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The Radiation Pattern of a Parabolic-Reflector Antenna in the Lateral and Backward Directions*

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Summary

Radiation pattern of a parabolic antenna is calculated by means of geometrical method for diffraction. The far field in the lateral and rear directions is described in terms of the diffracted waves by the edges of the reflector in the first approximation. In the front direction the reflected wave is taken into account in addition to the diffracted waves, however, the former wave should be considered in rather different manner from the usual ray concept, because the field intensity is not uniform over the aperture plane of the reflector and therefore the ray may be spread from the limited region in the front. The parabolic reflector is regarded as the limit of many narrow ribbon plates. The field produced by this spread of the ray, which is calculated by the above consideration, becomes to small magnitude in the lateral or backward directions, therefore the field is expressed only by the diffracted waves by the edges. The computed results are compared with the experimental results obtained from a paraboloid reflector. A good agreement is obtained, particularly a valley be tween a main beam and a first sidelobe is fairly filled up.

I. Introduction

Far field patterns of a parabolic antenna have been calculated by the aperture distribution method, and in order to obtain a maximum gain and low side-lobe levels, an optimum aperture field distribution has been studied.[1] When the aperture distribution method, however, is applied to calculation of a field intensity in the lateral or rear direction of an antenna, it would not yield good agreements with experimental results.

Recently a new geometrical method for diffraction has been developped by J. B. Keller[2] and authors.[3),4] Results computed by this method show good agreements with results obtained by an exact solution or experiments in cases of a ribbon plate, a circular plate[2] and a corner reflector antenna.[5] Similarly, far field patterns of a parabolic antenna can be calculated by the geometrical method and the far field, except the front direction of the antenna, is given in terms of diffracted

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waves by edges of the parabolic reflector and a direct wave. In the front direction, reflected waves by the reflector are also considered.

II. Geometry of a parabolic antenna

Consider a cylindrical parabolic reflector AB infinitely long along the z-axis, which is perpendicular to the paper. The focal length and the width of the reflector are f and d respectively. A dipole source F, whose axis is parallel to the z-axis, is located at a point z' on the focus line of the parabolic reflector as shown in Fig. 1. Two straight edges of the reflector are labelled as A and B. and quantities concerning to these edges are respectively specified by subscripts A and B.

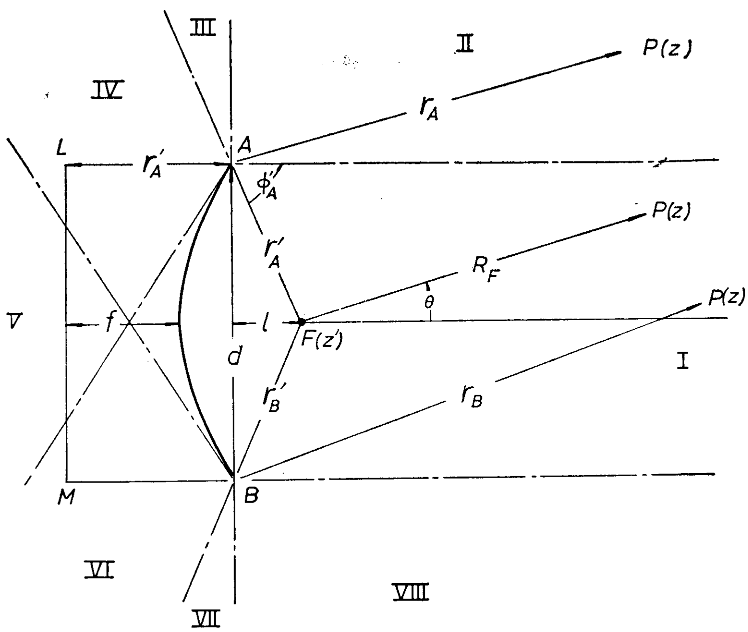

Fig. 1. Geometry of a parabolic antenna.

The waves reflected by the cylindrical surface behaves as if they come from sources distributed on the directrix LM of the parabola. Therefore images of the dipole may be considered on this line.

At an arbitrary point $P(z)$, the Hertzian vector of the direct wave from the dipole is described by

$$I_F = \frac{e^{-jkR_F}}{R_F}, \tag{1}$$

where $k=2\pi/\lambda$ (λ=wave length), a time dependence exp $(j\omega t)$ is omitted and R_F is a distance between F and P.

Using the geometrical method for diffraction, the whole space is divided into regions I, II, …… VIII (Fig. 1). An angle θ is measured from the front direction of the antenna.

