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Sub Title	
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Publisher	慶応義塾大学藤原記念工学部
Publication year	1960
Jtitle	Proceedings of the Fujihara Memorial Faculty of Engineering Keio University Vol.13, No.49 (1960.) ,p.49(1)- 57(9)
JaLC DOI	
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Notes	
Genre	Departmental Bulletin Paper
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=KO50001004-00130049-0001

Consideration on the Control Wheel Truing of a Centerless Grinder

(Received June 2, 1960)

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Abstract

In the centerless grinding of cylindrical work by the through feed method, it is desirable to have the control wheel trued so that it will be in contact with the work along the full width of the face. I have shown the formula of the control wheel surface in the form of the envelope of the work cylinders revolving about the axis of the control wheel, and then the equations for calculating the swiveled angle of the truing guide and the distance of the truing diamond from the center of the holder, corresponding to the relative location of the work for the control wheel. Moreover, I have considered the errors in the truing and been able to show some fundamental guides for the design of the truing device and centerless grinding.

I. Introduction

When cylindrical work is ground on a centerless grinding machine by the through feed method, the spindle of the control wheel is inclined to the work axis, and it is desirable for the stability of the rotation of the work and its movement in the axial direction as well as for its finishing precision that the control wheel have a curved surface of rotation which will enable it to keep in contact with the work all along its width. An ordinary control wheel truing device most often belongs to the type which "trues" the control wheel into a hyperboloid of one sheet of revolution created a straight generating line. Moreover, the work is ground with its center held at a certain height above the line connecting the centers of the grinding wheel and the control wheel[1]. Tsueda and Inoue[2] previously reported their research results on the approximate shape and the truing of the control wheel in cases where the work-holding height is 0. This writer has drawn a theoretical and an approximate formula giving the shape of the control wheel for the grinding of the work at a certain height as well as an equation giving the contact curve, and considered the conditions required when the control wheel is "trued" into a hyperboloid of one sheet of revolution instead of the shape so determined and also the main errors expected to result from the substituion.

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II. Axis of coordinates and symbols

In Fig. 1,

xyz: coordinates with center axis of work taken as z,

$x'y'z'$: coordinates with center axis of control wheel taken as z', z' and z being parallel to each other,

$x''y''z''$: coordinates obtained by turning $xy'z'$ around x' axis by angle θ,

XYZ: coordinates obtained by turning $x''y''z''$ around z'' axis by angle ϕ,

O: point where $x'y'$ plane crosses z axis (origin of xyz coordinate system),

O' (O'', O): origin of $x'y'z'$ ($x''y''z''$, XYZ) coordinate system,

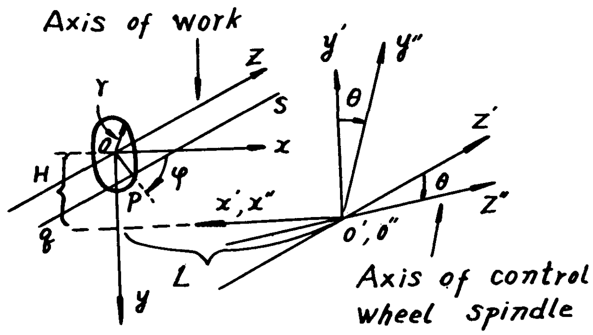

Fig. 1.

H: distance between x axis and x' axis (work-holding height),

L: distance between y axis and y' axis,

r: radius of cylindrical work,

qps: generating line of work,

φ: angle formed by op and x axis,

R: radius of control wheel (function of Z)

R_0: radius of gorge circle of control wheel.

The direction of the arrow in each coordinate system is called positive.

