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Abstract	

An Analysis on the Two-dimensional Jet Flow into an Unequal Pressure Field

(Particularly on the case in which the nozzle has a moderate convergent angle and the nozzle ends are on the same equi-potential line)

(Received February 7, 1966)

Tsuneyo ANDO*

Abstract

Two-dimensional jet flow into an unequal pressure field through a convergent nozzle with straight walls is analized here. Although the analysis is carried out with the object of getting fundamental data to the performance of annular jet type GEM, the existence of the ground is neglected here, because of tremendous difficulty in calculus, which arises by taking the ground into account from the outset. It is also assumed, for mathematical simplicity, that the convergent angle of the nozzle is a moderate one, so that there is neither positive pressure gradient along the nozzle walls, nor inflexion point along the free stream lines, and that the nozzle wall ends are situated on the same equi-potential line of flow.

The calculation is pursued by expansion of power function, making use of the relation γ (the nozzle convergent angle) $\ll \pi$. As the results, geometric patterns of flow, volumetric flow and power are obtained.

The results obtained here, are of ideal character, especially so in neglecting the existence of the ground. Nevertheless, they will serve as criterions for the actual effect of jet in annular jet type GEM, and the procedure of approximation developed here may be extended to the cases in which above assumptions are not set up, and further to the cases in which the ground exists.

I. Introduction

In the annular jet type GEM, sometimes it happens that the air flow separates at the inner side (high pressure side) nozzle wall, if a parallel nozzle is used, and the performance is injured. This phenomenon is understood, as shown in Fig. 1, that the flow in the nozzle becomes non-uniform, because of unequality of pressures in both sides of jet, and there occurs positive pressure gradient along the inside wall, so air particles cannot go through. As a remedy to this, it is often done to give some convergent angle to the nozzle, as shown in Fig. 2.

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As theoretical studies on annular jet type GEM, we can find several analyses on two dimensional jet flow into an unequal pressure field,[1)2)3)4)] but it seems that they are cofined to the case in which the nozzle is a parallel one. If the nozzle is convergent, the analysis may become extremely difficult.

When we imagine two-dimensional jet flow into an unequal pressure field, from a nozzle with straight walls, disregarding the existence of the ground, it will be, as shown in Fig. 3, of some unactual character. It may be, nevertheless, an interesting object from a theoretical standpoint, and also may serve as one of the criterions to actual phenomena. The author tried to make some analysis on this type of flow.

Flows of this type are devided into three cases, according to the pressure gradient on the nozzle wall and shape of free stream line, namely

a) positive pressure gradient exists (Fig. 4)

b) positive pressure gradient does not exist and free stream line has no inflexion point (Fig. 5)

c) positive pressure grandient does not exist and free stream line has an inflexion point (Fig. 6).

Here, the case b) is treated, with the condition that both nozzle wall ends are situated on the same equi-potential line.

II. Fundamental relations

When a flow is assumed to be two-dimensional and potential one, we have well known relation

$$\frac{d\chi}{dz} = w \tag{1}$$

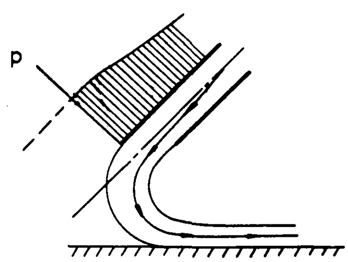

Fig. 1.

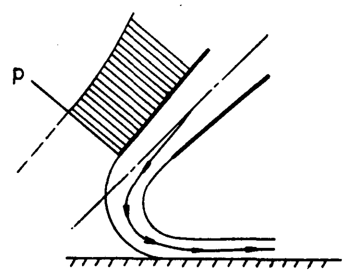

Fig. 2.

1) H. R. Chaplin; David Taylor Model Basin, Rep. 1373, Aero Report 923, July 1957.
2) R. W. Pinnes; NAVAER Research Division, Rep. DR—1958, April 1959.
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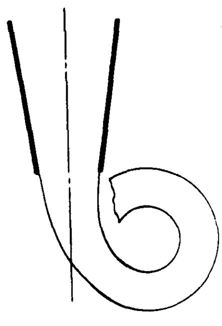

Fig. 3.

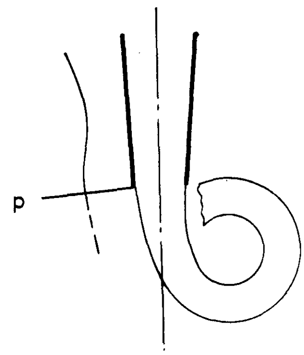

Fig. 4.

