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Effects of Collision on the Propagation of Electromagnetic Wave in a Plasma

—I. Homogeneous Isotropic Plasma—

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Abstract

Propagation constant, attenuation constant, index of refraction, and velocity of energy transfer are calculated for an isotropic homogeneous plasma.

It is explicitly shown that in the lossy plasma not the group velocity but the velocity of energy transfer satisfies the relativity requirement. These two velocities coincide only when the collision is absent.

I. Introduction

In recent years considerable attention has been paid to the theory of electromagnetic wave propagation in a plasma [1)~4)], in connection with the problems of radio communication with a high-speed space vehicles, and controlled thermonuclear fusion. In our present series of papers we will focus the special attention on the effects of collision on the propagation of the electromagnetic wave. The effects of the collision was taken into account by Balwanz [2)], and Heald and Wharton [4)]. The former has given graphs of absorption, phase shift and reflection coefficients. The latter has given graphs of refractive and attenuation indices as a function of

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1) A. B. Cambel; *Plasma Physics and Magnetofluidmechanics* (1960), McGraw-Hill, New York.
2) W. W. Balwanz; NRL Report 5388 (1959).
3) V. L. Ginzburg; *The Propagation of Electromagnetic Waves in Plasmas*, Translated by J. B. Sykes and R. J. Tayler (1964), Pergamon Press, New York.
4) M. A. Heald and C. B. Wharton; *Plasma Diagnostics with Microwaves* (1965), John Wiley and Sons Inc., New York.

$(\omega_p/\omega)^2$, ω_p and ω being the plasma frequency and the electromagnetic wave frequency, respectively.

In the present paper, propagation and attenuation constants in a homogeneous isotropic plasma are evaluated numerically as a function of ω/ω_p, the collision frequency being considered to be a parameter. These are presented and discussed in section II. In section III, velocity of energy transfer is calculated and compared with the group velocity, and shown explicitly to be less than the light velocity.

II. Propagation and Attenuation Constants

For a homogeneous isotropic plasma, Maxwell's equations are given by

$$rot\,\boldsymbol{H} = \sigma \boldsymbol{E} + i\omega\varepsilon \boldsymbol{E}, \tag{1}$$

$$rot\,\boldsymbol{E} = -i\omega\mu \boldsymbol{H}, \tag{2}$$

$$div\,\boldsymbol{E} = 0, \tag{3}$$

$$div\,\boldsymbol{H} = 0, \tag{4}$$

where $\boldsymbol{E}$ and $\boldsymbol{H}$ are the electric and magnetic field vectors which are supposed to be harmonic functions of time, i. e. being proportional to $e^{i\omega t}$. The dielectric constant, magnetic permeability and the electrical conductivity of the plasma are expressed by ε, μ and σ respectively. Using the equations (1), (2), (3) and the identity $rot\,rot\,\boldsymbol{E} = -\Delta \boldsymbol{E} + grad\,div\,\boldsymbol{E}$, we have the wave equation

$$\Delta \boldsymbol{E} + \omega^2 \mu \left(\varepsilon - i\frac{\sigma}{\omega}\right) \boldsymbol{E} = 0. \tag{5}$$

If we consider the oscillation of the type

$$\begin{aligned} \boldsymbol{E} &= \boldsymbol{E}_0\, e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})}, \\ \boldsymbol{H} &= \boldsymbol{H}_0\, e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})}, \end{aligned} \tag{6}$$

we have, from (5) and (6), the dispersion relation

$$k^2 = \omega^2 \mu \left(\varepsilon - i\frac{\sigma}{\omega}\right). \tag{7}$$

If we consider the case of vacuum, we have

$$k^2 = \omega^2 \mu_0 \varepsilon_0, \qquad \frac{\omega^2}{k^2} = \frac{1}{\mu_0 \varepsilon_0} = c^2, \tag{8}$$

where μ_0, ε_0 and c are respectively the magnetic permeability, the dielectric constant and the velocity of light in the vacuum. In the plasma, $\mu = \mu_0$, so the dispersion relation is rewritten as

$$k^2 = \frac{\omega^2}{c^2}\left(\frac{\varepsilon'}{\varepsilon_0}\right), \quad \varepsilon' = \varepsilon - i\frac{\sigma}{\omega}, \tag{9}$$

where we have introduced the complex dielectric constant ε'. This is obtained as follows.

The equation of motion of an electron subjected to the electric field $\boldsymbol{E}=\boldsymbol{E}_0 e^{i\omega t}$ is given by

$$m\frac{d\boldsymbol{v}}{dt}=-e\boldsymbol{E}_0 e^{i\omega t}-\nu m\boldsymbol{v}, \tag{10}$$

where m, $-e$ and $\boldsymbol{v}$ are the mass, the charge and the velocity of the electron and ν is the collision frequency between electrons and other heavy particles (neutrals and ions) in the plasma. By solving (10) we have the total current $\boldsymbol{J}_t$

$$\boldsymbol{J}_t=-en\boldsymbol{v}=\frac{ne^2}{m}\frac{1}{i\omega+\nu}\boldsymbol{E}=-\frac{\omega_p{}^2}{\omega^2-i\omega\nu}i\omega\varepsilon_0\boldsymbol{E}, \quad \omega_p{}^2=\frac{ne^2}{m\varepsilon_0}, \tag{11}$$

here ω_p is the frequency of the plasma oscillation and n the number density of the electron. The total current $\boldsymbol{J}_t$ is expressed in another way as the sum of the conduction current $\boldsymbol{J}_t$ and the current due to polarization $\partial\boldsymbol{P}/\partial t$, $\boldsymbol{P}$ being the electric polarization ;

$$\boldsymbol{J}_t=\boldsymbol{J}+\frac{\partial\boldsymbol{P}}{\partial t}=\sigma\boldsymbol{E}+i\omega(\varepsilon\boldsymbol{E}-\varepsilon_0\boldsymbol{E})=\left(\frac{\varepsilon'}{\varepsilon_0}-1\right)i\omega\varepsilon_0\boldsymbol{E}. \tag{12}$$

