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Forming Mechanism of Cylindrical Work in Centerless Grinding

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Abstract

The author has showed a theoretical formula on the profile for cylindrical work ground by a centerless grinder, expressed by Fourier's series in tangential polar co-ordinate, taking into consideration of the geometrical conditions of the three point supports for the work periphery, and the variation of form in grinding process. And the mode of variation, as to the amplitude and the phase of the higher harmonics of the out-of-roundness curve, has been discussed under a few conditions.

I. Introduction

Regarding the formation of Gleichdicke shapes in centerless grinding, "Principles of Centerless Grinding"[1] published by Cincinnati Grinders Inc. and other books[2][3] indicate that it is closely related to the work supporting conditions among the various work conditions. However, due to the lack of empirical or theoretical grounds to support this, it has not been clear in what shapes, Gleichdicke or otherwise, the work is generally finished, i. e., what is the forming mechanism of work in centerless grinding.

In his first report[4] this writer reported the results of his systematic examination of centerless grinding characteristics through harmonic analysis of the work's cross section profile, and clarified that the relationship between the higher harmonic amplitudes of the pre-grinding and post-grinding out-of-roundness curves is closely related to the work supporting height. In the report, by paying special attention to the geometrical conditions of work supporting in ideal, vibration-free centerless grinding, and also taking into consideration the variations in the shape of the work due to grinding, he geometrically analyzed the forming mechanism and drew a formula expressing the out-of-roundness curve of the work after centerless grinding.

II. Geometrical analysis of the forming mechanism

For the centerless grinding of cylindrical work, the in-feed method was employed,

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and a formula representing the shape of the work after ideal, vibration-free grinding was drawn on the assumption that the stock removal at a given rotation angle of the work is proportionate to the depth of cut of the grinding wheel, which is geometrically determined by the three radiuses of the work and these angular position, for its points of contact with the grinding wheel, feed grinder and the support. The time required for the cut-in motion of the grinding wheel rising from 0 to the specified value at the beginning of the in-feeding, was assumed to be approximately 0 sec to simplify the calculations.

Now, let us imagine that, as in Fig. 1, the work W is fed in and ground into

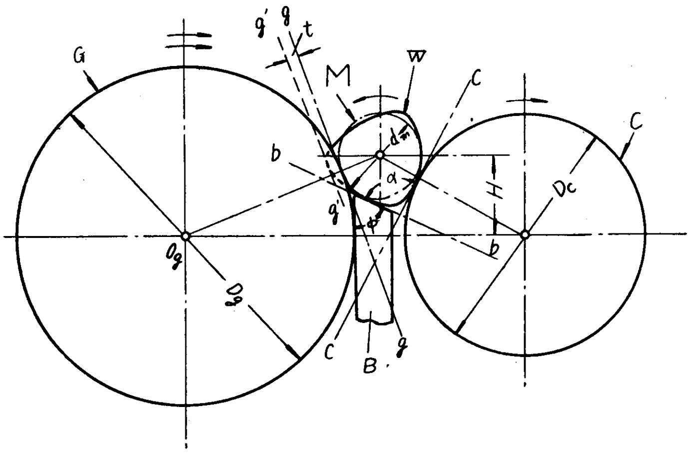


Fig. 1.

the position of an imaginery cylinder M with a diameter d_m (radius a_m) held by the grinding wheel G, regulating wheel C and the blade B at its three points of contact with them.

In this case, if the diameter of the work is small compared with the diameters of the two wheels G and C, ane if the depth of cut is small compared with the diameter of the work, it may be considered that the working planes of G and C are along the common tangents gg and cc drawn at their points of contact with M, respectively. Then the work, as it were, turns in a V block formed by the working plane bb of the blade and the working plane cc of the regulating wheel, and the distance between gg and tangent $g'g'$ of the work W drawn parallel to gg on G's side may be regarded as the depth of cut of the grinding wheel if the geometrical supporting conditions along are considered. Let us first consider the depth of cut as defined above.