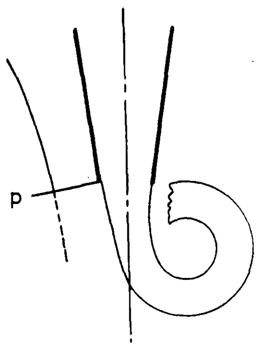

Fig. 5.

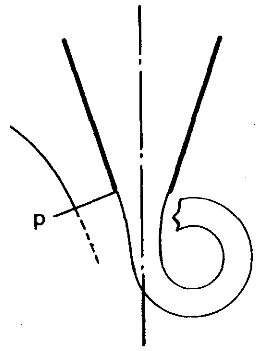

Fig. 6.

where $\chi=\Phi+i\Psi$ is complex velocity potential, $z=x+iy$ stands for coordinates on physical plane where the flow actually occurs, and $w=u-iv$ is complex velocity. Rearranging we have

$$dz=\frac{1}{w}d\chi\,. \tag{2}$$

Then, if it is possible to express w and χ as functions of the same complex variable t, the flow pattern is to be got integrating

$$dz=\frac{1}{w(t)}d\chi(t)\,. \tag{2}'$$

III. Mapping relations in differetial form

We introduce here a new complex variable

$$\Omega=\log(q_j/w)=\log(q_j/q)+i\theta \tag{3}$$

where

q_j=normal jet speed (real, positive constant)
q=speed of flow $(=\sqrt{u^2+v^2})$
θ=direction angle of flow $(=\tan^{-1}[v/u])$.

In the case of two-dimensional flow, on which we are to study here, z-, w-, Ω-, and χ-plane are such as shown in Fig. 7 to Fig. 10.

The inner domains of the images on Ω- and χ-plane can be mapped onto the upper half of the t plane, as shown in Fig. 11. Here a, b, j, b' stand for points themselves corresponding to A, B, J, B', and, at the same time, denote the numerical values of coordinates of them. We confine that $a=0$, $j=\infty$ for convenience. Thus the flow is mapped onto the t plane as a radial flow. Further we assume that both wall ends of the nozzle are on the same equi-potential line, by which we get $|b'|=b$.

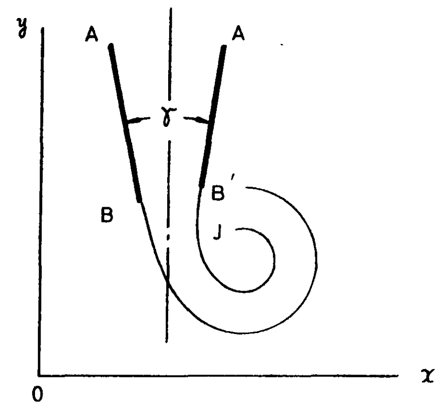

Fig. 7. z-plane

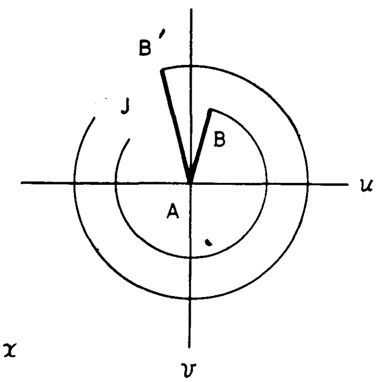

Fig. 8. $w=u-iv$

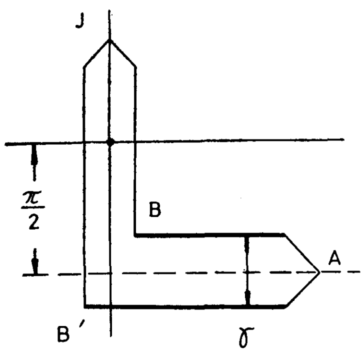

Fig. 9. $\Omega=\log\frac{q_J}{q}+i\theta$

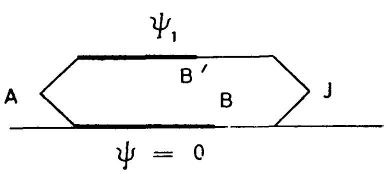

Fig. 10. $\chi=\Phi+i\Psi$

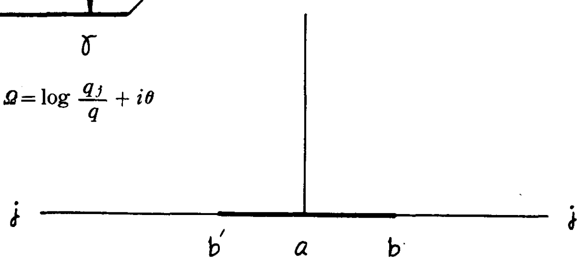

Fig. 11. t-plane

Relations $\Omega \sim t$ and $\chi \sim t$ in differential form are

$$d\Omega = \frac{M'\sqrt{t-b}}{(t-a)\sqrt{t-b'}}dt = \frac{M'\sqrt{t-b}}{t\sqrt{t+b}}dt\,, \tag{4}$$