III. Shape of curved surface of control wheel

In Fig. 1, the work cylinder is expressed by the following equations:

$$x=r\cos\varphi\,,\qquad y=r\sin\varphi\,,\qquad z=z \tag{1}$$

or

$$x'=L-r\cos\varphi\,,\qquad y'=H-r\sin\varphi,\qquad z'=z \tag{1'}$$

or

$$x''=L-r\cos\varphi\,,\qquad y''=(H-r\sin\varphi)\cos\theta+z\sin\theta$$
$$z''=-(H-r\sin\varphi)\sin\theta+z\cos\theta \tag{1''}$$

Now, if the envelope of the cylindrical surfaces obtained by turning this coordinate system around the shaft z'' of the control wheel is taken as the curved surface of the control wheel, the control wheel ought to be able to keep in touch with the work all along its width. Namely,

$$\left.\begin{aligned} X&=(L-r\cos\varphi)\cos\phi+\{(H-r\sin\varphi)\cos\theta+z\sin\theta\}\sin\phi\\ Y&=-(L-r\cos\varphi)\sin\phi+\{(H-r\sin\varphi)\cos\theta+z\sin\theta\}\cos\phi\\ Z&=-(H-r\sin\varphi)\sin\theta+z\cos\theta \end{aligned}\right\} \tag{2}$$

In the above equations for the cylindrical curved surface group,

$$\begin{vmatrix} \partial X/\partial\varphi & \partial X/\partial\phi & \partial X/\partial z\\ \partial Y/\partial\varphi & \partial Y/\partial\phi & \partial Y/\partial z\\ \partial Z/\partial\varphi & \partial Z/\partial\phi & \partial Z/\partial z \end{vmatrix}=0^{3)}.$$

From these we obtain the following condition for the contact curve:

$$\tan\varphi=(H+z\tan\theta)/L \tag{3}$$

Therefore, from eq. (2) and (3), we obtain the following equations for the curved surface of the control wheel when $L=R_0+r$:

$$\left.\begin{aligned} X&=(R_0+r-r\cos\varphi)(\cos\phi+\cos\theta\tan\varphi\sin\phi)\\ Y&=(R_0+r-r\cos\varphi)(-\sin\phi+\cos\theta\tan\varphi\cos\phi)\\ Z&=-H/\sin\theta+r\sin\theta\sin\varphi+(R_0+r)\cos\theta\cot\theta\tan\varphi \end{aligned}\right\} \tag{4}$$

When, in particular, $r=0$,

$$X^2/R_0^2+Y^2/R_0^2-(Z+H/\sin\theta)^2/(R_0^2\cot^2\theta)=1 \tag{5}$$

which gives a hyperboloid of one sheet.

As for the axial section of the control wheel, we obtain the following if $Y=0$ in eq. (4):

$$\left.\begin{aligned} X&=R=(R_0+r-r/\sqrt{1+m^2})\sqrt{1+\cos^2\theta\, m^2}=f(m)\\ Z&=-H/\sin\theta+r\,m\sin\theta/\sqrt{1+m^2}+(R_0+r)\,m\cos\theta\cot\theta=F(m) \end{aligned}\right\} \tag{6}$$

where $m=\tan\varphi$.

Here, we adopt the following method for the convenience of calculating R for a given value of Z:

Namely, if the approximate root of $F(m)=Z$ is called m' and its difference from the true value m, $m-m'=\delta m$,

$$\begin{aligned} Z&=F(m)=F(m'+\delta m)\\ &=F(m')+\frac{\delta m}{1!}F'(m')+\frac{(\delta m)^2}{2!}F''(m')+\cdots\cdots \end{aligned}$$

As δm is very small, we will take the terms down to the second only and obtain

$$\delta m=\{Z-F(m')\}/F'(m') \tag{7}$$

Also, $X=f(m'+\delta m)$

$$=f(m')+\frac{\delta m}{1!}f(m')+\frac{(\delta m)^2}{2!}f''(m')+\cdots\cdots$$

Similarly taking down to the second term and substituting (7), we obtain

$$X=f(m')+\{Z-F(m')\}f'(m')/F'(m') \tag{8}$$

Therefore, if the root of $F(m)$ when $m/\sqrt{1+m^2}$ is regarded as equal to m is called m', eq. (6), (7) and (8) will lead to the following:

$$\left.\begin{aligned} &X=R_0\sqrt{1+\cos^2\theta\, m'^2}+r(1+m'^2-\sqrt{1+m'^2}/\sqrt{1+\cos^2\theta\, m'^2}) \\ &\quad \text{where } m'=(H+Z\sin\theta)/(r+R_0\cos^2\theta) \end{aligned}\right\} \tag{9}$$

As may be seen from (6) or (9), the shape of the control wheel is determined by r, R_0 and θ and remains the same for all H values excepted that the position of the gorge circle deviates from the origin by $Z=-H/\sin\theta$.

