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Radiation from a Current Filament in an Anisotropic Plasma

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Abstract

The radiation field from a current filament is calculated. The filament is located in a homogeneous, anisotropic and unbounded plasma medium and carries travelling electric or magnetic current. The equation satisfied by propagation constants in the radial direction is obtained. Two sets of wave modes are found corresponding to two propagation constants. The field from the current filament is constructed by the superposition of two sets of wave modes in both cases of electric and magnetic current filaments.

I. Introduction

For several years the propagation of electromagnetic waves through a plasma medium has been of interest to many authors[1,2,3]. A plasma in an external magnetic field has anisotropic properties. As a result, two propagation constants are possible in propagation through the plasma medium, that is, the plasma medium has a doubly refractive property such as a crystal for light waves. Therefore problems of propagation in anisotropic media are rather more complicated than those in isotropic media.

According to some simplified assumptions for the plasma, Maxwell's equations for the plasma medium may be obtained by replacing a scalar dielectric constant ε for isotropic medium by a tensor permittivity.

The radiation from antennas in a plasma medium and the scattering of electromagnetic waves by a plasma medium have become very important in connection with reentry communication between a space vehicle and a terminal station. Several papers on these problems have been published[4,5,6]. Using a dyadic Green's function, Kuehl has treated the radiation from a dipole in an anisotropic plasma. Using a similar method in the two dimensional case, the radiation from an electric or a magnetic current filament, which is infinitely long and clad by an anisotropic plasma sheath, has been calculated by the author[7].

In this paper the radiation from an infinitely long filament, which carries a

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progressive electric or magnetic current in a plasma medium, is calculated. As is expected, two propagation constants are obtained. The radiation fields in both cases are obtained, by the superposition of two modes corresponding to two propagation constants. These propagation modes are the same as those given Epstein[8]. These modes may be very useful for problems of scattering by a plasma column or plasma sheath at oblique incidence of electromagnetic waves.

II. Maxwell's equations in a magnetized plasma medium

Consider an electrically neutral, homogeneous and unbounded plasma. An electron in the plasma under an external magnetic field will be subjected to a Lorentz force. As usual, a plasma frequency ω_p and a cyclotron frequency ω_c are defined. In order to describe collisions between particles in the plasma, a collision frequency ν is introduced. The electric displacement $\vec{D}$ in the plasma is related to an electric field $\vec{E}$ by

$$\vec{D}=\overrightarrow{\overrightarrow{\varepsilon}}\cdot\vec{E},$$

where $\overrightarrow{\overrightarrow{\varepsilon}}$ is the tensor permittivity defined in terms of these frequencies. When the plasma is magnetized homogeneously in the z-direction of rectangular coordinates, $\overrightarrow{\overrightarrow{\varepsilon}}$ is written as[9]

$$\overrightarrow{\overrightarrow{\varepsilon}}=\begin{pmatrix} \varepsilon & -jg & 0 \\ jg & \varepsilon & 0 \\ 0 & 0 & \eta \end{pmatrix}, \tag{1}$$

where

$$\varepsilon=\left[1-j\frac{\omega_p{}^2\,(j\omega+\nu)}{\{(j\omega+\nu)^2+\omega_c{}^2\}\omega}\right]\varepsilon_0,$$

$$g=\frac{\omega_p{}^2\omega_c}{\{(j\omega+\nu)^2+\omega_c{}^2\}\omega}\,\varepsilon_0,$$

$$\eta=\left\{1-j\frac{\omega_p{}^2}{(j\omega+\nu)\,\omega}\right\}\varepsilon_0.$$

Thus Maxwell's equations in the anisotropic plasma medium are

$$\left.\begin{aligned} \nabla\times\vec{H}&=\vec{J}+j\omega\overrightarrow{\overrightarrow{\varepsilon}}\cdot\vec{E}, \\ \nabla\times\vec{E}&=-j\omega\,\mu_0\vec{H}, \end{aligned}\right\} \tag{2}$$

where $\vec{E}$ and $\vec{H}$ are the field intensities, $\vec{J}$ is an impressed current density, and μ_0 is the magnetic permeability of the plasma which equals that of vacuum. In the above vector quantities, a time dependence $\exp(j\omega t)$ is assumed.

