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Author	水野, 正夫(Mizuno, Masao)
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The Effect of Number of Pitches of Test Pieces on the Fatigue Life of Roller-Chains *

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Masao MIZUNO**

Abstract

The anthor intended to explain the relations between the strength of machine-elements and the strength of their units by means of the statistical method. The theoritical results gave a considerably good fit to the observed values, in spite of a small number of observations, in the case of fatigue life of roller-chains.

I. Introduction

There are many machine-elements and machines which are composed of a multiple numbers of identical units, for example riveted joints, coiled springs, gears, ball and roller-bearings, chains and chain-sprockets, internal combustion engines with multiple cylinders and turbine rotors with many identical blades etc., and the fatigue tests of which are generally done with some of the units of the whole elements. But, from the view point of practical strength of these machine-elements and machines under service condition, the breaking of such a unit means ont of service of the whole machine-element or of the mechanism as a whole. The author intends to explain here the relations between the strength of machine-elements or machines and the strength of their units by means of the statistical method, considering the case of fatigue life of roller-chains. This method may also be extended to the cases of other machine-elements, which are under almost uniform condition of stresses with respect to their length, and the fatigue tests are carried for a part the whole length, for example, leaf springs, ropes and belts etc.[1]

II. Experiments about the fatigue life of roller-chains

II・1. Ordinal properties of the roller-chains used

The ordinal properties of the roller-chains used are as follows:
Dimentions of the roller-chains used are shown in the Fig. 1, where

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**水 野 正 夫 Dr. Eng., professor at Keio University.

[1]M. Mizuno, "Der Einfluss der probenlänge bei der Lebensdauerprüfung von Federmaterial", Proc. Faculty of Eng., Keio Univ., Tokyo, vol., 12, no. 47, 1959.

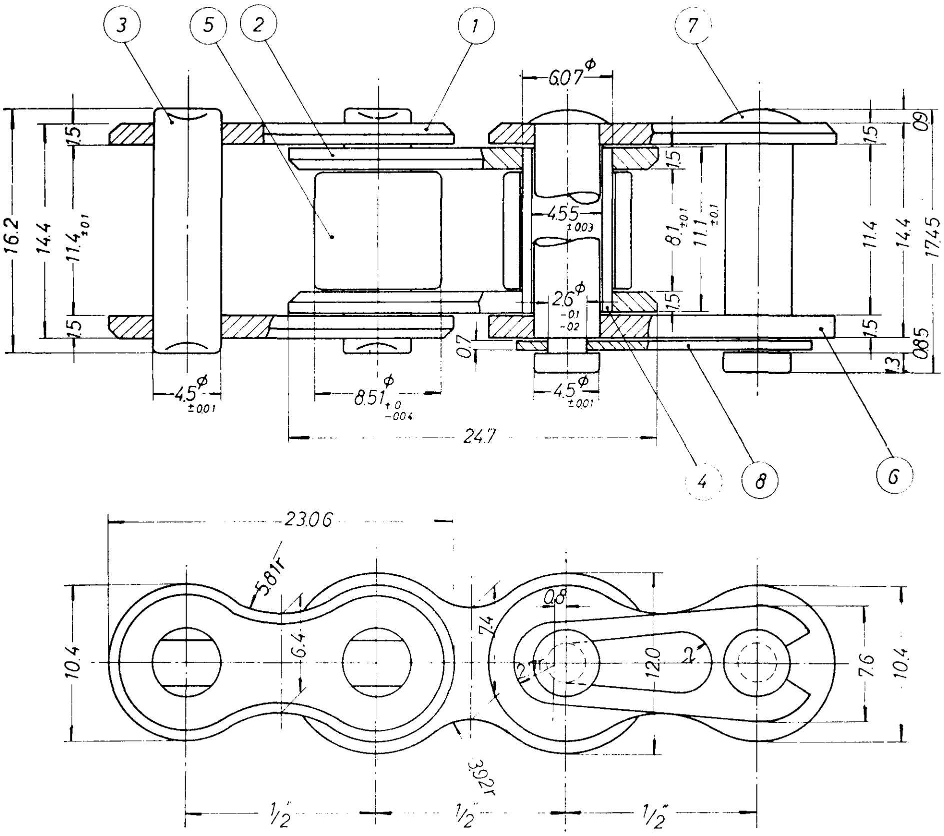

Fig. 1.