For example, the axial section shapes of the control wheel for $\theta=6°$, $R_0=100$ mm; $r=0$, 5, 10 mm; $H=0$, 1, 5, 10, 20 mm are shown in Fig. 2. The vertical chain lines represent the positions of the gorge circle.

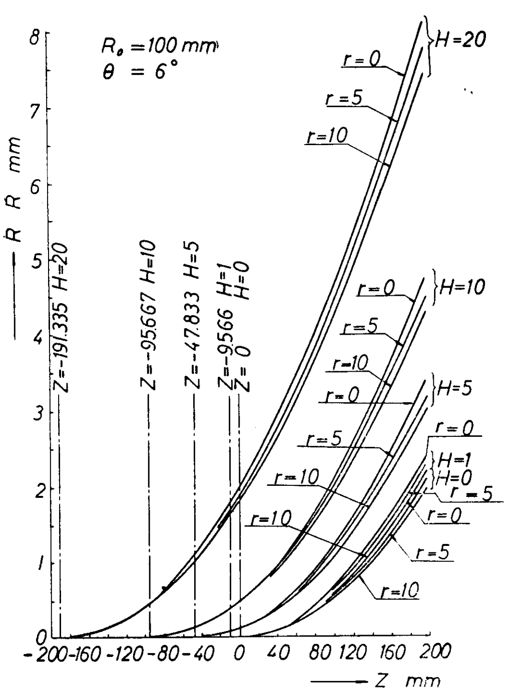

Fig. 2.

IV. Line of contact between control wheel and cylindrical work

When $L=R_0+r$, the line of contact between the control wheel and the cylindrical work is expressed as follows from (1) and (3):

$$\left.\begin{aligned} &x=r\cos\varphi,\quad y=r\sin\varphi, \\ &\varphi=\tan^{-1}\{(H+z\tan\theta) \\ &\qquad /(R_0+r)\} \end{aligned}\right\} \tag{10}$$

showing a space curve which would be a tangential curve if the work cylinder were developed on a plane. Along this curve the work receives feeding motion from the control wheel. If, in particular, $r=0$, eq. (10) are turned into the following:

$$X=0,\quad Y=0,\quad Z=z \tag{11}$$

showing a straight line.

For example, the projections of the contact line on the zy plane under the same conditions as in the last section are shown in Fig. 3, showing that the smaller the work radius is for the same work holding height, the closer the contact line becomes to a horizontal straight line.

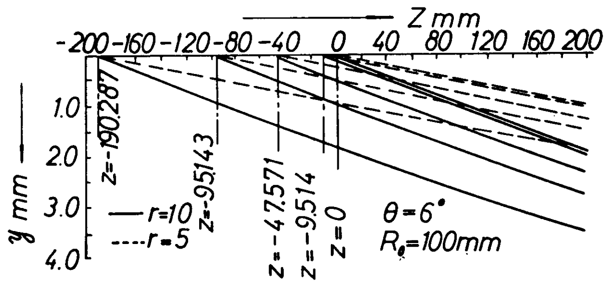

Fig. 3.

V. Truing of control wheel and truing errors

The ordinary truing device for the conventional centerless grinding machine is of the type in which the truing diamond performs the truing through a linear motion along a guide surface parallel to the spindle of the control wheel. The surface of the control wheel thus trued, therefore, becomes a one-sheet hyperboloid. Also, when, as in common practice, the inclination of the guide is set at the same angle, θ, as the control wheel and the sliding distance of the diamond from the center of the holder, made equal to the work-holding height, H, the point of the diamond will make a truing motion along the straight line of eq. (11), with the result that the surface of the control wheel will be trued into a one-sheet hyperboloid as expressed by eq. (5), in which the radius of the work is disregarded. Thus, as in apparent from Fig. 2, a trued curved surface in which $r=10$mm at $Z=200$mm and $r=0$mm for $H=20$mm will be subject to an error of approximately 0.7mm, and the direction of the work feeding will not be constant, either. We will therefore consider conditions in which the trued shape can be brought closer to the theoretical shape by changing the truing method alone.

