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# Measurement of Input-Impedance of Helical Type Delay Line

(Received June 2, 1965)

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## Abstract

The characteristic impedance of traveling wave tube is characterized by the ratio of the square of the electric field to the power. But in this paper, the input impedance of helix defined as the ratio of the voltage to the current is considered and measured. This impedance plays a major role in the matching of helical type delay lines. The input impedance of helix has been analyzed in our previous paper. But the measurements for it, are not so easy that the experimental researches are very rare, because of their poor reproducibility and large errors. The mean diameter of the helix used in the measurement is 4.0mm, and values of the ratio of the conductor diameter to the pitch are between 2 to 5. Frequencies for the measurement are in the 4000 Mc band, and the measuring apparatus are all of the coaxial type. In order to prevent the axial component of the electric field of the helix from being disturbed, the diameter of the outer cylindrical conductor is chosen to be more than three times as large as that of the helix. The input impedance of the helix is much larger than the impedance of a coaxial type standing wave detector. Accordingly it is possible to make the standing wave ratio smaller by using an impedance transformer. The results of experiments show that the real part of the input impedance decreases with increased frequency, and its imaginary part has the same characteristics as those of the real part but are negative in value. The polarity of the imaginary part obtained by experiments coincides with that theoretically obtained through equivalent analysis for the distribution line, while in its frequency characteristics no coincidence is observed between the two. The tendency of the experimental values coincides with that of the theoretical ones. Experiments show that the measurement has a good reproducibility of its results, the maximum error for the real part being within 15 percent. This data is useful in designing the junction between a waveguide and a traveling wave tube, which will be discussed in the next report.

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### I. Introduction

Two physically significant parameters of the traveling wave tube are the power flow in the circuit and the electric field associated with it which acts on the electron stream. The ratio of the square of the electric field to the power can be evaluated by physical measurement even when it cannot be calculated. These parameters are used by C. Cuttler and J. R. Pierce.<sup>1)</sup> In this paper, however, the input-impedance of the helices are evaluated by experiments, and then compared with the result of our theoretical analysis of the delay circuits.<sup>2)</sup>

The definition for the input impedance of the helix is the ratio of the voltage to the current at the input terminal. Using this definition, the equivalent circuit is composed easily, when the helix is connected to the other circuits, for instance, coaxial line, wave guide etc.. The difficulties of analysis of the helix end is caused by the unsymmetric distribution of the mutual inductances and capacitances.

In Fig. 1, it is assumed that the voltage and the current on the helix are  $v(x)$ ,  $i(x)$  at the point  $x$ ,  $v(\xi)$ ,  $i(\xi)$  at the point  $\xi$  and affects each other between two points by mutual inductance and capacitance. Then referring to Fig. 1, one can accordingly write the equations

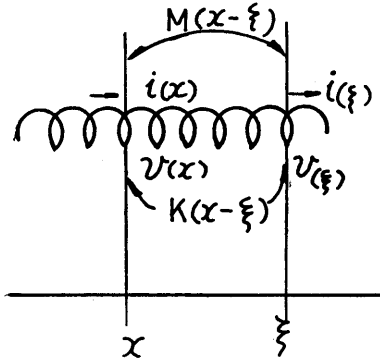


Fig. 1. Voltage and current distribution on the helix.

$$-\frac{\partial v(x)}{\partial x} = \int_0^{\infty} L(x-\xi) \frac{\partial i(\xi)}{\partial t} d\xi, \quad (1)$$

$$-\frac{\partial i(x)}{\partial x} = C(x) \frac{\partial v(x)}{\partial t} + \int_0^{\infty} K(x-\xi) \frac{\partial \{v(x) - v(\xi)\}}{\partial t} d\xi, \quad (2)$$

where  $L(x-\xi)$  is the mutual inductance between point  $x$  and point  $\xi$ ,  $K(x-\xi)$  is the capacitance between  $x$  and point  $\xi$ ,  $C(x)$  is the self capacitance at point  $x$ ,  $v(x)$ ,  $i(x)$  are voltage and current at point  $x$ ,  $v(\xi)$ ,  $i(\xi)$  are voltage and current at point  $\xi$ .

1) J. R. Pierce; *Traveling-Wave Tubes*, D. Van. Nostrand Co., Inc., New York N. Y.; 1950.

2) H. Fujita, H. Kashiwagi; *Theoretical Study of the Electromagnetic Wave Propagation and Termination Concerning the Helical Type Delay Line*, Proc. of the Fujihara Memorial Faculty of Eng., Keio Univ. Tokyo, JAPAN Vol. 15, No. 56, 1962, pp. 1~11.

Now, it is assumed that the voltage and current varies sinusoidally  $e^{j\omega t}$ . The distributions of the mutual inductance  $L(x-\xi)$  and the capacitance  $K(x-\xi)$  are as follows :

$$L(x-\xi) = L_0 e^{-a|x-\xi|},$$

$$K(x-\xi) = K_0 e^{-b|x-\xi|}.$$

The distribution of self capacitance is given by

$$C(x) = C_0 + D(x),$$

$$D(x) = D_0 e^{-qx}.$$

where,

The following result is obtained by substituting these assumptions into fundamental equations and by Laplace transformation,

$$z_{in} = \lim_{x \rightarrow 0} v(x)/i(x) = \sqrt{\eta - \omega^2 \xi^2} - j\omega \xi, \quad (3)$$

where,

$$\xi = \frac{L_0}{2(C_0 + D_0)} \left\{ \frac{C_0}{a^2} + \frac{D_0}{a(a+q)} - \frac{2K_0}{b(a^2 - b^2)} \right\}$$

$$\eta = \frac{L_0}{C_0 + D_0} \left\{ 1 + \frac{1}{a} \right\}$$

For comparing the experimental results with the theoretical values, the input impedance of the helices are measured by using a coaxial type standing wave detector. In this case, the load impedance  $z_L$  is given by

$$z_L = \frac{\rho - j \tan \frac{2\pi}{\lambda_g} \cdot x_{\max}}{1 - j \rho \tan \frac{2\pi}{\lambda_g} \cdot x_{\max}} \cdot z_0, \quad (4)$$

where  $z_0$  is the characteristic impedance of the measuring system,  $\rho$  is the voltage standing wave ratio,  $x_{\max}$  is the length from the load to the maximum point of the voltage standing waves, and  $\lambda_g$  is the guide wavelength.

