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Studies on the Noise Reduction Effects of Potential Minimum Plane in a Electron Beam

Tomoo FUJIOKA (藤岡 和夫)

There are many papers treating the noise reduction effects of potential minimum plane in an electron beam. However, all of them are based on the one dimensional model. In this paper the author analyzed three dimensional effects of potential minimum plane, which no one has not analyzed, though those effects would be prominent.

In chapter 1, the historical backgrounds of this paper are reviewed. And in chapter 2 Whinnery's one dimensional theory is summarized, on which three dimensional theory is based.

In chapter 3, the three dimensional theory is presented. The electron beam model to be analyzed is cylindrical one with axial symmetry, surrounded by metal tube and confined by magnetic field of infinite strength, as shown in Fig. 1. The section of the beam is divided into N layers, of which areas are all equal, that is \( r_n = \sqrt{n+1} r_0 \). In any layer all physical quantities, current, voltage and, etc. are assumed to behave one-dimensionally, but correlations between layers are considered. The dc potential distributions in Z direction in all layers are assumed to follow Fry-Langmuir curve.

![Fig. 1. Model of electron beam used in the analysis.](image)

![Fig. 2. Potential fluctuation \( h_1^{(m)}(r) \) due to perturbing charge and \( h_1^{(m\alpha)}(r) \) due to compensation charge.](image)

If a charge sheet is ejected as a noise in one layer, it moves following to dc potential in that layer and ac electric field due to the charge sheet distributes in whole layers. In any layer the potential minimum plane is disturbed by the electric field of noise, and the compensation currents are ejected toward the cathode and
the anode from the potential minimum planes. After mathematical analysis, the following integral equation is obtained in MKS units.

\[
\frac{kT_c}{eI_{[m]}^n} \Delta I_{[n]}^m = \frac{1}{\varepsilon_0} \left\{ Q H_0^{[m]}(t) + \sum_{n=0}^{N-1} \int_{t'}^t \Delta I_{[m]}^n(t') H_1^{[m]}(t-t') dt' \right\}
\tag{1}
\]

Here, \(\Delta I_{[n]}^m\) is the compensation current in \(m\) layer, \(Q\) is the total charge of the ejected charge sheet, \(H_1^{[m]}(t)\) represents the effects of the compensation current in \(n\) layer on the potential minimum plane in \(m\) layer, and \(H_0^{[m]}(t)\) represents the effects of noise in \(l\) layer on the potential minimum plane in \(m\) layer. Both \(H_0^{[m]}(t)\) and \(H_1^{[m]}(t)\) can be obtained by integrating the three dimensional electric field following the movement of charge.

The examples are shown in Fig. 2 in normalized forms, \(h_0^{[m]}(r)\) and \(h_1^{[m]}(r)\).

By making Fourier transformation of Eq. (1), the following simultaneous equations are obtained.

\[
\sum_{n=0}^{N-1} \left( 2 \sqrt{\pi} \frac{\partial}{\partial a} + 2\pi g_1^{[m]}(a^{[m]}) \right) g^{[m]}(a^{[m]}) = g_0^{[m]}(a^{[m]})
\tag{2}
\]

Here, \(a^{[m]}\) is the frequency normalized by twice the plasma frequency at the potential minimum in \(m\) layer, and \(g^{[m]}(a^{[m]})\), \(g_1^{[m]}(a^{[m]})\), \(g_0^{[m]}(a^{[m]})\) are Fourier transforms of \(j^{[m]}(r)\) (normalized), \(h_1^{[m]}(r)\), \(h_0^{[m]}(r)\), respectively. When the noise charge is ejected in \(l\) layer with any initial velocity, \(g_0^{[m]}(a)\) and \(g_1^{[m]}(a)\) can be calculated for every \(m\) and \(n\), so the compensation currents in every layers can be obtained through Eq. (2).

After \(g^{[m]}(a^{[m]})\)'s are obtained the noise reduction factors \(S_a\) (noise charge goes back to the cathode from the potential minimum) and \(S_{\beta}\) (noise charge reaches the anode) are obtained as follows

\[
S_a^{[m]} = 4\pi |g^{[m]}(a^{[m]})|^2
\tag{3}
\]

\[
S_{\beta}^{[m]} = \left| \frac{\partial}{\partial a} - \frac{2\pi g^{[m]}(a^{[m]})}{e^{-ja^{[m]}/\tau_0^{[l]}}} \right|^2
\tag{4}
\]

An example of calculation for \(N=1\) is shown in Fig. 3. The curve is wavy in comparison with Whinnery's one dimensional calculation in order that the electric field created by a charge sheet has radial component.

\(S^{[m]}\) for \(N=2\) are shown in Fig. 4. \(S^{[1]}\) and \(S^{[0]}\) are noise reduction factor in usual sense, but \(S^{[1]}\) and \(S^{[0]}\) have different meaning. That is, they show that if a noise is ejected in one layer, the additional noise is ejected in another layer. For this reason, the hollow electron beam seems better than the solid beam in the view point of the noise.

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In chapter 5, the experimental results are presented. Using low noise travelling wave tube, the noise reduction factor was obtained through the measurements of noise figure. The experimental results agree with the above mentioned theoretical one, which confirms the reality of the theory.

In the last chapter, the author describes the possibility of further studies, especially on the semiconductor devices.