$$d\chi = \frac{M}{t-a}dt = \frac{M}{t}dt\,. \tag{5}$$

From the conditions which must be satisfied when $t \to a (=0)$, we get

$$M' = i\gamma/\pi\,, \tag{6}$$

$$M = \Psi_1/\pi\,. \tag{7}$$

Therefore $d\Omega$ and $d\chi$ are written as

$$d\Omega = \frac{i\gamma}{\pi} \cdot \frac{\sqrt{t-b}}{t\sqrt{t+b}}dt\,, \tag{8}$$

$$d\chi = \frac{\Psi_1}{\pi} \cdot \frac{1}{t}dt\,. \tag{9}$$

IV. Intergation of mapping relations

In Eq. (8), we put arbitrarily $b = |b'| = 1$, which makes following calculus simplest, i.e.

$$d\Omega = \frac{i\gamma}{\pi} \cdot \frac{\sqrt{t-1}}{t\sqrt{t+1}}dt\,. \tag{10}$$

Integrating Eq. (10), we get

$$\Omega = -\frac{2\gamma}{\pi}\{i \log(\sqrt{t+1} - \sqrt{t-1}) + \log(\sqrt{t+1} + i\sqrt{t-1}) - \log\sqrt{t}\} + \text{const}\,. \tag{11}$$

The constant in the right hand side is got as

$$\text{const.} = \frac{\gamma}{2} + \frac{\gamma}{\pi}\log 2 + \frac{i\gamma}{\pi}\log 2 - \frac{\pi-\gamma}{2}i \tag{12}$$

from the condition $\Omega_B + \Omega_{B'} = -\pi i$, if we choose $q_j^2 = q_B q_{B'}$.

Then

$$\Omega = \frac{\gamma}{2} + \frac{\gamma}{\pi}\log 2 + \frac{i\gamma}{\pi}\log 2 - \frac{\pi-\gamma}{2}i - \frac{2\gamma}{\pi}\Big\{i \log(\sqrt{t+1} - \sqrt{t-1}) + \log(\sqrt{t+1} + i\sqrt{t-1}) - \log\sqrt{t}\Big\} \tag{13}$$

or

$$e^{\Omega} = e^{\frac{\gamma}{2}} 2^{\frac{\gamma}{\pi}} e^{-\frac{\pi-\gamma}{2}i} 2^{\frac{\gamma}{\pi}i} \times (\sqrt{t+1} - \sqrt{t-1})^{-\frac{2\gamma}{\pi}i} (\sqrt{t+1} + i\sqrt{t-1})^{-\frac{2\gamma}{\pi}} (\sqrt{t})^{\frac{2\gamma}{\pi}}\,. \tag{14}$$

V. Physical plane

From the relation (3), we have

$$1/w=e^{\Omega}/q_j. \tag{15}$$

Substituting (15) into (2)′

$$dz=\frac{e^{\Omega}}{q_j}\cdot\frac{d\chi}{dt}dt. \tag{16}$$

Again substituting (14) and (9) into (16), we get

$$dz=\frac{\Psi_1}{\pi q_j}e^{\frac{\gamma}{2}}2^{\frac{\gamma}{\pi}}e^{-\frac{\pi-\gamma}{2}i}2^{\frac{\gamma}{\pi}i}$$

$$\times(\sqrt{t+1}-\sqrt{t-1})^{-\frac{2\gamma}{\pi}i}(\sqrt{t+1}+i\sqrt{t-1})^{-\frac{2\gamma}{\pi}}(\sqrt{t})^{\frac{2\gamma}{\pi}}\frac{dt}{t}. \tag{17}$$

We may know the flow pattern on the physical plane z, if we carry out the integration of (17).

VI. Integration

The right hand side of Eq. (17) contains $\sqrt{t+1}$, $\sqrt{t-1}$, $\sqrt{t}$, and, morover, power of their combination, so it is impossible to know the rigid integrated value. We have to search some suitable method to integrate it approximately. Firstly, we need to know the nozzle form, which is equivalent to seek the value $z_B'-z_B$. This is done by selecting some suitable integration path, connecting b and b', and pursuing integration along it. As the origine a corresponds to the infinity on z-plane, taking a path which lies near the origine is not advantageous. We may take semi-unit-circle, and define θ as

$$t=e^{i\theta},\quad 0\leq\theta\leq\pi. \tag{18}$$

Using this relation, factors in right hand side of Eq. (17) become

$$(\sqrt{t+1}-\sqrt{t-1})^{-\frac{2\gamma}{\pi}i}$$

$$=2^{-\frac{\gamma}{\pi}i}(\cos\phi+\sin\phi-\sqrt{2\sin\phi\cos\phi})^{-\frac{\gamma}{\pi}i}$$