Then equating (11) to (12), it follows that

$$\frac{\varepsilon'}{\varepsilon_0}=1-\frac{\omega_p{}^2}{\omega^2-i\omega\nu}=1-\frac{\omega_p{}^2}{\omega^2+\nu^2}-i\frac{\nu}{\omega}\frac{\omega_p{}^2}{\omega^2+\nu^2},$$

and

$$\varepsilon=\varepsilon_0\left(1-\frac{\omega_p{}^2}{\omega^2+\nu^2}\right), \quad \sigma=\varepsilon_0\omega_p{}^2\frac{\nu}{\omega^2+\nu^2}, \tag{13}$$

where use has been made of the definition of ε' which is given by (9). The index of refraction n' of the plasma is defined by

$$n'=\frac{c}{v_p}, \quad v_p=\frac{\omega}{k},$$

v_p being the phase velocity. Using (9), we have

$$(n')^2=\left(\frac{ck}{\omega}\right)^2=\frac{\varepsilon'}{\varepsilon_0}. \tag{14}$$

Since $\varepsilon'/\varepsilon_0$ is a complex quantity, we can devide n' and k into real and imaginary parts*

* From (13) it follows that

i) $\sigma\geqq 0$, $\varepsilon\geqq 0$, for $\omega^2+\nu^2\geqq\omega_p{}^2$,

ii) $\sigma\geqq 0$, $\varepsilon<0$, for $\omega^2+\nu^2<\omega_p{}^2$,

then ε' which is defined by (9) lies in the 4 th quadrant of the complex ε' plane when $\omega^2+\nu^2\geqq\omega_p{}^2$ and in the 3 rd quadrant when $\omega^2+\nu^2<\omega_p{}^2$.

Thus $\sqrt{\varepsilon'}$ is in the 2 nd quadrant. Then the real and the imaginary parts of $\sqrt{\varepsilon'}$ have different signs.

$$n' = n_r - i n_i, \quad k = \beta - i\alpha,$$
$$n_r = \frac{c\beta}{\omega}, \quad n_i = \frac{c\alpha}{\omega}, \quad (n_r,\ n_i,\ \beta,\ \alpha = \text{positive}). \tag{15}$$

Taking the direction of the vector $\boldsymbol{k}$ as the z-axis, we can write the field $\boldsymbol{E}$ as

$$\boldsymbol{E} = \boldsymbol{E}_0\, e^{\mp\alpha z}\, e^{i(\omega t \mp \beta z)}$$
$$= \boldsymbol{E}_0\, e^{\mp \frac{\omega n_i}{c} z}\, e^{i(\omega t \mp \frac{\omega n_r}{c} z)}. \tag{16}$$

The $\mp$ signs in (16) correspond to waves which propagate in the direction of the positive and negative z-axis respectively. The constants α and β are called the attenuation constant and the propagation constant respectively. From (9), (14) and (15) we have

$$n_r = \sqrt{\frac{1}{2}\left[\frac{\varepsilon}{\varepsilon_0} + \sqrt{\left(\frac{\varepsilon}{\varepsilon_0}\right)^2 + \left(\frac{\sigma}{\varepsilon_0\omega}\right)^2}\right]},$$
$$n_i = \sqrt{\frac{1}{2}\left[-\frac{\varepsilon}{\varepsilon_0} + \sqrt{\left(\frac{\varepsilon}{\varepsilon_0}\right)^2 + \left(\frac{\sigma}{\varepsilon_0\omega}\right)^2}\right]}, \tag{17}$$

and

$$\beta = \frac{\omega}{c}\sqrt{\frac{1}{2}\left[\frac{\varepsilon}{\varepsilon_0} + \sqrt{\left(\frac{\varepsilon}{\varepsilon_0}\right)^2 + \left(\frac{\sigma}{\varepsilon_0\omega}\right)^2}\right]},$$
$$\alpha = \frac{\omega}{c}\sqrt{\frac{1}{2}\left[-\frac{\varepsilon}{\varepsilon_0} + \sqrt{\left(\frac{\varepsilon}{\varepsilon_0}\right)^2 + \left(\frac{\sigma}{\varepsilon_0\omega}\right)^2}\right]}, \tag{18}$$

$$\frac{\varepsilon}{\varepsilon_0} = 1 - \frac{\omega_p^2}{\omega^2 + \nu^2}, \quad \frac{\sigma}{\varepsilon_0\omega} = \frac{\nu}{\omega}\frac{\omega_p^2}{\omega^2 + \nu^2}. \tag{13'}$$

The expression (17) gives the real and the imaginary parts of the index of refraction, n_r and n_i. The expression (18) gives the propagation and attenuation constants β and α in terms of the constitutive parameters of the plasma.