Fig. 2 (a, b) show the geometrical interrelationship of W, G, B and C in Fig. 1. Suppose, in Fig. 2 (a), W is in contact with the line $g'g'$ at a and with the supporting plane bb of the blade B and the working plane cc of the regulating weel at the points b and c, respectively. In W; choose optionally the origin 0 near the center, and draw on W in an optional direction the original line OX fixed on W. Let us consider that the profile curve along the circumference of W is expressed by tangential polar coordinates in terms of the radius vector r and the angle θ it forms with the original line OX, and that this is expanded into a Fourier series in the frequency 2π, as follows:

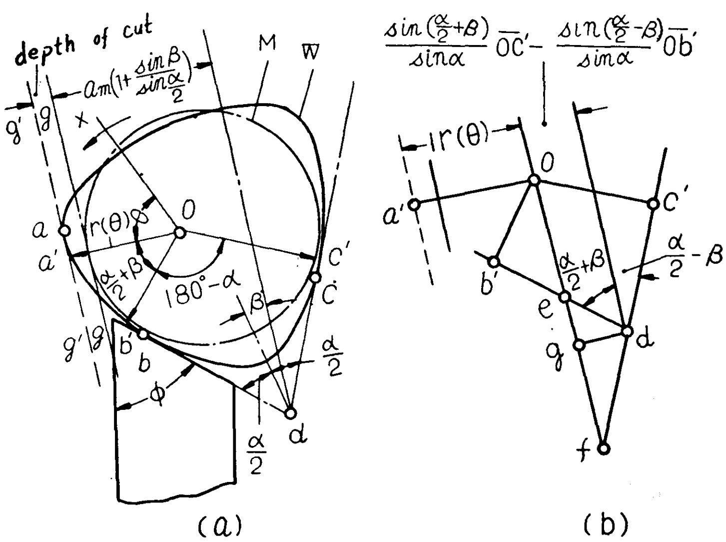


Fig. 2.

$$r(\theta)=a_0+\sum_{i=1}^{\infty} C_i \cos(i\theta+\varphi_i) \tag{1}$$

In Fig. 2 (a), again, draw perpendiculars from 0 to the lines $g'g'$, bb and cc and call their feet a', b', and c', respectively; and if the relationships as shown in Fig. 2 (b) are taken into consideration, the depth of cut of the grinding wheel may be expressed as follows:

$$(depth\ of\ cut)=\overline{Oa'}+\overline{dg}-a_m\left(1+\frac{\sin\beta}{\sin\frac{\alpha}{2}}\right)$$

where $\overline{Oa'}$ is the radius of an unground part of the work, *i. e.*

$$\overline{Oa'}=r(\theta)$$

And

$$\overline{ef}=\overline{dg}\cdot\left\{\cot\left(\frac{\alpha}{2}-\beta\right)+\cot\left(\frac{\alpha}{2}+\beta\right)\right\}=\overline{dg}\cdot\frac{\sin\alpha}{\sin\left(\frac{\alpha}{2}-\beta\right)\cdot\sin\left(\frac{\alpha}{2}+\beta\right)}$$

Also,

$$\overline{ef}=\overline{of}-\overline{oe}=\frac{\overline{Oc'}}{\sin\left(\frac{\alpha}{2}-\beta\right)}-\frac{\overline{Ob'}}{\sin\left(\frac{\alpha}{2}+\beta\right)}$$

From the above two,

$$\overline{dg}=\frac{\sin\left(\frac{\alpha}{2}+\beta\right)}{\sin\alpha}\cdot\overline{Oc'}-\frac{\sin\left(\frac{\alpha}{2}-\beta\right)}{\sin\alpha}\cdot\overline{Ob'}$$

