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Abstract	In recent years, the Helix properties are analized by solving Maxwell's equation, but the determination of the terminal condition is very complicated. Usually we use the Helix combined with other microwave circuits, such as wave-guides or coaxial cables. So, it is important that terminal impedance should be defined. The helix has a continuous and a periodical properties. In this paper, two approximation methods are discussed. One is the distributed-line method, of which equivalent circuit has a continuous property, but not periodical. The other is the lumped-circuit method, of which equivalent circuit has the contrary property to the distributed-line. The Helix properties would not be perfectly represented by both methods but characteristic impedance, propagation constant and terminal condition can be easily determined by these approximation methods.
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Theoretical study of the Electromagnetic Wave Propagation and Termination Concerning the Helical Type Delay Line

(Received November 20, 1962)

Hiroichi FUJITA*
Hiroshi KASHIWAGI**

Summary

In recent years, the Helix properties are analized by solving Maxwell's equation[1)-3)], but the determination of the terminal condition is very complicated. Usually we use the Helix combined with other microwave circuits, such as wave-guides or coaxial cables. So, it is important that terminal impedance should be defined.

The helix has a continuous and a periodical properties. In this paper, two approximation methods are discussed. One is the distributed-line method, of which equivalent circuit has a continuous property, but not periodical. The other is the lumped-circuit method, of which equivalent circuit has the contrary property to the distributed-line.

The Helix properties would not be perfectly represented by both methods but characteristic impedance, propagation constant and terminal condition can be easily determined by these approximation methods.

I. Introduction

The Helix used in various H. F. devices, for example T. W. T[1)], antenna[4)], cavity, filter and wave-guide.

The purpose of this paper is to discuss the case of delay circuit. ($\lambda \gg$ circumference of the Helix, i.e. quasi-stational condition is satisfied in 1 turn of the Helix.) This condition is satisfied circuit constants could be defined.

At the end of the Helix, the caluculations to get these propaties are complicated, because:

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1) J. R. Pierce, "Traveling-Wave Tubes" D. Van. Nostrand Co., Inc.. New York N. Y.; 1950.
2) C. C. Cutler, Proc. I. R. E. Vol. 36, pp. 230-233, Feb. 1948.
3) S. Sensiper, Proc. I. R. E. Vol. 43, pp. 149-161, Feb. 1955.
4) J. D. Kraus, "Antenna" Mc. Graw-Hill Book Co. Inc., New York. N. Y.; 1950.

1. The Mutual Inductance and the Mutual Capacitance (in this paper, axial component of the capacitance is called the Mutual Capacitance) are distributed not symmetrically.
2. The Self Capacitance (ordinate component of the capacitance is called the Self Capacitance) of the Helix increases nearer to the end.
3. The multi-reflection waves raise at the vicinity of the end.

II. Determination of the characteristic impedance and the propagation constant

Approximation by the infinite line with distributed parameters has symmetric distribution of the Mutual Inductance and the Mutual Capacitance.

II · 1. Fundamental equations

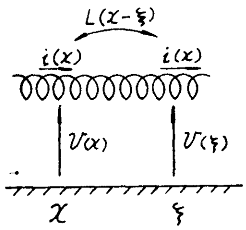

Fig. 1.
Model of the Helix.

The model of the Helix is shown in Fig. 1.

The current at the point ξ induces the voltage at the point x, as follows.

$$v_{(x)} = L\,(x-\xi)\,\frac{\partial i(\xi,t)}{\partial t} \tag{1-1}$$

Where $L(x-\xi)$ is the Mutual Inductance $[H/m^2]$, between the point x and the point ξ, $L(x-\xi)$ decreases with the distance $|x-\xi|$ the Mutual Inductance is an even function of it. Therefore, the induced voltage at the point x is given by an integration of Eq. (1-1) from $-\infty$ to ∞,

$$-\frac{\partial v_{(x)}}{\partial x} = \int_{-\infty}^{\infty} L\,(x-\xi)\,\frac{\partial i(\xi,t)}{\partial t}\,d\xi \tag{1-2}$$