The dimensionless propagation and attenuation constants $c\beta/\omega_p$ and $c\alpha/\omega_p$, and the indexes of refractions n_r and n_i are numerically computed as a function of normalized frequency ω/ω_p, on the electronic computer K—1 of Keio University. The normalized collision frequency ν/ω_p is taken as a parameter ranging from 0 to 0.5. Table 1 gives the numerical values of these constants. Fig. 1 and Fig. 2 are their graphical representations. These are equivalent respectively to the graph 12 and 5 of Balwanz [2].

From these graphs we can see the following behaviours. The propagation constant approaches asymptotically to that of vacuum when $\omega \to \infty$. This is because the electrons in the plasma become to be unable to follow the field as the frequency increases. In the lossless plasma no propagation occurs for $\omega < \omega_p$. in the lossy plasma, however, the wave can propagate even for $\omega < \omega_p$, but accompanied by attenuation. In Fig. 2 we note that the values of $c\alpha/\omega_p$ at $\omega = 0$ differ by unity depending on $\nu = 0$ or $\nu \neq 0$. The limiting value of α when $\omega \to 0$ and $\nu \to 0$ does

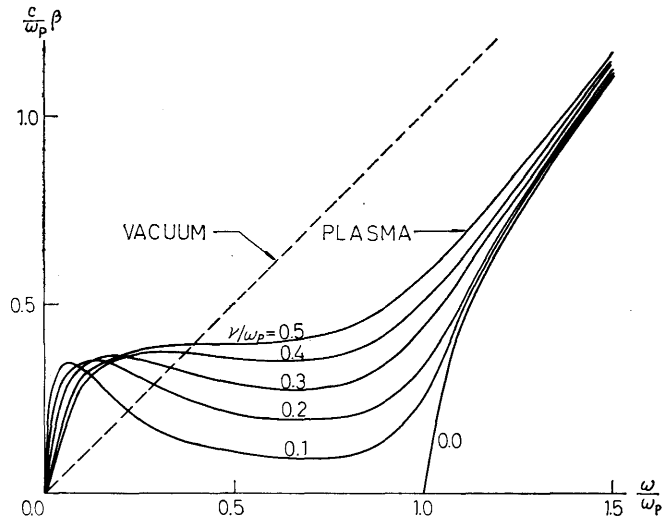

Fig. 1. Dimensionless propagation constant $c\beta/\omega_p$ as a function of ω/ω_p.

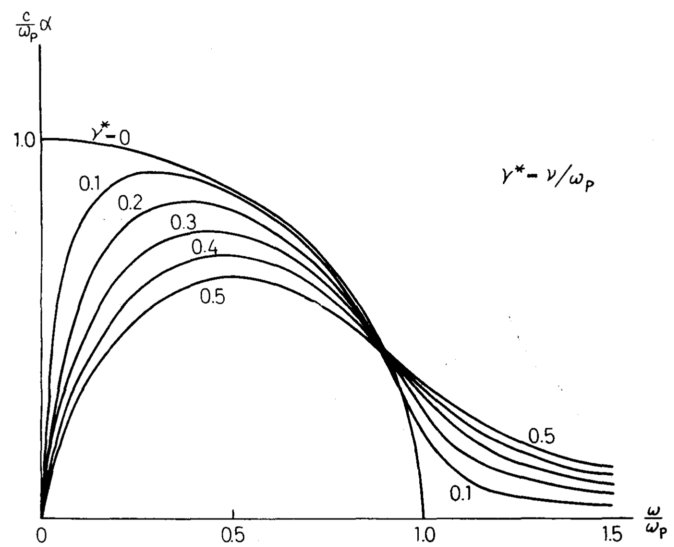

Fig. 2. Dimensionless attenuation constant $c\alpha/\omega_p$ as a function of ω/ω_p.

not exist, because the limit depends on the way of making $\omega \to 0$ and $\nu \to 0$. Neverthless when we make $\omega \to 0$ in the lossless plasma, the electrons are accelerated indefinitely and the Lorentz force cannot be negligible in the evaluation of ε and σ. Hence for lossless plasma, i. e. $\nu = 0$, the present theory becomes invalid near $\omega = 0$.

Fig. 3 gives the qualitative features of phase and group velocities v_p and v_g. When the collision exists the group velocity has the infinite value at two points of ω/ω_p. At a glance we may consider that the relativity requirement is not fulfilled. In order to clarify this we will calculate the velocity of energy transfer in the next section.