Measurable values in Eq. (4) are  $\rho$ ,  $x_{\max}$  and  $\lambda_g$ . Comparing characteristic impedance to input impedance  $z_{in}$ , when the difference between two is extremely large,  $\rho$  takes inaccurate values. Usually, the double minimum method is used for high VSWR but in this experiment, an impedance transformer is used, for reason of its simplicity and accuracy. The impedance transformer is designed to have a standing wave ratio from 1.5 to 5.0.

## II. Design of the impedance transformer

When designing an impedance transformer between two lines with different impedances, generally  $1/4 \cdot \lambda$  length line is used, where the line is terminated by the load  $z_b = r + jx$ . If a length from the generator to the reference plane is  $l$ , the input impedance is given by

$$(3)$$

$$z = R_0 \left( \frac{z_L + R_0 \tanh \gamma l}{R_0 + z_L \tanh \gamma l} \right), \quad (5)$$

where  $\gamma$  is the propagation constant. If it is a lossless line, the propagation constant  $\gamma$  is equal to  $j\beta$ . Then Eq (5) is as follows:

$$z = R \left[ \frac{r/R_0 \left\{ \cot^2 \left( \frac{\pi}{2} \cdot \frac{\lambda_{g0}}{\lambda_g} \right) + 1 \right\} + j \left[ x/R_0 \cdot \left\{ \cot^2 \left( \frac{\pi}{2} \cdot \frac{\lambda_{g0}}{\lambda_g} \right) - 1 \right\} \right]}{\left\{ \cot \left( \frac{\pi}{2} \cdot \frac{\lambda_{g0}}{\lambda_g} \right) - x/R_0 \right\}^2 + (r/R_0)^2} + \frac{\left\{ 1 - (x/R_0)^2 - (r/R_0)^2 \right\} \cot \frac{\pi}{2} \cdot \frac{\lambda_{g0}}{\lambda_g}}{\left[ \right]} \right], \quad (6)$$

where the wavelength of the center frequency and the wavelengths of the optional frequencies are  $\lambda_{g0}$  and  $\lambda_g$  respectively. The length of the transformer is  $l = 1/4 \cdot \lambda_{g0}$ ,

$$\text{than } \beta l = \frac{2\pi}{\lambda_g} \cdot \frac{\lambda_{g0}}{4} = \frac{\pi}{2} \cdot \frac{\lambda_{g0}}{\lambda_g} \quad (7)$$

For the particular case, in which the load has no reactive component and when the frequency is equal to the center frequency,  $x=0$ ,  $\cot \left( \frac{\pi}{2} \cdot \frac{\lambda_{g0}}{\lambda_g} \right) = 0$ .

Substituting them into Eq. (6), the following relation is obtained

$$z = R_0^2 / r \quad (8)$$

When the load is pure resistance, the impedance transformer which has a characteristic impedance  $R_0$  and length of  $1/4 \cdot \lambda_g$  placed between  $z$  and  $r$ . The efficiency of the transmission should be maximum.<sup>3)</sup> On the other hand, if the load has the imaginary part and the frequencies are varied,  $z$  is very complicated, but the difficulty to calculate such impedance is overcome by use of the Smith Chart. This will be explained later.

Design plan is as follows:

- 1) The complete system has the same diameter for the outer cylinder, 19.2 mm, and transformation of the impedance depends only on the diameter of the inner cylinder.
- 2) The center frequency is 4,000 Mc. The real part of the input impedance of the helix is 500  $\Omega$  theoretically. It is extremely larger than the imaginary part. Therefore it is considered that the value of imaginary part is negligible.
- 3) The diameter of inner conductors are defined by

$$z_0 = \frac{138}{\sqrt{\epsilon}} \log_{10} b/a, \quad (9)$$

where  $a$  and  $b$  are the diameters of the inner and outer conductors respectively,

3) H. Nukiyama; Electromagnetism, Maruzen Co., Inc., Tokyo, JAPAN, 1962.

and  $\epsilon$  is the dielectric constant of the medium. From Eq. (9),  $b/a=13.99$  is obtained, then the diameter of the inner conductor in the transformer is  $a=1.37$  mm and its length is  $\lambda_{g0}/4=18.8$  mm.

**III. Method of support of the helix and its termination**

For the reliability and reproducibility of experiments, the supporting method of the helix is very importance. Various supporting methods were considered, but in this case the helix is placed on two parallel quartz rods, and the ends are supported by a Teflon piece as shown in Fig. 2. To prevent the reflecting power from the helix end, some resistive materials are used.

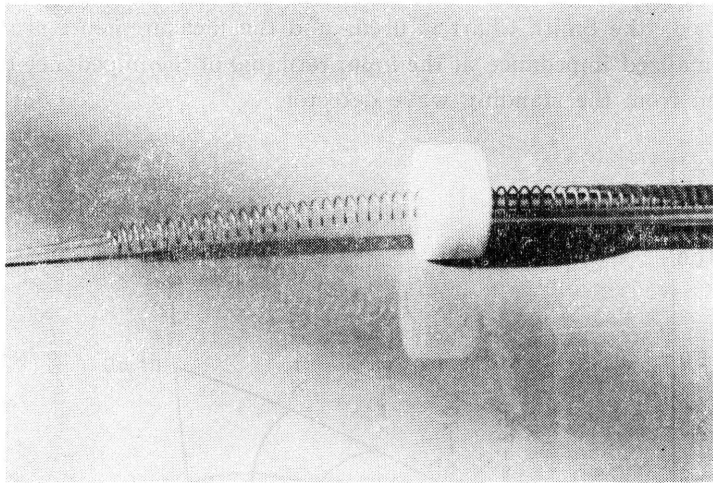


Fig. 2. Supporting method of the helix.