① Pin-link plate, (or outer-link plate),
② Roller-link plate, (or inner-link plate),
③ Pin,
④ Bushing,
⑤ Roller,
⑥ Loose-link plate,
⑦ Pin for clip-link,
⑧ Clip-spring.

Chemical composition and mechanical properties of link-plates are given in the Table 1 and 2.

Table 1. Chemical Composition of Link-Plates

C	Si	Mn	P	S
0.45-0.55	0.10-0.45	0.50-0.80	0.045	0.045

Table 2. Mechanical Properties of Link-Plates

1) Statical Breaking Loads : mean = 1,764 kg, range = 60 kg.
2) Hardness (R_a) Oil quenched from 830°C : mean = 81, range = 1,
Annealed at 420°C 3 hr.: mean = 71, range = 2.

II · 2. Method of experiments

The Haig's electro-magnetic fatigue testing machine of the capacity ± 1 ton was used.

The Photo 1 and 2 show the method of experiments, and the Photo 3 shows the breaking point of link-plate.

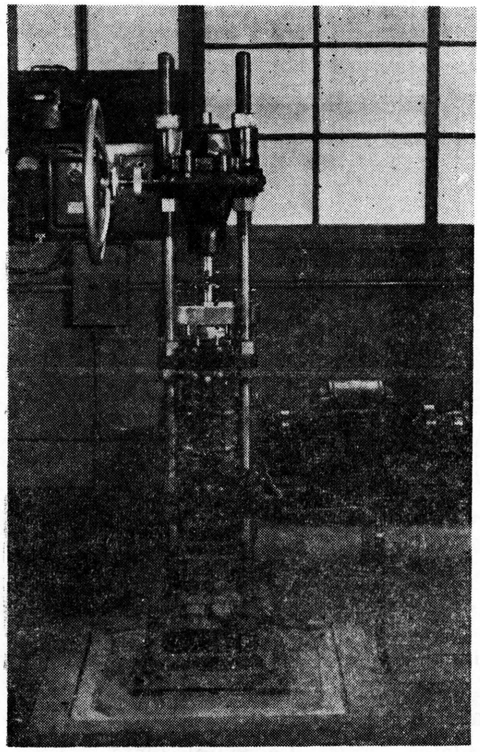

Photo 1

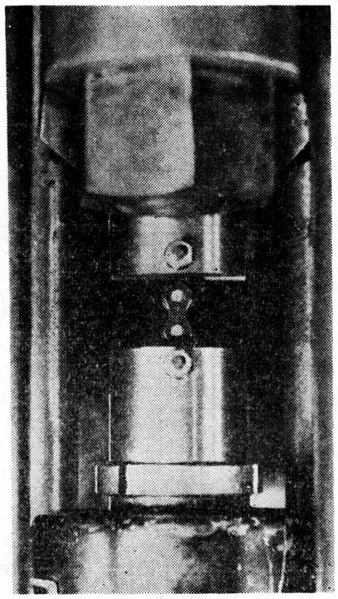

Photo 2

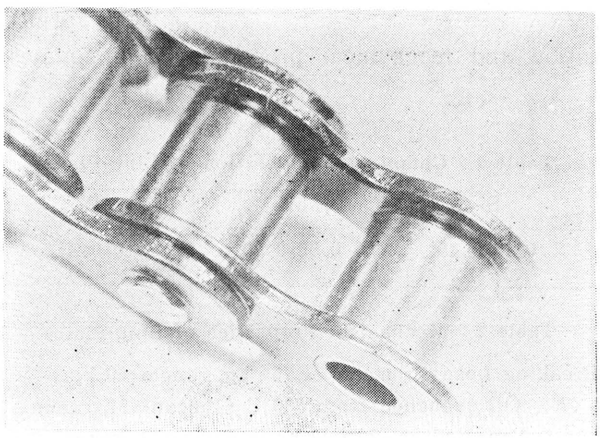

Photo 3

II・3. Results of experiments

The $S-N$ curve of the roller-chain with 5 links under the condition of stress amplitude $S_a=$ mean stress S_m is shown in the Fig. 2.