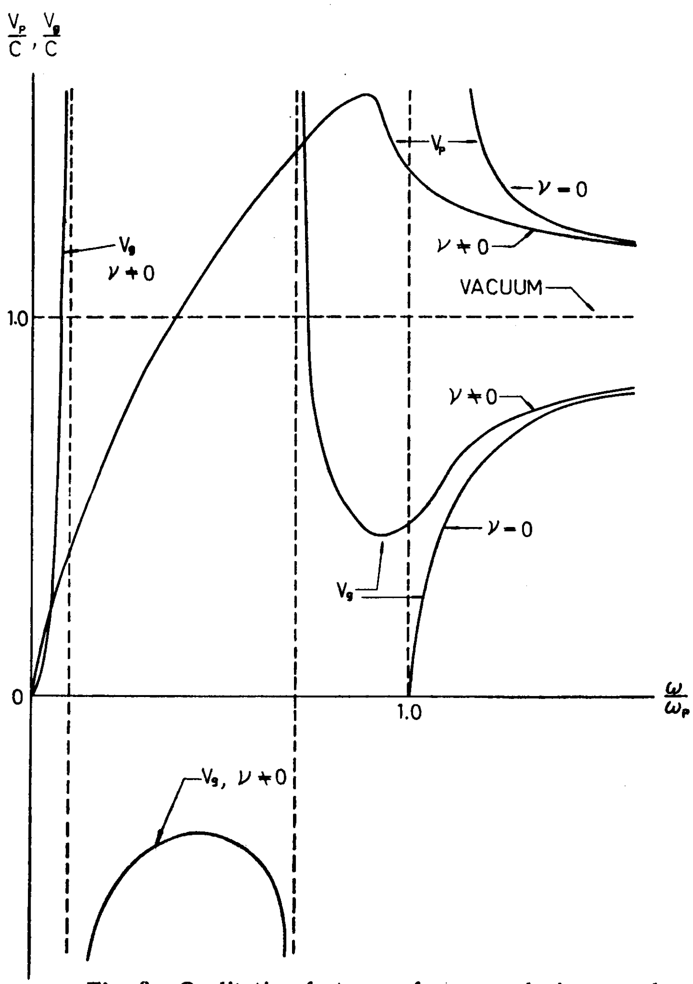

Fig. 3. Qualitative features of group velocity v_g and phase velocity v_p as functions of ω/ω_p.

III. Velocity of Energy Transfer

In this section the velocity of energy transfer is calculated and shown to fulfill the relativity requirement. This velocity is shown to agree with the group velocity in the absence of the collision.

From (2) and (6) it follows that

$$H = \frac{1}{\omega\mu} k \times E. \tag{19}$$

From this it is apparent that the vectors $\boldsymbol{E}$, $\boldsymbol{H}$ and $\boldsymbol{k}$ form the right-handed co-ordinate system. We have taken the direction of the vector $\boldsymbol{k}$ as the z-axis, so the directions of $\boldsymbol{E}$ and $\boldsymbol{H}$ are in the directions of x and y-axes. From the expression (16) and (19) we have

$$\begin{aligned} E_x &= E_{0x}\, e^{-\alpha z}\, e^{i(\omega t-\beta z)}, \\ H_y &= \frac{k}{\omega\mu} E_x. \end{aligned} \tag{20}$$

With use of the relations (15), H_y becomes

$$H_y = \frac{1}{\mu c}(n_r - i n_i)\, E_x. \tag{21}$$

The poynting vector $\boldsymbol{S}=\boldsymbol{E}\times\boldsymbol{H}$ has only z-component $S_z=E_xH_y$ in our case. With the bar notation we express the time averaged value.

Using (20) and (21), we have

$$\overline{S}_z = \frac{1}{2} R_e E_x H_y^* = c\, n_r \frac{\varepsilon_0}{2} |E_x|^2, \tag{22}$$

where use has been made of the assumption $\mu=\mu_0$.
The instantaneous $S_z(t)$ is given by

$$\begin{aligned} S_z(t) &= (R_e E_x)(R_e H_y) \\ &= 2c\cos\theta\,(n_r\cos\theta + n_i\sin\theta)\frac{\varepsilon_0}{2} E_{0x}^2\, e^{-2\alpha z}, \end{aligned} \tag{22'}$$

with

$$\theta = \omega t - \beta z. \tag{23}$$

The energy density W consists of three parts;

$$W = W_E + W_H + W_K, \tag{24}$$

here W_E and W_H are the energy densities due to the electric and magnetic fields respectively and W_K is the kinetic energy of the electrons. They are given by

$$W_E = \frac{\varepsilon_0}{2} E_x^2, \quad W_H = \frac{\mu_0}{2} H_y^2, \quad W_K = \frac{1}{2} m v_x^2 N, \tag{25}$$

where v_x and N being the velocity and the number density of the electron. By using (20), (21) and (10) we have

$$\overline{W}_E = \frac{\varepsilon_0}{4} |E_x|^2, \tag{26}$$

$$W_E = \cos^2\theta\, \frac{\varepsilon_0}{2} E_{0x}^2\, e^{-2\alpha z}, \tag{26'}$$

$$\overline{W}_H = (n_r^2 + n_i^2)\frac{\varepsilon_0}{4} |E_x|^2, \tag{27}$$

$$W_H = (n_r \cos\theta + n_i \sin\theta)^2 \frac{\varepsilon_0}{2} E_{0x}^2 e^{-2\alpha z}, \tag{27'}$$

$$\overline{W}_K = \frac{\omega_p{}^2}{\omega^2 + \nu^2} \frac{\varepsilon_0}{4} |E_x|^2, \tag{28}$$

$$W_K = \frac{\omega_p{}^2}{(\omega^2+\nu^2)^2} (\nu \cos\theta + \omega \sin\theta)^2 \frac{\varepsilon_0}{2} E_{0x}^2 e^{-2\alpha z}. \tag{28'}$$