$$\times e^{-\frac{2\gamma}{\pi}\tan^{-1}\frac{\sqrt{2\sin\phi\cos\phi}}{1+\cos\phi-\sin\phi}}$$

$$=2^{-\frac{\gamma}{\pi}i}e^{-\frac{\gamma i}{\pi}\log(\cos\phi+\sin\phi-\sqrt{2\sin\phi\cos\phi})}$$

$$\times e^{-\frac{2\gamma}{\pi}\tan^{-1}\frac{\sqrt{2\cos\phi\sin\phi}}{1+\cos\phi-\sin\phi}} \tag{19}$$

$$(\sqrt{t+1}+i\sqrt{t-1})^{-\frac{2\gamma}{\pi}}$$

$$=2^{-\frac{\gamma}{\pi}}(\cos\phi+\sin\phi-\sqrt{2\sin\phi\cos\phi})^{-\frac{\gamma}{\pi}}$$

$$\times e^{-\frac{2\gamma i}{\pi}\tan^{-1}\frac{2\sin\phi\cos\phi+\sqrt{2\sin\phi\cos\phi}}{(\cos\phi-\sin\phi)(1+\cos\phi+\sin\phi)}}$$

$$=2^{-\frac{\gamma}{\pi}}e^{-\frac{\gamma}{\pi}\log(\cos\phi+\sin\phi-\sqrt{2\sin\phi\cos\phi})}$$

$$\times e^{-\frac{2\gamma i}{\pi}\tan^{-1}\frac{2\sin\phi\cos\phi+\sqrt{2\sin\phi\cos\phi}}{(\cos\phi-\sin\phi)(1+\cos\phi+\sin\phi)}} \tag{20}$$

$$(\sqrt{t})^{\frac{2\gamma}{\pi}}=e^{\frac{2\gamma i}{\pi}\phi} \tag{21}$$

$$\frac{dt}{t}=2id\phi \tag{22}$$

where $\phi=\theta/2$.

Substituting these four relations into (17) and rearranging, we obtain

$$dz=Ke^{-\frac{\gamma}{\pi}(X_1+iY_1)} \tag{23}$$

where

$$K=(2\Psi_1/\pi q_j)e^{\gamma/2}e^{\gamma i/2}$$

$$X_1=a_1+b_1,\quad Y_1=a_1-c_1+d_1$$

$$a_1=\log(\cos\phi+\sin\phi-\sqrt{2\sin\phi\cos\phi})$$

$$b_1=2\tan^{-1}\frac{\sqrt{2\sin\phi\cos\phi}}{1+\cos\phi-\sin\phi}$$

$$c_1=2\phi$$

$$d_1=2\tan^{-1}\frac{2\sin\phi\cos\phi+\sqrt{2\sin\phi\cos\phi}}{(\cos\phi-\sin\phi)(1+\cos\phi+\sin\phi)}$$

and $b_1+c_1=d_1,\quad X_1=Y_1$.

If the convergent angle is small, we have

$$\left|\frac{\gamma}{\pi}(X_1+iY_1)\right|\ll 1,$$

and (23) can be written as

$$dz=K\left\{1-\frac{\gamma}{\pi}X_1-i\frac{\gamma}{\pi}Y_1+\left(\frac{\gamma}{\pi}\right)^2X_2+i\left(\frac{\gamma}{\pi}\right)^2Y_2\right.$$

$$\left.-\left(\frac{\gamma}{\pi}\right)^3X_3-i\left(\frac{\gamma}{\pi}\right)^3Y_3+\left(\frac{\gamma}{\pi}\right)^4X_4+i\left(\frac{\gamma}{\pi}\right)^4Y_4\right\}d\phi \tag{24}$$

where

$$X_2=(1/2)(X_1^2-Y_1^2),\quad Y_2=X_1Y_1$$

$$X_3=(1/6)X_1(X_1^2-3Y_1^2),\quad Y_3=(1/6)Y_1(3X_1^2-Y_1^2)$$

$$X_4=(1/6)(X_2^2-Y_2^2),\quad Y_4=(1/3)X_2Y_2.$$

In Eq. (24), X_1, Y_1; ......; X_4, Y_4 are functions only of ϕ, and can be integrated independent of γ. For $z_B'-z_B$, they are to be integrated oxer $0\leq\phi\leq\frac{\pi}{2}$.