III. Far field in the lateral and backward directions

By the geometrical method for diffraction, the field in the region II is given by the superposition of a direct wave and diffracted waves by edges A and B. Therefore the total Hertzian vector Π_z is

$$\Pi_z=I_F-I_{FA}-I_{FB}-I_{LA}+I_{MB}, \tag{2}$$

where I_F is given by (1) and

$$I_{FA}=\frac{e^{-jkR_{FA}}}{R_{FA}}f(T_{FA}),$$

$$R_{FA}^2=(r_A'+r_A)^2+(z-z')^2,$$

$$f(T)=\frac{1}{2}e^{jT^2}[\{1-C(T)-S(T)\}+j\{S(T)-C(T)\}],$$

$$=\frac{1}{2}e^{jT^2}\left[1-2e^{j\cdot\pi/4}\sqrt{\frac{1}{\pi}}T\right], \qquad T\ll 1 \tag{3}$$

$$=\frac{1}{2T}\sqrt{\frac{1}{\pi}}e^{-j\cdot\pi/4}, \qquad T\gg 1, \tag{4}$$

$$T_{FA}=\sqrt{2kr_A'}\left|\sin\frac{\pi-\phi_A'-\theta}{2}\right|,$$

r_A, r_A' and ϕ_A' are shown in Fig. 1. Functions $S(T)$ and $C(T)$ are Fresnel integrals. I_{FB}, I_{LA} and I_{MB} are expressed by the same form as I_{FA}. The Hertzian vectors in other regions are respectively given as follows : in the region III.

$$\Pi_z=I_F-I_{FA}-I_{LA}, \tag{5}$$

in the region IV,

$$\Pi_z=I_{FA}-I_{LA}, \tag{6}$$

in the region V,

$$\Pi_z=I_{FA}+I_{FB}-I_{LA}-I_{MB}. \tag{7}$$

The formulas for regions VI, VII and VIII are omitted. In (2), (5), (6) and (7), multiply diffracted waves and waves, which are reflected by the parabolic surface after diffraction by an edge, are omitted for a first approximation.

Using (4), because T_{LA}, T_{MB}, T_{FA} and T_{FB} are larger than unity in a lateral direction on the H-plane, (2) is rewritten for the z-component of electric field

$$E_z=k^2\frac{e^{-jkR_F}}{R_F}\Big[1-e^{-jk(r'+l\cos\theta)+j\cdot\pi/4}\frac{d}{\sqrt{\lambda r'}}\Big\{\sqrt{\frac{1+\cos\theta}{2}}\frac{\sin\left(\frac{\pi d}{\lambda}\sin\theta\right)}{\frac{\pi d}{\lambda}\sin\theta}$$

$$+\frac{\sin\left(\frac{\pi d}{\lambda}\sin\theta\right)\sin\frac{\phi'}{2}\sin\frac{\theta}{2}-j\cos\left(\frac{\pi d}{\lambda}\sin\theta\right)\cos\frac{\phi'}{2}\cos\frac{\theta}{2}}{\frac{\pi d}{\lambda}(\cos\phi'+\cos\theta)}\Big\}\Big], \tag{8}$$

where $r'=r_A'=r_B'$, $\phi'=\phi_A'=\phi_B'$ and the length l is shown in Fig. 1. The first term in the curly bracket of (8) coincides with the expression obtained by the aperture distribution method, except the power of the factor $\left(\frac{1+\cos\theta}{2}\right)$. The second term is a contribution of diffracted waves which are produced when the direct waves from the dipole hit two edges respectively. Although (8) is the expression in case of $T_{LA}\gg 1$ and $T_{MB}\gg 1$, it may be extended to the case of $T_{LA}\ll 1$ and $T_{MB}\ll 1$, i.e. to the expression in the direction of $\theta\doteqdot 0$. Using (5), (6) and (7), the electric field in other directions is calculated respectively.

For a paraboloid reflector, the field in the H-plane may be approximately expressed by the same form, because the contribution of the diffracted waves in the H-plane is limited only to those by two parts where the circular edge intersects the H-plane, and these diffracted waves may be very similar to those by the edges of the cylindrical reflector.

The direct wave from a dipole is usually suppressed in an antenna design. Neglecting the direct wave in (8), radiation pattern in the H-plane is computed as shown in Fig. 2, and this result is compared with experimental one.[1] The dimensions of this antenna are given in the title of Fig. 2. Although the feed device of the experiment is a double dipole, the agreement is very good. Particularly a valley between the main beam and the first side-lobe is fairly filled up because of the contribution of diffracted waves from the dipole.