As for the current along the axis, when we consider the electro-static capacity $K\,(x-\xi)$, the ordinate displacement current I_k is given by

$$I_k = \int_{-\infty}^{\infty} K\,(x-\xi)\,\frac{\partial\{v_{(x)}-v_{(\xi)}\}}{\partial t}\,d\xi \tag{1-3}$$

Where $K(x-\xi)$ decreases with the distance $|x-\xi|$ and it is an even function of it. At the point x, abscissa current through the self capacitance C_0 is

$$I_c = C_0\,\frac{\partial v_{(x)}}{\partial t} \tag{1-4}$$

hence, the total current varing with distance is given by following equation.

$$-\frac{\partial i(x)}{\partial x} = C_0\frac{\partial v_{(x)}}{\partial t} + \int_{-\infty}^{\infty} K\,(x-\xi)\left\{\frac{\partial v_{(x)}}{\partial t} - \frac{\partial v_{(\xi)}}{\partial t}\right\}d\xi \tag{1-5}$$

Eq. (1-2) and Eq. (1-5) are the fundamental equation of the line.

II・2. Solution of the equation

Before the caluculation of the fundamental equation, we set up the following assumption;

i) $i(x,t)$, $v(x,t)$ sinusoidally varies with time $(e^{j\omega t})$
ii) $i(x,t)$, $v(x,t)$ varies with distance. $(e^{-\gamma x})$
iii) Definition of the characteristic impedance $Z_0(x)$ is shown

$$Z_0(x)=\frac{\partial v_{(x)}}{\partial i(x)} \tag{1-6}$$

where $Z_0(x)$ is the function of the distance x. For the usual line, the characteristic impedance Z_0 has a constant value, but when the distribution of impedance is dissymmetrical, Z_0 varies with distance.

iv) The properties of the mutual inductance $L(x-\xi)$ and Mutual capacifance $K(x-\xi)$ are shown in Fig. 2. and the followings.

a) Mutual Inductance

$$L(\eta)=M_0e^{-a\eta} \quad \text{for} \quad \eta=\xi-x>0$$
$$L(\eta)=L_0\delta(\eta) \quad \text{for} \quad \eta=0$$
$$L(\eta)=M_0e^{a\eta} \quad \text{for} \quad \eta<0$$

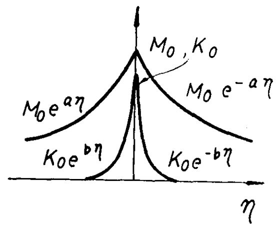

Fig. 2. Distribution of $L(x-\xi)$ and $K(x-\xi)$

where δ is Delta function. Physical meanings of these functions are as follows from the Neuman's formula at $\eta=0$, $L(0)=\infty$ then right hand of Eq. (1-2) is

$$j\omega L(x-\xi)\, i(x)\rightarrow\infty,$$

Delta function must be used for this reason.

b) Mutual Capacitance

$$K(\eta)=K_0e^{-b\eta} \quad \text{for} \quad \eta>0$$
$$K(\eta)=K_0e^{b\eta} \quad \text{for} \quad \eta<0$$

Physical meanings of $K(\eta)$ are as follows, $K(\eta)$ has an infinite value at $\eta=0$, but in Eq. (1-5), $\{v_{(x)}-v_{(\xi)}\}$ tends to zero, then the products of them will have a finite value.

