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Measurement of Flowability of Powder with Rotating Horizontal Cylinder[1)]

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Abstract

An attempt was made to evaluate the flowability of powder using a rotating horizontal cylinder. Material constants corresponding to the viscosity of liquids were obtained for granular materials by means of photographs of the profile of the flowing powder bed in the cylinder. This powder bed was assumed to be in a steady state and behaves as a Newtonian fluid. It was found that these constants indicate the flowability of powder agreeable with the results obtained by Ford cup tests.

Laminarly flowing portion of the powder bed in the rotating horizontal cylinder was found to consist of two parts: one is moving as a mass and the other flowing with a velocity gradient in itself. This made our assumption incorrect. Flow of powder always comprises these static and flowing parts. Therefore, more precise analysis may not be done until both the correlation between these parts and the behavior of particles in each part are made clear.

I. Introduction

Flow properties of granular materials have been estimated with such methods as measurements of angle of repose and/or Ford cup test. All these methods are based on the observation of powder not in a dynamic state, but in a transient state from a static state to a dynamic state or *vice versa*.

The authors tried to determine a quantity for granular materials corresponding to the viscosity of fluids by keeping the granular materials in a dynamic state and assuming that they behave as Newtonian liquids.

Powder or liquid sample was put into a rotating horizontal cylinder in order to keep it in a dynamic state. The free surface of the fluid was observed through a clear plastic wall attached to one end of the cylinder. With some proper assumptions, a theoretical relation among the angle of inclination of the free surface of

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fluid against the horizontal plane, the rate of rotation, the amount of the fluid and the viscosity of fluid was derived. This relation was verified by the measurement on liquids and applying this relation to powder particles we obtained a quantity which may be called the viscosity of powder.

The viscosity thus obtained were considered to bear some regular relations to the flowability of powders, but it was also noticed that the flow behavior of powder sample in a cylinder did not fit our assumption. Although our description on the flowing behavior of the powder is far from our satisfaction, we are going to present it since it is believed to be one method for evaluating flowability of powdery materials in a dynamic state.

II. Experimental

II. 1. Apparatus

A cylinder, 80 mm in diameter, 95 mm in length, made of transparent plastics, was attached to the rotating shaft, keeping the axis horizontal and driven by a motor through a pulley, as given in Fig. 1. The rate of rotation was measured with a rotation counter and a watch. The profile of the free surface was observed by photographs, as shown in Figs. 2 and 3. Though the shape of the liquid surface differs from that of powder one, we defined the angle of inclination, Θ, as drawn in Figs. 2 and 3. Amount of fluid, i. e., liquid or powder, is expressed by the ratio, a, of maximum depth at rest, d, to inner radius of the cylinder, R, that is, $a=d/R$.

Viscosity of liquid was determined with the Ostwald-type viscosimeter for reference.

II. 2. Sample

Silicon oil (300 and 100 c. s.) and paraffin oil were used as liquid samples. Polymethylmethacrylate sphere (40–80 mesh), glass sphere (0.1 mm in dia. and 60–100 mesh), crushed glass powder (65–80 mesh) and Carborundum were used as powder samples.

III. Derivation of the theoretical relation

Several approximate equations of liquid flow in a rotating horizontal cylinder were obtained according to different models; the equation given below, together with its derivation, is one of them, whose solution agreed most closely with the experimental results obtained by the measurements with a Ostwald-type viscosimeter on liquids.

In this approximation, motions of liquid in a cylinder were replaced by those in a parallel trough $a'b'c'd'$, as shown in Fig. 4. The base of this trough, $b'c'$, is parallel with the free surface of the liquid in the cylinder, the depth of the trough equals to the maximum depth, d, of the liquid. The width of the trough, $b'c'$, is determined so that the cross sectional area of the liquid in the trough and in the

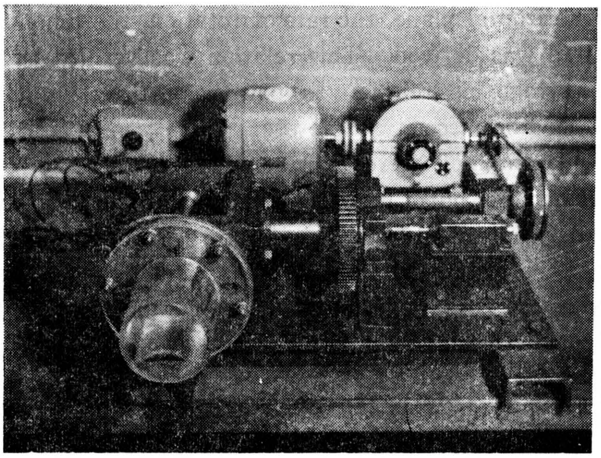

Fig. 1. Device of the horizontal rotating cylinder.