Therefore

$$(\text{dpth of cut})=\left\{r(\theta)+\frac{\sin\left(\frac{\alpha}{2}+\beta\right)}{\sin\alpha}\cdot\overline{Oc'}-\frac{\sin\left(\frac{\alpha}{2}-\beta\right)}{\sin\alpha}\cdot\overline{Ob'}\right\}-a_m\left(1+\frac{\sin\beta}{\sin\frac{\alpha}{2}}\right) \qquad (2)$$

where

$$\left.\begin{aligned}\alpha&=\sin^{-1}\left(\frac{2H}{D_c+d_m}\right)+\phi\\ \beta&=\frac{\alpha}{2}-\sin^{-1}\left(\frac{2H}{D_c+d_m}\right)-\sin^{-1}\left(\frac{2H}{D_g+d_m}\right)\end{aligned}\right\} \qquad (3)$$

Thus, $\overline{Oc'}$ and $\overline{Ob'}$ determined for a given θ will yield the depth of cut.

If $\overline{Ob'}$ and $\overline{Oc'}$ are the radiuses of ground parts of the work, their pregrinding radiuses minus their grinding amounts ought to give their post-grinding radiuses. And when the stock removal is expressed in terms of radial length, the depth of cut of the grinding wheel does not agree with the stock removal in practice due to the elastic deformrtion of the regulating wheel, evasion of the work and so-call cutting residue of the grinding wheel, and it is difficult to theoretically determine all this. Here we will assume, for the simplicity of calculation, that the actual stock removal is proportionate to the depth of cut expressed by eq. (2), and consequently that

$$\frac{(\text{cut-in depth of cut})-(\text{actual stock removal})}{(\text{depth of cut})}=K \qquad (K<1) \qquad (4)$$

with the constant of proportionality $(1-K)$ assumed. Also, during the centerless grinding, the stock removal in the second or a later round is assumed to have been caused by a new depth of cut which is the residue from the last round, i. e., (depth of cut)−(actual stock removal in last round). This is considered appropriate for the early stages of grinding where the stock removal is comparatively large.[5]

On these assumptions, eq. (1) for rhe work's radius r, eq. (2) and (4) for the depth of cut will lead to the following equation expressing the post-grinding value of the radius $\overline{Oa}$ within the first one rotation of the work.

$$\text{(radius of ground part)}=r(\theta)-(1-K)\cdot\text{(depth of cut)} \quad (5)$$

Now, $\overline{Ob'}$ and $\overline{Oc'}$ on the right side of eb. (2) are both the radiuses of unground parts, i. e.,

$$\overline{Ob'}=r\left(\theta+\frac{\alpha}{2}+\beta\right),\quad Oc'=r\left(\theta-180^\circ-\frac{\alpha}{2}+\beta\right)$$

If, in this case, the depth of cut is specially called $t(\theta)$,

$$t(\theta)=(a_o-a_m)\left\{1+\frac{\sin\beta}{\sin(\alpha/2)}\right\}+\sum_{i=2}^{\infty}C_i\left\{(1+e_i)\cos(i\theta+\varphi_i)+f_i\sin(i\theta+\varphi^i)\right\} \quad (6)$$

where

$$\left.\begin{aligned}
e_i=&\frac{\sin\beta}{\sin\frac{\alpha}{2}}\text{eos}\,i\left(90^\circ+\frac{\alpha}{2}\right)\cos i\left(-90^\circ+\beta\right)\\
&+\frac{\cos\beta}{\cos\frac{\alpha}{2}}\sin i\left(90^\circ+\frac{\alpha}{2}\right)\sin i\left(-90^\circ+\beta\right)\\
f_i=&-\frac{\sin\beta}{\sin\frac{\alpha}{2}}\cos i\left(90^\circ+\frac{\alpha}{2}\right)\sin i\left(-90^\circ+\beta\right)\\
&+\frac{\cos\beta}{\cos\frac{\alpha}{2}}\sin i\left(90^\circ+\frac{\alpha}{2}\right)\cos i\left(-90^\circ+\beta\right)
\end{aligned}\right\} \quad (7)$$

In this case, the actũal stock removal is $(1-K)\cdot t(\theta)$.