**IV. Measuring system**

The measuring system is shown in Fig. 3. The klystron 2K54 is used as the microwave source and is modulated by 1kc square wave. Output power is adjusted

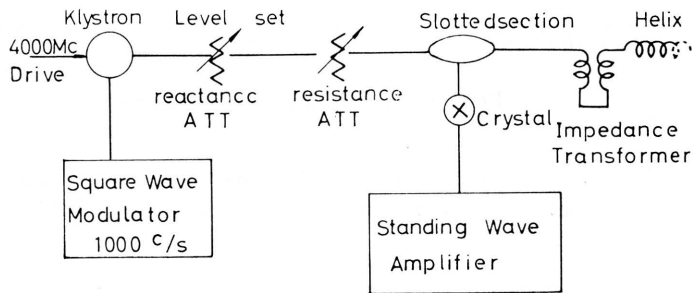


Fig. 3. Block diagrams of Measuring apparatus.

by a reactance attenuator. The microwave power is supplied to the helix through the coaxial type standing wave detector and impedance transformer. The reflections from the helix end and other equipment are so small that they are negligible, then the reflected power is only from the input terminal of the helix.

### V. Measuring method (I)

Two methods are considered: One is the method obtained from the relation of Eq. (8). It is represented by the relation of the normalized impedance at the input terminal of the transformer to the load impedance, then the calculating values of Eq. (8), Table 1, and the transferring chart, Fig. 4, 5, 6, 7, and 8 are obtained. This method is more accurate than the other method but not so practical. In the other practical method, the Smith Chart is used, and the measurements are performed by this. Normalized impedance at the input terminal of the impedance transformer  $z_{NA}$  is obtained from the standing wave detector.

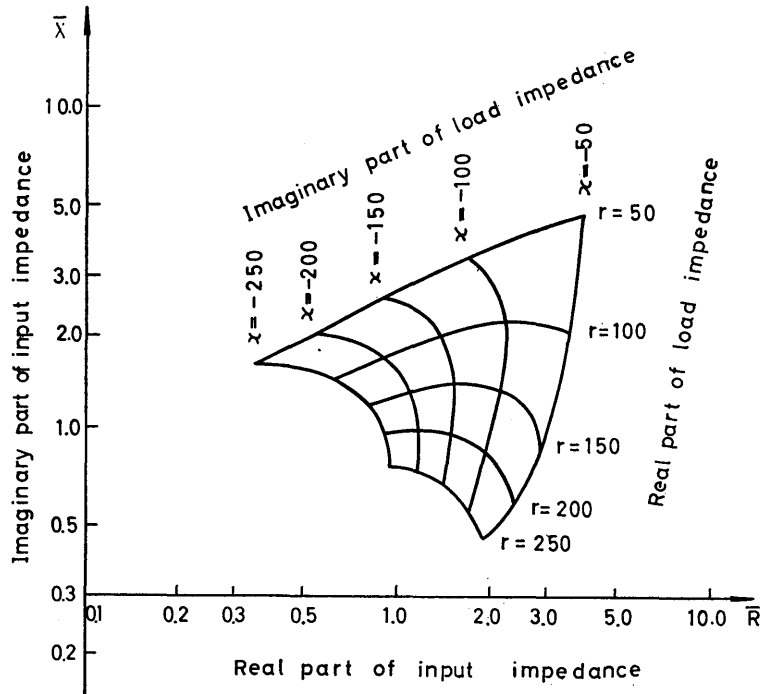


Fig. 4. Plots of normalized input impedance vs. load impedance at the entrance of the transformer. Frequency  $f=3,800$  Mc. [ref. eq. (8)]

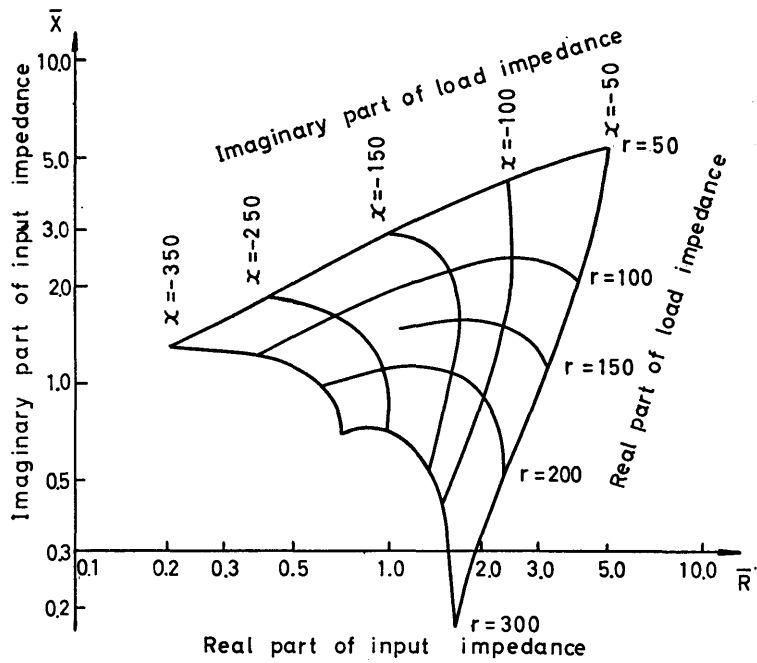


Fig. 5. The case of the frequency 3,900 Mc.