Substituting the above assumption into Eqs. (1-2) and (1-5)

$$\gamma Z_0e^{-\gamma x}=j\omega\int_{-\infty}^{\infty} L(x-\xi)e^{-\gamma\xi}\,d\xi \tag{1-7}$$

$$\gamma e^{-\gamma x}=j\omega C_0Z_0e^{-\gamma x}+j\omega Z_0\int_{-\infty}^{\infty} K(x-\xi)e^{-\gamma x}\,d\xi-j\omega Z_0\int_{-\infty}^{\infty} K(x-\xi)e^{-\gamma\xi}\,dx \tag{1-8}$$

Multiplying $e^{\gamma x}$, then eqs. (1-7) and (1-8) are

$$\gamma Z_0 = j\omega \int_{-\infty}^{\infty} L(x-\xi)\, e^{-\gamma(\xi-x)}\, d\xi = j\omega\, L(\gamma) \tag{1-9}$$

$$\gamma = j\omega Z_0 \{C_0 + \int_{-\infty}^{\infty} K(x-\xi)\, d\xi - K(\gamma)\} \tag{1-10}$$

where $L(\gamma)$ and $K(\gamma)$ are Fourier Transformations of $L(x-\xi)$ and $K(x-\xi)$. Eliminate Z_0, then from Eqs. (1-9) and (1-10) then

$$\gamma^2 = -\omega^2 L(\gamma)[C_0 + \int_{-\infty}^{\infty} K(x-\xi)\, d\xi - K(\gamma)] \tag{1-11}$$

The above equation is the determinantial equation of the propagation constant γ, and from Eq. (1-9)

$$Z_0 = \frac{1}{\gamma} j\omega\, L(\gamma)$$

$L(\gamma)$, $K(\gamma)$ are determined by the assumption (iv) as follows.

$$L(\gamma) = L_0\left\{1 + \frac{M_0}{L_0}\frac{2a}{a^2-\gamma^2}\right\} \tag{1-13}$$

$$K(\gamma) = \frac{2bK_0}{b^2-\gamma^2} \tag{1-14}$$

and

$$\int_{-\infty}^{\infty} K(\eta)\, d\eta = \frac{2K}{b} \tag{1-15}$$

Substituting the above relation into Eqs. (1-11) and (1-12), then

$$\gamma^2 = -\left(\frac{\omega}{\omega_0}\right)^2 \left\{1 + k\frac{2a}{a^2-\gamma^2}\right\}\left\{1 - \frac{2K\gamma^2}{b(b^2-\gamma^2)}\right\} \tag{1-16}$$

where

$$\omega_0{}^2 = 1/L_0C_0, \quad k = M_0/L_0, \quad K = K_0/C_0$$

Usually $b^2 \gg \gamma^2$; γ is pure imaginary or pure real number. hence

$$2K\gamma^2/b(b^2-\gamma^2) \simeq 2K\gamma^2/b^3 \tag{1-17}$$

solving the Eq. (1-16) for the propagation constant γ. it will be shown as follows

$$\gamma^2 = \frac{1}{2(1-2\Omega^2 K_0')}[a^2 - \Omega^2\{1 - 2K_0' a(a+2k)\} \pm \sqrt{[a^2-\Omega^2\{1-2K_0' a(a+2k)\}]^2 + 4\Omega^2 a(a+2k)(1-2K_0'\Omega^2)} \tag{1-18}$$

where $\Omega = \omega/\omega_0, \quad K_0' = K/b^3$

For

$$\Omega^2<\frac{1}{2K_0}\qquad\left(\omega<\sqrt{\frac{b^3}{2L_0K_0}}\right)$$

$$a^2-\Omega^2\{1-2K_0'a(a+2k)\}<\sqrt{[a^2-\Omega^2\{1-2K_0'a(a+2k)\}]^2+4\Omega^2a(a+2k)(1-2K_0'\Omega^2)}$$

there are two waves. The one is a propagation wave, the other is an attenuation wave.

