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Author	大場, 勇治郎(Oba, Yujiro)
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Scattering of a Plane Wave from an Anisotropic Plasma Column at Oblique Incidence

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Yujiro OHBA*

Abstract

The scattering of a plane wave by a circular plasma cylinder is calculated in the case of an oblique incidence. The plasma cylinder is lossy and is magnetized in the direction of the cylinder axis. The solution is derived for the magnetic field vector of the incident plane wave transverse to the cylinder axis. The corresponding result for the electric vector transverse to the axis is also obtained from symmetry of the electric and magnetic vectors in the Maxwell's equations. The modes of the field in the plasma are specified by propagation constants in the radial direction. These constants S_1 and S_2 are calculated for several sets of values of plasma frequency, cyclotron frequency and collision frequency.

I. Introduction

The diffraction of electromagnetic waves by a plasma medium has been of interest to many authors. Platzman and Ozaki treated the scattering of a plane wave from a circular cylinder of a homogeneous and anisotropic plasma at normal incidence and they gave the rigorous solution in terms of a series of Bessel function[1]. Using the Born approximation, Midzuno treated the similar problems in two cases of the plane wave and spherical wave incident upon the non-uniform plasma cylinder[2,3]. These solutions are useful not only for the determination of an electron density and a collision frequency of the plasma[4,5], but also for a reentry communication between a space vehicle and a terminal station. In the latter, the problem of diffraction should be solved in the case of the oblique incidence of the plane wave. This problem is very similar to that of the scattering by an isotropic dielectric circular cylinder. Wait solved the latter problem and showed that the scattered field is expressed as a superposition of a set of *TE* and *TM* modes even though the incident wave is purely *TE* or *TM* mode[6].

In this paper the problem of diffraction of a plane wave by the circular cylinder of a homogeneous and anisotropic plasma at the oblique incidence is treated. The solution is derived for the magnetic vector of the incident wave transverse to the axis of the cylinder, that is, *TM* mode. The corresponding result for *TE* mode is

* 大 場 勇治郎 School of Engineering, Keio University, Koganei, Tokyo, Japan.

also obtained from symmetry. The results are rather more complicated than those for isotropic cylinder. The field in the plasma can be computed after two propagation constants in the plasma medium are determined. These constants are given by complex numbers. These numbers are calculated for the sets of values of plasma frequency, cyclotron frequency, and collision frequency of the plasma cylinder.

II. Cylindrical waves in anisotropic plasma medium

Consider an electrically neutral and unbounded plasma, which is homogeneously magnetized in the z-direction of rectangular coordinates. The permittivity tensor of the plasma is written as

$$\overrightarrow{\overrightarrow{\varepsilon}} = \begin{pmatrix} \varepsilon & -jg & 0 \\ jg & \varepsilon & 0 \\ 0 & 0 & \eta \end{pmatrix}, \tag{1}$$

where the explicit expressions of tensor components can be found in many places[7].

When the cylindrical coordinates (ρ, ϕ, z) are taken in the plasma medium, three components of the electric field for a wave mode, which has a periodicity $\exp(-j\gamma z)$ in the z-direction, are written as[7]

$$\left.\begin{aligned} E_\rho &= -j\Big\{(F_m)\gamma\, S_m B_n'(S_m\rho) - (G)\frac{n}{\rho}B_n(S_m\rho)\Big\}\exp j(n\phi-\gamma z), \\ E_\phi &= \Big\{(F_m)\frac{n}{\rho}\gamma\, B_n(S_m\rho) - (G)S_m B_n'(S_m\rho)\Big\}\exp j(n\phi-\gamma z), \\ E_z &= S_m^2(K_m)B_n(S_m\rho)\exp j(n\phi-\gamma z). \end{aligned}\right\} \tag{2}$$

where a time dependence $\exp(j\omega t)$ is suppressed, $B_n(S_m\rho)$ and $B_n'(S_m\rho)$ are respectively the n-th order Bessel function and its derivative, and factors (F_m), (G) and (K_m) are given by

$$\gamma(F_m) = -(\gamma^2 + S_m^2 - \omega^2\mu_0\varepsilon),$$

$$(G) = \omega^2\mu_0 g,$$

$$\gamma(K_m) = -\frac{\varepsilon}{\eta}(S_m^2+\gamma^2) + \omega^2\mu_0\frac{\varepsilon^2-g^2}{\eta}.$$

As is well known, two propagation constants are possible in propagation through the plasma medium. In case of cylindrical waves these constants correspond to S_m ($m=1$ and 2) which are two roots of the equation

$$(\varepsilon S^2 + \gamma^2\eta - \omega^2\mu_0\varepsilon\eta)(S^2+\gamma^2-\omega^2\mu_0\varepsilon) + \omega^2\mu_0 g^2(S^2-\omega^2\mu_0\eta) = 0. \tag{3}$$

Therefore two independent sets of wave modes are obtained by putting as S_1 and S_2 for S_m in equation (2). The general solution of electric field in the plasma medium can be expressed by a linear combination of two independent sets of wave modes.