III. Dyadic Green's function for the anisotropic medium

From equation (2), one obtains

$$\nabla\times\nabla\times\vec{E}-\omega^2\mu_0\overrightarrow{\overrightarrow{\varepsilon}}\cdot\vec{E}=-j\omega\,\mu_0\vec{J}. \tag{3}$$

Introducing a dyadic Green's function $\overrightarrow{\overrightarrow{\Gamma}}(\vec{r},\vec{r}')$, the electric field $\vec{E}(\vec{r})$ at an observing point is

$$\vec{E}(\vec{r})=\int_V \overrightarrow{\overrightarrow{\Gamma}}(\vec{r},\vec{r}')\cdot\vec{J}(\vec{r}')\,dV', \tag{4}$$

where $\vec{r}$ and $\vec{r}'$ are coordinate vectors of observing and source points respectively, and the integration is performed over the volume V containing the source currents. Substituting equation (4) into equation (3), one obtains

$$\int_V[\nabla\times\nabla\times\overrightarrow{\overrightarrow{\Gamma}}(\vec{r},\vec{r}')-\omega^2\mu_0\overrightarrow{\overrightarrow{\varepsilon}}\cdot\overrightarrow{\overrightarrow{\Gamma}}(\vec{r},\vec{r}')]\cdot\vec{J}(\vec{r}')\,dV'=-j\omega\mu_0\vec{J}(\vec{r}). \tag{5}$$

$\vec{J}(\vec{r})$ on the right-hand side of equation (5) is given by

$$\vec{J}(\vec{r})=\int_V\overrightarrow{\overrightarrow{U}}\cdot\vec{J}(\vec{r}')\,\delta(\vec{r}-\vec{r}')\,dV', \tag{6}$$

where $\overrightarrow{\overrightarrow{U}}$ is the unit dyadic and $\delta(\vec{r}-\vec{r}')$ is the Dirac delta-function. From equations (5) and (6), one obtains

$$(-\nabla^2\overrightarrow{\overrightarrow{U}}+\nabla\nabla-\omega^2\mu_0\overrightarrow{\overrightarrow{\varepsilon}})\cdot\overrightarrow{\overrightarrow{\Gamma}}(\vec{r},\vec{r}')=-j\omega\mu_0\overrightarrow{\overrightarrow{U}}\delta(\vec{r}-\vec{r}'). \tag{7}$$

Taking the Fourier transform of equation (7) with respect to $\vec{r}$, one obtains

$$\overrightarrow{\overrightarrow{\lambda}}\cdot\overrightarrow{\overrightarrow{\Gamma}}(\vec{p},\vec{r}')=-j\omega\mu_0\overrightarrow{\overrightarrow{U}}\,e^{j\vec{p}\cdot\vec{r}'}, \tag{8}$$

where $\overrightarrow{\overrightarrow{\Gamma}}(\vec{p},\vec{r}')$ is the transform of $\overrightarrow{\overrightarrow{\Gamma}}(\vec{r},\vec{r}')$,

$$\overrightarrow{\overrightarrow{\lambda}}=p^2\overrightarrow{\overrightarrow{U}}-\vec{p}\vec{p}-\omega^2\mu_0\overrightarrow{\overrightarrow{\varepsilon}}, \tag{9}$$

$$\vec{p}=p_x\vec{i}+p_y\vec{j}+p_z\vec{k}, \tag{10}$$

and $\vec{i}$, $\vec{j}$ and $\vec{k}$ are unit vectors in x, y and z axes, respectively. Premultiplying by the inverse dyadic $\overrightarrow{\overrightarrow{\lambda}}^{-1}$, equation (8) becomes

$$\overrightarrow{\overrightarrow{\Gamma}}(\vec{p},\vec{r}')=-j\omega\mu_0\overrightarrow{\overrightarrow{\lambda}}^{-1}e^{j\vec{p}\cdot\vec{r}'}.$$