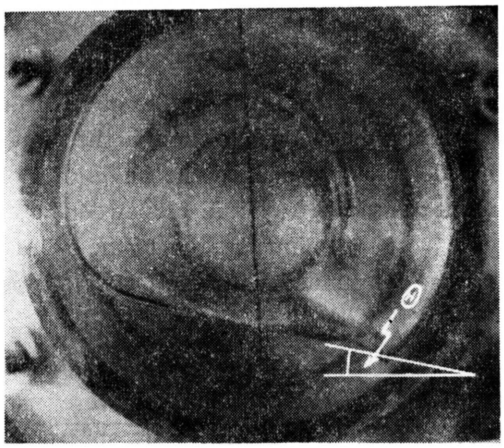

Fig. 2. Profile of a liquid in the rotating cylinder.

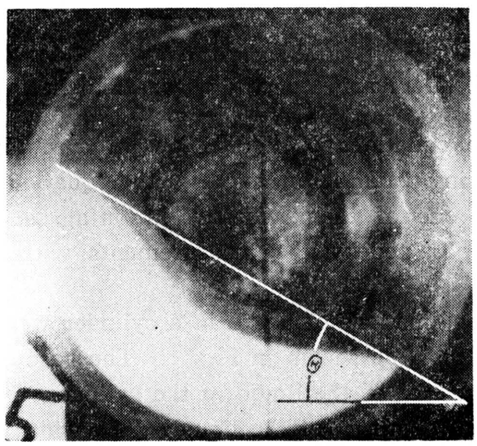

Fig. 3. Profile of a powder in the rotating cylinder.

cylinder be equal.

Thus the motion of fluid in a cylinder is approximated to that in a parallel trough, the three walls of which move in the same velocity and direction as those of the cylinder wall, as indicated in Fig. 4. The coordinates x, y, r and θ are taken as given in Fig. 4.

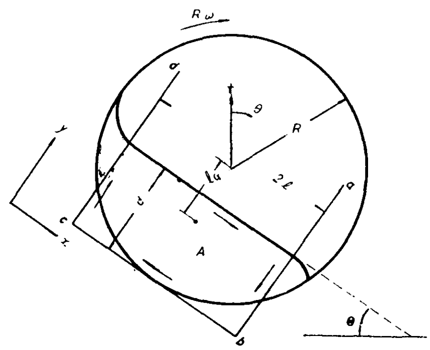

Fig. 4. Diagram of the system. R: radius of the cylinder, A; cross sectional area of the fluid, d: maximum depth of the fluid, l_G: distance from the centre of the gravity of fluid to the centre of cylinder, ω: angular velocity of the cylinder.

Fluids are assumed to be Newtonian and exhibit creeping-flow. If a streaming function, ψ, is the function of y only, the equation of flow along the wall $b'c'$ is, with the boundary conditions,

$$\begin{cases} y=0,\ \psi=0 \text{ and } d\psi/dy=v_x=R\omega \\ y=d,\ \psi=0 \text{ and } \tau_{xy}=0 \end{cases}$$

$$\psi=(R\omega/2d^2)\,y^3-(3R\omega/2d)\,y^2+R\omega y \tag{1}$$

Similarly, the equation of flow along the walls $a'b'$ and $c'd'$ is,

$$\begin{cases} x=l,\ \psi=0 \text{ and } d\psi/dx=v_y=R\omega \\ x=-l,\ \psi=0 \text{ and } d\psi/dx=v_y=-R\omega \end{cases}$$

$$\psi=(R\omega/2l)\,x^2-R\omega l/2 \tag{2}$$

where, ψ, is again assumed to be a function containing x as the single variable in this case.

The shear stress $\tau_{r\theta}$ is, by definition

$$\tau_{r\theta}=\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)+\frac{1}{r}\frac{\partial v_r}{\partial\theta}\right] \tag{3}$$

where μ is the viscosity of the liquid.