The velocity of energy transfer v_{en} which is defined by

$$\bar{v}_{en} = \frac{\overline{S}_z}{\overline{W}}, \quad v_{en} = \frac{S_z}{W}, \tag{29}$$

is easily written down using (24) – (29)

$$\bar{v}_{en} = \frac{2n_r}{1 + n_r{}^2 + n_i{}^2 + \dfrac{\omega_p{}^2}{\omega^2+\nu^2}} c, \tag{30}$$

$$v_{en} = \frac{2A\cos\theta}{\cos^2\theta + A^2 + B^2} c, \tag{30'}$$

where we have introduced

$$A = n_r \cos\theta + n_i \sin\theta,$$
$$B = \frac{\omega_p}{\omega^2+\nu^2} (\nu\cos\theta + \omega\sin\theta). \tag{31}$$

As a matter of course the velocity of energy transfer does not exceed the light velocity. This is proved with use of the following inequalities

$$1 + n_r{}^2 + n_i{}^2 + \frac{\omega_p{}^2}{\omega^2+\nu^2} - 2n_r = (1-n_r)^2 + n_i{}^2 + \frac{\omega_p{}^2}{\omega^2+\nu^2} > 0,$$
$$\cos^2\theta + A^2 + B^2 - 2A\cos\theta = (\cos\theta - A)^2 + B^2 > 0. \tag{32}$$

By using the definitions of n_r and n_i, we can change the expression (30) into

$$\bar{v}_{en} = \frac{n_r}{1+n_i{}^2} c. \tag{33}$$

Ginzburg [3] also derived the same expression by the different method. But this expression is less convienient than (30) for proving the inequality $v_{en} < c$.

The group velocity v_g derived by differentiation of β with ω is (See Appendix)

$$v_g = \left(\frac{d\beta}{d\omega}\right)^{-1} = n_r c \frac{n_r{}^2 + n_i{}^2}{n_r{}^2} \frac{\omega^2+\nu^2}{\omega^2(1-n_i{}^2) + \nu^2 n_r{}^2}. \tag{34}$$

Further manipulation yields

$$v_g = n_r c \frac{n_r{}^2 + n_i{}^2}{n_r{}^2} \frac{(1-n_r{}^2+n_i{}^2)^2 + 4n_r{}^2 n_i{}^2}{(1-n_i{}^2)(1-n_r{}^2+n_i{}^2)^2 + 4n_r{}^4 n_i{}^2}. \tag{35}$$

These expressions differ from that of v_{en}. In order to obtain the relation between v_g and v_{en}, we put $v_g = v_{en}$, then we have, from (33) and (35),

$$n_i^2 [(1+2n_r^2+n_i^2)(1-n_r^2+n_i^2)^2 + 4n_r^2\{(1+n_i^2)n_i^2+n_r^2(1-n_r^2+n_i^2)\}]=0. \tag{36}$$

Taking account of an inequality $1-n_r^2+n_i^2>0$ which is given by (A. 3), we note that the expression in the bracket of (36) is positive definite. From this fact we can conclude that (36) is satisfied only when $n_i^2=0$. In other word, the group velocity coincides with the velocity of energy transfer only when the collision is absent.

Fig. 4 gives the velocity of energy transfer as a function of ω/ω_p. We note that the value of v_{en} increases when the collision exists.

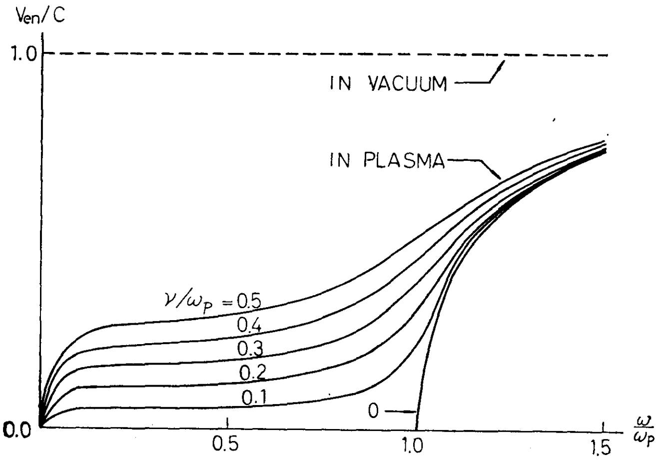

Fig. 4. Velocity of energy transfer v_{en} as a function of ω/ω_p.

Appendix Derivation of the Group Velocity (34) and (35)

By using $\beta=\omega n_r/c$, we have

$$\frac{d\beta}{d\omega}=\frac{1}{4n_r c}\left[4n_r^2+\omega\frac{d(2n_r^2)}{d\omega}\right]. \tag{A.1}$$