And if, as the work turns, its ground-part radius comes into contact with the blade and then with the regulating wheel, the depth of cut of the grinding wheel is not given by (6); it must be considered that the radiuses $\overline{Ob'}$ and $\overline{Oc'}$ have been reduced by the stock removal. Let us consider this below.

Looking at the form of ϵq. (2), we find that the formula of depth of cut is linear with respect to $\overline{Ob'}$ and $\overline{Oc'}$. Namely, for the variations of $\overline{Ob'}$ and $\overline{Oc'}$,

$$\text{(depth of cut)}+\Delta\text{(depth of cut)}=r(\theta)+\frac{\sin\left(\frac{\alpha}{2}+\beta\right)}{\sin\alpha}(\overline{Oc'}+\Delta\overline{Oc'})$$

$$-\frac{\sin\left(\frac{\alpha}{2}-\beta\right)}{\sin\alpha}(\overline{Ob'}+\Delta\overline{Ob'})-a_m\left(1+\frac{\sin\beta}{\sin\frac{\alpha}{2}}\right)$$

Therefore, the actual stock removal in this case can be determined from the following:

$$(1-K)\text{(depth of cut)}+\Delta\text{(depth of cut)}$$

By this means let us figure out the actual stock removal for increasing values of the work rotation angle one after another.

Fig. 3 shows the relationship between the work rotation angle ($\theta<0$) and the perpendiculars drawn from the origin 0 to the working planes *gg*, *bb* and *cc* of the grinding wheel *G*, blade *B* and the regulating wheel *C*. In the figure, the original line *OX* turns with the work, and it is assumed that the perpendiculars $\overline{Oa'}$, $\overline{Ob'}$ and $\overline{Oc'}$ drawn from the origin 0 have the lengths of the pre-grinding radiuses or the pre-grinding radiuses minus the actual stock removal, respectively, depending on the rotation angle of the work. In this case, as the actual stock removal is very small in comparison with work diameter, it may be assumed that the directions of $\overline{Oa'}$, $\overline{Ob'}$ and $\overline{Oc'}$ are constant regardless of the position of the origin 0. Also, $\angle a'Ob'$ and $\angle b'Oc'$ are $(\alpha/2+\beta)$ and $(180°-\alpha)$, respectively. Now, let us say $\angle b'Oc'=n\cdot\angle a'Ob'$, i. e.,

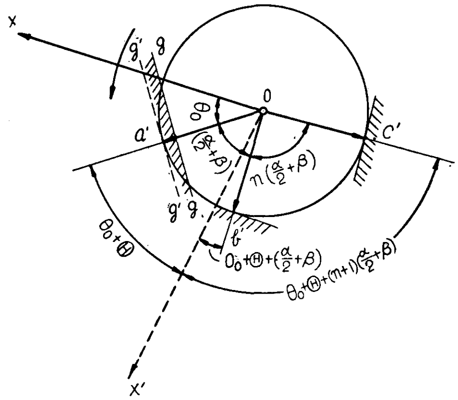


Fig. 3.

$$180°-\alpha=n\left(\frac{\alpha}{2}+\beta\right) \tag{8}$$

As the work turns by the argle $\theta\,(\leqq 0)$ after the grinding began with $\theta=\theta_0$, the original line *OX* turns to *OX'*, and $\overline{Oa'}$, $\overline{Ob'}$ and $\overline{Oc'}$ form with *OX'* the angles $\theta_0+\theta$, $\theta_0+\theta+(\alpha/2+\beta)$ and $\theta_0+\theta+(n+1)(\alpha/2+\beta)$, respectively.

