) and (4)

$$\frac{E_i}{\omega_i}+\frac{E_r}{\omega_r}=\frac{E_t}{\omega_t} \tag{17}$$

holds between the amplitudes of electric fields and frequencies.
The amplitude of the reflected wave is

$$\frac{E_r}{E_i}=\frac{-(1+\beta n_1\cos\theta_i)(n_2\cos\theta_t+\beta n_2^2)+(1+\beta n_2\cos\theta_t)(n_1\cos\theta_1+n_1^2\beta)}{(1-\beta n_1\cos\theta_r)(n_2\cos\theta_t+\beta n_2^2)+(1+\beta n_2\cos\theta_t)(n_1\cos\theta_r-\beta n_1^2)} \tag{18}$$

The amplitude of the transmitted wave is

$$\frac{E_t}{E_i}=\frac{(1-\beta n_1\cos\theta_r)(n_1\cos\theta_i+\beta n_1^2)+(1+\beta n_1\cos\theta_i)(n_1\cos\theta_r-n_1^2\beta)}{(1-\beta n_1\cos\theta_r)(n_2\cos\theta_t+\beta n_2^2)+(1+\beta n_2\cos\theta_t)(n_1\cos\theta_r-\beta n_1^2)} \tag{19}$$

The power reflection coefficient is

$$R=\left(\frac{E_r}{E_i}\right)^2\frac{\cos\theta_r}{\cos\theta_i} \tag{20}$$

The power transmission coefficient is

$$T=\left(\frac{E_t}{E_i}\right)^2\frac{\cos\theta_t}{\cos\theta_i}\frac{n_2}{n_1} \tag{21}$$

(*b*) TM Wave

The incident wave, the reflected wave and the transmitted wave are represented by

$$\begin{aligned}&H_{iz}=H_i\exp j(k_i\,x+k_{iy}y-\omega_i t)\\&E_{iz}=0\end{aligned} \tag{22}$$

$$\begin{aligned} H_{rz} &= H_r \exp j(k_{rx}x + k_{ry}y - \omega_r t) \\ E_{rz} &= 0 \end{aligned} \tag{23}$$

$$\begin{aligned} H_{tz} &= H_t \exp j(k_{tx}x + k_{ty}y - \omega_t t) \\ E_{tz} &= 0 \end{aligned} \tag{24}$$

The angle of reflection and transmission, the frequencies of reflected wave and transmitted wave are expressed by equations (12)—(16), just like in case of TE wave. From the moving boundary conditions expressed by equations (3) and (4),

$$\frac{H_i}{\omega_i} + \frac{H_r}{\omega_r} = \frac{H_t}{\omega_t} \tag{25}$$

holds between the amplitudes of magnetic fields and frequencies. The amplitude of the reflected wave is

$$\frac{H_r}{H_i} = \frac{-(1+\beta n_1 \cos\theta_i)(\cos\theta_t + /n_2 + \beta) + (1+\beta n_2 \cos\theta_t)(-\cos\theta_i/n_1 + \beta)}{(1-\beta n_1 \cos\theta_r)(\cos\theta_t/n_2 + \beta) + (1+\beta n_2 \cos\theta_t)(\cos\theta_r/n_1 - \beta)} \tag{26}$$

The amplitude of the transmitted wave is

$$\frac{H_t}{H_i} = \frac{(1-\beta n_1 \cos\theta_r)(\cos\theta_i/n_1 + \beta) + (1+\beta n_1 \cos\theta_i)(\cos\theta_r/n_1 - \beta)}{(1-\beta n_1 \cos\theta_r)(\cos\theta_t/n_2 + \beta) + (1+\beta n_2 \cos\theta_t)(\cos\theta_r/n_1 - \beta)} \tag{27}$$