(1) Generating line for truing

If the movement plane of the trying diamond is expressed as $Y=R_0$, the generating line for truing the control wheel can be obtained from eq. (4), and if this is projected on the XZ plane, we obtain the following:

$$\left.\begin{aligned} X &= \sqrt{(R_0+r-r/\sqrt{1+m^2}\,)^2(1+m^2\cos^2\theta)-R_0{}^2} \\ Z &= -H/\sin\theta+m\,r\sin\theta/\sqrt{1+m^2}+(R_0+r)\,m\cos\theta\cot\theta \end{aligned}\right\} \qquad (12)$$

showing a curve with the characteristics described in Table 1.

Also, if the direction coefficient of the straight line passing two points $(0, -H/\sin\theta)$, (X', Z') on the generating line for truing is called $\tan\theta_t$,

$$\tan\theta_t = X'/(Z'+H/\sin\theta) \qquad (13)$$

where X' and Z' are in the eq. (12) relationship.

Table 1.

m	$0 \rightarrow \infty$
Z	$-H/\sin\theta \rightarrow \infty$
X	$0 \rightarrow \infty$
dX/dZ	$\tan\theta\sqrt{R_0/(R_0+r\sec^2\theta)} \rightarrow \tan\theta$
d^2X/dZ^2	positive
Generating line for truing	(point of inflection) convex downward

Moreover, within the range of practicality, m for the various θ, R_0, r, H and Z values is very small in comparison with unity. By transforming the equation for X in eq. (12), therefore, we obtain

$$X=(R_0+r)\cos\theta\, m\sqrt{1+\frac{r}{R_0+r}\left\{\left(\frac{1}{\cos^2\theta}-\frac{3}{4}\frac{m^2}{\cos^2\theta}+\cdots\cdots\right)+\left(-2+m^2-\frac{3}{4}m^4+\cdots\cdots\right)\right\}-\left(\frac{r}{R_0+r}\right)^2\tan^2\theta\,(1-m^2+m^4-\cdots\cdots)}\ .$$

If, in the above equation, m' in eq. (9) is substituted for m and the minute terms of higher orders within $\sqrt{\quad}$ are omitted,

$$X=\tan\theta_t(Z+H/\sin\theta)$$

where

$$\tan\theta_t=\tan\theta\,\frac{\sqrt{R_0/(R_0+r)+r\tan^2\theta/(R_0+r)}}{1+r\tan^2\theta/(R_0+r)} \tag{14}$$

If, moreover, the term $r\tan^2\theta/(R_0+r)$ is ignored,

$$X=\tan\theta_t(Z+H/\sin\theta)$$

where

$$\tan\theta_t=\tan\theta\sqrt{R_0/(R_0+r)}\,^{4)} \tag{15}$$

or

$$\theta_t\doteqdot\theta\left(1-\frac{1}{2}\frac{r}{R_0+r}\right)\ . \tag{15$'$}$$

Thus, the theoretical generating line for truing shown in eq. (12) can be substituted, within the range of practicality, by the straight line represented by eq. (14) or eq. (15).

For example, Fig. 4 shows θ_t's calculated by eq. (13), (14) and (15) for $\theta=6°$, $R_0=100$ mm, $r=100$ mm, $H=200$ mm. Namely, within a range of 400 mm for the control wheel width, the θ_t's calculated by the equations independent of Z and H show agreement down to a minute with that calculated by eq. (13), which means that eq. (15) may be used for all practical purposes.

(2) Determining truing dimensions

If the distance between the motion plane of the point of the truing diamond and the axis of the control wheel is called R_0, the inclination angle of the guide plane, θ_t, the distance from the center of the carrier of the diamond to its sliding position H_t, and the radius of the control wheel to be trued, R_t,

$$R_t=\sqrt{R_0^2+(Z+H_t/\sin\theta_t)^2\tan^2\theta_t} \tag{16}$$

where H_t is the length measured in the direction at right angles to the moving direction of the truing diamond.

