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Methods of Selecting the Optimum Combination of Projects*

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Shizuo SENJU**

Abstract

There are three types of problems in selecting the optimum mix of projects from several proposals. The first type is seen when all proposals are mutually exclusive, the second one is seen when they are mutually independent, and the last one is seen when two different types of proposals are mixed.

This paper summarizes the method already developed for the first and second types, and proposes a new method of selection for the third type.

1. Introduction

In general, the problem of selecting the optimum mix of projects from several alternative ones should begin with the clarification of the interrelationship among those projects. The interrelationship of projects is broken down into the following three types:

i) projects are mutually exclusive,

ii) projects are mutually independent,

iii) the two different types of projects are mixed.

i) Selecting from mutually exclusive projects.

This type can be seen in the case where once a particular project is selected, the other alternatives are necessarily discarded. Examples can be seen in the problems of selecting plant layout, machines to be installed in a building, and deciding how to use vacant space in a plant, etc.

ii) Selecting from mutually independent projects.

In this case, projects are taken to be independent each other. For instance, when investment for a drilling machine, replacement of a crane and the revision of the plant layout are under consideration at the same time. The problem is to decide which of them to be selected. If each of these projects is in-

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* This is a condensed material from the following papers (in Japanese):

1. S. Senju, " On the Selection from Mutually Exclusive Projects ", Industrial Engineering (Japan), Vol. 7, No. 1, pp. 89-93, 1965.
2. S. Senju, " On the Selection from Mixed Projects," Industrial Engineering (Japan), Vol. 7, No. 2, pp. 181-185, 1965.

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dependent, after the best one of them is chosen, the next one or more might be further chosen as long as profitable.

iii) Selecting from mixed projects.

However, problems normally encountered in a business are not always of either type, but a mixture of the two types mentioned above.

For example, suppose the following: one lathe is to be selected from alternatively proposed lathes $A_1, A_2, \ldots A_l$, one drilling machine from drilling machines $B_1, B_2, \ldots B_m$, one crane from cranes $C_1, C_2, \ldots C_n$.

Considering the amount of fund available, the problem is to select the combination of machines which gives the most effective return on investment.

These three types of selecting problems exist not only in equipment investment problems but also in OR, IE and QC problems. A decision for optimality should be considered after the type has been defined. This paper summarizes the method already known for the first and second types, and proposes a new method of selection for the third type.

2. Selecting from Mutually Exclusive Projects—Part I

Assume that there are two alternative methods A and B in producing screws.

For method A, the cost for purchasing and installing of a machine, C_A, is $10,000, and that for operation, O_A, is $2,840 a year. For method B, the cost for purchasing and installing of a machine, C_B, is $20,000, and for operation, O_B, is $970. The rate of interest for borrowed money is 8% and both machines are expected to maintain effective operation for eight years.

In this case, as already mentioned, the problem is to select the better method from the two alternatives which serve the same function. If we restrict this problem within the economic aspect, we only have to select the method having less cost. However, for the case having different timing for disbursements, the cost should be evaluated with consideration of the time value of money. Let us consider several approaches to this problem under the assumption that funds necessary for investment are available at a fixed rate of interest.

One of the ways to solve it is based on the present worth method. It is based on the comparison of total costs by converting expenses at different times for each alternative to present worth.

Assume the sum of present worth of operation costs on method A is $P_{A,O}$, that of capital cost is $P_{A,C}$, then the total present worth $P_{A,T}$ is

$$P_{A,T}=P_{A,C}+P_{A,O}=C_A+O_A\times(\text{uspwf}), \tag{1}$$

where the factor (uspwf) means the " uniform-series present-worth factor ".

Similarly for method B,

$$P_{B,T}=P_{B,C}+P_{B,O}=C_B+O_B\times(\text{uspwf})\ . \tag{2}$$

The evaluation of the above shows that

$$P_{A,T}>P_{B,T}\ . \tag{3}$$

Consequently, method B is found to be more profitable. However, for a case when the interest rate is greater than 10%, then $P_{A,T}<P_{B,T}$.