For

$$\Omega^2>\frac{1}{2K_0'}\qquad\left(\omega>\sqrt{\frac{b^3}{2L_0K_0'}}\right)$$

$$a^2-\Omega^2\{1-2K_0'a(a+2k)\}>\sqrt{[a^2-\Omega^2\{1-2K_0'a(a+2k)\}]^2+4\Omega^2a(a+2k)(1-2K_0'\Omega^2)}$$

Moreover the denominator of Eq. (1–18) is negative, So, there are only attenuation waves. On the other hand, from the Eq. (1–12) the characteristic impedance is determined as follows:

$$Z_0^2=-\omega^2L_0^2\frac{1}{\gamma^2}\left\{1+\frac{M_0}{L_0}\frac{2a}{a^2-\gamma^2}\right\}\tag{1–19}$$

Eqs. (1–18) and (1–19) are determinantial equations of the propagation constant γ and charcteristic impedance Z_0. These properties are shown in Figs. 3 and 4. It is evident that the charcteristic impedance Z_0 is a function of the frequency. This result will be useful for the termination.

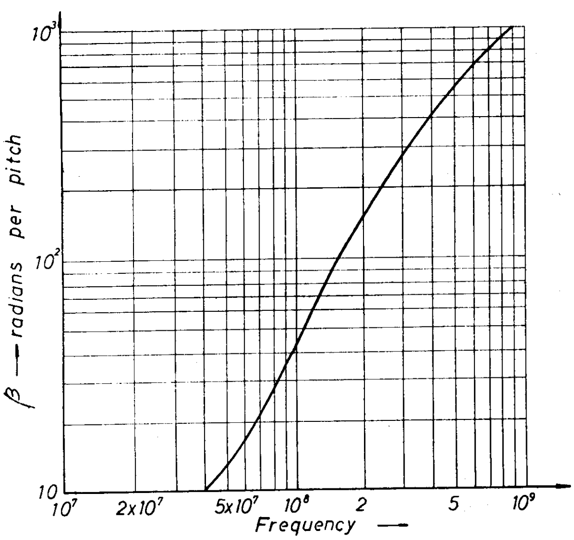

Fig. 3. Frequency characteristics of the Phase Constant β.

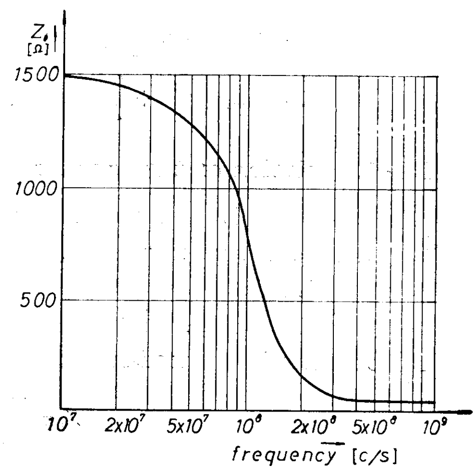

Fig. 4. Frequency characteristics of the characteristic impedance properties.

III. Approximation by lumped constant circuits

III・1. Fundamental equation

Equivalent circuits are shown in Fig. 5.

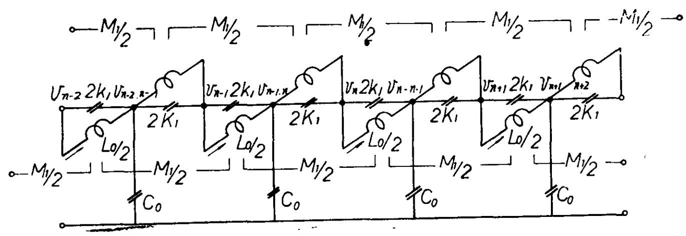

Fig. 5. Equivalent Circuits of the Helix.

In Fig. 5, the magnetic field on the Helix is represented by inductances, and the electric field is represented by electrostatic capacitances, where self-inductance L_0 and self-capacitance C_0 are indicated by vertical components of magnetic field and electric field. Similarly, mutual inductances and mutual capacitances are represented by the axial components of magnetic field and electric field. The value of L_0 and C_0, is given by every turn of the Helix and M_1 and K_1 is given by the distance between neighbore-turns and their shape.