From (17) we have the following relations

$$\frac{\varepsilon}{\varepsilon_0}=1-\frac{\omega_p^2}{\omega^2+\nu^2},$$
$$\frac{\sigma}{\omega\varepsilon_0}=\frac{\nu}{\omega}\frac{\omega_p^2}{\omega^2+\nu^2}=\frac{\nu}{\omega}\left(1-\frac{\varepsilon}{\varepsilon_0}\right) \tag{A.2}$$

$$\frac{\varepsilon}{\varepsilon_0}=n_r^2-n_i^2, \quad 1-n_r^2+n_i^2=\frac{\omega_p^2}{\omega^2+\nu^2}>0, \tag{A.3}$$

$$\sqrt{\left(\frac{\varepsilon}{\varepsilon_0}\right)^2+\left(\frac{\sigma}{\varepsilon_0\omega}\right)^2}=n_r^2+n_i^2, \quad \left(\frac{\sigma}{\varepsilon_0\omega}\right)^2=4n_r^2n_i^2. \tag{A.4}$$

$$\omega\left(\frac{\varepsilon}{\varepsilon_0}\right)'=\frac{2\omega^2}{\omega^2+\nu^2}\left(1-\frac{\varepsilon}{\varepsilon_0}\right), \tag{A.5}$$

$$\omega\left(\frac{\sigma}{\varepsilon_0\omega}\right)'=-\left(1+\frac{2\omega^2}{\omega^2+\nu^2}\right)\frac{\sigma}{\varepsilon_0\omega}, \tag{A.6}$$

where prime means to make differentiation in terms of ω. Differentiating n_r^2 which is given by (17) with respect to ω and using (A.4) we have

$$\omega\frac{d(2n_r^2)}{d\omega}=\frac{1}{n_r^2+n_i^2}\left[2n_r^2\omega\left(\frac{\varepsilon}{\varepsilon_0}\right)'+\omega\left(\frac{\sigma}{\varepsilon_0\omega}\right)\left(\frac{\sigma}{\varepsilon_0\omega}\right)'\right]. \tag{A.7}$$

Substitution of (A.5) and (A.6) into (A.7) yields

$$\omega\frac{d(2n_r^2)}{d\omega}=\frac{4n_r^2}{n_r^2+n_i^2}\frac{1}{\omega^2+\nu^2}[\omega^2(1-n_i^2-n_r^2)-n_i^2(\omega^2+\nu^2)]. \tag{A.8}$$

From this and (A.1) we have

$$v_g=\left(\frac{d\beta}{d\omega}\right)^{-1}=n_rc\frac{n_r^2+n_i^2}{n_r^2}\frac{\omega^2+\nu^2}{\omega^2(1-n_i^2)+\nu^2n_r^2}. \tag{A.9}$$

From (A.2), (A.3) and (A.4), it follows that

$$\left(\frac{\nu}{\omega}\right)^2=\frac{\left(\frac{\sigma}{\varepsilon_0\omega}\right)^2}{\left(1-\frac{\varepsilon}{\varepsilon_0}\right)^2}=\frac{4n_r^2n_i^2}{(1-n_r^2+n_i^2)^2}. \tag{A.10}$$

With use of this expression, (A.9) becomes

$$v_g=n_r c\frac{n_r^2+n_i^2}{n_r^2}\frac{(1-n_r^2+n_i^2)^2+4n_r^2n_i^2}{(1-n_i^2)(1-n_r^2+n_i^2)^2+4n_r^4n_i^2}. \tag{A.11}$$

Table 1. Dimensionless propagation constant $c\beta/\omega_p$, attenuation constant $c\alpha/\omega_p$, refractive indexes n_r and n_i and velocity of energy transfer v_{en} as functions of ω/ω_p.

$\nu/\omega_p=0.$

ω/ω_p	$c\alpha/\omega_p$	$c\beta/\omega_p$	n_r	n_i	v_{en}
0.01	0.99995	0.	0.	99.99500	0.
0.02	0.99980	0.	0.	49.99000	0.
0.03	0.99955	0.	0.	33.31833	0.
0.04	0.99920	0.	0.	24.97999	0.
0.05	0.99875	0.	0.	19.97498	0.
0.06	0.99820	0.	0.	16.63664	0.
0.07	0.99755	0.	0.	14.25067	0.
0.08	0.99679	0.	0.	12.45994	0.
0.09	0.99594	0.	0.	11.06602	0.
0.10	0.99499	0.	0.	9.94987	0.
0.12	0.99277	0.	0.	8.27312	0.
0.14	0.99015	0.	0.	7.07251	0.
0.16	0.98712	0.	0.	6.16948	0.
0.18	0.98367	0.	0.	5.46481	0.
0.20	0.97980	0.	0.	4.89898	0.
0.30	0.95394	0.	0.	3.17980	0.
0.40	0.91652	0.	0.	2.29129	0.
0.50	0.86603	0.	0.	1.73205	0.
0.60	0.80000	0.	0.	1.33333	0.
0.70	0.71414	0.	0.	1.02020	0.
0.80	0.60000	0.	0.	0.75000	0.
0.90	0.43589	0.	0.	0.48432	0.
1.00	0.	0.	0.	0.	0.
1.10	0.	0.45826	0.41660	0.	0.41660
1.20	0.	0.66332	0.55277	0.	0.55277
1.30	0.	0.83066	0.63897	0.	0.63897
1.40	0.	0.97980	0.69985	0.	0.69985
1.50	0.	1.11803	0.74536	0.	0.74536