III. Solution

Consider a plasma column of radius a. The column is infinitely long and is magnetized in the direction of its axis (z-axis). A plane wave impinges on this column making an angle θ with the negative z-axis as shown in Fig. 1. Consider

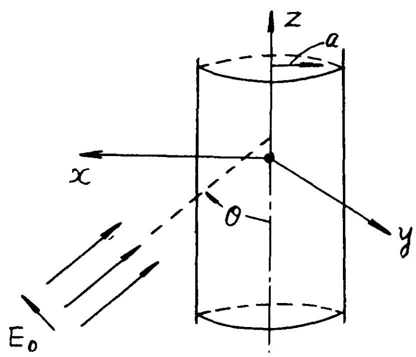

Fig. 1.

the case that the electric vector is parallel to the xz-plane and $H_z=0$. The incident wave can be calculated by the z-component of the Hertzian vector which has the factor

$$\sum_{n=-\infty}^{\infty} J_n(k_x\rho)\, j^{-n} \exp j(n\phi-k_z z),$$

where a time dependence $\exp(j\omega t)$ is suppressed and

$$k_x=-k_0 \sin\theta, \quad k_z=k_0\cos\theta, \quad k_0{}^2=\omega^2\varepsilon_0\mu_0$$

and ε_0 and μ_0 are dielectric constant and permeability of vacuum. The $\varphi-$ and z-components of the electric and magnetic vectors of the incident waves respectively written as

$$\left.\begin{aligned} E_\varphi{}^i &= E_0 \sum_{n=-\infty}^{\infty} \frac{n}{\rho} k_z J_n(k_x\rho)\, Z_n, \\ E_z{}^i &= E_0 \sum_{n=-\infty}^{\infty} k_x{}^2 J_n(k_x\rho)\, Z_n, \\ H_\varphi{}^i &= -\frac{j}{\omega\mu_0} E_0 k_0{}^2 \sum_{n=-\infty}^{\infty} k_x J_n'(k_x\rho)\, Z_n, \\ H_z{}^i &= 0, \end{aligned}\right\} \tag{4}$$

where

$$Z_n=j^{-n}\exp j(n\phi-k_z z).$$

As is shown by J. R. Wait in case of a dielectric column, the scattered field are given by the superposition of sets of TM and TE modes. Since the scattered field must vary according to the factor exp $(-jk_z z)$, the TM mode is expressed by the z-component of the Hertzian vector which has the factor

$$R_{1n}H_n^{(2)}(k_x\rho)\,Z_n,$$

and the TE mode is expressed by the z-component of the Fitzgerald vector which has the factor

$$R_{2n}H_n^{(2)}(k_x\rho)\,Z_n,$$

Therefore the $\varphi-$ and z-components of the scattered field are given by

$$\left.\begin{aligned}
E_\varphi^s &= \sum_{n=-\infty}^{\infty}\left\{R_{1n}\frac{n}{\rho}k_zH_n^{(2)}(k_x\rho)-R_{2n}k_xH_n^{(2)\prime}(k_x\rho)\right\}Z_n,\\
E_z^s &= \sum_{n=-\infty}^{\infty}R_{1n}k_x^2H_n^{(2)}(k_x\rho)\,Z_n,\\
H_\varphi^s &= -\frac{j}{\omega\mu_0}\sum_{n=-\infty}^{\infty}\left\{R_{1n}k_0^2k_xH_n^{(2)\prime}(k_x\rho)-R_{2n}\frac{n}{\rho}k_zH_n^{(2)}(k_x\rho)\right\}Z_n,\\
H_z^s &= \frac{j}{\omega\mu_0}\sum_{n=-\infty}^{\infty}R_{2n}k_x^2H_n(k_x\rho)\,Z_n.
\end{aligned}\right\}\quad(5)$$