Taking the inverse Fourier transform of the above equation, the dyadic Green's function is obtained as

$$\overrightarrow{\overrightarrow{\Gamma}}(\vec{r},\vec{r}')=-\frac{j\omega\mu_0}{8\pi^3}\iiint_{-\infty}^{\infty}\overrightarrow{\overrightarrow{\lambda}}^{-1}e^{-j\vec{p}\cdot(\vec{r}-\vec{r}')}\,dp_x\,dp_y\,dp_z. \tag{11}$$

As calculated from equations (9) and (10), each of nine components of $\overleftrightarrow{\lambda}^{-1}$ is the quotient of two polynomials in p_x, p_y and p_z. Because the polynomial in the numerator of each component operates on the term $\exp[-j\vec{p}\cdot(\vec{r}-\vec{r}')]$, p_x, p_y and p_z in the numerator may be replaced by

$$p_x=j\frac{\partial}{\partial x},\qquad p_y=j\frac{\partial}{\partial y},\qquad p_z=j\frac{\partial}{\partial z}.$$

Interchanging the order of integration and differentiation, one obtains

$$\overleftrightarrow{\Gamma}(\vec{r},\vec{r}')=-\frac{j\omega\mu_0}{8\pi^3}\overleftrightarrow{D}\iiint_{-\infty}^{\infty}\frac{1}{|\overleftrightarrow{\lambda}|}e^{-j\vec{p}\cdot(\vec{r}-\vec{r}')}\,dp_x\,dp_y\,dy_z, \tag{12}$$

where $|\overleftrightarrow{\lambda}|$ is the determinant of $\overleftrightarrow{\lambda}$ and $\overleftrightarrow{D}$ is the dyadic operator. From equations (9) and (10), one obtains

$$|\overleftrightarrow{\lambda}|=\begin{vmatrix} p^2-p_x^2-\omega^2\mu_0\varepsilon & -p_xp_y+j\omega^2\mu_0 g & -p_xp_z \\ -p_yp_x-j\omega^2\mu_0 g & p^2-p_y^2-\omega^2\mu_0\varepsilon & -p_yp_z \\ -p_zp_x & -p_zp_y & p^2-p_z^2-\omega^2\mu_0\eta \end{vmatrix}$$

$$=-p_z^4\omega^2\mu_0\eta+p_z^2\omega^2\mu_0[2\omega^2\mu_0\varepsilon\eta-(\varepsilon+\eta)(p_x^2+p_y^2)]$$
$$-\omega^2\mu_0[(p_x^2+p_y^2)-\omega^2\mu_0\eta][\varepsilon(p_x^2+p_y^2)-\omega^2\mu_0(\varepsilon^2-g^2)], \tag{13}$$

$$\overleftrightarrow{D}=\begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{pmatrix}, \tag{14}$$

$$D_{xx}=\begin{vmatrix} \left(\frac{\partial^2}{\partial y^2}-\nabla^2-\omega^2\mu_0\varepsilon\right) & \left(\frac{\partial^2}{\partial y\partial z}\right) \\ \left(\frac{\partial^2}{\partial z\partial y}\right) & \left(\frac{\partial^2}{\partial z^2}-\nabla^2-\omega^2\mu_0\eta\right) \end{vmatrix}, \tag{15a}$$

$$D_{xy}=-\begin{vmatrix} \left(\frac{\partial^2}{\partial x\partial y}+j\omega^2\mu_0 g\right) & \left(\frac{\partial^2}{\partial x\partial z}\right) \\ \left(\frac{\partial^2}{\partial z\partial y}\right) & \left(\frac{\partial^2}{\partial z^2}-\nabla^2-\omega^2\mu_0\eta\right) \end{vmatrix}, \tag{15b}$$