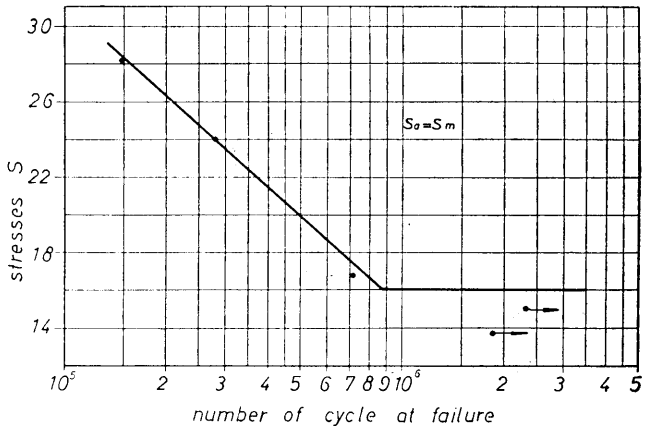

Fig. 2.

The fatigue lives of the roller-chains under the sterss condition

$$S_a=S_m=24\ \mathrm{kg/mm^2}$$

are shown in the Table 3.

Table 3. Fatigue Lives of the Roller-Chains

5 links Testpieces with 3 roller-links & 2 pin-links		3 links Testpieces with 2 roller-links & One pin-link	
no. of testpieces	fatigue lives in 10^4	no. of testpieces	fatigue lives in 10^4
5−1	25.02	3−1	9.96
5−2	18.78	3−2	143.67
5−3	29.52	3−3	27.69
5−4	16.35	3−4	63.69
5−5	8.08	3−5	36.60
5−6	37.21	3−6	22.65
5−7	16.91	3−7	41.42
5−8	50.73	3−8	59.65
5−9	72.48	3−9	20.51

But, in the case of 3 links testpieces, 4 coses are omitted from the Table 3, because the pin-link plates are broken.

III. Statistical treatments

III · 1. Statistics of extremes[2)]

The aim of the statistical theory of extreme values is to analyse the observed extremes and to forecast further extremes. These extremes are not fixed, but are new statistical variates, depending on the initial distribution and on the sample size. However, certain results, which are distribution free may be attained.

There are some difficulties to study the exact distribution of the extremes as a function of sample size. Consequently, it is often to discuss about the asymptotic distributions, where the sample sixe $n \rightarrow \infty$. And since, the asymptotic distributions of smallest values are linked by the symmetric principle to the asymptotic distributions of the largest values (see the transformations in the Table 4), it is, in general, sufficient to say about one of them.

R. A. Fisher and L. H. C. Tippet derived the three asymptotic distributions considering N samples each of size n, taken from the same population and in each sample there are a largest value, and the largest value in the Nn observations is the largest of N largest values taken from samples of size n. And the distribution of the largest value in Nn observations will tend to the same asymptotic expression as the distribution of the largest value in sample size n, provided that such asymptote exists. Consequently, the asymptote must be such that the largest value of a sample size n taken from it must have same asymptotic distribution.

Table 4 summerises the results of asymptotic theory of extreme values, where we can see the properites of three asymptotic probabilities of extreme values.

III · 2. Fatigue of roller-links of chains[3)]

W. Weibull derived in numerous papers the third asymptotic distribution of smallest values in an empirical way and applied it to the analysis of dynamical breaking strength. Applying these to the fatigue failures of innerlinks (roller-links) of roller-chains, the third asymptotic probability of smallest values,

$$1(x) = \exp\left[-\left(\frac{x-e}{v-e}\right)^{\alpha},\right] \tag{1}$$

is chosen for the probability 1 (x) of survival of stress cycles x, where, the location parameters v & e, which are in the functions of number of cycles, decreasing with increasing stress levels. The parameter v is known as characteristic number at failure, while e is known as minimum life, the number of cycles before which no failure occur.