The field in the plasma column can be obtained by the linear combination of two independent sets of modes shown by equation (2), in which the Bessel's function and its derivative are of the first kind, and $\gamma=k_z$. Thus

$$\left.\begin{aligned}
E_\varphi^t &= \sum_m^{1,2}\sum_{n=-\infty}^{\infty}T_{mn}\left\{(F_m)\frac{n}{\rho}k_zJ_n(S_m\rho)-(G)\,S_mJ_n'(S_m\rho)\right\}Z_n,\\
E_z^t &= \sum_m^{1,2}\sum_{n=-\infty}^{\infty}T_{mn}S_m^2(K_m)\,J_n(S_m\rho)\,Z_n,\\
H_\varphi^t &= -\frac{j}{\omega\mu_0}\sum_m^{1,2}\sum_{n=-\infty}^{\infty}T_{mn}\left\{(K_m)S_mJ_n'(S_m\rho)\,\omega^2\mu_0\eta\right.\\
&\qquad\left.-(G)\frac{n}{\rho}k_zJ_n(S_m\rho)\right\}Z_n,\\
H_z^t &= \frac{j}{\omega\mu_0}\sum_m^{1,2}\sum_{n=-\infty}^{\infty}T_{mn}S_m^2(G)J_n(S_m\rho)\,Z_n.
\end{aligned}\right\}\quad(6)$$

The boundary conditions at $\rho=a$, written symbolically, are

$$\left.\begin{aligned} &E_\varphi{}^i+E_\varphi{}^s=E_\varphi{}^t,\\ &E_z{}^i+E_z{}^s=E_z{}^t,\\ &H_\varphi{}^i+H_\varphi{}^s=H_\varphi{}^t,\\ &H_z{}^i+H_z{}^s=H_z{}^t, \end{aligned}\right\} \tag{7}$$

From equation (7) the coefficient R_{1n}, R_{2n}, T_{1n} and T_{2n} are obtained as

$$\left.\begin{aligned} R_{1n}&=\frac{D_1}{D}, & R_{2n}&=\frac{D_2}{D},\\ T_{1n}&=\frac{D_3}{D}, & T_{2n}&=\frac{D_4}{D}, \end{aligned}\right\} \tag{8}$$

where

$$D=\begin{vmatrix} \frac{n}{\rho}k_zH_n{}^{(2)}(k_xa) & -k_xH_n{}^{(2)\prime}(k_xa) & X_n(S_1a) & X_n(S_2a)\\ k_x{}^2H_n{}^{(2)}(k_xa) & 0 & S_1{}^2(K_1)J_n(S_1a) & S_2{}^2(K_2)J_n(S_2a)\\ k_0{}^2k_xH_n{}^{(2)\prime}(k_xa) & -\frac{n}{\rho}k_zH_n(k_xa) & Y_n(S_1a) & Y_n(S_2a)\\ 0 & k_x{}^2H_n{}^{(2)}(k_xa) & S_1{}^2(G)J_n(S_1a) & S_2{}^2(G)J_n(S_2a) \end{vmatrix}, \tag{9}$$

$$X_n(S_ma)=(F_m)\frac{n}{\rho}k_zJ_n(S_ma)-(G)\,S_mJ_n'(S_ma),$$

$$Y_n(S_ma)=(K_m)\,S_mJ_n'(S_ma)\,\omega^2\mu_0\eta-(G)\frac{n}{\rho}k_zJ_n(S_ma),$$

and determinants D_r ($r=1$, 2, 3 and 4) is obtained by replacing the r-th column of the determinant (9) by

$$\begin{vmatrix} -E_0\frac{n}{\rho}k_zJ_n(k_xa)\\ -E_0\,k_x{}^2J_n(k_xa)\\ -E_0\,k_0{}^2k_xJ_n'(k_xa)\\ 0 \end{vmatrix}.$$

In case that the incident magnetic field vector is parallel to the xz-plane and $E_z=0$, the $\varphi-$ and $z-$ components of the incident wave are written as

$$\left.\begin{aligned} &E_\varphi{}^i=H_0\sum_{n=-\infty}^{\infty}k_xJ_n'(k_x\rho)\,Z_n,\\ &E_z{}^i=0,\\ &H_\varphi{}^i=-\frac{j}{\omega\mu_0}H_0\sum_{n=-\infty}^{\infty}\frac{n}{\rho}k_zJ_n(k_x\rho)\,Z_n, \end{aligned}\right\} \tag{10}$$

$$H_z^i = -\frac{j}{\omega\mu_0} H_0 k_x^2 \sum_{n=-\infty}^{\infty} \frac{n}{\rho} J_n(k_x\rho)\, Z_n.$$