$$D_{xz}=\begin{vmatrix} \left(\frac{\partial^2}{\partial x\partial y}+j\omega^2\mu_0 g\right) & \left(\frac{\partial^2}{\partial x\partial z}\right) \\ \left(\frac{\partial^2}{\partial y^2}-\nabla^2-\omega^2\mu_0\varepsilon\right) & \left(\frac{\partial^2}{\partial y\partial z}\right) \end{vmatrix}, \tag{15c}$$

$$D_{yx}=-\begin{vmatrix} \left(\frac{\partial^2}{\partial y\partial x}-j\omega^2\mu_0 g\right) & \left(\frac{\partial^2}{\partial y\partial z}\right) \\ \left(\frac{\partial^2}{\partial z\partial x}\right) & \left(\frac{\partial^2}{\partial z^2}-\nabla^2-\omega^2\mu_0\eta\right) \end{vmatrix}, \tag{15d}$$

$$D_{yy}=\begin{vmatrix} \left(\frac{\partial^2}{\partial x^2}-\nabla^2-\omega^2\mu_0\varepsilon\right) & \left(\frac{\partial^2}{\partial x\partial z}\right) \\ \left(\frac{\partial^2}{\partial z\partial x}\right) & \left(\frac{\partial^2}{\partial z^2}-\nabla^2-\omega^2\mu_0\eta\right) \end{vmatrix}, \tag{15e}$$

$$D_{yz}=-\begin{vmatrix} \left(\frac{\partial^2}{\partial x^2}-\nabla^2-\omega^2\mu_0\varepsilon\right) & \left(\frac{\partial^2}{\partial x\partial z}\right) \\ \left(\frac{\partial^2}{\partial x\partial y}-j\omega^2\mu_0 g\right) & \left(\frac{\partial^2}{\partial y\partial z}\right) \end{vmatrix}, \tag{15f}$$

$$D_{zx}=\begin{vmatrix} \left(\frac{\partial^2}{\partial y\partial x}-j\omega^2\mu_0 g\right) & \left(\frac{\partial^2}{\partial y^2}-\nabla^2-\omega^2\mu_0\varepsilon\right) \\ \left(\frac{\partial^2}{\partial z\partial x}\right) & \left(\frac{\partial^2}{\partial z\partial y}\right) \end{vmatrix}, \tag{15g}$$

$$D_{zy}=-\begin{vmatrix} \left(\frac{\partial^2}{\partial x^2}-\nabla^2-\omega^2\mu_0\varepsilon\right) & \left(\frac{\partial^2}{\partial x\partial y}+j\omega^2\mu_0 g\right) \\ \left(\frac{\partial^2}{\partial z\partial x}\right) & \left(\frac{\partial^2}{\partial z\partial y}\right) \end{vmatrix}, \tag{15h}$$

$$D_{zz}=\begin{vmatrix} \left(\frac{\partial^2}{\partial x^2}-\nabla^2-\omega^2\mu_0\varepsilon\right) & \left(\frac{\partial^2}{\partial x\partial y}+j\omega^2\mu_0 g\right) \\ \left(\frac{\partial^2}{\partial y\partial x}-j\omega^2\mu_0 g\right) & \left(\frac{\partial^2}{\partial y^2}-\nabla^2-\omega^2\mu_0\varepsilon\right) \end{vmatrix}. \tag{15i}$$

Substituting equation (12) to equation (4), the electric field $\vec{E}(\vec{r})$ can be obtained for a given distribution of source current density $\vec{J}(\vec{r}')$.