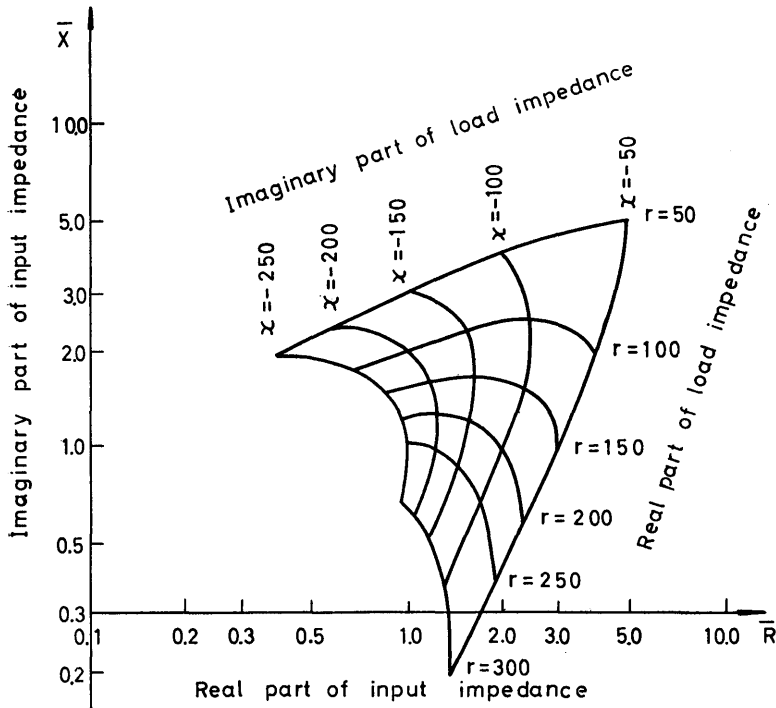


Fig. 6. The case of the frequency 4,000 Mc.



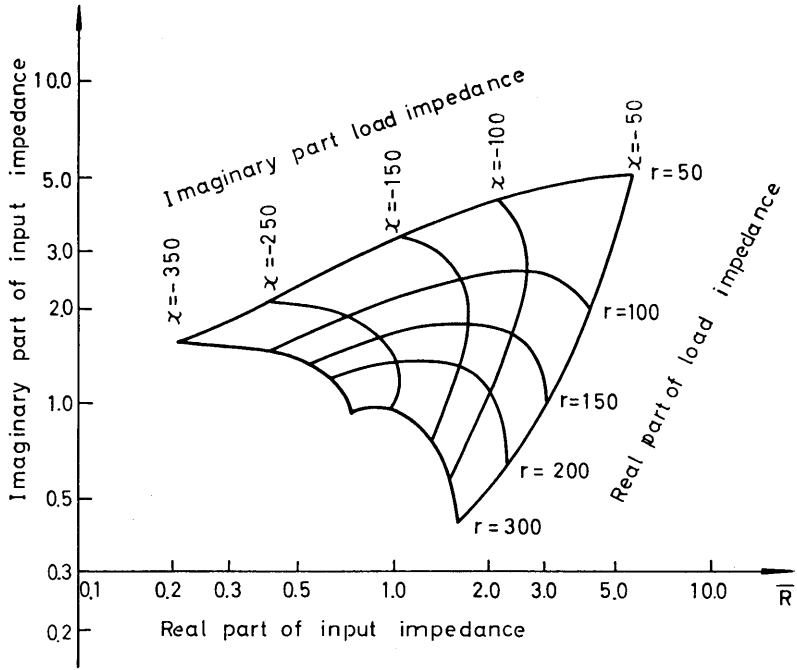


Fig. 7. The case of the frequency 4,100 Mc.

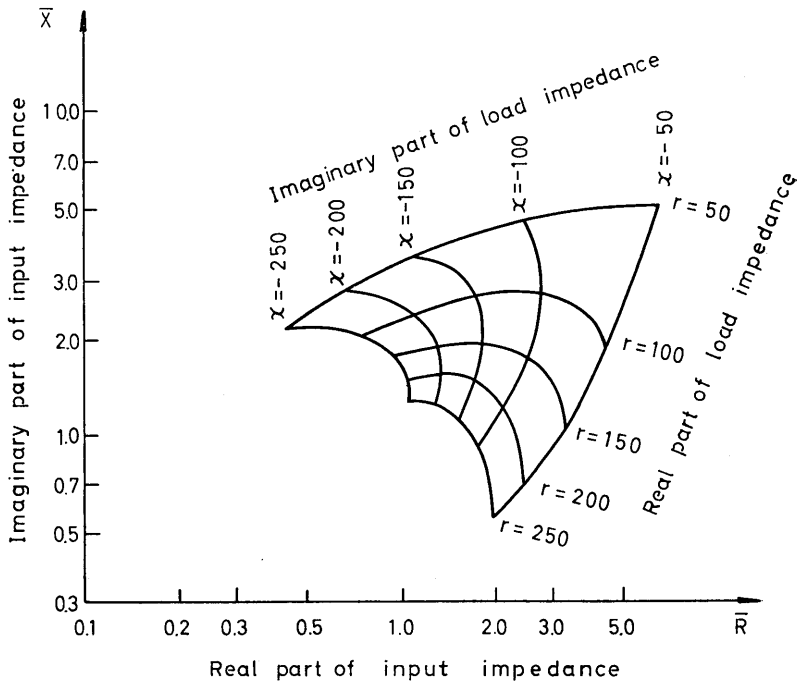


Fig. 8. The case of the frequency 4,200 Mc.

Table 1. Calculating value of Eq. (8).