$\nu/\omega_p=0.1$

ω/ω_p	$c\alpha/\omega_p$	$c\beta/\omega_p$	n_r	n_i	v_{en}
0.01	0.23377	0.21177	21.17697	23.37678	0.03868
0.02	0.34213	0.28105	14.05243	17.10627	0.04786
0.03	0.42939	0.32049	10.68285	14.31315	0.05189
0.04	0.50355	0.34239	8.55987	12.58882	0.05367
0.05	0.56727	0.35256	7.05126	11.34549	0.05436
0.06	0.62200	0.35465	5.91076	10.36660	0.05449
0.07	0.66881	0.35122	5.01739	9.55449	0.05437
0.08	0.70872	0.34415	4.30183	8.85897	0.05412
0.09	0.74263	0.33478	3.71981	8.25140	0.05384
0.10	0.77139	0.32409	3.24092	7.71386	0.05357
0.12	0.81639	0.30121	2.51006	6.80323	0.05308
0.14	0.85865	0.27866	1.99043	6.06182	0.05273
0.16	0.87175	0.25778	1.61112	5.44845	0.05250
0.18	0.88818	0.23899	1.32771	4.93434	0.05238
0.20	0.89968	0.22230	1.11151	4.49838	0.05234
0.30	0.91481	0.16397	0.54656	3.04938	0.05307
0.40	0.89359	0.13166	0.32914	2.23398	0.05494
0.50	0.85106	0.11298	0.22596	1.70212	0.05798
0.60	0.78963	0.10268	0.17114	1.31605	0.06264
0.70	0.70697	0.09901	0.14145	1.00995	0.07002
0.80	0.59605	0.10324	0.12905	0.74506	0.08299
0.90	0.43975	0.12479	0.13866	0.48861	0.11194
1.00	0.21166	0.23389	0.23389	0.21166	0.22386
1.10	0.09459	0.47660	0.43327	0.08599	0.43009
1.20	0.06164	0.67134	0.55945	0.05136	0.55798
1.30	0.04577	0.83545	0.64265	0.03520	0.64186
1.40	0.03615	0.98305	0.70218	0.02582	0.70171
1.50	0.02962	1.12040	0.74694	0.01975	0.74664

$\nu/\omega_p=0.2$

ω/ω_p	$c\alpha/\omega_p$	$c\beta/\omega_p$	n_r	n_i	v_{en}
0.01	0.16175	0.15417	15.41735	16.17506	0.05870
0.02	0.23342	0.21209	10.60446	11.67078	0.07729
0.03	0.29096	0.25210	8.40320	9.69864	0.08840
0.04	0.34108	0.28191	7.04776	8.52698	0.09562
0.05	0.38618	0.30465	6.09291	7.72354	0.10046
0.06	0.42738	0.32200	5.36663	7.12296	0.10373
0.07	0.46529	0.33506	4.78660	6.64705	0.10594
0.08	0.50030	0.34462	4.30779	6.25370	0.10740
0.09	0.53264	0.35129	3.90322	5.91820	0.10835
0.10	0.56251	0.35555	3.55550	5.62509	0.10893
0.12	0.61544	0.35842	2.98684	5.12870	0.10939
0.14	0.66019	0.35580	2.54146	4.71567	0.10937
0.16	0.69774	0.34956	2.18476	4.36086	0.10914
0.18	0.72899	0.34105	1.89470	4.04994	0.10888
0.20	0.75479	0.33122	1.65609	3.77394	0.10865
0.30	0.82496	0.27973	0.93245	2.74987	0.10891
0.40	0.83508	0.23950	0.59874	2.08770	0.11174
0.50	0.81074	0.21266	0.42533	1.62147	0.11720
0.60	0.76083	0.19715	0.32859	1.26806	0.12599
0.70	0.68668	0.19234	0.27477	0.98096	0.14003
0.80	0.58454	0.20127	0.25158	0.73067	0.16402
0.90	0.44634	0.23723	0.26358	0.49593	0.21155
1.00	0.28076	0.34247	0.34247	0.28076	0.31745
1.10	0.16916	0.52021	0.47292	0.15378	0.46199
1.20	0.11694	0.69333	0.57777	0.09745	0.57234
1.30	0.08850	0.84909	0.65315	0.06808	0.65013
1.40	0.07053	0.99246	0.70890	0.05038	0.70710
1.50	0.05810	1.12732	0.75154	0.03874	0.75042

$\nu/\omega_p=0.3$

ω/ω_p	$c\alpha/\omega_p$	$c\beta/\omega_p$	n_r	n_i	v_{en}
0.01	0.13100	0.12709	12.70861	13.09991	0.07363
0.02	0.18777	0.17673	8.83663	9.38872	0.09912
0.03	0.23283	0.21262	7.08744	7.76098	0.11575
0.04	0.27187	0.24093	6.02325	6.79681	0.12762
0.05	0.30703	0.26408	5.28166	6.14058	0.13645
0.06	0.33934	0.28336	4.72259	5.65567	0.14317
0.07	0.36939	0.29953	4.27896	5.27702	0.14833
0.08	0.39754	0.31313	3.91410	4.96927	0.15234
0.09	0.42403	0.32454	3.60604	4.71139	0.15545
0.10	0.44900	0.33407	3.34073	4.49004	0.15788
0.12	0.49488	0.34839	2.90328	4.12402	0.16123
0.14	0.53582	0.35759	2.55423	3.82730	0.16323
0.16	0.57225	0.36280	2.26748	3.57659	0.16441
0.18	0.60453	0.36489	2.02719	3.35849	0.16509
0.20	0.63295	0.36459	1.82297	3.16473	0.16549
0.30	0.72684	0.34395	1.14651	2.42282	0.16689
0.40	0.76119	0.31530	0.78824	1.90298	0.17057
0.50	0.75536	0.29203	0.58406	1.51073	0.17794
0.60	0.71925	0.27807	0.46345	1.19875	0.19017
0.70	0.65642	0.27579	0.39399	0.93774	0.20964
0.80	0.56649	0.29018	0.36272	0.70812	0.24158
0.90	0.44900	0.33407	0.37119	0.49889	0.29722
1.00	0.31998	0.43007	0.43007	0.31998	0.39012
1.10	0.22152	0.57297	0.52089	0.20138	0.50059
1.20	0.16234	0.72469	0.60391	0.13528	0.59306
1.30	0.12596	0.86973	0.66902	0.09689	0.66280
1.40	0.10172	1.00710	0.71936	0.07265	0.71558
1.50	0.08448	1.13824	0.75883	0.05632	0.75643