Kirchhoff's equations for the n-th junction and the n-th loop will be described as follows:

$$i_n+2K_1\frac{d}{dt}\{v_n-v_{n,n+1}\}=i_{n+1}+C_0\frac{d}{dt}v_{n,n+1}+2K_1\frac{d}{dt}(v_{n,n+1}-v_{n+1}) \tag{2-1}$$

$$v_n=v_{n,n+1}+\frac{L_0}{2}\frac{d}{dt}i_n+\frac{M_1}{2}\frac{d}{dt}(i_{n-1}+i_{n+1}) \tag{2-2}$$

$$v_{n,n+1}=v_{n+1}+\frac{L_0}{2}\frac{d}{dt}i_{n+1}+\frac{M_1}{2}\frac{d}{dt}(i_n+i_{n+2}) \tag{2-3}$$

III・2. Solution of the equations

In order to solve the equations following properties are assumed:

1) Current and voltage varies
   with time, $(e^{j\omega t})$
   with section. $(e^{2\gamma n}$ $(n=\text{integer}))$
2) Characteristic impedance (internal impedance) is defined as follows,

$$Z_0\equiv\frac{V_n}{I_n}=\frac{V_0e^{j\omega t-2\gamma n}}{I_0e^{j\omega t-2\gamma n}} \tag{2-4}$$

Substituting the assumption 1), 2) into the fundamental equations, then following results are obtained:

i) $\omega\neq 0$

$$\cosh 2\gamma=\frac{1}{4\Omega^2k\kappa}[[\Omega^2\{k+\kappa(2k-1)\}+1]$$
$$\pm\sqrt{[\Omega^2\{k+\kappa(2k-1)\}+1]^2+4\Omega^2\kappa k[\Omega^2(1+2\kappa)-2]}] \tag{2-5}$$

ii) $\omega=0$

$$\cosh 2\gamma=1 \tag{2-6}$$

iii) ω tends to infinity; in this case, Eq. (5) has a limited value,

$$\cosh 2\gamma=\frac{1}{4k\kappa}\left[\{k+\kappa(2k-1)\}\pm\sqrt{\{k+\kappa(2k-1)\}^2+4\kappa k(1+2\kappa)}\right] \tag{5-7}$$

The characteristic impedance is given by following equation:

$$Z_0{}^2=\frac{L_0}{C_0}\left[\left\{1-\Omega^2\frac{1+4\kappa}{4}\right\}+k\{2-\Omega^2(1+4\kappa)\cosh 2\gamma\}-k^2\Omega^2(1+4\kappa)\cosh^2 2\gamma\right] \tag{2-8}$$

In the Eq. (5), the right hand has a value which is larger than 1. So γ is a real number, and there are attenunation waves only.

If $\cosh 2\gamma$ has a value between -1 and 1, γ is an imaginary number, and so there are propagation waves. When $\cosh 2\gamma$ is equal to 1, this frequency is called cutoff frequency.

Let, $k=0$, $K=0$. This is the wellknown L_0, C_0—ladder network. If $k\neq 0$ $K=0$, this is called Pierce's delay circuit.

The properties of the propagation constant and the characteristic impedance are shown in Figs. 6, and 7.

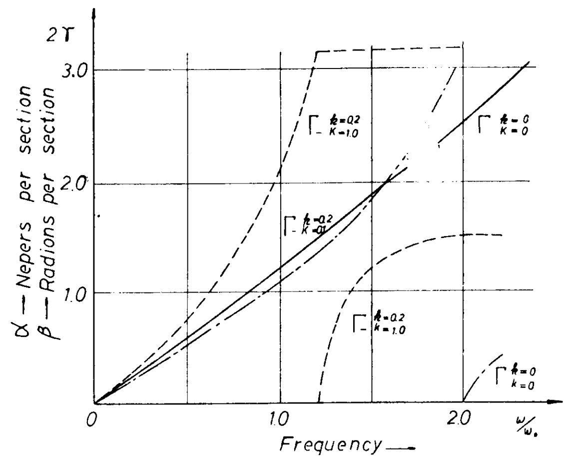

Fig. 6. Property of the propagation constant.