As shown above, the present worth method is subject to the rate of interest, thus the profitability of a project depends on its value. Now multiplying capital recovery factor (crf) for eight years to both sides of equation (3), we get

$$P_{A,T}\times(\text{crf})>P_{B,T}\times(\text{crf})\ .$$

Each side of this equation is the adjusted average of the sum of rate of interest and operating cost. Since the operating life of both methods is 8 years, the values of the capital recovery factors on both sides are equal. Let the value of each side in the above equation be $R_{A,T}$ and $R_{B,T}$, then $R_{A,T}>R_{B,T}$ holds as long as $P_{A,T}>P_{B,T}$ holds, and vice versa. Namely, in this case, the present worth method and the adjusted average method show the same result.

Putting (2) and (1) into (3) and rearranging it, we get the following equation:

$$(O_A-O_B)\times(\text{uspwf})>C_B-C_A\ .$$

Multiplying both sides by the capital recovery factor, we obtain

$$O_A-O_B>(C_B-C_A)\times(\text{crf})\ . \tag{4}$$

Formula (4) means that an adjusted average value (right hand side) of the additional investment (C_B-C_A) in method B compared with method A is smaller than the saved value (O_A-O_B) of operating cost. Also, it is obvious that formula (4) always holds if (3) does. That is to say, the yearly saving of cost is larger than the adjusted average value of cost required for additional investment. This type of comparison, the additional investment method, draws the same conclusion as the present worth factor method and adjusted average method, in the case of comparing several projects having the same lives and functions.

This rule holds even for a problem of selecting the most profitable one from any number of projects. It, however, is not applicable to the case where the lives of machines are different. The additional investment method is not convenient in this case.

3. Selecting from Mutually Exclusive Projects—Part II

Let us consider a method for comparison based on gain or profit, using the same case cited earlier. Assume the work mentioned earlier is done manually, and operating cost is \$5,000 a year, as shown in Fig. 1. Method A will save cost by \$2,160

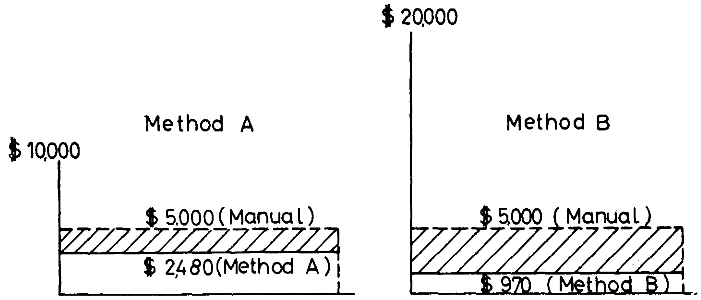

Fig. 1. Relative Profits Compared with Manual Operation

(=\$5,000−\$2,840) with an investment of \$10,000. Method B will save cost by \$4,030 (=\$5,000−\$970) with an investment of \$20,000. If the yearly savings obtained are reinvested at the interest rate of 8%, the sum of the present worth $P'_{A,T}$ of yearly net gains for Machine A is expressed as follows:

$$P'_{A,T}=\{(\$5{,}000-2{,}840)\times(\text{uspwf})-\$10{,}000\}$$
$$=\{\$5000\times(\text{uspwf})\}-\{\$2{,}840\times(\text{uspwf})+\$10{,}000\}\,. \qquad (5)$$

Since the second term of the right hand side of the above equation is equal to $P_{A,T}$ of equation (1), and the first term is the same for both methods A and B, thus if $P_{A,T}<P_{B,T}$, then $P'_{A,T}>P'_{B,T}$, and vice versa. Namely, as long as the yearly gains obtained are reinvested at a fixed rate, "Maximizing of gains" by present worth method" and "minimizing of cost," will yield the same result.

However, the method of maximizing the "internal rate of return" is not always the same as maximizing the amount of net gains", thus it entails a risk to use the internal rate of return as a criterion of selection. Tentatively, calculating the rates of return r_A, r_B on methods A, B for this problem yields $r_A=14\%$, $r_B=12\%$, which shows the wrong solution.