VII. Integration of X_i, Y_i (i=1, 2, 3, 4)

Fig. 12 to Fig. 15 show X_i, Y_i (i=1, 2, 3, 4) as functions of ϕ, except those which are identically zero. In order to obtain $z_B'-z_B$, we have to know the areas of these patterns, abscissa being measured in radians. For the interval $\phi=0°\sim80°$,

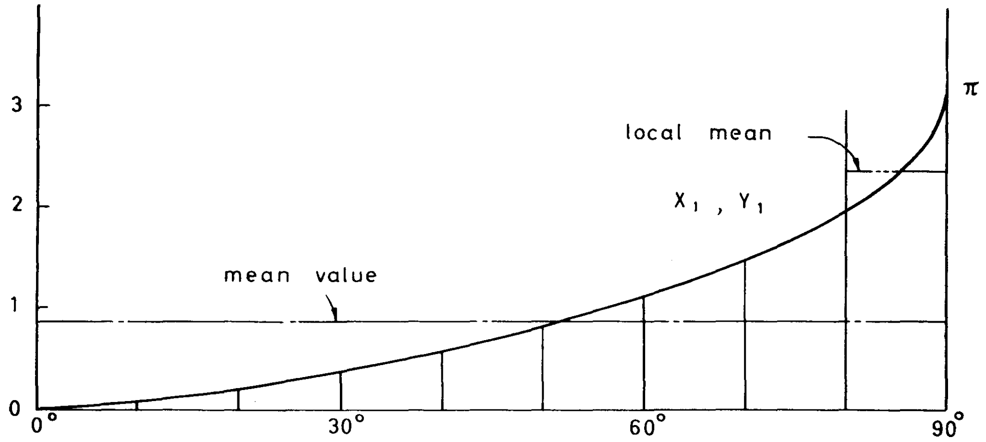

Fig. 12.

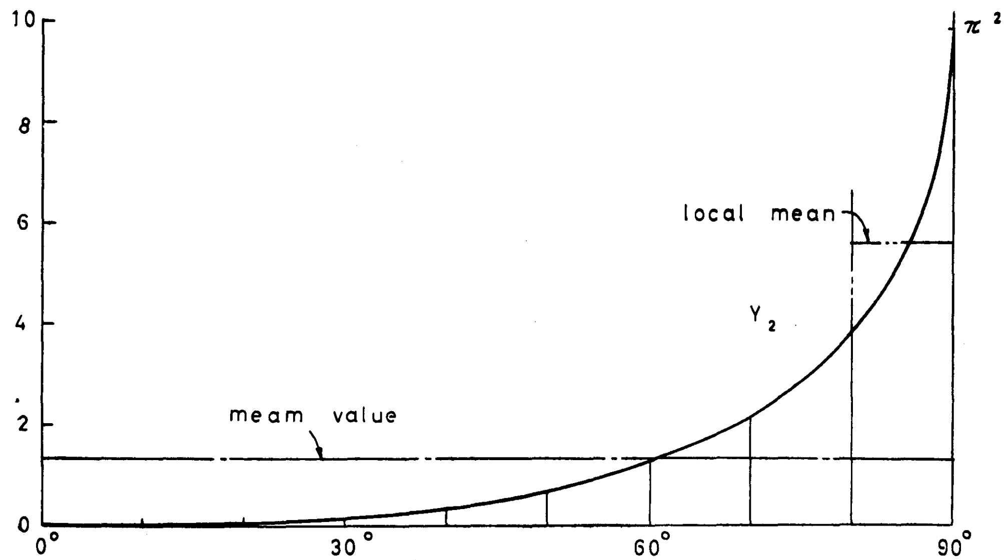

Fig. 13.

