Title	Interpretation of Tetmajer's empirical formula for column design
Sub Title	
Author	水野, 正夫(Mizuno, Masao)
Publisher	慶応義塾大学藤原記念工学部
Publication year	1967
Jtitle	Proceedings of the Fujihara Memorial Faculty of Engineering Keio University (慶応義塾大学藤原記念工学部研究報告). Vol.20, No.77 (1967.) ,p.54(54)- 56(56)
JaLC DOI	
Abstract	The Tetmajer's empirical formula for column design is interpreted, using a simplified model of a rigid-plastic column with rectangular cross-section, assuming eccentricity in load application depending on the length of the column.
Notes	
Genre	Departmental Bulletin Paper
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=KO50001004-00200077-0054

Interpretation of Tetmajer's Empirical Formula for Column Design

(Received June 24, 1967)

Masao MIZUNO*

Abstract

The Tetmajer's empirical formula for column design is interpreted, using a simplified model of a rigid-plastic column with rectangular cross-section, assuming eccentricity in load application depending on the length of the column.

I. Introduction

A discussion is still in progress over the theory of buckling of columns at stresses greater than the proportional limit, the start of which is associated with the names of F. Engesser, F. S. Yasinski, and T. von Karman. The reduced modulus theory was considered to be correct theory of inelastic column action until 1946 when F. R. Shanley showed that it represented a paradox. It is now clear for an ideal (straight) column in the inelastic range that the Engesser-Shanley's tangent modulus theory gives the load which is considered as the practical upper limit for column strength.[1)]

On the other hand, in the discussion of application of theoretical formula in column design, it is indicated that the principal difficulty lies in evaluating for the various imperfections such as eccentricity in load application, initial curvature, nonhomogeneity of the material and unavoidable variation in the cross-sectional area of the column.[2)] From these reasons, empirical formulas are still used in practical column design.

In this paper, L. von Tetmajer's empirical formula is interpreted, using a simplified model of a rigid-plastic column with rectangular cross-section, assuming inaccuracies depending on the length of the column.

II. Interpretation of Tetmajer's formula

Let *ABC* be a center-line of a rigid-plastic column in post-buckling state, which has a rectangular cross-section $b\times 2h$ and a eccentricity in load application e, as

*水 野 正 夫 Professor, Faculty of Engineering, Keio Uuiversity.

1) F. R. Shanley, "Strength of Materials", 1957, McGraw Hill § 24. 7.

2) S. P. Timoshenko & J. M. Gere, "Theory of Elastic Stability", 2nd ed., 1961, McGraw-Hill, § 4. 4.

shown in Fig. 1. At a plastic hinge at C, the distribution of the stresses is shown in Fig. 2. Let the both ends are hinged and the column length $2l$.

From the condition of equilibrium of forces,

$$b\,\{(h+h_s)-(h-h_s)\}\,\sigma_y=2bh_s\sigma_y=P,$$

or,

$$\frac{P}{A\sigma_y}=\frac{h_s}{h} \tag{1}$$

where, $A=2bh$ is the cross-sectional area.

From the condition of equilibrium of moments about point O,

$$b(h+h_s)\left\{l\sin\gamma+e-h+\frac{h+h_s}{2}\right\}$$

$$=b(h-h_s)\left\{l\sin\gamma+e+h-\frac{h-h_s}{2}\right\},$$

or,

$$2h_s(l\sin\gamma+e)=h^2-h^2_s.$$

$$\therefore h_s=\sqrt{(l\sin\gamma+e)^2+h^2}-(l\sin\gamma+e)>0.$$

At $\gamma=0$,

$$h_{s0}=\sqrt{e^2+h^2}-e<h. \tag{2}$$

Substituting the eq. (2) in the eq. (1),

$$\frac{P_0}{A\sigma_y}=\sqrt{\left(\frac{e}{h}\right)^2+1}-\frac{e}{h}\,. \tag{3}$$

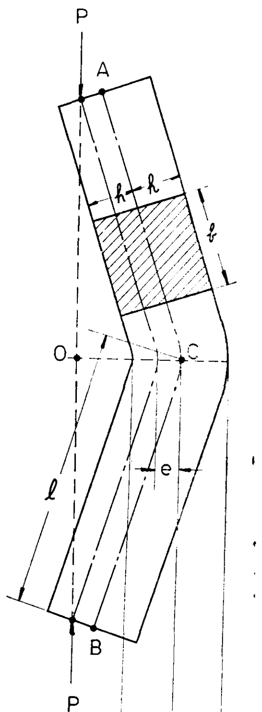

Fig. 1.

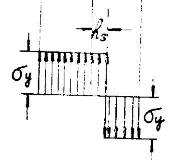

Fig. 2.

Assuming, reduced inaccuracies of column be expressed eccentricity in load application e which is proportional to the column length,[2)]

$$e=2\sqrt{3}\,kl, \tag{4}$$

or $k=e/2\sqrt{3}\,l$ is constant, the geometrical radius of gyration $r=h/\sqrt{3}$, then the slenderness ratio

$$\lambda=2l/r=2\sqrt{3}\,l/h, \qquad \text{or} \quad h=2\sqrt{3}\,l/\lambda,$$

and

$$\frac{e}{h}=\frac{2\sqrt{3}\,kl\lambda}{2\sqrt{3}\,l}=k\lambda. \tag{5}$$

Putting the eq. (5) into the eq. (3)

$$\frac{P_0}{A\sigma_y}=\sqrt{k^2\lambda^2+1}-k\lambda\fallingdotseq 1-k\lambda+\frac{1}{2}k^2\lambda^2, \tag{6}$$

Provided $1>k\lambda$.

III. Discussion

The eq. (6) has the same form as L. von Tetmajer's empirical formula for column design.[3)]

$$\frac{\sigma_{cr}}{\sigma_D}=1-C_1\lambda+C_2\lambda^2 \tag{7}$$

Where,

$$C_1=0.01546, \qquad C_2=0.00007,$$

for cast iron, and $C_2=0$ for the other materials because of C_1 is one order smaller than that for cast iron.

If we calculate according to the eq. (6), assuming

$$k=C_1=0.01546,$$

C_2 must be equal to $1/2\ k^2=0.00012$, and the difference from the Tetmajer's formula eq. (7) may be neglected. And, for the other materials, C_2 is negligible beause of $C_1\lambda=k\lambda\ll 1$ in the eq. (6).

3) L. von Tetmajer, "Die Gesetze der Knickungs-und der zusammengesetzen Druckfastigkeit der technisch wichtigsten Baustoffe", Mitt. d. Mat. Anstalt a. schweiz. Poly, in Zurich, Heft 4, 1890; Heft 8, 1896; republished Leipzig, 1903.