III · 3. Fatigue life of roller-chains

Now, we consider a roller-chain of 2m or 2m−1 pitches, (where m is an integer)

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2) E. J. Gumbel, "Statistics of Extremes", Columbia Unv. Press, 1958, New York.

3) W. Weibull, "Fatigue Testing and Analysis of Results", Pergamon Press, 1961.

Table 4. Asymptotic Probabilities of Extreme Values

Types	Lergest Values	Boundaries	Smallest Values	Boundaries	Symmetry
1.) Exponential	$\phi^{(1)}(x)=\exp\left\{-e^{[-a_n(x-u_n)]}\right\}$	$a_n>0$	$1-\phi_{(1)}(x)=\exp\left[-e^{a_1(x-u_1)}\right]$	$a_1>0$	$a_1=a_n$ $u_1=-u_n$
explanation of perameters	$\phi^{(1)}(u_n)=1/e=0.36788$ $\phi^{(1)}(u_n-1/\alpha_n)=0.06599$		$=1-\phi_{(1)}(u_1)$ $=1-\phi_{(1)}(u_1+1/a_1)$		
2.) Cauchy	$\phi^{(2)}(x)=\exp\left[-\left(\frac{v_n-e}{x-e}\right)^{k_n}\right]$	$k_n>0$ $x\geq e$ $v_n>e\geq 0$	$1-\phi_{(2)}(x)=\exp\left[-e\left(\frac{w-v_1}{w-x}\right)^{k_1}\right]$	$k_1>0$ $x\leq w$ $v_1<w$	$v_1=-v_n$ $k_1=k_n$ $w=-e$
expianation of parameters	$\phi^{(2)}(e)$ $\phi^{(2)}(v_n)={}^1/e$	$=0$ $=0.36788$	$=1-\phi_{(2)}(w)$ $=1-\phi_{(2)}(v_1)$		
3.) Baunded	$\phi^{(3)}(x)=\exp\left[-\left(\frac{w-x}{w-v_n}\right)^{k_n}\right]$	$x\geq w$ $v_n<w$ $k_n>0$	$1-\phi_{(3)}(x)=\exp\left[-\left(\frac{x-e}{v_1-e}\right)^{k_1}\right]$	$x\geq e$ $k_1>0$ $v_1>e\geq 0$	$v_1=v_n$ $k_1=k_n$ $w=-e$
explanation of paramsters	$\phi^{(3)}(w)$ $\phi^{(3)}(v_n)={}^1/e$	$=1$ $=0.36788$	$=1-\phi_{(3)}(e)$ $=1-\phi_{(3)}(v_1)$		
Transformations	*from type* 1.) to 2.) *from type* 2.) to 3.)		$x=ln\ (x-e),\ u=ln\ (v-e),\ a=k$ $x=-x,\ e=-w,\ v=-v,\ k=-k$		

includes m roller-links, and fatigue failure occurs at the weakest of these roller-links. Thus, the probability $1_m(x)$ of survival of x cycles for these roller-chains is,

$$1_m(x)=\left\{\exp\left[-\left(\frac{x-e}{v-e}\right)^{\alpha}\right]\right\}^m=\exp\left[-m\left(\frac{x-e}{v-e}\right)^{\alpha}\right], \quad (2)$$

because, the probability of survival to the m independent observations is obtained from,

$$1_m(x)=[1\ (x)]^m \quad (3)$$

In the Fig. 3, the round points are plotted from the observed number of cycles x at failure of roller-chains of $2m-1=5$ or $m=3$ under the condition of $S_a=S_m=24\text{kg/mm}^2$, and these show a practical linear survivorship function on the probability paper. Thus it may be assumed that no lower limit for the number of cycles at breakage exists and that the failure starts with the first cycle, because in the case $e=0$,

$$1_m(x)=\exp\left[-m\left(\frac{x}{v}\right)^{\alpha}\right], \quad (4)$$

$$y=1n[-1n(1_m)]=\alpha'[\log x-\log v]+1n\ m \quad (5)$$

where, $\alpha'=2.3026\alpha$
and,

$$\log x=\log v+1/\alpha'(y-1n\ m), \quad (6)$$

which is the equation of straight line on the probability papers.