Now, as was explained previously, let us say, for a given θ,

$$\overline{Oa'}=r(\theta_0+\theta),\quad \overline{Ob'}=r(\theta_0+\theta+\alpha/2+\beta),\quad \overline{Oc'}=r(\theta_0+\theta-180°-\alpha/2+\beta)$$

Then the actual stock removal by the working plane *gg* of the griding wheel may be expressed by means of eq. (6) as follows:

$$\left.\begin{array}{ll}\text{When} & 0\geqq\theta>-360° \\ \text{(actual stock removal)} & =(1-K)\cdot t(\theta_0+\theta)\end{array}\right\} \tag{9}$$

However, spposing that, when $-(\alpha/2+\beta)\geqq\theta>-360°$, Ob' alone is reduced by the actual stock removal given by the above equation, $\overline{Ob'}=r(\theta_0+\theta+\alpha/2+\beta)-(1-K)\cdot t(\theta_0+\theta+\alpha/2+\beta)$. Therefore, let us express the variation of $\overline{Ob'}$ as follows:

$$\Delta(\overline{Ob'})=-(1-K)\cdot t(\theta_0+\theta+\alpha/2+\beta)$$

And considering that eq. (2) is linear, the depth of cut of the *gg* plane may be obtained by correcting the depth of cut $t(\theta_0+\theta)$ for the case without an $\overline{Ob'}$ variation by the following amount:

$$-\frac{\sin(\alpha/2-\beta)}{\sin\alpha}\cdot\Delta(\overline{Ob'})$$

Consequently, the actual stock removal in this case needs the following correction from $(1-K)\cdot t(\theta_0+\theta)$:

$$-(1-K)^2\cdot\frac{\sin\left(\frac{\alpha}{2}-\beta\right)}{\sin\alpha}\cdot t\left(\theta_0+\theta+\frac{\alpha}{2}+\beta\right)$$

Similarly, when $-2(\alpha/2+\beta)\geqq\theta>-360°$, considering that, for $r(\theta_0+\theta+\alpha/2+\beta)$, $\overline{Ob}$ is subject to the above actual stock removal, i. e.,

$$-(1-K)\cdot t(\theta_0+\theta+\alpha/2+\beta)-(1-K)^2\cdot\frac{\sin\left(\frac{\alpha}{2}-\beta\right)}{\sin\alpha}\cdot t\{\theta_0+\theta+2(\alpha/2+\beta)\}$$

the actual stock removal is figured out by correcting $(1-K)\cdot t(\theta_0+\theta)$ by the following amount:

$$(1-K)^2\frac{\sin\left(\frac{\alpha}{2}-\beta\right)}{\sin\alpha}\cdot t\left(\theta_0+\theta+\frac{\alpha}{2}+\beta\right)+(1-K)^3\cdot\left\{\frac{\sin\left(\frac{\alpha}{2}-\beta\right)}{\sin\alpha}\right\}^2\cdot t\left\{\theta_0+\theta+2\left(\frac{\alpha}{2}+\beta\right)\right\}$$

In the similar manner, successive $\overline{Ob'}$ values may be expressed by $r(\theta_0+\theta+\alpha/2+\beta)$ corrected only by the above variation due to the depth of cut, while $\overline{Oc'}$ may be expressed simply by $r(\theta_0+\theta-180°-\alpha/2+\beta)$. Then, within the following range of θ, i.e.,

$$-\left(\frac{\alpha}{2}+\beta\right)\geqq\theta=-(L+l)\left(\frac{\alpha}{2}+\beta\right)>-360° \qquad (10)$$

where L=positive integer, and $0\leqq l<1$

the actual stock removal by the *gg* plane needs the following correction for $(1-K)\cdot t(\theta_0+\theta)$.

$$\Delta t_1=\sum_{s=1}^{L}(1-K)^{S+1}\cdot\left\{\frac{\sin\left(\frac{\alpha}{2}-\beta\right)}{\sin\alpha}\right\}^S\cdot t\left\{\theta_0+\theta+S\left(\frac{\alpha}{2}+\beta\right)\right\} \qquad (11)$$