The power reflection coefficient is

$$R_{TM} = (H_r/H_i)^2 (\cos\theta_r/\cos\theta_i) \tag{28}$$

The power transmission coefficient is

$$T_{TM} = (H_t/H_i)^2 (\cos\theta_t/\cos\theta_i)(n_1/n_2) \tag{29}$$

(*c*) Numerical computation and discussion

Results of numerical computation which were obtained from equations (12)—(29) are shown in Fig. 2—Fig. 13. We took θ_i(angle of incidence) as variable and β as parameter. These are the angle of reflection (θ_r), the angle of transmission (θ_t), the frequency of the reflected wave (ω_r), the frequency of the transmitted wave (ω_t), the amplitude of reflected electric field (E_r), the amplitude of transmitted electric field (E_t), the amplitude of reflected magnetic field (H_r), the amplitude of transmitted magnetic field (H_t), the power reflection coefficient (R), and the power transmission coefficient (T). The angle of reflection, the angle of transmission, the frequencies of reflection and transmission are common to TE wave and TM wave in these results. In these figures, figures (*a*) correspond to the condition that the relative permittivities of each media are $\varepsilon_1/\varepsilon_0=1.001$, $\varepsilon_2/\varepsilon_0=2$, and figures (*b*) $\varepsilon_1/\varepsilon_0=2$, $\varepsilon_2/\varepsilon_0=1.001$. The marks (⊙) in figures (*a*) show the points which are not satisfied by

$$k_{ry}/\omega_r = \sqrt{\varepsilon_1 \mu_0}\,\cos\theta_r > 0 \tag{30}$$

When (30) is not satisfied, reflected wave doesn't exist. The marks (⊙) in figures (*b*) are the critical angles. When the angle of incidence is greater than the critical angles, the transmitted wave is damped. The amplitudes, the power reflection coefficients and power transmission coefficients of TE wave are shown in Fig. 6—Fig. 9. Those of TM wave are shown in Fig. 10—Fig. 13.

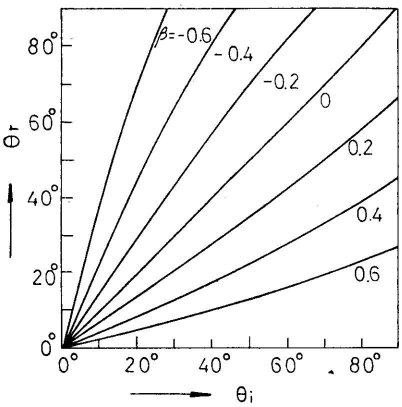

Fig. 2. (*a*) Relation between angles of incidence and those of reflection. ($\varepsilon_1/\varepsilon_0=1.001$, $\varepsilon_2/\varepsilon_0=2$)

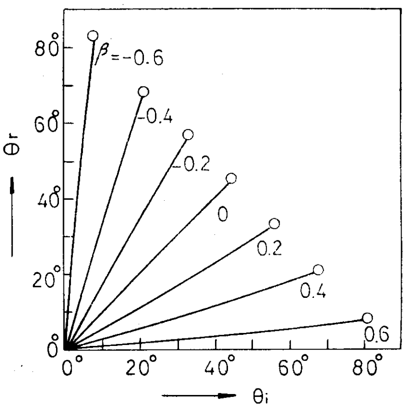

Fig. 2. (*b*) Relation between angles of incidence and those of reflection. ($\varepsilon_1/\varepsilon_0=2$, $\varepsilon_2/\varepsilon_0=1.001$)

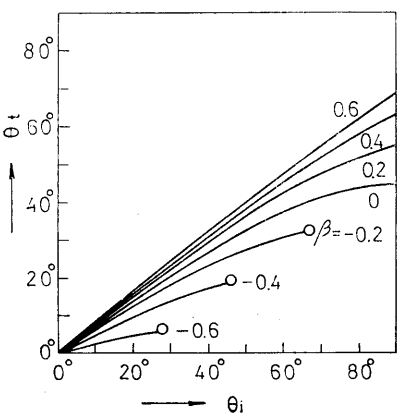

Fig. 3. (*a*) Relation between angles of incidence and those of transmission. ($\varepsilon_1/\varepsilon_0=1.001$, $\varepsilon_2/\varepsilon_0=2$)

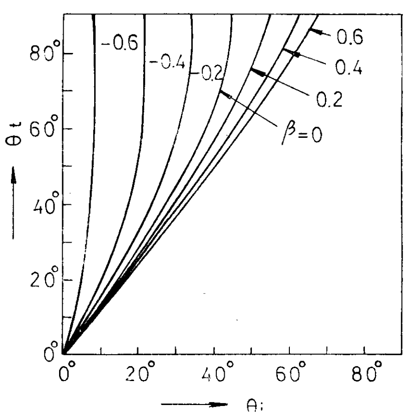

Fig. 3. (*b*) Relation between angles of incidence and those of transmission. ($\varepsilon_1/\varepsilon_0=2$, $\varepsilon_2/\varepsilon_0=1.001$)

The Snell's law in stationary boundary

$$\theta_i=\theta_r \qquad n_1 \sin\theta_i=n_2 \sin\theta_t \tag{31}$$

is extended in moving boundary. The relations given in equations (12) and (13) represent extended Snell's law. In extreme cases, the followings may be obtained.