Let us take a simple example. Suppose there are three projects H, J and K (mutually exclusive), as shown in Fig. 2. Necessary amount for those investments is \$12,000, \$22,000 and \$32,000 each. Annual returns are expected to be \$1,700, \$2,700 and \$3,400, each. Fund required is available at an interest rate of 8%. Then the internal rates of return for those projects, r_H, r_J, r_K, are calculated as follows:

$$r_H=\$1{,}700\div\$12{,}000=14.2\%$$
$$r_J=\$2{,}700\div\$22{,}000=12.3\%$$
$$r_K=\$3{,}400\div\$32{,}000=10.6\%$$

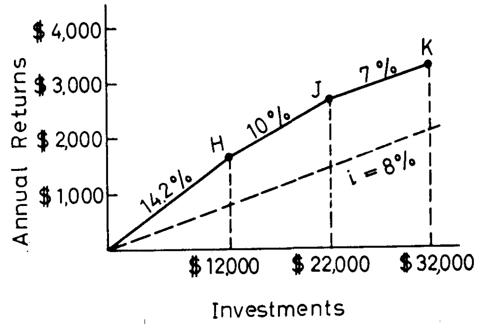

Fig. 2. Alternative Projects H. J and K

The absolute amounts of return, R_H, R_J and R_K, after subtracting interests are

$$R_H = \$1700 - \$12{,}000 \times 0.08 = \$740/\text{year},$$
$$R_J = \$2700 - \$22{,}000 \times 0.08 = \$940/\text{year},$$
$$R_K = \$3400 - \$32{,}000 \times 0.08 = \$840/\text{year}.$$

This shows that J is the most profitable. The same conclusions may be drawn when calculations are made by the present worth method. Moreover, the so-called additional investment method will give the same conclusion. Therefore, if fund is availabe at a fixed rate of interest, the present worth method, the adjusted average method and the additional investment method essentially give the same correct solution. But, as shown above, the selection by internal rate of return might lead to a wrong solution. The same can be said for the case of selecting from mutually exclusive projects having different purposes. In this case, the cost minimization method can not be used because of different purposes, but only profit maximization method is reliable from the standpoint of economic analysis.

4. Selecting from Independent Projects

Let us consider the problem of selecting from independent projects. Assume there are projects to buy (1) a lathe A, (2) a drilling machine B, (3) a crane C, and/or (4) a measuring machine D. These projects are considered to be independent of each other.

Let us assume that their costs are \$25,000, \$15,000, \$30,000 and \$24,000, their rates of return on investment are 16%, 14%, 17% and 20%, their lives can be considered as infinite, and the return from each project is obtained at the end of each year. Figure 3 shows the rate of return on investments in descending order. The horizontal axis is to show the amount of investment for each project, and the vertical axis the internal rate of return for each project. The area of each rectangle is proportional to the amount of return from each project.

If the interest rate is 8%, all of them are acceptable. If the available amount

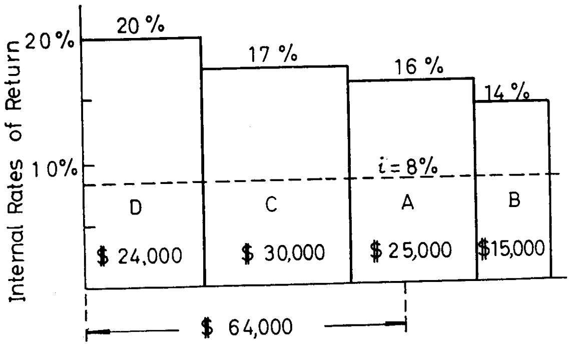

Fig. 3. Independent Projects A, B, C and D

of capital is limitted to $ 64,000, the purpose is to realize the maximum return within this limit. The possible combinations of projects in this case are as follows:

The first combination: Buying machines D and C, and investing the remaining fund $ 10,000 at interest rate of 8%.

The second combination: Buying machines D, A and B, and dropping C.