Next, within the following range of θ, i.e.,

$$-(n+1)\left(\frac{\alpha}{2}+\beta\right)\geqq\left\{\begin{matrix}\theta=-(L'+l'+n)\left(\frac{\alpha}{2}+\beta\right)\\ =-(L+l)\left(\frac{\alpha}{2}+\beta\right)\end{matrix}\right\}>-360° \qquad (12)$$

where (L', L) = positive integers, and $0\leqq(l', l)<1$

$\overline{Ob'}$ and $\overline{Oc'}$ are considered to be subject to the above variations due to the depth of cut for $r(\theta_0+\theta+\alpha/2+\beta)$ and $r(\theta_0+\theta-180°-\alpha/2+\beta)$, respectively; then the actual stock removal by the gg plane needs the following correction for $(1-K)\cdot t(\theta_0+\Theta)$:

$$\Delta t_1+\Delta t_2=\sum_{S=1}^{L}(1-K)^{S+1}\left\{\frac{\sin\left(\frac{\alpha}{2}-\beta\right)}{\sin\alpha}\right\}^S\cdot t\left\{\theta_0+\Theta+S\left(\frac{\alpha}{2}+\beta\right)\right\}$$

$$-\sum^{L'}S\cdot(1-K)^{S+1}\cdot\frac{\sin\left(\frac{\alpha}{2}+\beta\right)}{\sin\alpha}\cdot\left\{\frac{\sin\left(\frac{\alpha}{2}+\beta\right)}{\sin\alpha}\right\}^{S-1}\cdot t\left\{\theta_0+\Theta\right.$$

$$\left.+(n+S)\left(\frac{\alpha}{2}+\beta\right)\right\} \tag{13}$$

Here, Δt_2 represents thc second term on the right side of the above equation. Also, within the following range of Θ, i. e.

$$-(2n+2)\left(\frac{\alpha}{2}+\beta\right)\geqq\left\{\begin{array}{l}\Theta=-(L''+l''+2n+1)\left(\frac{\alpha}{2}+\beta\right)\\ \quad=-(L'+l'+n)\left(\frac{\alpha}{2}+\beta\right)\\ \quad=-(L+l)\left(\frac{\alpha}{2}+\beta\right)\end{array}\right\}>-360° \tag{14}$$

where (L'', L', L) = positive integers, and $0\leqq(l'', l', l)<1$

$\overline{Ob'}$ and $\overline{Oc'}$ are considered subject to the above variations due to the depth of cut for $r(\theta_0+\Theta+\alpha/2+\beta)$ and $r(\theta_0+\Theta-180°-\alpha/2+\beta)$, respectively; then the actual stock removal by the gg plane needs the following correction for $(1-K)\cdot(\theta_0+\Theta)$:

$$\Delta t_1+\Delta t_2+\Delta t_3=\sum_{S=1}^{L}(1-K)^{S+1}\cdot\left\{\frac{\sin\left(\frac{\alpha}{2}-\beta\right)}{\sin\alpha}\right\}^S\cdot t\left\{\theta_0+\Theta+S\left(\frac{\alpha}{2}+\beta\right)\right\}$$

$$-\sum_{S=1}^{L'}S\cdot(1-K)^{S+1}\cdot\frac{\sin\left(\frac{\alpha}{2}+\beta\right)}{\sin\alpha}\cdot\left\{\frac{\sin\left(\frac{\alpha}{2}-\beta\right)}{\sin\alpha}\right\}^{S-1}\cdot t\left\{\theta_0+\Theta+(n+S)\left(\frac{\alpha}{2}+\beta\right)\right\}$$

$$+\sum_{S=1}^{L''}\frac{(1+S)S}{2}\cdot(1-K)^{S+2}\cdot\left\{\frac{\sin\left(\frac{\alpha}{2}+\beta\right)}{\sin\alpha}\right\}^2\left\{\frac{\sin\left(\frac{\alpha}{2}+\beta\right)}{\sin\alpha}\right\}^{S-1}\cdot t\left\{\theta_0+\Theta\right.$$

$$\left.+(2n+1+S)\left(\frac{\alpha}{2}+\beta\right)\right\} \tag{15}$$

Here, Δt_3 represents the third term on the right side of the above equation.