$\varepsilon_1<\varepsilon_2$			$\varepsilon_1>\varepsilon_2$		
	θ_r	θ_t		θ_r	θ_t
$\beta\to 1/n_2$	0°	θ_i	$\beta\to 1/n_1$	0°	θ_i
$\beta\to -1/n_2$	90°	0°	$\beta\to -1/n_1$	90°	90°

The expression for critical angle (θ_0) is also extended. When the boundary is stationary

$$\sin\theta_0=n_2/n_1 \tag{32}$$

But when the boundary moves,

$$\cos\theta_0=\left(\frac{n_2}{n_1}\right)^2\left(-\beta n_1+\sqrt{(\beta^2n_1^2-n_1^2/n_2^2)(1-n_1^2/n_2^2)}\right) \tag{33}$$

The frequencies of reflected wave and transmitted wave are not equal to that of incident wave. The reason is not moving media but moving boundary. Even if the media move, it holds that $\omega_i=\omega_r=\omega_t$ in stationary boundary. The equations for Doppler's shifts are given in equations (16) and (17).

When the speed of the boundary is light speed in medium 2, the absolute value of the amplitude of the reflected wave is equal to that of the incident wave and is independent of the angle of incidence.

When the boundary moves,

$$\begin{aligned} R_{TE}+T_{TE}&\fallingdotseq 1\\ R_{TM}+T_{TM}&\fallingdotseq 1 \end{aligned} \tag{34}$$

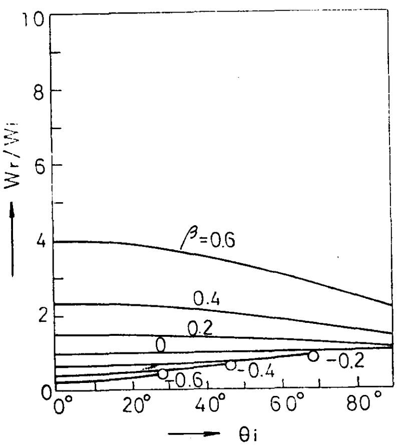

Fig. 4. (*a*) Relation between angles of incidence and frequencies of reflection. ($\varepsilon_1/\varepsilon_0=1.001$, $\varepsilon_2/\varepsilon_0=2$)

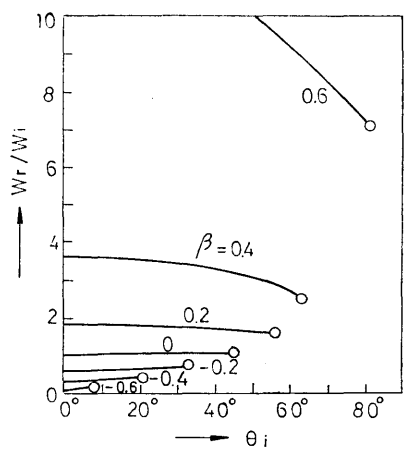

Fig. 4. (*b*) Relation between angles of incidence and frequencies of reflection. ($\varepsilon_1\varepsilon/_0=2$, $\varepsilon_2/\varepsilon_0=1.0001$)

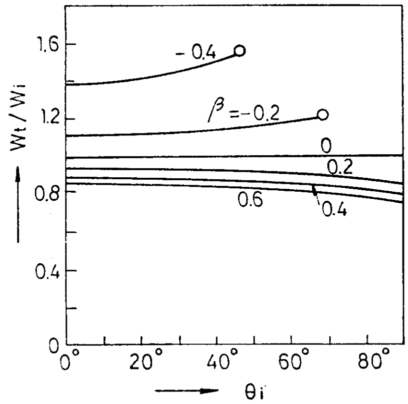

Fig. 5. (a) Relation between angles of incidence and frequencies of transmission.
($\varepsilon_1/\varepsilon_0=1.001$ $\varepsilon_2/\varepsilon_0=2$)

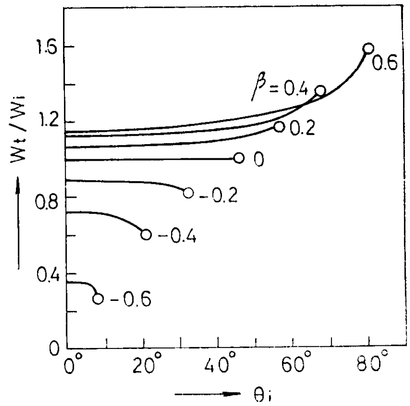

Fig. 5. (b) Relation between angles of incidence and frequencies of transmission.
($\varepsilon_1/\varepsilon_0=2$, $\varepsilon_2/\varepsilon_0=1.001$)