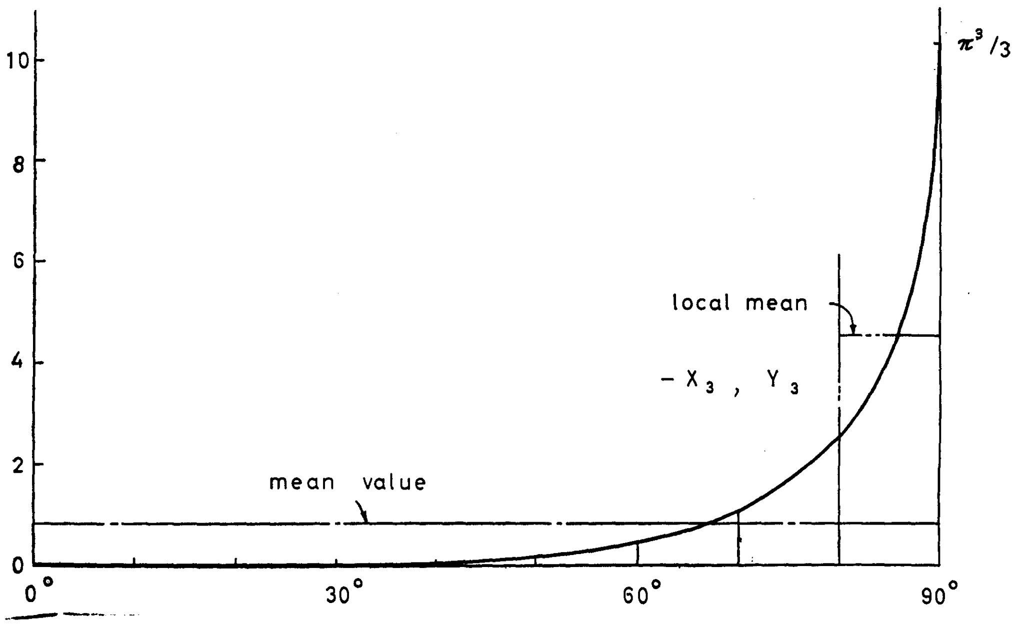

Fig. 14. ——→ ϕ

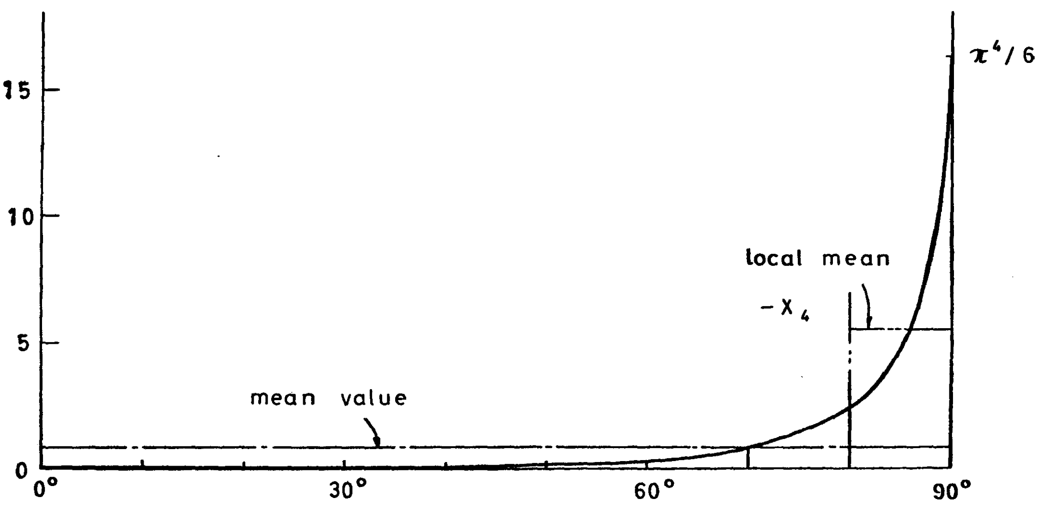

Fig. 15. ——→ ϕ

Simpson's formula is available, and fairly correct values are got, by dividing the interval into 8 equal sub-intervals. For the interval $\phi=80°\sim90°$, where the curves have vertical tangents, approximation

$$X_1=Y_1=\pi-2\sqrt{2}\,\psi^{\frac{1}{2}} \tag{25}$$

where $\psi=\pi/2-\phi$

will be used. Accuracy of this Eq. (25), is to about 1/2,000 for $\phi=80°$ ($\phi=10°=\pi/18$), and 1.3/2,000 in case of X_4. Thus integrated values are

$$\int_0^{\pi/2} X_1 d\phi = \int_0^{\pi/2} Y_1 \, d\phi = 1.379$$

$$\int_0^{\pi/2} Y_2 \, d\phi = 2.087$$

$$-\int_0^{\pi/2} X_3 \, d\phi = \int_0^{\pi/2} Y_3 \, d\phi = 1.295$$

$$\int_0^{\pi/2} X_4 \, d\phi = -1.348.$$

VIII. $z_{B'} - z_B$

For the value $\gamma/\pi=0.1$, 0.2, 0.3, values of $z_B' - z_B$ are as in Table 1.

Table 1.

γ/π	0.1	0.2	0.3
$\dfrac{z_B' - z_B}{2\Psi_1/\pi q_j}$	$1.679+0.126\,i$	$1.783+0.287\,i$	$1.876+0.487$

IX. Breadth of jet, B_j

Breadth of the jet B_j is obtained by integration of Eq. (17), taking a semicircle with sufficiently large radius R as integration path. Substituting $t=Re^{i\theta}(R\gg1)$ and rearranging, we have

$$\int dz = \frac{\Psi_1}{\pi q_j} e^{\frac{\gamma}{2}} e^{\frac{\gamma i}{\pi}(\log 2+\log R)} \int e^{-\frac{\gamma}{\pi}\theta} d\theta\,, \tag{26}$$

that is

$$B_j = \left| \int_{\theta=0}^{\pi} dz \right| = \frac{\Psi_1}{\pi q_j} e^{\frac{\gamma}{2}} \int_0^{\pi} e^{-\frac{\gamma}{\pi}\theta} d\theta$$

$$= \frac{2\Psi_1}{\gamma q_j} \sinh \frac{\gamma}{2}\,. \tag{27}$$

Numerically, values of B_j are as in Table 2.