The returns from the first and second combinations, R_1 and R_2, are calculated as follows:

$$R_1=24{,}000\times 0.2+30{,}000\times 0.17+10{,}000\times 0.08=10{,}900\ (\$)$$
$$R_2=24{,}000\times 0.2+25{,}000\times 0.16+15{,}000\times 0.14=10{,}980\ (\$)$$

Therefore, the second combination is more profitable. Project D which produces the highest rate of return is to be selected in both combinations. But, as the analysis shows, the best combination made above is different from the one in which projects are selected in descending order of rates of return. Therefore, in the case that fund required for each project is large relative to the amount of available fund, there is no reliable method except for investigating on all the possible combinations of machines which are being considered.

But, if there are a lot of projects to be considered, and if we may neglect a small error in selecting the best combination, then we may pick up projects in the order of higher rate of return up to the limit of available fund for investment. (Refer to Fig. 4.)

In the case where fund is available at an interest rate of 8% without limit, all projects up to project K in Fig. 4 are acceptable, and the remaining projects under L are to be rejected. If a limit on fund exists as in Fig. 4, all projects up to I are acceptable. In actual enterprises, limit on fund is not as "rigid" as explained above. If we need more fund, we can stretch the limit to some extent by paying higher rate of interest as shown by a dotted line in Fig. 4. Let us call

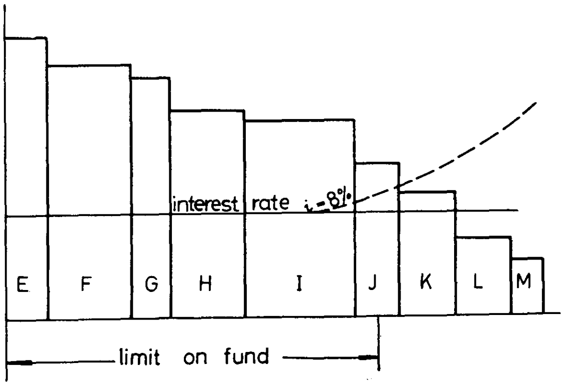

Fig. 4. Independent Projects

this case the "flexible" restriction. Even in this case, there is no difference in the basic concept. Namely, putting each project into descending order from left to right concerning their rates of return and comparing the relation between the rates of return and interest. It becomes clear that all projects up to *J* are acceptable while the projects on the right side of *K* are unacceptable.

The cases which have been explained so far are those where machine lives are considered as infinite. The same can be said even when machine lives are not infinite, only if their lives are equal and the yearly returns on investment are uniform.

For the cases where these conditions mentioned above do not hold, we have to calculate the returns on all the possible combinations of projects. But it is hard to estimate the cost and efficiency of future machines correctly. In fact, strictly speaking, there is no way to solve this kind of problem distinctly. Therefore, we should compromise to some extent by making some approximation. As far as the problem on "selecting from typical independent projects" is concerned, it might be good enough to consider the internal rate of return as a criterion.

5. "Screening" of Mutually Exclusive Projects

Before proceeding to the next section, we have to consider the "screening" of a given group of mutually exclusive projects in order to obtain an "economically significant" group among them.

Suppose that it is suggested to improve an assembly process which is currently performed manually (A_0), and that there are mutually exclusive eight proposals, A_1, A_2, . . . , A_8, having the same life. Assume that the initial cost of proposal A_1 is $ 5,000 and realizes a return of $ 2,000 per year. Initial costs and annual returns concerning the other proposals A_2, A_3, . . . , A_8 will not be shown here, since it is immaterial to describe the numerical values in detail for the purpose of this section. Let us show the relations between them in Fig. 5, where the horizontal

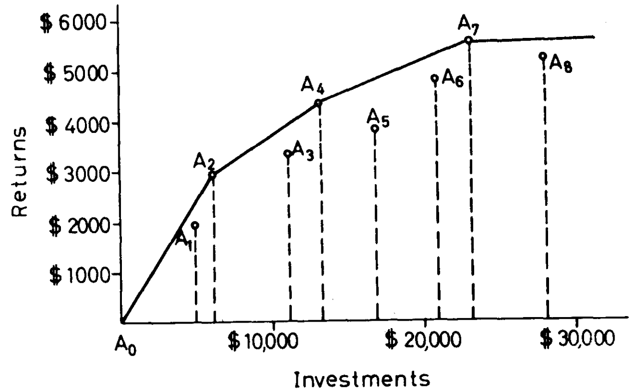

Fig. 5. Relations Between Returns and Investment

axis is to show the initial investments and the vertical axis the annual returns on them.