As the scattered field and the field in the plasma are expressed by equations (5) and (6) respectively, four coefficients R_{1n}, R_{2n}, T_{1n} and T_{2n} are written as equation (8), and the determinant of numerator D_r ($r=1$, 2, 3 and 4) can be obtained by replacing the n-th column of determinant (9) by

$$\begin{vmatrix} -H_0 k_x J_n'(k_x a) \\ 0 \\ -H_0 \dfrac{n}{\rho} k_z J_n(k_x a) \\ H_0 k_x^2 J_n(k_x a) \end{vmatrix}.$$

IV. Propagation constants in the plasma

Putting $\gamma = k_z$, roots of Eq. (3) are calculated as

$$S^2 = \frac{-A \pm \sqrt{A^2-4B}}{2}, \tag{11}$$

where

$$\left.\begin{aligned} A &= \left\{\left(1+\frac{\eta}{\varepsilon}\right)(k_z^2-\omega^2\mu_0\varepsilon)+\omega^2\mu_0\frac{g^2}{\varepsilon}\right\}, \\ B &= \frac{\eta}{\varepsilon}\,\{k_z^2-\omega^2\mu_0(\varepsilon+g)\}\,\{k_z^2-\omega^2\mu_0(\varepsilon-g)\}. \end{aligned}\right\} \tag{12}$$

The elements of tensor permeability (1) are shown as

$$\frac{\varepsilon}{\varepsilon_0} = 1 - j\frac{p^2(j+\nu)}{(j+\nu)^2+c^2},$$

$$\frac{\eta}{\varepsilon_0} = 1 - j\frac{p^2}{j+\nu},$$

$$\frac{g}{\varepsilon_0} = \frac{p^2 c}{(j+\nu)^2+c^2},$$

where p, c and ν are plasma frequency, cyclotron frequency and collision frequency, respectively, divided by the angular frequency of the incident plane wave. Using Eqs. (11) and (12), the propagation constants are calculated in the following cases:

Case 1; $\nu=0$, $k_z=0$ (normal incidence of $\theta=90°$)

$$\left.\begin{aligned} S_1^2 &= \omega^2\mu_0\varepsilon_0(1-p^2), \\ S_2^2 &= \omega^2\mu_0\varepsilon_0\left[1+\frac{p^2(1-p^2)}{c^2+p^2-1}\right]. \end{aligned}\right\} \tag{13}$$

These constants coincide with those in the case of the direction of propagation perpendicular to the static magnetic field. S_1 and S_2 are pure imaginary or pure real numbers.

Case 2; $\nu=0$, $k_z^2=\frac{1}{2}\omega^2\mu_0\varepsilon_0$ (oblique incidence of $\theta=45°$)

$$S_{1,2}^2=\omega^2\mu_0\varepsilon_0\left[\frac{1}{2}-p^2+\frac{3c^2p^2}{4(p^2+c^2-1)}\pm\frac{p^2c\sqrt{c^2+8-8p^2}}{4(p^2+c^2-1)}\right]. \tag{14}$$

When $c^2+8-8p^2<0$, the propagation constants are complex numbers. When static magnetic field equals to zero ($c=0$), the plasma cylinder is isotropic and two propagation constants equal each other and are obtained as[6]

$$S_{1,2}^2=\omega^2\mu_0\varepsilon_0\left(\frac{1}{2}-p^2\right).$$

Case 3; $\nu\neq 0$, $c=0$, $k_z^2=\frac{1}{2}\omega^2\mu_0\varepsilon_0$. (15)

$$S^2=\omega^2\mu_0\varepsilon_0\left[\frac{1}{2}-\frac{1+j\nu}{\nu^2+1}p^2\right].$$

The plasma cylinder is the same as an isotropic and lossy dielectric cylinder.

Case 4; $\nu\neq 0$, $c=1$, $k_z^2=\frac{1}{2}\omega^2\mu_0\varepsilon_0$.

$$S_{1,2}^2=\omega^2\mu_0\varepsilon_0\left[\frac{1}{2}-\frac{1+j\nu}{\nu^2+1}p^2+\frac{p^2(\nu-j)}{4V(1+\nu^2)}\{3j\pm\sqrt{8\nu^2+p^2-9+\nu j(16-p^2)}\}\right], \tag{16}$$

where

$$V=\nu^2+p^2+j\nu(2-p^2).$$

In general, two propagation constants are complex numbers.

V. Conclusion

The scattered field by the plasma and the field in the plasma are given by expressions (5) and (6) respectively. It is found that these expressions are very complicated in a practical computation even when the propagation constants are computed. Computational results will be published in the near future.

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