Using these values of B_j, values of $z_B' - z_B$ in B_j are got as in Table 3.

Table 2.

γ/π	0.1	0.2	0.3
$\frac{B_j}{2\Psi_1/\pi q_j}$	1.577	1.596	1.630

Table 3.

γ/π	0.1	0.2	0.3
$\frac{z_B'-z_B}{B_j}$	$1.065+0.080i$	$1.117+0.180i$	$1.151+0.299i$

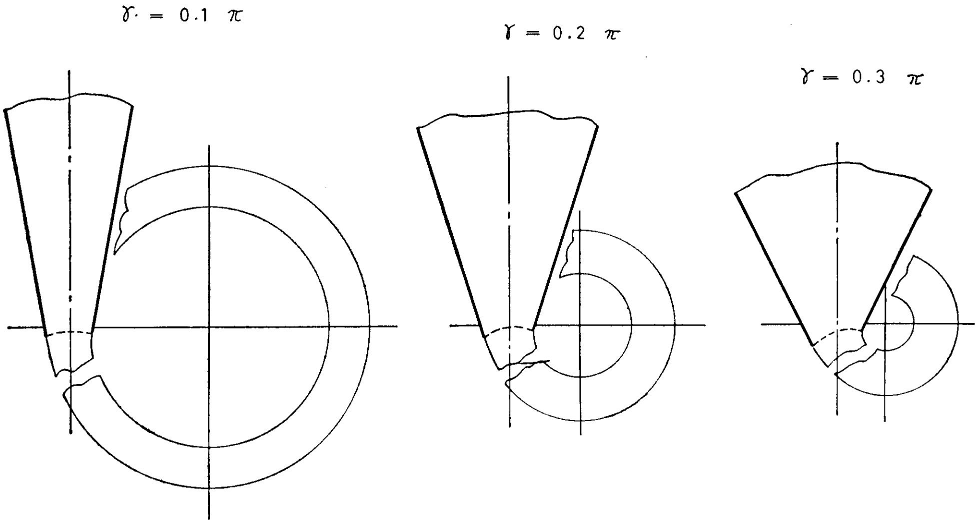

Fig. 16.

X. Ratio of outer and inner radius of jet

Finally, outer and inner extreme stream lines make circles, and the ratio of their radii is obtained, noting that the flow is potential one, and that the pressure is constant along any stream line. That is

$$\frac{r_0}{r_i}=\frac{q_{ji}}{q_{j0}}=\frac{q_B'}{q_B} \tag{28}$$

From the definition (3)

$$\left.\begin{aligned} q_B&=q_j\,e^{-R(\Omega_B)}=q_j\,e^{-\frac{\gamma}{2}} \\ q_B'&=q_j\,e^{-R(\Omega_B')}=q_j\,e^{\frac{\gamma}{2}} \end{aligned}\right\} \tag{29}$$

therefore

$$\frac{r_0}{r_i}=e^{\gamma}. \qquad (30)$$

Numerical results are such as shown in Table 4.

Table 4.

γ/π	0.1	0.2	0.3
r_0/r_i	1.369	1.874	2.566
r_0/B_j	3.71	2.144	1.639
r_i/B_j	2.71	1.144	0.639

XI. Volumetric flow and power

The flow makes finally an annular flow, with velocity distribution same as a free vortex. Using here the relationship between velocity and radius of a streamline

$$qr=q_{j0}\,r_0=q_{ji}\,r_i=\text{const}., \qquad (31)$$

the flow quantity Ψ_1 is obtained as

$$\Psi_1=\frac{B_j\log(q_{ji}/q_{j0})}{1/q_{j0}-1/q_{ji}}=\frac{B_j\gamma}{(1/q_B)(1-e^{-\gamma})}=\frac{B_j\gamma}{(1/q_B')(e^{\gamma}-1)}. \qquad (32)$$

From Eq. (32),

$$\left.\begin{aligned} q_B&=(1-e^{-\gamma})\Psi_1/B_j\gamma\\ q_B'&=(e^{\gamma}-1)\Psi_1/B_j\gamma.\end{aligned}\right\} \qquad (33)$$

On the other hand, from Bernoulli's Equation

$$\frac{1}{2}\rho q_{j0}^2+p_0=\frac{1}{2}\rho q_{ji}^2+p_i=p_A, \qquad (34)$$

or

$$\frac{1}{2}\rho q_B^2+p_0=\frac{1}{2}\rho q_B'^2+p_i=p_A, \qquad (34)'$$

so substituting (33) into (34)′, we have

$$\frac{1}{2}\rho\left\{\frac{(1-e^{-\gamma})\Psi_1}{B_j\gamma}\right\}^2+p_0=\frac{1}{2}\rho\left\{\frac{(e^{\gamma}-1)\Psi_1}{B_j\gamma}\right\}+p_i=p_A. \qquad (35)$$