At first, we have to find out a point among A_i's (i=1,2, . . . 8), which has the steepest slope against A_0 (point A_2 is chosen in this case), then to link A_0 and A_2 by a straight line.

Next, find out a point (A_4 in this case), which has the greatest slope against A_2, and link A_2 and A_4 by a straight line. Similarly, link A_4 and A_7.

Comparing A_2 with A_1, the former is more profitable than the latter in terms of absolute value of return and rate of return. From the principles of economic analysis, it is obvious that A_2 dominates A_1. Similarly, it shows that A_4 dominates A_3 and A_5, and A_7 dominates A_6 and A_8. So, the candidates which are worth considering for further analysis are only A_0, A_2, A_4 and A_7, and it is obvious that the slopes of the straight lines drawn by linking them are monotonously decreasing.

Now, let us call these four mutually exclusive projects obtained by the above process the " screened " projects.

6. Selecting from Mixed Projects

Assume, in a plant, that there are some mutually exclusive proposals concerning some processes.

Concerning an automatic machine process, three proposals A_1, A_2, A_3 are expected to improve it. The present method A_0 and three proposals A_1, A_2, A_3 are mutually exclusive, and they are already " screened ". Likewise, there are five methods B_0, B_1, B_2, B_3, B_4 for assembly process, four methods C_0, C_1, C_2, C_3 for inspection process and three methods D_0, D_1, D_2, for materials handling. Several methods for each process are mutually exclusive, and they are already " screened ".

Methods with subscript "0" means present methods.

Suppose that initial cost for each proposals and annual return comparing with the present method was shown in Table 1.

Let us consider the problem to select the optimal combination of proposals to get the greatest return whithin a limit of $ 400,000 for available fund. As already explained, it entails a risk to consider the absolute value of annual return itself or internal rate of return itself.

The author proposes a reliable and practical method to solve the problem presented above, that is a kind of "the rate of return on the additional investment" methods. This method is easy to be explained with the example shown above.

At first, we have to break down proposals A_2 and A_3 concerning the automatic machine process into the imaginary proposals A_2' and A_3', according to the concept of additional investment. That is to say, the difference between the initial investment for A_1 and A_2 is regarded as the initial investment for the imaginary proposal A_2'. Likewise, the difference between the initial investment for A_2 and A_3 is regarded as the initial investment for the imaginary proposal A_3'. It can be considered that $ 40,000 for A_1 is the additional investment to A_0.

Annual returns should also be divided into the additional contribution.

Then, we can consider that there are three independent imaginary proposals A_1, A_2', A_3', as shown in Table 2, concerning automatic machine process, instead of three mutually exclusive proposals A_1, A_2 and A_3. The rates of return for those three independent proposals A_1, A_2' and A_3', are monotonously decreasing because proposals A_1, A_2 and A_3 are mutually exclusive and already "screened".

In the same way for the other processes, we can make a list (Table 3) which shows the relations between amounts of investment and annual returns concerning the independent imaginary proposals derived from the real proposals shown in Table 1.

As all proposals in Table 1 are converted into equivalent independent ones, we can easily make the optimum decision in selecting the appropriate proposals under given conditions.

If our available fund for investment is limited to $ 300,000, we should select the proposals D_1, B_1, B_2', D_2', A_1, B_3' as shown in Fig. 6. From our premise mentioned earlier, this means to select proposals A_1, B_3 $(=B_1+B_2'+B_3')$, C_0, D_2 $(=D_1+D_2')$.