IV. Field in the front direction

An image of the dipole F with respect to the parabolic reflector can not be put at infinity, because the intensity of reflected waves is not uniform over the aperture of the reflector. Therefore images are distributed on the directrix and reflected waves may not propagate only in the front direction but spread over other directions. This fact carries a difficulty for a treatment of the usual geometrical method.

A parabolic reflector AB is approximated by three ribbon plates AC, CD and DB as shown in Fig. 3. Three images L, O and M of the dipole F with respect to three ribbon plates are considered on the directrix of the parabola. Fixing our

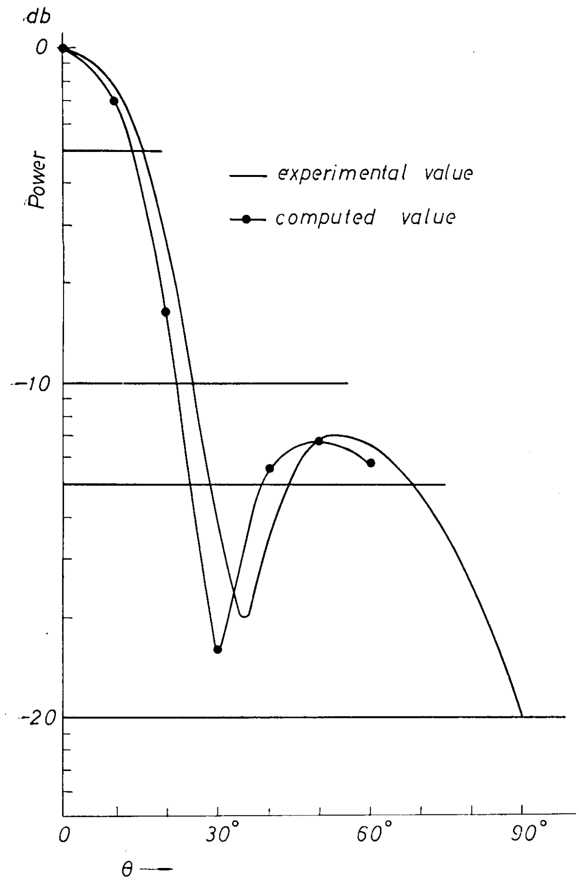

Fig. 2. Comparison of computed and experimental values for the H-plane radiation pattern of a paraboloid antenna. Diameter of paraboloid 8″, focal length 2″. Experimental values after reference 1.

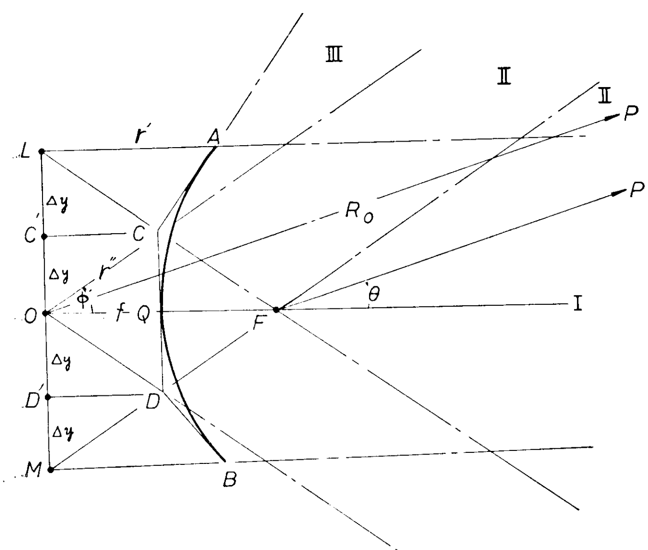

Fig. 3. Approximation of a parabolic reflector by three ribbon plates. $LC'=C'O=OD'=D'M=\Delta y$, $FP=R_F$, $OP=R_0$, $OQ=f$, $OC=r''$, $LA=r'$, $AB=d$.

attention only on the waves from these images, the Hertzian vector in the direction $\theta=0$ is described by

$$\Pi_z=-(I_O+I_L+I_M-I_{OC}-I_{OD}-I_{LC}-I_{MD}-I_{LA}-I_{MB}), \tag{9}$$

where I_O, I_{OC}, …… are given in the previous sections. Assuming that Δy, shown in Fig. 3, is small and then T_{OC}, T_{OD}, …… are small as compared with unity, (9) is transformed as

$$\Pi_z=-\frac{e^{-jkR_O}}{R_O}e^{j\cdot\pi/4}\frac{1}{\sqrt{\pi}}(T_{OC}+T_{OD}+T_{LC}+T_{LA}+T_{MD}+T_{MB}). \tag{10}$$