The above-explained actual stack removal corrections, Δt_1, $\Delta t_1+\Delta t_2$, $\Delta t_1+\Delta t_2+\Delta t_3$, are for $350°<3(n+1)(\alpha/2+\beta)$, and they are shown against the work rotation angle Θ in Fig. 4.

Namely, $(1-K)\cdot t(\theta_0+\Theta)$ is shown by the horizontal arrow line (I) for the Θ range of $0\sim360°$; for this the actual stock removal correction is Δt, consisting of the horizontal arrow lines (II_1), (II_2),…… which represent the terms for $s=1, 2,$…… inside Σ on the right side of oq. (11) and have phase differences of $-(\alpha/2+\beta)$, $-2(\alpha/2+\beta)$,

......, respectively, from $\theta=0$. In Δt_2, the horizontal arrow lines (III_1), (III_2),...... represent the terms for $s=1, 2,$...... inside Σ in the second term on the right side of eq. (13) and have phase differences of $-(n+1)(\alpha/2+\beta)$, $-(n+2)(\alpha/2+\beta)$ ...... from $\theta=0$. Similarly, in Δt_3, the horizontal arrow lines ((IV_1), (IV_2), ... represents the terms for $s=1, 2,$ ...... inside Σ in the third term on the right side of eq. (15) and have phase differences of $-(2n+2)(\alpha/2+\beta)$, $-(2n+2)(\alpha/2+\beta)$, ... from $\theta=0$.

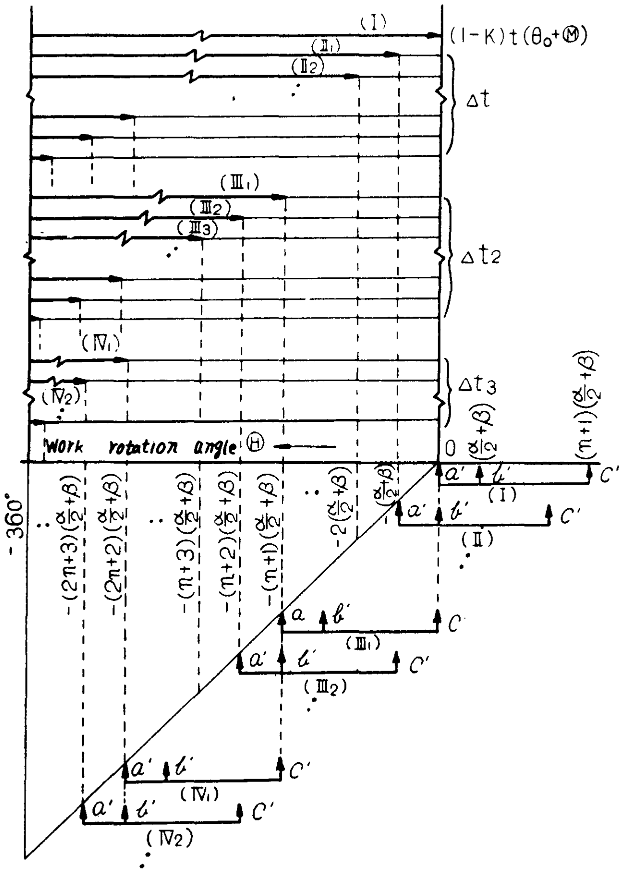


Fig. 4.

In short, the actual stock removal for a given work rotation angle θ is the total of the horizontal arrow lines crossing the perpendicular drawn to the horizontal axis at θ. In the figure, also $\underline{\uparrow a'\uparrow b'\uparrow c'}$ is the expansion of $\overline{Oa'}$, $\overline{Ob'}$ and $\overline{Oc'}$ in Fig. 3; b' and c' have phase differences from a' of $(\alpha/2+\beta)$ and $(n+1)(\alpha/2+\beta)$, respectively, and as $\underline{\uparrow a\uparrow b\uparrow c}$ moves leftward, it shows the relationship between the work radius changes in b' and c' or the work radius changes due to the actual stock removal on one hand and the actual stock removal in a' based on these on the other. For instance, in $\underline{\uparrow a'\uparrow b'\uparrow c'}$, b' and c' are unground part radiuses, and a' has a actual stock removal for $\Theta=0$, represented by the horizontal arrow line (I); similarly, in $\uparrow a'\uparrow b'\uparrow c'$, c' alone is an unground part radius, and b' is the unground part radius minus the actual stock removal represented by the part of the horizontal arrow line (I) corresponding to the position of b', whereas in a' there occurs actual stock removal represented by the parts of (I) and (II_1) correspending to $\Theta=-(\alpha/2+\beta)$.