I 3800 Mc/s				II 3900 Me/s				III 4000 Me/s				IV 4100 Mc/s				V 4200 Mc/s				
$\gamma(\varphi)$	$x(\varphi)$	$\bar{R}$	$\bar{X}$	$\gamma(\varphi)$	$x(\varphi)$	$\bar{R}$	$\bar{X}$	$\gamma(\varphi)$	$x(\varphi)$	$\bar{R}$	$\bar{X}$	$\gamma(\varphi)$	$x(\varphi)$	$\bar{R}$	$\bar{X}$	$\gamma(\varphi)$	$x(\varphi)$	$\bar{R}$	$\bar{X}$	
50	-50	3.932	4.661	50	-50	4.987	5.472	50	-50	5.000	5.000	50	-50	5.668	5.088	50	-50	6.42	5.06	
100		3.620	2.012	100		4.000	2.119	100		4.000	2.000	100		4.202	1.964	100		4.40	1.90	
150		2.859	0.941	200		2.355	0.536	150		3.000	1.000	150		3.007	1.002	150		3.16	1.038	
200		2.292	0.467	300		1.623	0.179	200		2.353	0.588	200		2.389	0.647	200		2.42	0.705	
250		1.894	0.448	50	-150	0.999	2.991	250		1.923	0.355	300		1.634	0.363	250		1.96	0.544	
50	-100	1.661	3.487	100		1.538	2.273	50	-100	2.00	4.00	50	-100	2.216	4.282	50	-100	2.47	4.58	
100		2.222	2.249	200		1.600	1.123	100		2.50	2.50	100		2.664	2.623	100		2.85	2.74	
150		2.147	1.361	300		1.334	0.569	150		2.35	1.47	150		2.400	1.625	150		2.50	1.71	
200		1.911	0.826	50	-250	0.385	1.842	200		2.00	1.00	200		2.053	1.087	200		2.11	1.17	
250		1.674	0.504	100		0.690	1.639	250		1.724	0.650	300		1.521	0.600	250		1.792	0.08	
50	-150	0.871	2.580	200		0.976	1.123	50	-150	1.000	3.000	50	-150	1.040	3.231	50	-150	1.074	3.479	
100		1.353	1.997	300		0.984	0.569	100		1.538	2.308	100		1.633	2.472	100		1.739	2.641	
150		1.544	1.423	50	-350	0.200	1.298	150		1.667	1.667	150		1.740	1.792	150		1.822	1.920	
200		1.516	0.999	100		0.378	1.217	200		1.600	1.200	200		1.651	1.311	200		1.708	1.423	
250		1.415	0.670	200		0.616	0.970	250		1.470	0.882	300		1.358	0.775	250		1.545	1.099	
50	-200	0.528	1.995	300		0.706	0.712	50	-200	0.588	2.353	50	-250	0.404	2.096	50	-200	0.668	2.753	
100		0.913	1.690	100	-100	2.499	2.524	100		1.000	2.000	100		0.721	1.883	100		1.114	2.338	
150		1.116	1.231	150		2.308	1.507	150		1.200	1.600	150		0.917	1.614	150		1.309	1.885	
200		1.182	1.007	200		2.001	0.938	200		1.250	1.250	200		1.007	1.352	200		1.338	1.504	
250		1.168	0.744	300		1.501	0.406	250		1.219	0.976	300		1.005	0.941	250		1.288	1.215	
50	-250	0.353	1.601	150	-50	3.001	0.998	50	-250	0.385	1.923	50	-350	0.207	1.551	50	-250	0.427	2.277	
100		0.638	1.425		-150	1.661	1.608	100		0.690	1.724	100		0.391	1.467	100		0.757	2.048	
150		0.826	1.196					150		0.882	1.470	150		0.534	1.348	150		0.956	1.763	
200		0.924	0.964					200		0.976	1.219	200		0.633	1.212	200		1.044	1.488	
250		0.957	0.756					250		1.000	1.000	300		0.722	0.951	250		1.057	1.254	
				350	-50	1.400	0.200													
					-100	1.321	0.377													
					-200	1.007	0.615													

$$\bar{R} + j\bar{X} = \frac{R + jX}{50 \varphi}$$

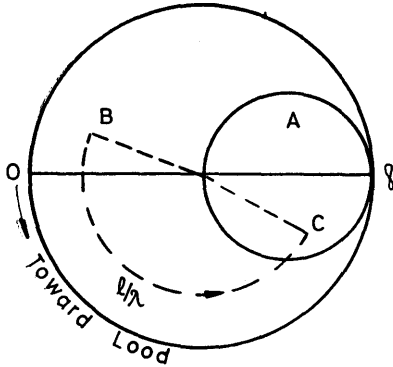


Fig. 9. Transferring method on the Smith Chart.

When  $z_{NA}$  is point A in Fig. 9,

$$z_{NA} = R_{NA} + jX_{NA} \tag{10}$$

is assumed where the suffix N indicates the normalized value. The characteristic impedance of the standing wave detector is  $z_0$ , and the characteristic impedance of the impedance transformer is  $R_0$ , therefore the impedance at the entrance of the impedance transformer  $z_{NB}$  is as follows :

$$z_{NB} = \frac{z_0}{R_0} (R_{NA} + jX_{NA}) . \tag{11}$$

This is point B on the Smith Chart.

The input impedance of the helix is given by rotating  $z_{NB}$  along curve B toward the load by the length  $l/\lambda_0$ , then it is point C on the Smith Chart and assumed as follows :

$$z_{NC} = R_{NC} + jX_{NC} . \tag{12}$$

Hence the input impedance of the helix  $z_h$  is

$$z_h = R_0 (R_{NC} + jX_{NC}) . \tag{13}$$

### VI. Measuring method (II)

.....Setting up the equivalent reference plane .....

It is important problem to set up the reference plane for measuring the impedance. Usually the geometrical reference plane does not coincide with the practical one. In this paper, the latter is used. In Fig. 10 where the short plate is placed on the plane 2-2', the input impedance from the plane 1-1' is

$$z_R = jR_0 \tan \beta l , \tag{14}$$

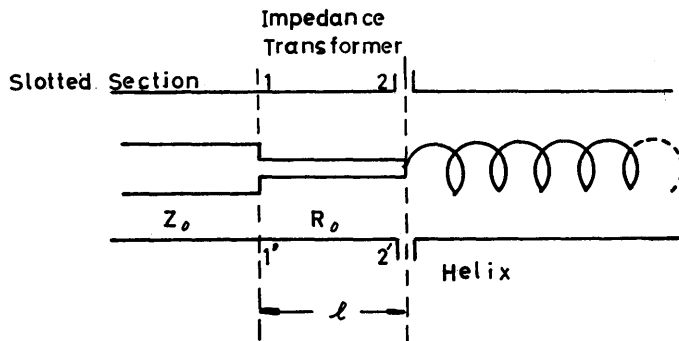


Fig. 10. Cross section of the measuring system.