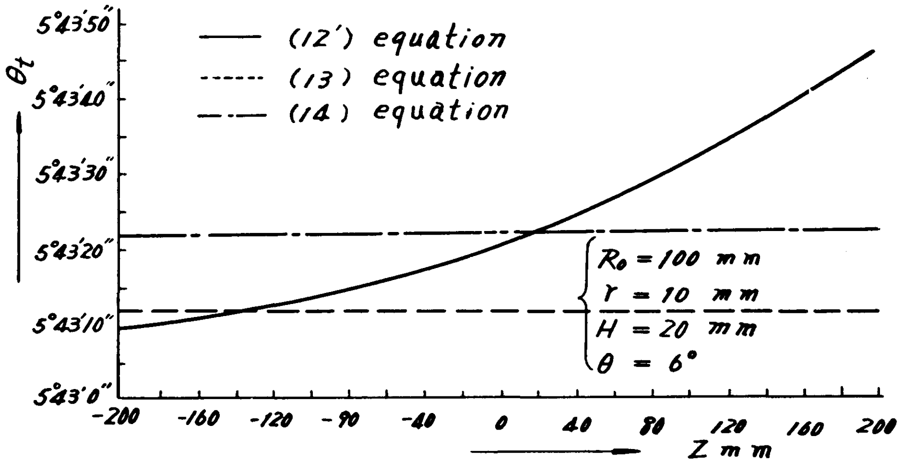

Fig. 4.

θ_t is calculated by using either eq. (13) or (14) or (15) or (15′). Then, from

$$H_t=H\sin\theta_t/\sin\theta \tag{17}$$

or

$$H_t\doteqdot H\left(1-\frac{1}{2}\frac{r}{R_0+r}\right). \tag{17$'$}$$

H_t is determined, which, for the reason explained in the last section, gives a truing motion route close to the theoretical generating line for truing (Fig. 5).

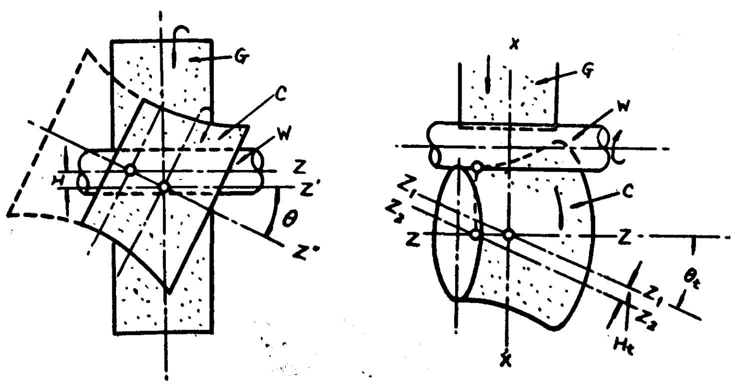

Fig. 5.

In practice, depending on the width and inclination of the control wheel and the radius and work-holding it sometimes happens that the position of the gorge circle finds itself outside the width of the control wheel. If, for the purpose of minimizing the amount of grinding for truing, it is desirable to calculate R_0 beforehand from the radius of control wheel, it can be done by solving the following cubic equation derived from eq. (16), (17) and (15):

$$R_0^3+rR_0^2+\{(Z_i+H/\sin\theta)^2\tan^2\theta-R_{ti}^2\}R_0-rR_{ti}^2=0 \tag{18}$$

where R_{ti} is the radius for the known Z_i.

(3) Truing errors

Truing under the conditions described in the last section is subject to the following main errors:

i) Shape difference between (16) and (6): δR_t

ii) Shape error resulting from δH_t error, if any: $\delta R_{Ht}=\pm\frac{\partial R_t}{\partial H_t}\delta H_t$

iii) Shape error resulting from $\delta\theta_t$ error, if any: $\delta R_{\theta t}=\pm\frac{\partial R_t}{\partial\theta_t}\delta\theta_t$

iv) Shape error resulting from δR_0 error, if any: $\delta R_{R0}=\pm\frac{\partial R_t}{\partial R_0}\delta R_0$

These four errors are conceivable.