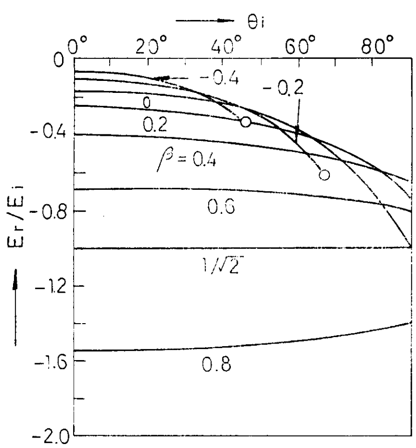

Fig. 6. (a) Relation coefficient of TE wave.
($\varepsilon_1/\varepsilon_0=1.001$, $\varepsilon_2/\varepsilon_0=2$)

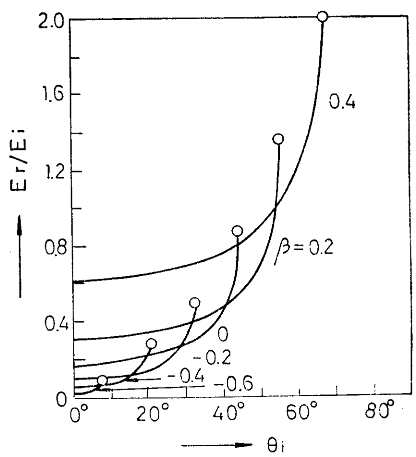

Fig. 6. (b) Reflection coefficient of TE wave.
($\varepsilon_1/\varepsilon_0=2$, $\varepsilon_2/\varepsilon_0=1.001$)

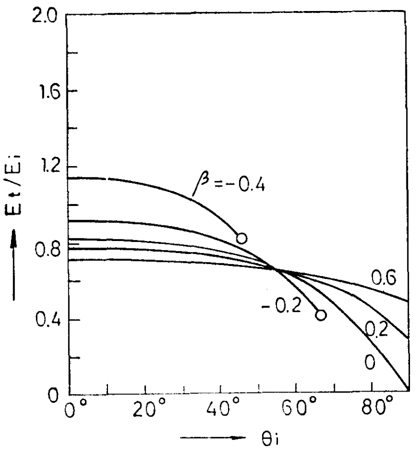

Fig. 7. (a) Tranmission coefficient of TE wave.
($\varepsilon_1/\varepsilon_0=1.001$, $\varepsilon_2/\varepsilon_0=2$)

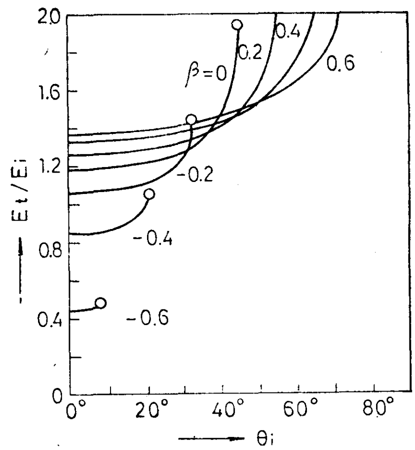

Fig. 7. (b) Transmission coefficient of TE wave.
($\varepsilon_1/\varepsilon_0=2$, $\varepsilon_2/\varepsilon_0=1.001$)

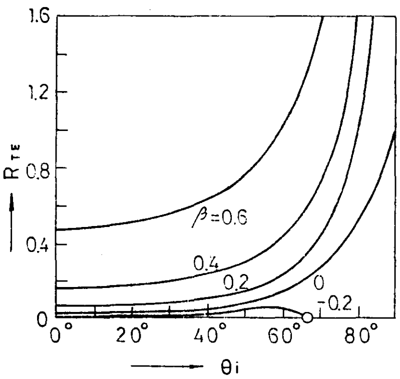

Fig. 8. (a) Power reflection coefficient of TE wave.
($\varepsilon_1/\varepsilon_0=1.001$, $\varepsilon_2/\varepsilon_0=2$)