where  $R_0$  is the characteristic impedance of the transformer ( $158.1 \Omega$ ),  $l$  is the length of the transformer ( $l = \lambda_{4GC}/4 = 18.8 \text{ mm}$ ). When the frequency is 4,000 Mc,  $z_R$  approach infinity, but at other frequencies is finite. Generally, the reflection coefficient  $\Gamma$  on the plane 1-1' in Fig. 10 is given by

$$\begin{aligned} \Gamma_R &= |\Gamma_R| e^{j\varphi} = \frac{z_R - z_0}{z_R + z_0} = \frac{jR_0 \tan \beta l - z_0}{jR_0 \tan \beta l + z_0} \\ &= \angle \exp j \left[ \tan^{-1} \left( -\frac{R_0}{z_0} \tan \beta l \right) - \tan^{-1} \left( \frac{R_0}{z_0} \tan \beta l \right) \right], \end{aligned} \quad (15)$$

where  $z_0$  is the characteristic impedance ( $z_0 = 50 \Omega$ ) of the standing wave detector. The voltages on the line can be expressed in the following form:

$$V(x) = V_i e^{j\gamma x} + V_r e^{-j\gamma x}, \quad (16)$$

where  $V_i$  is the incident voltages,  $V_r$  is the reflection voltages and  $V(x)$  is the composed voltages.

Using reflection coefficient  $\Gamma = \frac{V_r}{V_i} e^{-2j\gamma x} = \Gamma_R e^{-2j\gamma x}$  and  $\gamma = j\beta$  to rewrite Eq. (16), the following relation is obtained,

$$\begin{aligned} V(x) &= V_i e^{j\beta x} (1 + \Gamma) = V_i e^{j\beta x} (1 + \Gamma_R e^{-2j\beta x}) \\ &= V_i e^{j\beta x} (1 + |\Gamma_R| e^{j(\varphi - 2\beta x)}), \end{aligned} \quad (17)$$

then

$$\left| \frac{V(x)}{V_i} \right| = 1 + |\Gamma_R| e^{j(\varphi - 2\beta x)}. \quad (18)$$

Fig. 11. shows the vector diagram of the reflection coefficient and the phase from Eq. (18).  $|V(x)/V_i|$  is the absolute value of the vector. When  $e^{j(\varphi - 2\beta x)} = 1$ , the value of  $|V(x)/V_i|$  is maximum and  $e^{j(\varphi - 2\beta x)} = -1$ , the value of it is minimum. Hence, if the distance  $x$  is satisfied the following relation

$$|\varphi - 2\beta x| = n\pi \quad (n = 0, 1, 2, \dots)$$

then,

- 1) it has maximum value when  $n$  is an even number, or
- 2) it has minimum value when  $n$  is an odd number.

When the probe is moved toward the generator, it is decided from  $n$  that the first extremum is maximum or minimum when

$$1^\circ \quad \varphi \geq 0, \quad \begin{cases} \varphi < \pi, \quad \varphi - 2\beta x = 0, \rightarrow \text{max.}, \\ \varphi > \pi, \quad \varphi - 2\beta x = \pi, \rightarrow \text{min.}, \end{cases}$$

(11)

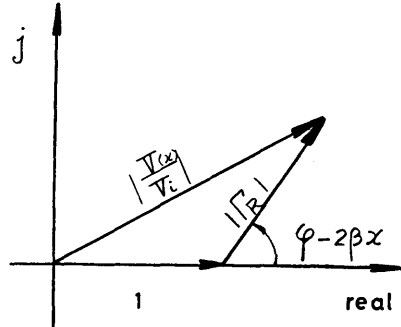


Fig. 11. The relation of the reflection coefficient and the phase.

$$2^\circ \varphi \leq 0, \begin{cases} |\varphi| < \pi, & |\varphi - 2\beta x| = \pi, \rightarrow \min., \\ |\varphi| > \pi, & |\varphi - 2\beta x| = 2\pi, \rightarrow \max.. \end{cases}$$

From Eq. (15) and (18)  $l_{\min}$  is obtained theoretically. In the practical frequency band, 3,700~4,300 Mc, the measured values of  $l_{\min}$  are shown in Table 2.

Table 2. Relation of the length of minimum point to frequencies.

Freq. (Mc)	3,700	3,800	3,900	4,000	4,100	4,200	4,300
$\varphi$ (°)	4.3	2.8	1.4	0	-1.4	-2.8	-4.3
$l_{\min}$ (mm)	20.76	20.04	19.38	18.75	18.16	17.58	17.03

In practical cases, it is necessary to use the correction of  $\Delta L = L - l_{\min}$ .

## VII. The result

The sizes of helices that have been used for experiment are as follows (ref. Fig. 12).

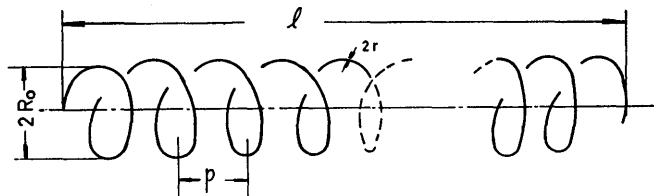


Fig. 12. Notations for the helix.

Mean diameter of the helix  $2R_0 = 4.0$  mm,  
 mean diameter of the wire  $2r = 0.4$  mm,  
 pitches  $p = 0.8, 1.2, 1.6, 2.0$  mm,  
 the length of the helix  $l \approx 150$  mm.

The frequency range is from 3.700 to 4.300 Mc, and measuring system is shown in Fig. 13.