The torque, T_1, caused by the motion of the wall of the cylinder is,

$$T_1 = R\int_0^S [\tau_{r\theta}]_{r=R}\, dS \tag{4}$$

where S is the length of the perimeter of the cylinder in contact with the fluid, and can be expressed as the function of the amount of the fluid, $a=d/R$. On the other hand, the inverse torque, T_2, caused by the gravitational force on the fluid is,

$$T_2 = \rho g A l_G \sin\Theta \tag{5}$$

where ρ is the density of the fluid and g is the acceleration of gravity. In the integration of eq. (4), the $x-$ and $y-$ components of the velocity of fluid, given in eqs. (1) and (2), were expressed by the polar coordinates and put into eq. (3).

At steady state these two torques must be equal, that is, $T_1 = T_2$, thus we obtain the following relation among the viscosity of fluids, the rate of rotation of the cylinder, the angle of inclination of the free surface and the amount of the fluids :

$$\mu = \frac{\rho g}{4R^3} \cdot \frac{\sin\Theta}{\omega} \cdot \frac{A\, l_G}{\frac{a}{d^2}\left\{3R\,(3-3a+a^2) + \frac{(2-a)}{2l}\,(3dl + d^2 - 3Rl)\right\}} \tag{6}$$

IV. Results

IV. 1. Liquids

In order to check the applicability of eq. (6), several liquids were used at various amounts of liquids and rates of rotation. In all liquids, straight lines through the origin were obtained in the plots of $\sin\Theta$ *vs.* the angular velocity of the cylinder, ω, for each amount of the liquid (Fig. 5). The viscosities calculated from the gradient $\sin\Theta/\omega$ by eq. (6) are shown in Table 1, together with those obtained from the measurement by Ostwald-type viscosimeter.

Calculated viscosities are in good agreement with observed values, though in a strict comparison, we can find that the less the amount of liquid the larger the discrepancy. We presume, however, eq. (6) is satisfactory at present step.

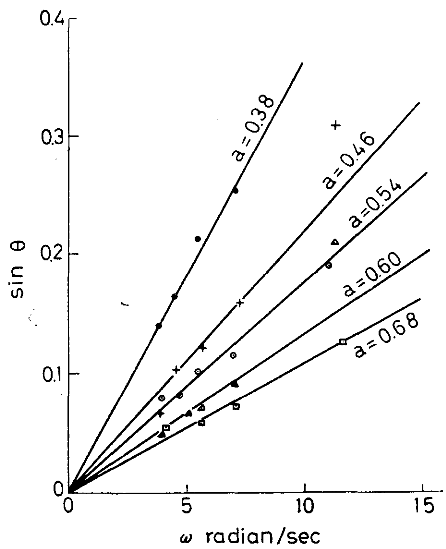

Fig. 5. Sih Θ *vs.* ω relation of silicon oil 100 c.s.

Table 1. Viscosities of liquids measured by Ostwald viscosimeter, μ_0, and calculated from Eq. (6), μ_c.

Liquids	a	μ_0, *poise*	μ_c *poise*	μ_0/μ_c
Paraffin oil	0.49	1.48	1.56	0.95
	0.64		1.78	0.83
	0.73		1.87	0.79
Silicon oil 300 c. s.	0.43	3.16	2.73	1.16
	0.50		2.59	1.22
	0.58		2.88	1.10
	0.64		2.67	1.18
Silicon oil 100 c. s.	0.38	1.16	0.94	1.23
	0.46		0.91	1.27
	0.54		0.98	1.18
	0.60		1.03	1.13
	0.68		0.98	1.18
Propylene Glycol	0.39	0.66	0.61	1.08
	0.49		0.86	0.77
	0.59		0.96	0.69
	0.65		1.03	0.64

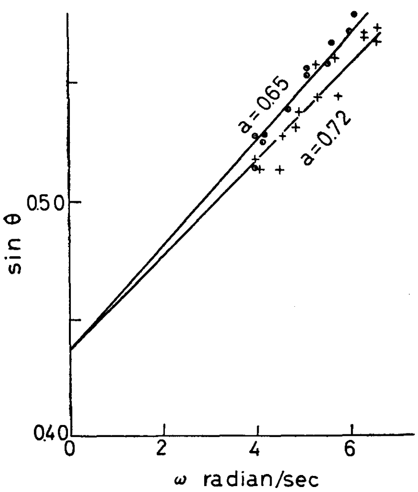

Fig. 6. Sin θ *vs.* ω relation of polymethyl methacrylate 48-80 mesh.

IV. 2. Powders

Figs. 6 to 8 show the results on powdery materials. The relation between sin θ and ω is, except for Carborundum, also linear as in the case of liquids, but the lines do not pass through the origin. The value of sin θ at $\omega=0$ indicates the sine of the angle of free surface inclination when the cylinder ceases rotation, that is, a kind of angle of repose for each amount of the powder. The intercepts obtained by extrapolation of the line and the value of sin θ at $\omega=0$ were found to agree fairly well.