$\nu/\omega_p=0.4$

ω/ω_p	$c\alpha/\omega_p$	$c\beta/\omega_p$	n_r	n_i	v_{cn}
0.01	0.11295	0.11060	11.06013	11.29480	0.08602
0.02	0.16127	0.15464	7.73187	8.06327	0.11712
0.03	0.19928	0.18713	6.23763	6.64252	0.13824
0.04	0.23201	0.21337	5.33433	5.80028	0.15398
0.05	0.26138	0.23544	4.70882	5.22751	0.16623
0.06	0.28832	0.25441	4.24009	4.80529	0.17600
0.07	0.31338	0.27091	3.87020	4.47691	0.18392
0.08	0.33692	0.28539	3.56744	4.21144	0.19040
0.09	0.35914	0.29815	3.31282	3.99044	0.19575
0.10	0.38022	0.30942	3.09421	3.80217	0.20019
0.12	0.41935	0.32816	2.73466	3.49461	0.20698
0.14	0.45494	0.34269	2.44778	3.24955	0.21175
0.16	0.48734	0.35379	2.21117	3.04585	0.21515
0.18	0.51680	0.36205	2.01142	2.87111	0.21761
0.20	0.54351	0.36798	1.83988	2.71757	0.21942
0.30	0.64060	0.37465	1.24884	2.13532	0.22463
0.40	0.68726	0.36376	0.90941	1.71815	0.23011
0.50	0.69494	0.35097	0.70194	1.38987	0.23943
0.60	0.67120	0.34381	0.57302	1.11867	0.25451
0.70	0.62008	0.34735	0.49621	0.88583	0.27804
0.80	0.54351	0.36798	0.45997	0.67939	0.31471
0.90	0.44555	0.41649	0.46277	0.49506	0.37168
1.00	0.34166	0.50464	0.50464	0.34166	0.45189
1.10	0.25632	0.62649	0.56954	0.23302	0.54020
1.20	0.19715	0.76083	0.63403	0.16429	0.61736
1.30	0.15702	0.89506	0.68851	0.12078	0.67861
1.40	0.12877	1.02570	0.73264	0.09198	0.72649
1.50	0.10802	1.15241	0.76828	0.07201	0.76431

$\nu/\omega_p=0.5$

ω/ω_p	$c\alpha/\omega_p$	$c\beta/\omega_p$	n_r	n_i	v_{cn}
0.01	0.10073	0.09923	9.92331	10.07325	0.09684
0.02	0.14344	0.13921	6.96030	7.17212	0.13273
0.03	0.17682	0.16905	5.63511	5.89408	0.15767
0.04	0.20542	0.19349	4.83717	5.13544	0.17671
0.05	0.23096	0.21434	4.28684	4.61924	0.19191
0.06	0.25433	0.23257	3.87611	4.23880	0.20436
0.07	0.27602	0.24873	3.55325	3.94318	0.21471
0.08	0.29637	0.26320	3.28996	3.70460	0.22344
0.09	0.31558	0.27624	3.06932	3.50645	0.23086
0.10	0.33381	0.28805	2.88046	3.33814	0.23721
0.12	0.36776	0.30852	2.57104	3.06470	0.24740
0.14	0.39882	0.32551	2.32510	2.84873	0.25508
0.16	0.42735	0.33962	2.12263	2.67096	0.26096
0.18	0.45359	0.35130	1.95169	2.51995	0.26553
0.20	0.47770	0.36092	1.80462	2.38850	0.26915
0.30	0.56974	0.38717	1.29057	1.89914	0.28015
0.40	0.62035	0.39317	0.98293	1.55086	0.28866
0.50	0.63601	0.39308	0.78615	1.27202	0.30028
0.60	0.62176	0.39550	0.65916	1.03626	0.31785
0.70	0.58116	0.40692	0.58132	0.83023	0.34412
0.80	0.51742	0.43431	0.54288	0.64678	0.38277
0.90	0.43655	0.48623	0.54026	0.48505	0.43736
1.00	0.35158	0.56886	0.56886	0.35158	0.50628
1.10	0.27814	0.67720	0.61563	0.25286	0.57864
1.20	0.22235	0.79835	0.66529	0.18529	0.64321
1.30	0.18151	0.92294	0.70995	0.13963	0.69638
1.40	0.15128	1.04690	0.74779	0.10805	0.73916
1.50	0.12832	1.16896	0.77931	0.08555	0.77364