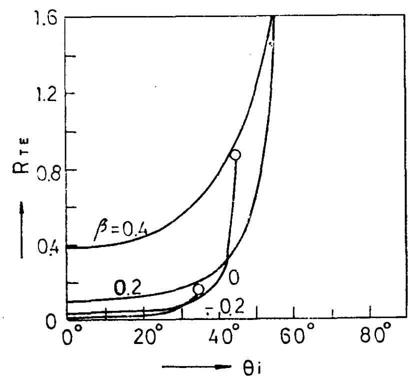

Fig. 8. (b) Power reflection coefficient of TE wave.
($\varepsilon_1/\varepsilon_0=2$, $\varepsilon_2/\varepsilon_0=1.001$)

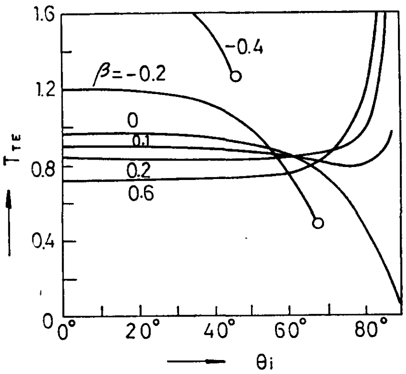

Fig. 9. (*a*) Power transmission coefficient of TE wave.
($\varepsilon_1/\varepsilon_0=1.001$, $\varepsilon_2/\varepsilon_0=2$)

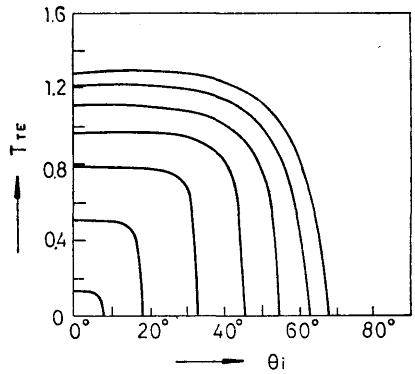

Fig. 9. (*b*) Power transmission coefficient of TE wave.
($\varepsilon_1/\varepsilon_0=2$, $\varepsilon_2/\varepsilon_0=1.001$)

When the media move but the boundary is stationary,

$$\begin{aligned} R_{TE}+T_{TE}&=1 \\ R_{TM}+T_{TM}&=1 \end{aligned} \tag{35}$$

It results that the energy conservation doesn't hold within electromagnetic energy of waves only, because the permittivities change.

The expression of the Brewster angle (θ_b) in stationary boundary is

$$\tan\theta_b=n_2/n_1 \tag{36}$$

But it is extended in moving boundary as follows.

$$\cos\theta_b=\frac{-\beta n_1+n_1/n_2(1+n_1^2/n_2^2-\beta^2 n_1^2)^{\frac{1}{2}}}{1+n_1^2 n_2^2} \tag{36}$$

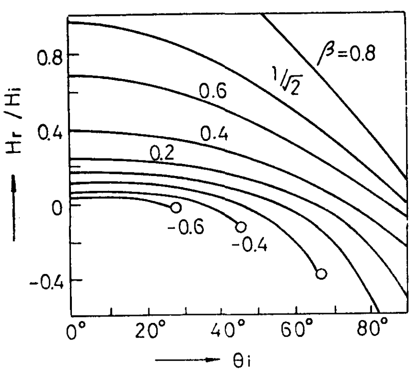

Fig. 10. (*a*) Reflection coefficient of TM wave.
($\varepsilon_1/\varepsilon_0=1.001$, $\varepsilon_2/\varepsilon_0=2$)

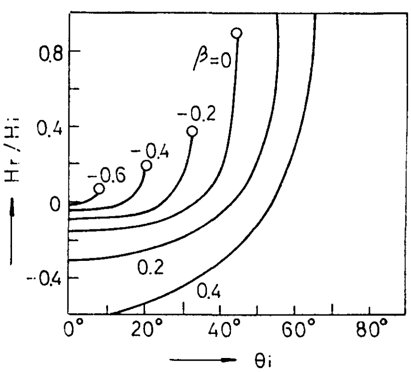

Fig. 10. (*b*) Reflection coefficient of TM wave.
($\varepsilon_1/\varepsilon_0=2$, $\varepsilon_2/\varepsilon_0=1.001$)