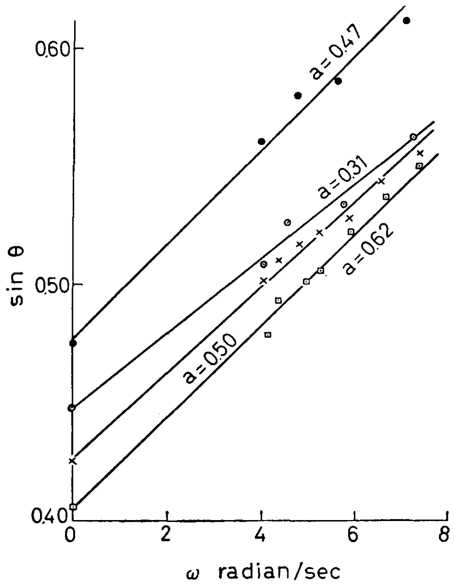

Fig. 7. Sin θ *vs.* ω relation of glass sphere 0.1 mm in dia., ●, ⊙, and 65-100 *mesh*, ×, ▣.

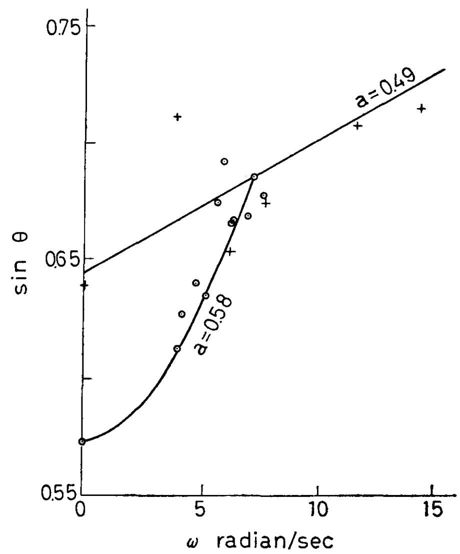

Fig. 8. Sin θ *vs.* ω relation of crushed glass powder 65-80 *mesh*, +, and Carborundum, ⊙.

V. Discussion

In the case of powder, a single value of $\sin \theta/\omega$ can not be obtained since the straight line in the plots of $\sin \theta$ *vs.* ω does not run through the origin.

If we assume the gradient of the line of Fig. 6 to 8 to be the value of $\sin \theta/\omega$, the viscosity can be calculated from eq. (6), where apparent bulk density of the powder bed was used as ρ. Values thus obtained are given in Table 2. For polymethylmethacrylate shere, μ_c seems to be constant regardless of the amount of

Table 2. Viscosities of powders calculated from Eq. (6).

Powders	a	Viscosity of powders, μ_c *poise*
Glass sphere 65—100 mesh	0.50	1.41
	0.62	2.27
Glass sphere 0.1 mm in dia.	0.31	0.38
	0.47	1.28
Polymethyl methacrylate 48—80 mesh	0.65	1.41
	0.72	1.48
Carborundum	0.49	2.51
Crushed glass powder 65—80 mesh	0.49	0.27

powder, but in other cases the effect of the amount is significant.

Table 3 shows calculated viscosities of various samples obtained from the experiments with similar values of a, that is, with similar amount of powder, together with the results of Ford cup tests. Samples with lower viscosity show the shorter exhausting time from the Ford cup, hence it is reasonably presumed that the viscosity thus obtained can be considered as one of the powder parameters expressing the flow property of powder, although the calculation is based on rough and conventional approximations.

Table 3. Viscosity of powder and other related properties.

Powders	a	Viscosity, μ_c *poise*	Flow-out time from Ford cup, *sec*	Angle of repose *degree*
Glass sphere 65—100 mesh	0.62	2.27	44.4	24.0
Polymetyl metacrylate 48—80 mesh	0.65	1.41	40.3	25.7
Carborundum	0.65	2.51	52.3	35.0

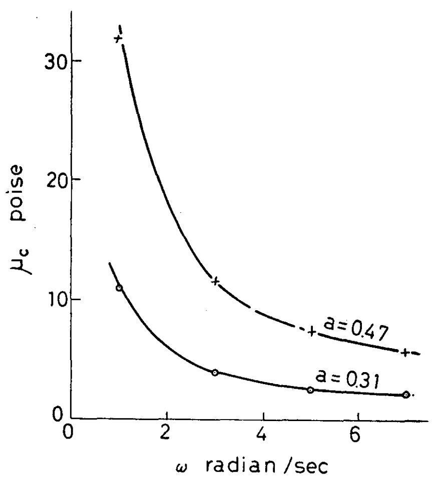

Fig. 9. Change of the calculated viscosity of glass sphere, 0.1 mm in dia., with the angular velocity of the cylinder.