IV. Field from an electric current filament

Consider a filament carrying a progressive electric current $I_0 \exp(-jk_z z+j\omega t)$ in the unbounded anisotropic plasma medium. The filament is located at a point (x_1, y_1) and it is parallel to the z-axis of the rectangular coordinate system. Suppressing the term $\exp(j\omega t)$, the current density $\vec{J}(\vec{r}')$ is written as

$$\vec{J}(\vec{r}')=I_0\delta(x'-x_1)\,\delta(y'-y_1)\exp(-jk_z z')\,\vec{k}. \tag{16}$$

Substituting equation (16) into equation (4) and performing integrations with respect to x' and y', one obtains

$$\vec{E}(\vec{r})=\int_{-\infty}^{\infty}\overrightarrow{\overrightarrow{\Gamma}}(\vec{r}, x_1, y_1, z')\,I_0\exp(-jk_z z')\cdot\vec{k}\,dz'. \tag{17}$$

Integrating equation (17) with respect to z', and then p_z after substitution of equation (11) into equation (17), one obtains

$$\vec{E}(\vec{r})=-\frac{j\omega\mu_0 I_0}{4\pi^2}\overrightarrow{\overrightarrow{D}}\iint_{-\infty}^{\infty}\frac{1}{|\overrightarrow{\overrightarrow{\lambda}}|}e^{-jp_x(x-x_1)jp_y(y-y_1)-jk_z z}\cdot\vec{k}\,dp_x\,dp_y. \tag{18}$$

Performing the transformation

$$p_x = q\cos\beta, \qquad p_y = q\sin\beta, \qquad p_x^2+p_y^2=q^2,$$

$$x-x_1=|\vec{\rho}-\vec{\rho}_1|\cos\alpha, \qquad y-y_1=|\vec{\rho}-\vec{\rho}_1|\sin\alpha,$$

equation (18) becomes

$$\vec{E}(\vec{r}) = -\frac{j\omega\mu_0 I_0}{4\pi^2}\overrightarrow{\overrightarrow{D}}\, e^{-jk_z z}\int_0^{\infty}\int_0^{2\pi}\frac{e^{-jq|\vec{\rho}-\vec{\rho}_1|\cos(\beta-\alpha)}}{|\overrightarrow{\overrightarrow{\lambda}}|}\, q\,dq\,d\beta\cdot\vec{k}.$$

As is well known, the integration with respect to β yields

$$\vec{E}(\vec{r}) = \frac{jI_0}{2\pi\omega\varepsilon}\overrightarrow{\overrightarrow{D}}\, e^{-jk_z z}\int_0^{\infty}\frac{J_0(q|\vec{\rho}-\vec{\rho}_1|)}{(q^2-S_1^2)(q^2-S_2^2)}\, q\,dq\cdot\vec{k}$$

$$= \frac{jI_0}{4\pi\omega\varepsilon}\overrightarrow{\overrightarrow{D}}\, e^{-jk_z z}\int_{-\infty}^{+\infty}\frac{qH_0^{(2)}(q|\vec{\rho}-\vec{\rho}_1|)}{(q^2-S_1^2)(q^2-S_2^2)}\, dq\cdot\vec{k}. \tag{19}$$

Taking into account equation (13), it is found that S_1^2 and S_2^2 in equation (19) are given by two roots of the equation

$$(\varepsilon S^2+k_z^2\eta-\omega^2\mu_0\varepsilon\eta)(S^2+k_z^2-\omega^2\mu_0\varepsilon)+\omega^2\mu_0 g^2(S^2-\omega^2\mu_0\eta)=0. \tag{20}$$