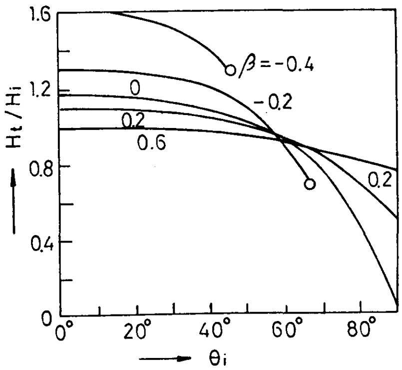

Fig. 11. (a) Transmission coefficient of TM wave.
($\varepsilon_1/\varepsilon_0=1.001$, $\varepsilon_2/\varepsilon_0=2$)

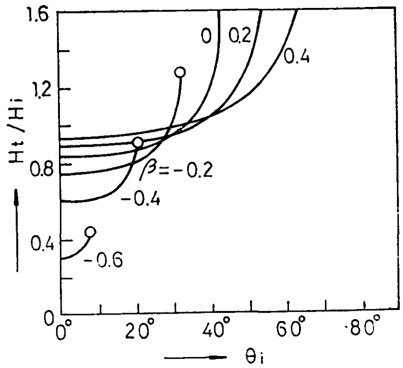

Fig. 11. (b) Transmission coefficient of TM wave.
($\varepsilon_1/\varepsilon_0=2$, $\varepsilon_2/\varepsilon_0=1.001$)

When the speed of the boundary is equal to the light speed in medium 2 the Brewster angle is 90, and when it exceeds the light speed in medium 2 the Brewster angle vanishes.

IV. Conclusion

We showed that the problem of reflection and transmission of a plane wave at a moving boundary has many features. As the assumption in this paper is very primitive, it is scarcely possible to apply this problem to physical problem. But we think that we can solve the real problem of moving boundary in this way.

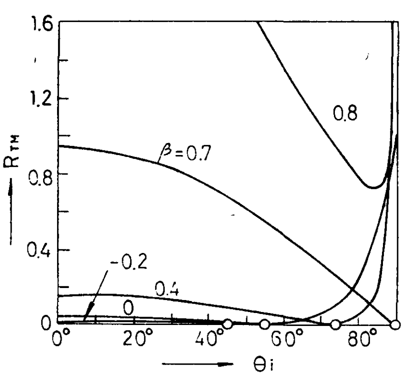

Fig. 12. (a) Power reflection coefficient of TM wave.
($\varepsilon_1/\varepsilon_0=1.001$, $\varepsilon_2/\varepsilon_0=2$)

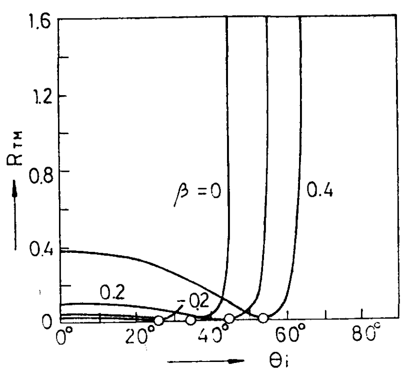

Fig. 12. (b) Power reflection coefficient of TM wave.
($\varepsilon_1/\varepsilon_0=2$, $\varepsilon_2/\varepsilon_0=1.001$)

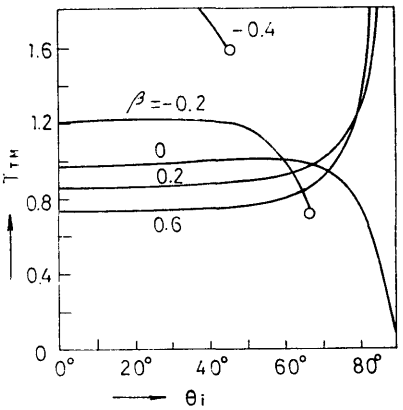

Fig. 13. (*a*) Power transmission coefficient of TM wave. ($\varepsilon_1/\varepsilon_0=1.001$, $\varepsilon_2/\varepsilon_0=2$)

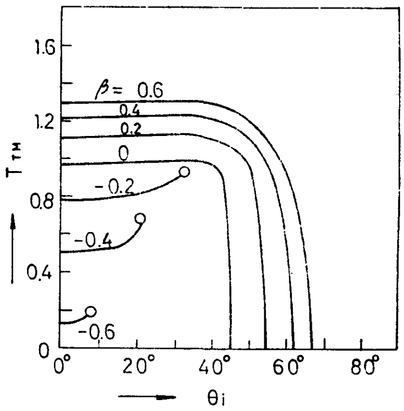

Fig. 13. (*b*) Power transmission coefficient of TM wave. ($\varepsilon_1/\varepsilon_0=2$, $\varepsilon_2/\varepsilon_0=1.001$)

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