On the other hand, μ can be calculated by putting each value of $\sin\Theta$ and ω into eq. (6). Typical results are given in Fig. 9, which indicates the dependence of μ_c, thus calculated, on ω, that is, μ_c decreases rapidly with ω from infinity to a definite value close to those given in Table 2. It is known by experience that violent flow of powder makes the packing looser, hence the powder becomes more flowable at higher angular velocity of the cylinder. The relation shown in Fig. 9 then seems to be supported qualitatively, though the physical meaning of μ_c remains an open question in this case. According to the above method, the viscosity at $\omega\to\infty$ must be equal to the viscosity given in Table 2.

As we felt a lot of questions on the behavior of powder particles in the rotating cylinder, the following experiment was performed in order to study the pattern of flow. A small amount of 0.5 mm white colored glass spheres were mixed with black-colored spheres of the same size and material, and then put into the cylinder, inner walls of which was coated with *AA*-40 cloth file lest powder should slip at the wall. The profiles of the powder bed in the rotating cylinder were photographed by long time exposure (1/30 sec). Fig. 10 shows the velocity distribution of the particle in the cylinder which was calculated from the length of arcs drawn by white spheres. Upper layers of powder in the cylinder flowed down in the opposite direction of rotation in a turbulent flow, which could not be followed quantitatively. Lower layers, on the other hand, showed laminar flow. The velocity distribution shown in Fig. 10 is that for the laminar flow portion. We can find velocity gradient on the left side of the point P in Fig. 10, which corresponds to the upper part of the laminar portion. Lower part, which is nearer to the wall, moves, however, at a constant angular velocity, smaller than that of the wall, indicating occurance of slippage at the wall of the cylinder. Powder bed in a rotating horizontal cylinder is then considered to be made of three parts : the first, particles are relatively in a static state, though they are moving along the wall of the cylinder as a bulk with a little slippage at the wall, the second, particles make laminar flow with a certain velocity gradient, and the third, the surface part of the bed where particles are flowing in a turbulent state.

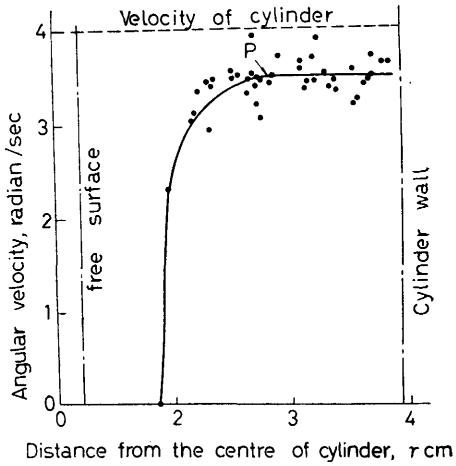

Fig. 10. Velocity distribution of the powder particles in the rotating cylinder.

Since the theoretical analysis used in this report is based on the assumption that the velocity gradient exists throughout the whole powder bed, the result is applicable only to the left side of the point P in Fig. 10. The radial distribution of velocity shown in Fig. 10 is only that at the deepest portion of the bed and in other portions the points corresponding to P might be considered to be located nearer to the wall, that is, the boundary surface, including these points, between the "static" part and the "flowing" part somewhat differs from a coaxial cylindrical surface of the cylinder. These situations make the analysis so complicated that it can not be accomplished now, and, furthermore, it is incongruous with our assumption that the upper part of the powder bed is in a turbulent state.

Though Fig. 10 shows an example of experimental results obtained at one constant velocity, the location of the point P should vary with rates of rotation of the cylinder, and the mode of this variation should depend on the characteristics of the powder particles. Deviation from the linearity observed with Carborundum in Fig. 8 might be partially interpreted by reasons given above.

It is also established from Fig. 10 that the powder bed slips at the wall of the cylinder. This slippage sometimes induces the vibration of the powder bed, which might have caused the fluctuation of plots in Fig. 8 for crushed glass powder.

The authors wish to express their appreciation to the Keio University and the Asahi Glass Co. for the grant of parts of the expence of this research.