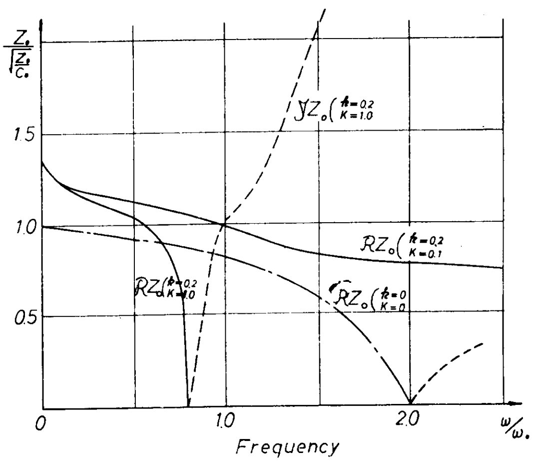

Fig. 7. Property of the Characteristic impedance.

IV. Determination of the input impedance

In the foregoing paragraph, the line has an infinite length, but in this section we will analyze semi-infinite distribution line (0 to ∞) and its input impedance.

IV · 1. Fundamental equations

The fundamental equation is led by substituting $C_{(x)}=C_0+D_0e^{-qx}$ into C_0. In Eq. (1–15), $C_{(x)}$ is the selfcapacitance about the Helix end. In this case the integrand is from 0 to ∞, hence

$$-\frac{\partial v_{(x)}}{\partial x}=\int_0^\infty L(x-\xi)\frac{\partial i(\xi,t)}{\partial t}d\xi \tag{4–1}$$

$$-\frac{\partial i(x)}{\partial x}=C_0\frac{\partial v_{(x)}}{\partial t}+D_{(x)}\frac{\partial v_{(x)}}{\partial t}+\int_0^\infty K(x-\xi)\frac{\partial v_{(x)}}{\partial t}d\xi-\int_0^\infty K(x-\xi)\frac{\partial v_{(\xi)}}{\partial t}d\xi \tag{4–2}$$

IV · 1. Solution of the equations

In respect of the equation, we set up the following assumptions;

i) $i\ (x,t)$ and $v\ (x,t)$ sinusoidally varies with the time. $(e^{j\omega t})$

ii) As the wave properties are unknown, the propagation constant is set γ. The aspect of the propagation is $e^{-\gamma x}$. So, the equation are solved by Laplace transformation.

From that formula, left hand of Eq. (4–1) is

$$-[\gamma V_{(\gamma)}-V_{(0)}]$$

where $V_{(\gamma)}$ is Laplace transformation of the function $V_{(x)}$

$V_{(0)}$ is voltage at the point $x=0$,

then right hand of Eq. (3–1) is

$$\int_0^\infty e^{-\gamma x}\,dx\int_0^\infty L(x-\xi)\ i(\xi)\ d\xi$$

And set $e^{-\gamma x}$ in the integrand of ξ,

$$\int_0^\infty\int_0^\infty L(x-\xi)\ i(\xi)\ e^{-\gamma x}\,d\xi\,dx$$

The procedure of integral can be changed as follows.

$$\int_0^\infty i(\xi)\,d\xi\int_0^\infty L(x-\xi)\ e^{-\gamma x}\,dx$$

Considering that the property of $L\ (x-\xi)$ is exponentially distributed (reff. §1–2, assumption iv), above equation leads to the following equation.

$$L_0\left[\left(1+\frac{2a}{a^2-\gamma^2}\right)I_{(\gamma)}-\frac{1}{a-\gamma}I_{(a)}\right]$$

Where $I_{(\gamma)}$ is Laplace transformation of the function $i(x)$, and $I_{(a)}$ is the value in the case of $\gamma=a$ in $I_{(\gamma)}$, then $I_{(a)}$ is a constant value.