Adding the first and the second side of this equation, and putting half of the sum is equal to the third side, we get

$$\frac{1}{4}\cdot\frac{\rho\Psi_1^2}{(B_j\gamma)^2}\left\{(1-e^{-\gamma})^2+(e^{\gamma}-1)^2\right\}+\frac{1}{2}(p_0+p_i)=p_A. \qquad (36)$$

Then Ψ_1 is obtained as

$$\Psi_1=B_j\frac{\gamma}{\sinh(\gamma/2)}\sqrt{\frac{p_A-(p_0+p_i)/2}{2\rho\cosh\gamma}}. \qquad (37)$$

For the case $\gamma \ll 1$ (this means that the pressur difference is very small), $\gamma/\sinh(\gamma/2)=2$, $\cosh\gamma=1$, and so,

$$\Psi_1 = B_j\sqrt{\frac{2\{p_A-(p_0+p_i)/2\}}{\rho}}. \tag{38}$$

This is the flow quantity of a jet, which flows into an equal pressure field, with pressure equal to $(p_0+p_i)/2$. Taking p_i as pressure standard, the power required to produce the jet flow becomes

$$L=(p_A-p_i)\Psi_1=\frac{1}{2}\rho q_{ji}^2\Psi_1=\frac{1}{2}\rho q_j^2 e^{\gamma}\Psi_1\,. \tag{39}$$

As the pressure difference between both sides of the jet is given from (29) as

$$p_0-p_i=\frac{1}{2}\rho(q_{ji}^2-q_{j0}^2)=\frac{1}{2}\rho q_j^2(e^{\gamma}-e^{-\gamma}) \tag{40}$$

and therefore

$$q_j^2=\frac{2}{\rho}\cdot\frac{p_0-p_i}{e^{\gamma}-e^{-\gamma}}\,. \tag{41}$$

Then Eq. (39) becomes

$$L=\frac{e^{\gamma}}{e^{\gamma}-e^{-\gamma}}(p_0-p_i)\Psi_1$$

$$=\frac{\gamma e^{\gamma}B_j}{2\sinh(\gamma/2)\sinh\gamma}\sqrt{\frac{p_A-(p_0+p_i)/2}{2\rho\cosh\gamma}}(p_0-p_i)\,. \tag{42}$$

From Eq. (34)

$$\left.\begin{aligned} p_A-p_i&=\rho q_{ji}^2/2=\rho q_j^2 e^{\gamma}/2 \\ p_A-p_0&=\rho q_{j0}^2/2=\rho q_j^2 e^{-\gamma}/2 \end{aligned}\right\} \tag{43}$$

and using Eq. (41)

$$p_A-\frac{p_0+p_i}{2}=\frac{1}{4}\rho q_j^2(e^{\gamma}+e^{-\gamma})=\frac{1}{2}(p_0-p_i)\coth\gamma\,. \tag{44}$$

Then (42) becomes

$$L=\frac{\gamma e^{\gamma}B_j}{4\sinh h(\gamma/2)\sinh\gamma}\sqrt{\frac{1}{\rho\sinh\gamma}}(p_0-p_i)^{3/2}. \tag{45}$$

Using the relation

$$B_j/r_0=(r_0-r_i)/r_0=1-r_i/r_0=1-e^{-\gamma}$$

or

$$B_j=r_0(1-e^{-\gamma}),$$

Eq. (45) is rewritten finally as

$$L=\frac{\gamma e^{\gamma/2}r_0}{2\sqrt{\rho}\,(\sinh\gamma)^{3/2}}(p_0-p_i)^{3/2}\,. \tag{46}$$

For $\gamma \ll 1$, $e^{\gamma/2}=1$, $\sinh \gamma = \gamma$, and

$$L=\frac{r_0}{2\sqrt{\rho\gamma}}(p_0-p_i)^{3/2} \tag{47}$$

which holds for small p_0-p_i.

XII. Conclusion

Here, two-dimensional jet flow into an unequal pressure field is analyzed, assuming that the nozzle has a moderate converging angle, and that the both ends of the nozzle walls are located on the same equi-potential line. Same method of approximation may be extended to the case in which nozzle wall ends are not on the same equi-potential line, and further, to the case in which the nozzle converging angle is small (case a), or large (case c). The author wishes this study would be utilized as a criterion to the actual cases. And the author wishes to express his deep thankfiness to Mr. T. Saito and Mr. M. Watabe for their good co-operation with the author.