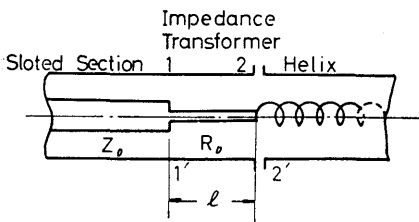
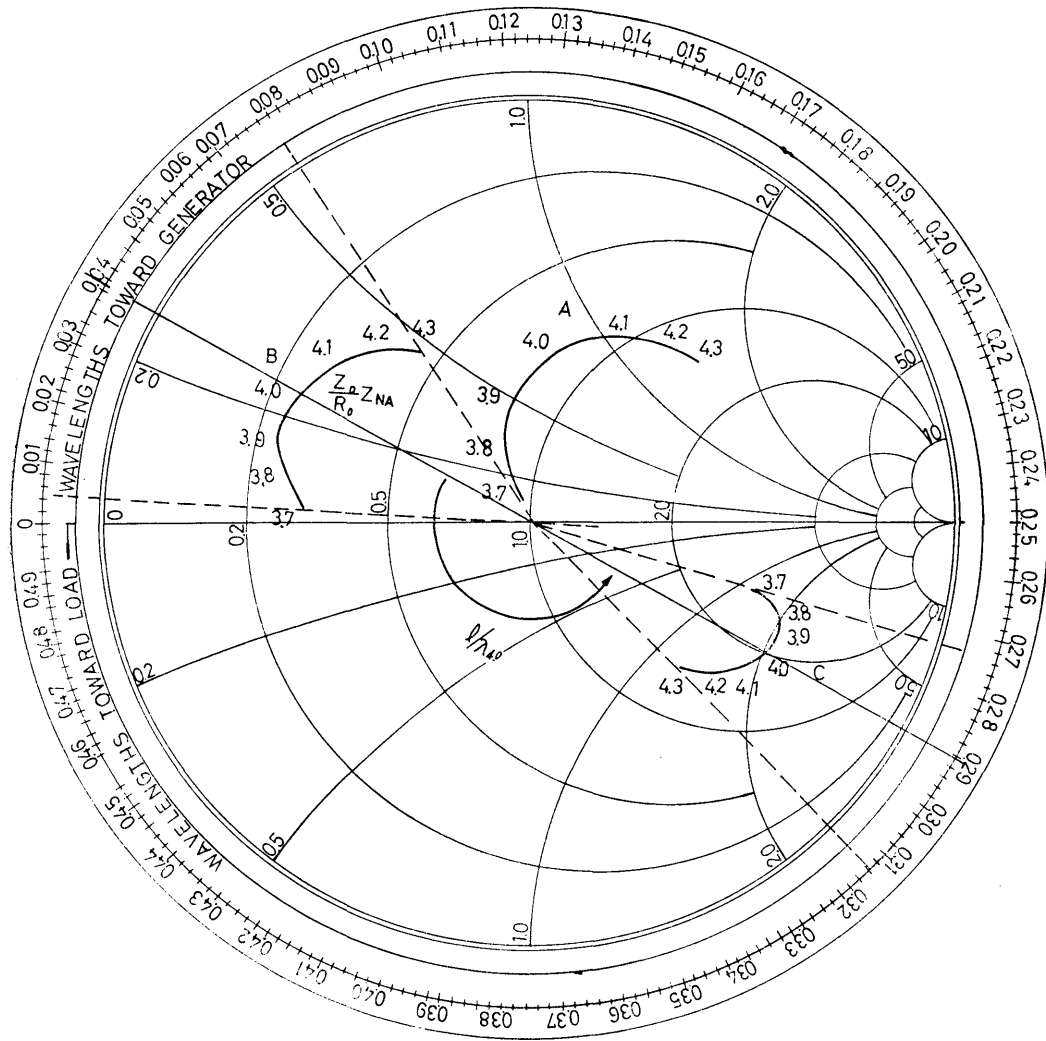
For 1.6 mm pitch, the processes for obtaining load impedance are illustrated in Fig. 13.

1. It is necessary to measure the VSWR,  $\rho$ , and voltage minimum points. If the value of standing wave detector scale was  $x_0$ , the distance from this point to reference plane is as follows:

$$x_{\min} = x_0 - L + l_{\min},$$

where  $L$  is the length of standing wave detector.

2. Using the value of  $\rho$ ,  $x_{\min}$ , point A on the Smith Chart, the input terminal of impedance transformer is obtained. (ref. Fig. 13)



- A.  $Z_{NA}$  the impedance from 1-1' toward the helix
- B.  $\frac{Z_0}{R_0} Z_{NA}$
- C. The impedance from 2-2' toward the helix

Fig. 13. Typical input impedance characteristics of a 3.7~4.3 kMc. [pitch  $p=1.6$  mm]  
 (Mean diameter of the helix : 4.0 mm $\phi$ )  
 (Mean diameter of the conductor : 0.4 mm $\phi$ )

3. If the value of point A was  $R_{NA} + jX_{NA}$  normalized impedance in the transformer is given by

$$(13)$$

$$z_{NB} = \frac{z_0}{R_0} (R_{NA} + jX_{NA}).$$

This is a point  $B$  on the Smith Chart.

4. Rotating point  $B$  toward the load by the length  $l/\lambda$ , where  $\lambda$  is the wavelength of the measuring frequency,  $l$  is the length of the impedance transformer (75/4 mm), the input impedance at the plane 2-2' is obtained by the point  $C$ . This is the normalized input impedance of the helix.

5. If the value of point  $C$  was  $R_L + jX_L$ , the input impedance of the helix  $z_h$  is as follows :

$$z_h = R_0(R_L + jX_L)$$

The input impedance of the helices on each pitch are shown in Fig. 14 and 15 with frequency characteristics. The impedance characteristic is as follows :