Accordingly, Laplace transformation of Eq. (4-1) is

$$-[\gamma V_{(\gamma)}-v_{(0)}]=j\omega L_0\left\{\left(1+\frac{2a}{a^2+\gamma^2}\right)I_{(\gamma)}-\frac{1}{a-\gamma}I_{(a)}\right\} \tag{4-3}$$

Similarly, Laplace transformation of Eq. (4-2) is as follows

$$-[\gamma I_{(\gamma)}-i_{(0)}]=j\omega\left[\left\{\left(C_0+\frac{2K_0}{b}\right)-\frac{2bK_0}{b^2-\gamma^2}\right\}V_{(\gamma)}+D_0 V_{(q+\gamma)}\right.$$

$$\left.-\frac{K}{b}V_{(\gamma+b)}+\frac{K}{b-\gamma}V_{(b)}\right] \tag{4-4}$$

Eq. (4-4) contains $V_{(q+\gamma)}$ and $V_{(\gamma+b)}$, then this is a finite difference equation. It is difficult to find the values of $I_{(\gamma)}$, $V_{(\gamma)}$ hence the characterstic impedance is determined as follows,

$$Z_{(0)}=\lim_{x\to 0}\frac{v_{(x)}}{i(x)} \tag{4-5}$$

Inverse transformation of Eq. (4-3) is

$$v_{(x)}=v_{(0)}-j\omega L_0\left[\mathcal{L}^{-1}\left\{\frac{1}{\gamma}+\frac{2a}{\gamma(a^2-\gamma^2)}\right)I_{(\gamma)}\right\}-\mathcal{L}^{-1}\left\{\frac{1}{\gamma(a-\gamma)}I_{(a)}\right\}\right]$$

$$=v_{(0)}-j\omega L_0\int_0^x f_1(x-\eta)\,i(\eta)\,d\eta-\frac{1}{a}(1-e^{ax})I_{(a)}\Big\} \tag{4-6}$$

where

$$f_1(x)=\mathcal{L}^{-1}\left\{\frac{1}{\gamma}+\frac{2a}{(a^2-\gamma^2)\gamma}\right\}$$

$$=\left(\frac{2}{a}+1\right)u_{-1}(x)-\frac{1}{a}(e^{-ax}+e^{ax})$$

and $u_{-1}(x)$ is Unit Step.

Similarly, $i_{(x)}$ is obtained from Eq. (4-4)

$$i_{(x)}=i_{(0)}-j\omega K\left[\int_0^x g_1(x-\eta)\,v(\eta)\,d\eta+\frac{D_0}{K_0}\int_0^x u_{-1}(x-\eta)\,v(\eta)\,e^{-q\eta}\,d\eta\right.$$

$$\left.-\frac{1}{b}\int_0^x u_{-1}(x-\eta)v(\eta)e^{-b\eta}d\eta+\frac{1}{b}(1-e^{bx})\,V_{(b)}\right] \tag{4-7}$$

then

$$Z_{(0)}=\lim_{x\to 0}\frac{L_0}{K_0}\Big\{\int_0^x f_1(x-\eta)\,i(\eta)\,d\eta-\frac{1}{a}(1-e^{ax})I_{(a)}\Big\}$$

$$\div\Big\{\int_0^x g_1(x-\eta)\,v(\eta)\,d\eta-\frac{1}{b}\int_0^x u'_{-1}(x-\eta)\,v(\eta)\,e^{-b\eta}\,d\eta$$

$$+\frac{D_0}{K_0}\int_0^x u_{-1}(x-\eta)\,e^{-q\eta}\,d\eta+\frac{1}{b}(1-e^{bx})\,V_{(b)}\Big\} \tag{4-8}$$

As Eq. (4-8) is the indefinite form, then we differentiate Eq. (4-8), and let $x=0$.

$$Z_{(0)}=\frac{L_0}{K_0}\,(i_{(0)}+I_{(a)})\Big/\Big\{\Big(\frac{C_0+D_0}{K_0}+\frac{1}{b}\Big)v_{(0)}-V_{(b)}\Big\} \tag{4-9}$$

Where $i\,(x)$, $v_{(x)}$ is unknown, then, $I_{(a)}$, $V_{(b)}$ will be determined by the approach.

$$V_{(b)}=\int_0^\infty v_{(\xi)}\,e^{-b\xi}\,d\xi \tag{4-10}$$

When the distance is large enough, $v_{(\xi)}$ vanishes.