Closing the contour in the lower half-plane of the complex q-plane and applying the residue theorem, equation (19) becomes

$$\vec{E}(\vec{r}) = E_e\overrightarrow{\overrightarrow{D}}e^{-jk_z z}\sum_{m=1,2}(-1)^m H_0^{(2)}(S_m|\vec{\rho}-\vec{\rho}_1|)\cdot\vec{k}, \tag{21}$$

where

$$E_e = \frac{I_0}{4\omega\varepsilon(S_2^2-S_1^2)}.$$

Using equations (14) and (15) for the dyadic operator $\overrightarrow{\overrightarrow{D}}$ in equation (21), three components of $\vec{E}(\vec{r})$ in rectangular coordinate system can be obtained. It is convenient, however, to describe the components of $\vec{E}(\vec{r})$ in the circular cylinder coordinate system (ρ, ϕ, z). Applying the addition theorem to the term $H_0^{(2)}(S_m|\vec{\rho}-\vec{\rho}_1|)$ in the case of $\rho>\rho_1$, three components can be calculated as

$$\left.\begin{aligned}
E_\rho &= E_e jk_z e^{-jk_z z}\sum_m^{1,2}\Big[(-1)^m\sum_{n=-\infty}^{\infty}\Big\{(F_m)S_m H_n^{(2)\prime}(S_m\rho) \\
&\quad -(G)\frac{jn}{\rho}H_n^{(2)}(S_m\rho)\Big\}J_n(S_m\rho_1)\,e^{jn(\phi-\phi_1)}\Big], \\
E_\phi &= E_e jk_z e^{-jk_z z}\sum_m^{1,2}\Big[(-1)^m\sum_{n=-\infty}^{\infty}\Big\{(F_m)\frac{jn}{\rho}H_n^{(2)}(S_m\rho) \\
&\quad +(G)S_m H_n^{(2)\prime}(S_m\rho)\Big\}J_n(S_m\rho_1)\,e^{jn(\phi-\phi_1)}\Big], \\
E_z &= E_e e^{-jk_z z}\sum_m^{1,2}\Big[(-1)^m S_m^2 k_z^2\frac{(F_m)}{(L_m)}\sum_{n=-\infty}^{\infty}H_n^{(2)}(S_m\rho)\,J_n(S_m\rho_1)\,e^{jn(\phi-\phi_1)}\Big],
\end{aligned}\right\} \tag{22}$$

where ρ_1 and ϕ_1 are circular cylinder coordinates of the source current and coefficients (F_m), (G) and (L_m) are written as

$$(F_m)=k_z^2+S_m^2-\omega^2\mu_0\varepsilon,$$
$$(G)=j\omega^2\mu_0 g,$$
$$(L_m)=S_m^2-\omega^2\mu_0\eta.$$

When the electric current is located at the origin, equation (22) becomes

$$\left.\begin{aligned} E_\rho&=E_e jk_z e^{-jk_z z}\sum_m^{1,2}(-1)^m(F_m)S_m H_0^{(2)\prime}(S_m\rho),\\ E_\phi&=E_e jk_z e^{-jk_z z}\sum_m^{1,2}(-1)^m(G)S_m H_0^{(2)\prime}(S_m\rho),\\ E_z&=E_e e^{-jk_z z}\sum_m^{1,2}(-1)^m S_m^2 k_z^2\frac{(F_m)}{(L_m)}H_0^{(2)}(S_m\rho).\end{aligned}\right\}\qquad(23)$$

V. Field from a magnetic current filament

Consider a filament which is carrying a progressive magnetic current $K_0\exp(-jk_z z+j\omega t)$, and is located along the z-axis of a cylindrical coordinate system. The magnetic current may be regarded as the limit, as the radius tends to zero, of a circular cylindrical electric current. Therefore the electric current density

$$\vec{J}(\vec{r})=I_0(-\sin\varphi'\vec{i}+\cos\varphi'\vec{j})\,e^{-jk_z z'}\,\delta(\rho'-\rho_1)\qquad(24)$$

is considered in order to express the magnetic current.