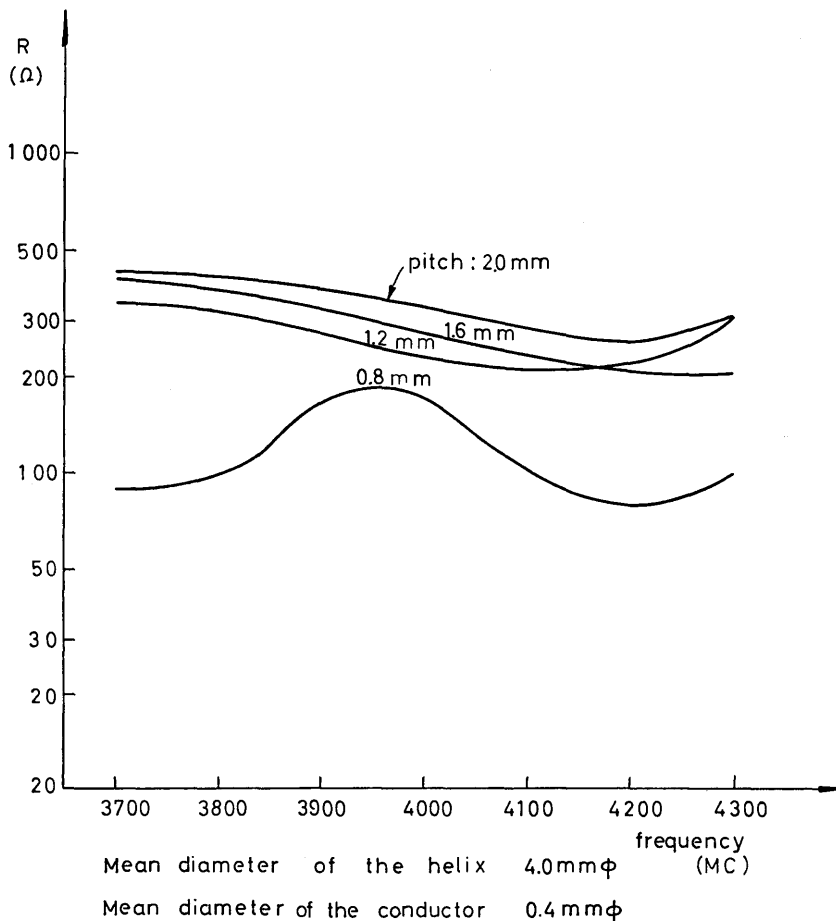


Fig. 14. The frequency characteristics of the real parts of helices input impedance ( $R - jX$ ).

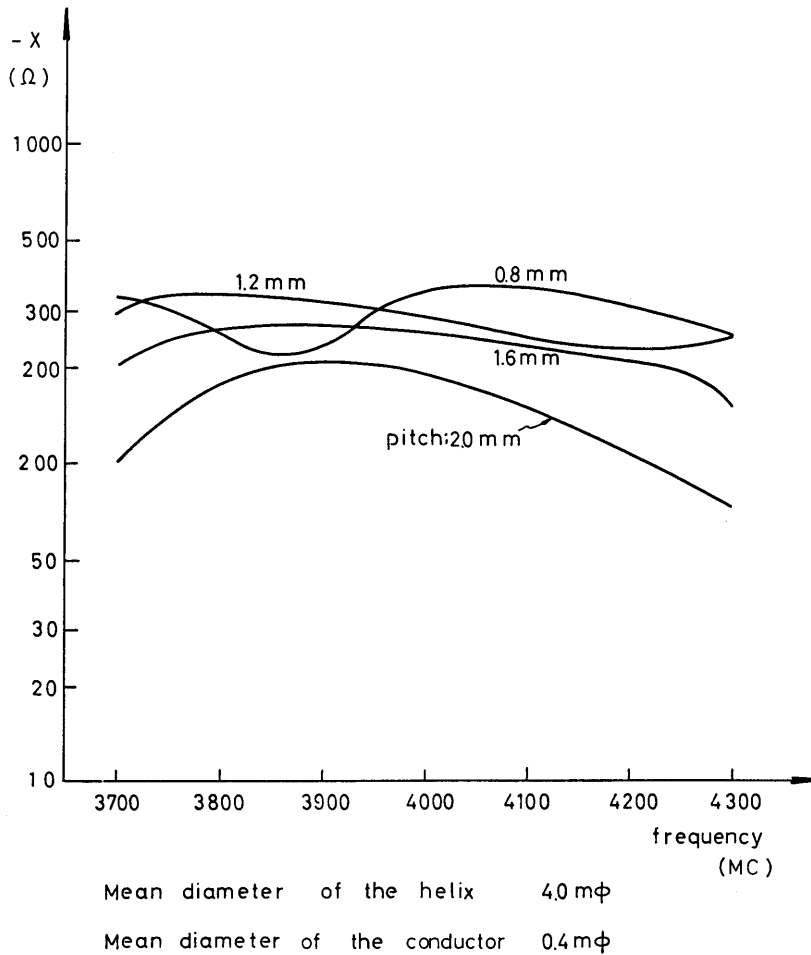


Fig. 15. The frequency characteristics of the imaginary part of helices input impedance ( $R-jX$ ).

i) The real part increases from  $80 \Omega$  to  $500 \Omega$  with increasing pitch and its value decreases from  $430 \Omega$  to  $210 \Omega$  with frequency when the pitch is small, for instance,  $0.8 \text{ mm}$  or at upper limit of oscillation  $4.300 \text{ Mc}$ , the data has considerable error.

ii) The imaginary part decreases gradually from  $400 \Omega$  to  $100 \Omega$  with increasing pitch and decreases with increasing frequency. The case in which the pitch is  $0.8 \text{ mm}$  and frequency is  $4,300 \text{ Mc}$  is the same as i).

### VIII. Discussion

As shown in Fig. 16, the curve of the  $0.8 \text{ mm}$  pitch helix is quite apart from any other curves. This is caused by the large load impedance and mechanical





Hence,

$$0.0123 \leq \delta \leq 0.0144 .$$

Fig. 16 shows the theoretical curves of helix impedance. It is assumed that  $q$  and  $D_0$  are proportional to self inductance and self capacitance. When the pitch is less than 1.2 mm, the measured impedance coincides with theoretical values.

### IX. Conclusion

It is possible to obtain stable and accurate values when the helix input impedance are measured. The tendency of the experimental value coincides with the theoretical value, but it is necessary to discuss the various parameters in detail. The real parts of the input impedance decrease in proportion to the frequency and are proportional to the pitch. The imaginary parts of the input impedance decrease in proportion to the frequency, their signs are negative, and the absolute value decreases with increasing pitch length.

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