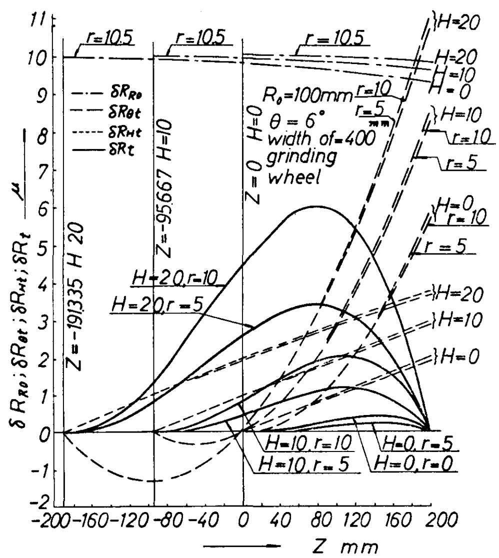

Fig. 6.

For instance, Fig. 6 shows the shape errors δR_t, δR_{Ht}, $\delta R_{\theta t}$ and δR_{R0} for $\theta=6°$, $R_0=100$ mm; $r=5$, 10 mm; $H=0$, 10, 20 mm, when there are $\delta H_t=10\mu$, $\delta\theta_t=30''$ and $\delta R_0=10\mu$ errors. However, θ_t is the value calculated from eq. (13) for $Z=200$ mm, or $\theta_t=5°43'45''$——that is, when the guide inclination is set so as to make the position of the gorge circle and the radius of the control wheel for $Z=200$ mm agree with those of the theoretical control wheel. From the figure, the following facts can be seen:

For the same work-holding height,

i) Among the errors due to the work radius, δR_{Ht}, $\delta R_{\theta t}$ and δR_{R0} are small whereas δR_t alone is comparatively large.

ii) δR_t is an inevitable error. In the present example, it is 0 at the gorge circle position and $Z=200$ mm, and negative when Z is larger than 200 mm. The magnitude of the error in such cases, however, is comparatively small.

iii) While δR_{Ht} increases generally in a straight line from the gorge circle position toward the positive direction of Z, $\delta R_{\theta t}$, although turning negative a little while, rapidly rises in the positive region. Meanwhile, δR_{R0} shows about the same amount of error as δR_0 throughout the range.

Conclusion

We have obtained eq. (4) as the exact solution showing the shape of the control wheel and eq. (9) as an approximation for figuring out the shape of its axial section. From these, the conditions for truing the control wheel into a shape very close to the theoretical shape by using the conventional device for truing it into a one-sheet hyperboloid have been found to be as follows: the inclination of the guide be determined by solving eq. (13), (14) or (15) or (15)′; the sliding distance of the truing diamond from the center of the carrier, by solving eq. (17) or (17)′; and the distance of the motion plane of the truing diamond point from the axis of the control wheel, by solving eq. (18) if it is desirable to calculate it beforehand. It has been shown that the various truing factors can thus be determined. Also, by examining the errors occurring in these cases, it is possible to find some guides in designing truing devices and studying truing methods.

The writer is sincerely grateful for the Scientific Research Fund of the Ministry of Education which made this study possible.

Notes

(1) Sachsenberg *u.* Kreher, Werkstattstech. *u.* Werksl., *Jg.* 33, *Ht* 11, s. 280 (1939) showed through experiments that a work-holding height of 20 mm in diameter, or a height equal to the work diameter for work 15 mm or less in diameter, improves the out-of-roundness of the work. Kenzo Nagasawa in **Grinding Work,** p. 187 (1935) sets the maximum at 1/2″, and Toshio Asaeda in his **Studies on Machinery**, Vol. 1, No. 8, p. 377 (1949) adopts 1/2～1/3 of the work diameter.

(2) Shosuke Tsueda and Ryoji Inoue: **Mechanical Society Papers**, Vol. 16, No. 53, p. 82 (1950).

(3) For instance. Shin-ichi Sato: **Gear Tooth Shapes and Screw Faces,** p. 156 (1949).

(4) Shosuke Tsueda and Ryuji Inoue: ibid., p. 85, show the same result as eq. (8)