Substituting equation (24) into equation (4), performing the integration with respect to ρ', z', p_z and then p_x and p_y, after using the same transformation as in the last section for variables p_x and p_y, one obtains

$$\vec{E}(\vec{r})=\frac{I_0\rho_1}{4\omega\varepsilon(S_2^2-S_1^2)}\int_0^{2\pi}\vec{\vec{D}}\,e^{-jk_z z}$$
$$\cdot(-\sin\varphi'\vec{i}+\cos\varphi'\vec{j})\sum_m^{1,2}(-1)^m H_0^{(2)}(S_m|\vec{\rho}-\vec{\rho}_1|)\,d\varphi'.$$

Performing the integration with respect to φ' after applying the addition theorem for $H_0^{(2)}(S_m|\vec{\rho}-\vec{\rho}_1|)$, one obtains

$$\vec{E}(\vec{r})=\frac{2\pi I_0\rho_1}{4\omega\varepsilon(S_2^2-S_1^2)}\sum_m^{1,2}(-1)^m J_1(S_m\rho_1)\,\vec{\vec{D}}e^{-jk_z z}\cdot(-\sin\varphi\vec{i}+\cos\varphi\vec{j})H_1^{(2)}(S_m\rho).$$

In the limit of $\rho_1\to 0$, the above equation becomes

$$\vec{E}(\vec{r})=\frac{K_0}{4j\omega^2\mu_0\varepsilon(S_2^2-S_1^2)}\sum_m^{1,2}(-1)^m S_m\vec{\vec{D}}e^{-jk_z z}\cdot(-\sin\varphi\vec{i}+\cos\varphi\vec{j})H_1^{(2)}(S_m\rho),$$

where K_0 is the amplitude of magnetic current and by definition

$$K_0 = j\omega\mu_0\pi\rho_1^2 I_0.$$

Using equations (14) and (15), three components of $\vec{E}(\vec{r})$ in the circular cylinder coordinate system are obtained as

$$\left.\begin{aligned} E_\rho &= E_0 e^{-jk_z z} \sum_m^{1,2} (-1)^m S_m (L_m) H_0^{(2)\prime}(S_m \rho), \\ E_\phi &= E_0 e^{-jk_z z} \sum_m^{1,2} (-1)^m S_m \frac{(G)(L_m)}{(F_m)} H_0^{(2)\prime}(S_m \rho), \\ E_z &= E_0(-jk_z) e^{-jk_z z} \sum_m^{1,2} (-1)^m S_m^2 H_0^{(2)}(S_m \rho), \end{aligned}\right\} \quad (25)$$

where

$$E_0 = \frac{gK_0}{4\varepsilon(S_2^2 - S_1^2)}.$$

IV. Conclusion

The field from a current filament, which is placed along the z-axis taken in an anisotropic plasma medium, has been obtained and is shown by expressions (23) and (25) in cases of electric and magnetic current filaments respectively. It is easily found that the expressions (23) and (25) degenerate to those for the field from a current filament in an isotropic medium by putting as $\varepsilon = \eta$, $g = 0$ and $S_1 = S_2$.

Two propagation constants S_1 and S_2 in the radial direction may be calculated from equation (20). They are given in terms of ω_p, ω_c, ν and k_z. Whether or not radiation into the plasma medium occurs, will depend on these parameters.

Two sets of wave modes for two propagation constants S_1 and S_2 are obtained. They agree with those shown by Epstein[8]. The field from the current filament is constructed from the superposition of two sets of wave modes. Consider only the modes for the propagation constant S_1. Taking ratios of ρ-, ϕ- and z-components of electric field for the electric current filament to those for the magnetic one respectively, it is easily found from equations (23) and (25) that all three ratios are equal to $jk_z \frac{E_e}{E_o} \frac{(F_m)}{(L_m)}$, that is, two electric field vectors for electric and magnetic currents are parallel to each others. The same relation is also found between two electric field vectors for the other propagation constant S_2. It is noted that the fact mentioned above is rather different from the relation between two sets of the electric field vectors for the electric and magnetic current filaments in isotropic media.

Two sets of these modes are very useful for the calculation of scattering by a plasma column or plasma-clad metal cylinder at oblique incidence of the incident waves.

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