Because $T_{OC}=T_{OD}=T_{LC}=T_{MD}\doteqdot\sqrt{\frac{\pi}{f\lambda}}\Delta y$ and $T_{LA}=T_{MB}=0$, (10) is rewritten as

$$\Pi_z=-\frac{e^{-jkR_o}}{R_o}e^{j\cdot\pi/4}\frac{d}{\sqrt{f\lambda}}, \tag{11}$$

In the region III, the Hertzian vector is

$$\Pi_z=-(I_{OC}-I_{OD}+I_{LA}-I_{LC}+I_{MD}-I_{MB}). \tag{12}$$

Assuming that T_{OC}, T_{OD}, …… are small, (12) is calculated as

$$\Pi_z=-\frac{e^{-jkR_o}}{R_o}e^{j\cdot\pi/4}\frac{d}{\sqrt{f\lambda}}\sqrt{\frac{1+\cos\theta}{2}}. \tag{13}$$

In the case of large T_{OC}, T_{OD}, …… (this corresponds to the lateral direction),

$$\Pi_z=-\frac{e^{-jkR_F}}{R_F}\sqrt{\frac{1+\cos\theta}{2}}\frac{e^{j\cdot\pi/4}}{\sqrt{\lambda}}\left[\frac{1}{\sqrt{r'}}e^{-jk(r'+l\cos\theta)}\frac{\sin\left(\frac{\pi d}{\lambda}\sin\theta\right)}{\frac{\pi d}{\lambda}\sin\theta}d\right.$$
$$\left.-j\sqrt{\frac{1}{r''}}\frac{e^{-jk(r''+f\cos\theta)}\cos(k\Delta y\sin\theta)}{\frac{\pi}{\lambda}(1-\cos\theta)}\frac{\Delta y}{r''}\right]. \tag{14}$$

An increase in the number of ribbon plates makes this ribbon plates construction approach to the parabolic reflector. As a result the summation in (10) changes to the integration, and the Hertzian vector in the front direction is obtained as

$$\Pi_z=-\frac{e^{-jkR_o}}{R_o}e^{j\cdot\pi/4}\frac{1}{\sqrt{f\lambda}}\int_{-d/2}^{d/2}\frac{dy}{\sqrt{1+\frac{y^2}{4f^2}}} \tag{15}$$

and in the lateral direction

$$\Pi_z=-\frac{e^{-jkR_F}}{R_F}\sqrt{\frac{1+\cos\theta}{2}}\frac{e^{j\cdot\pi/4}}{\sqrt{r'}}\left[\frac{1}{\sqrt{r'}}e^{-jk(r'+l\cos\theta)}\frac{\sin\left(\frac{\pi d}{\lambda}\sin\theta\right)}{\frac{\pi d}{\lambda}\sin\theta}d\right.$$
$$\left.-j\,e^{-jkf(1+\cos\theta)}\frac{\lambda}{\pi(1-\cos\theta)}\int_0^{d/2}\frac{e^{-jkx(1-\cos\theta)}}{(1+x)^{3/2}}\cos(ky\sin\theta)\,dy\right], \tag{16}$$

where $y^2=4fx$.

The first term in the bracket of (14) and (16) corresponds to the first term in curly bracket of (8), and it is easily found that the second term of (16) means the spread of the reflected waves caused by the parabolic reflector. If x is a small length in comparison with f, the second term is integrated and this contribution in (16) is the order of λ/r' of the first term. When (8) is extended to $\theta \fallingdotseq 0$, the first term in (8) becomes to the expression very close to (11), (13) and (15). Therefore (8) is considered to show the field intensity in the H-plane of the regions I and II as a first approximation.

V. Conclusion

The spread of the reflected waves by the parabolic reflector, which has not been considered in the usual geometrical optics, is calculated. The radiation pattern in the front and lateral direction of a parabolic antenna is obtained in terms of the diffracted waves by two edges and the spread of the reflected waves. It is shown that the far field can be approximately computed by the contribution of the former waves.

When a dipole is put at a point different from the focus of a parabolic reflector, the field can be also calculated in the similar way. In the usual parabolic antenna design, there are many kind of feed devices, i.e. horn feed, double dipole feed, etc.. In these cases a primary field from a feed is analyzed into fields from few dipoles which are distributed around the focus. Radiation field produced by the reflector and each dipole are superposed and the real radiation patterns can be computed.

In the previous analysis the waves reflected by the reflector after diffraction by an edge are omitted, however, the magnitude of these waves should be calculated exactly.

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