As b is large enough, $e^{-b\xi}$ is influenced by the vicinity of the origin, then

$$V_{(b)}\doteqdot\int_0^\infty v_{(0)}\,e^{-b\xi}d\xi=\frac{v_{(0)}}{b} \tag{4-11}$$

On the other hand a is not so large, that the similar way can not be used for the $I_{(a)}$. When x tends to zero, integral vanishes, if the integrand tends to zero, then this integral is neglected as a higher order of zero.

Therefore in the vicinity of the point $x=0$, Eq. (4-7) are

$$i_{(x)}=i_{(0)}-j\omega K_0\Big[\frac{C_0}{K_0}x+\frac{D_0}{K_0}\frac{1}{q}(1-e^{-qx})-\frac{1}{b^2}(1-e^{-bx})+\frac{1}{b^2}(1-e^{bx})\Big]v_{(0)} \tag{4-12}$$

and

$$I_{(a)}=\int_0^\infty i_{(x)}\,e^{-ax}\,dx$$

$$=\frac{i_{(0)}}{a}-j\omega K_0\Big[\frac{C_0}{K_0}\frac{1}{a^2}+\frac{D_0}{K_0}\frac{1}{a(a+q)}-\frac{2}{b(a^2-b^2)}\Big]Z_{(0)}\cdot i_{(0)} \tag{4-13}$$

substitute Eqs. (4-11) and (4-13) in Eq. (4-9), then

$$Z^2_{(0)}+2\omega\xi\,Z_{(0)}-\eta=0 \tag{4-14}$$

where

$$\xi=\frac{L_0}{2(C_0+D_0)}\Big[\frac{C_0}{a^2}+\frac{D_0}{a(a+q)}-\frac{2K_0}{b(a^2-b^2)}\Big]$$

$$\eta=\frac{L_0}{C_0+D_0}\Big\{1+\frac{1}{a}\Big\}$$

Therefore,

$$Z_{(0)} = -j\omega\xi - \sqrt{\eta - \omega^2\xi^2} \tag{4-15}$$

In Eq. (4–15), imaginary part is always negative, because $a \ll b$, hence input impedance of the Helix is capacitive.

When we take the termination with two terminal elements, the conjugate value of Eq. (4–15) should be used.

In Fig. 8 is shown the frequency characteristics of the input impedance.

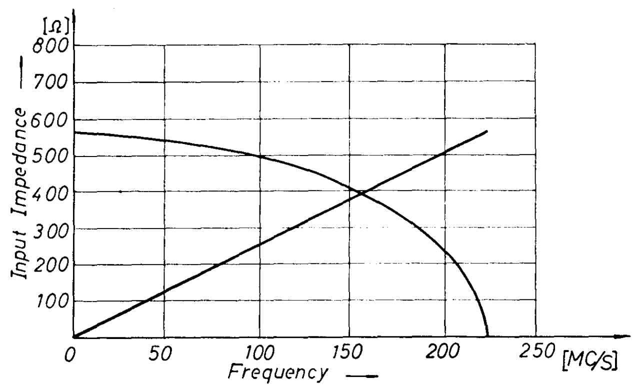

Fig. 8. Frequency characteristics of the input impedance.

V. Conclusion

We have discussed the Helix properties by the equivalent circuits. This is a good approach for the propagation constant and the characteristic impedance in a fair approximation.

We have caluculated the terminal impedance, which is capacitive value, but it is not satisfied nonreflection condition. Also the multi-terminal termination is necessary, it is very difficult to be determined under our caluculation.

In the next report, it will be explained the nonreflection termination in using the lumped constant network.

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