The estimated values of the parameters α' and v, which are the function of sample size n, are shown in the Table 5, with the help of Table 6.[2] And the theoritical line drawn in the Fig. 3 gives a considerably good fit to the observed points.

Table 5.
Parameters of Fatigue Failures of Roller-chains with 5 Pitches

Stress in kg/mm²	$S_a=S_m=24$
Mean Logarithm of Number of Cycles at Failure	$\overline{\log x}=5.3835$
Geometric Standard Deviation	$s(\log x)=0.305$
Scale Parameter	$1/\alpha'=0.328$
Logarithmic Charactaristic Number	$\log v=5.564$
Charactaristic Number of Cycles in 10^4	36.6

Table 6.
Means and Standard—Deviations of Reduced Extremes

N	$\bar{y}_N$	σ_N
9	.4902	.9288
19	.5220	1.0566
24	.5296	1.0864

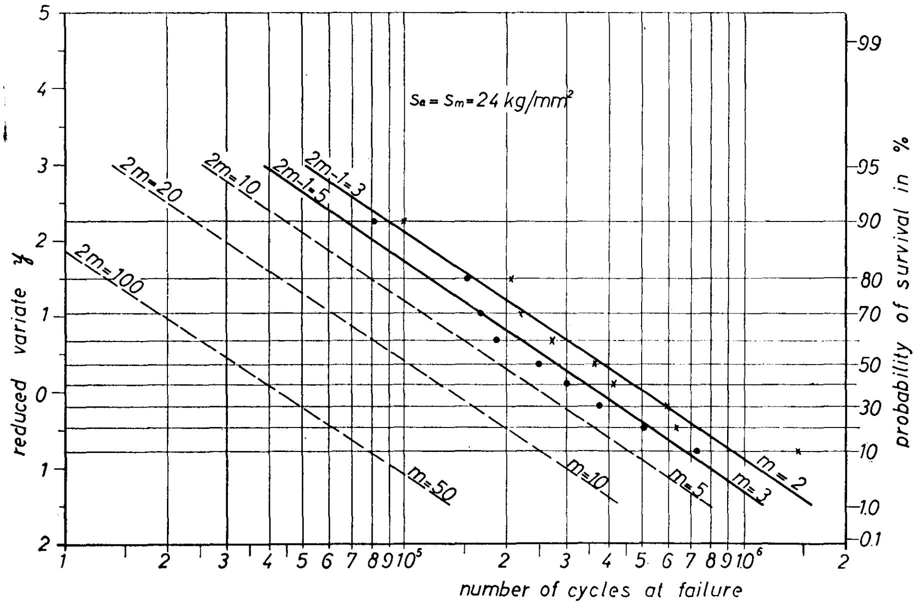

Fig. 3.

From the equation (6), we may have the theoritical line for 2m−1=3 or m=2, shifting by the amount of,

$$\ln 3-\ln 2=1.0986-0\text{-}6931=0.4055 \qquad (7)$$

to the direction of y−axis. The observed number of cycles at failures of roller-chains of 2m−1=3 are shown by cross points and compared with the theoritical line in the Fig. 3. We can see a good fit between the experimental line without the use of the observed values of 2m−1=3, in spite of a small number of observations.

With these methods, we may guess the fatigue strength of roller-chains with any pitches, as shown by the broken lines in the Fig. 3.

I think that a part of the values of safety-factors may be theoritically explained by these statistical treatments of the strength of the machine-parts.

Acknowledgments

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