If our available fund is limited to $ 400,000, we should select D_1, B_1, B_2', D_2', A_1, B_3', A_2' and C_1. This means to select A_2 $(=A_1+A_2')$, B_3 $(=B_1+B_2'+B_3')$, C_1, D_2 $(=D_1+D_2')$.

If the limit given to our fund is relaxed to $ 470,000, then the imaginary proposal A_3' will be further accepted, thus the final solution for the automatic machine process becomes A_3 instead of A_2.

Table 1.

		Amount of Investment ($)	Annual Return ($)
Automatic Machine Process	A_1	40,000	10,400
	A_2	95,000	21,400
	A_3	165,000	30,500
Assembly Process	B_1	30,000	11,100
	B_2	90,000	32,100
	B_3	175,000	50,800
	B_4	225,000	55,300
Inspection Process	C_1	45,000	7,200
	C_2	120,000	16,200
	C_3	210,000	21,600
Material Handling Process	D_1	20,000	8,000
	D_2	85,000	28,800

Table 2.

Amount of Investment ($)		Annual Return ($)	Rate of Return (%)
A_1	40,000	10,400	26
A_2'	55,000	11,000	20
A_3'	70,000	9,100	13

Table 3.

Amount of Investment ($)		Annual Return ($)	Rate of Return (%)
B_1	30,000	11,100	37
B_2'	60,000	21,000	35
B_3'	85,000	18,700	22
B_4'	50,000	4,500	9
C_1	45,000	7,200	16
C_2'	75,000	9,000	12
C_3'	90,000	5,400	6
D_1	20,000	8,000	40
D_2'	65,000	20,800	32

If our fund is restricted in terms of the interest rate, we have to pick up those independent proposals whose rates of return exceeds that rate of interest. For example, if the interest rate of fund is 10%, we should drop B_4' and C_3', thus the optimum combination is A_3, B_3, C_2 and D_2.

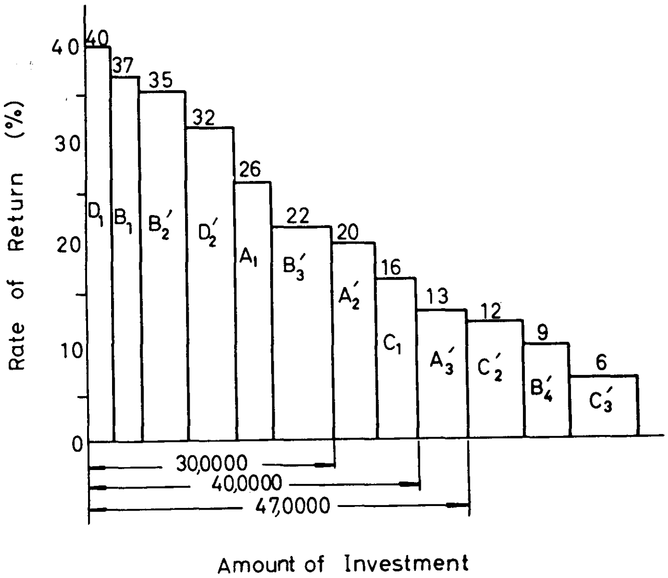

Fig. 6. List of Imaginary Proposals

When the rate of interest increases in accordance with the amount of fund for investment, we can take the advantage of graphical representation like Fig. 4, with the principle of selection remaining the same explained already.

7. Conclusions

When proposed projects are composed of the two different types of proposals, mutually exclusive type and independent type, it is very difficult to select the optimum combination of proposals just to meet the condition on the available fund.

But, if the mutually exclusive proposals are reconstructed into imaginary independent proposals from the standpoint of additional fund, all proposals and imaginary ones can be dealt with as if all were independent each other, thus it is not difficult to make the optimum selection under any given conditions on the fund for investment.

What was discussed here was restricted in the case where the machine lives are equal and the annual returns are uniform. Even when those assumptions do not hold, the method proposed here can be applied with small quantity of error in terms of profit in selecting the best combination.