Now, within the ordinary range of work supporting conditions, the constants, $\sin(\alpha/2-\beta)\sin\alpha$ and $\sin(\alpha/2+\beta)/\sin\alpha$, in the formulas representing the actual stock removal corrections in eq. (11), (13) and (15) are within the ranges of $-0.11\sim0.20$ and $0.90\sim1.06$, respectively, and K is smaller that 1. Therefore, the terms for $s=2$ and over inside Σ on the right side of eq. (1), (13) and (15) are omitted as negligibles and again it is assumed that $\theta_0+\Theta=\theta$; then the actual stock removal is as follows:

$$\left.\begin{array}{l}
[360^\circ+\theta_0\geqq\theta>360^\circ-\alpha/2-\beta+\theta_0] \text{ interval : } (1-K)\cdot t(\theta) \\
[360^\circ-\alpha/2-\beta+\theta_0\geqq\theta>180^\circ+\alpha/2+\beta+\theta_0] \text{ interval :} \\
\qquad (1-K)\cdot t(\theta)+(1-K)^2\cdot\dfrac{\sin\left(\frac{\alpha}{2}-\beta\right)}{\sin\alpha}\cdot t\left(\theta+\frac{\alpha}{2}+\beta\right) \\
[180^\circ+\alpha/2-\beta+\theta_0\geqq\theta>\theta_0] \text{ interval :} \\
\qquad (1-K)\cdot t(\theta)+(1-K)^2\cdot\dfrac{\sin\left(\frac{\alpha}{2}-\beta\right)}{\sin\alpha}\cdot t\left(\theta+\frac{\alpha}{2}+\beta\right) \\
\qquad -(1-K)^2\cdot\dfrac{\sin\left(\frac{\alpha}{2}+\beta\right)}{\sin\alpha}\cdot t\left(\theta-180^\circ-\frac{\alpha}{2}+\beta\right)
\end{array}\right\} \tag{16}$$

Therefore, if the radius of the work after one rotation is expressed as $r_1(\theta)$, $r(\theta)$ minus the actual stock removal figured out from the above for the pertinent range of θ will yield $r_1(\theta)$.

And since the grinding wheel usually gives a depth of cut which is larger than the height of bumps along the circumference of the cross section of the work, it is considered that in the $t(\theta)$ function the constant terms are larger than the variable ones. In eq. (16), therefore, only the constant ones among the terms have $(1-K)^2$ coefficients are considerd, and $r_1(\theta)$ for the $(\theta, 360^\circ+\theta_0)$ rarge is expanded into a Fourier series as follows:

$$r_1(\theta)=(a_0-a_m)\cdot\eta+a_m+\sum_{i=1}^{\infty}\{C_i\cdot\mu_i\cos(i\theta+\varphi_i+\delta_i) \\ +(a_0-a_m)D_i\cos(i\theta-i\theta_0-\varepsilon_i)\} \tag{17}$$

where

$$\eta=1-(1-K)\cdot\left(1+\frac{\sin\beta}{\sin\alpha/2}\right)+\frac{(1-K)^2}{4}\cdot\left(\frac{\sin\beta}{\sin\alpha/2}\right)\left\{\left(3-\frac{2\beta}{\pi}\right)\frac{\sin\beta}{\sin\alpha/2} -\left(1-\frac{\alpha}{\pi}\right)\frac{\cos\beta}{\cos\alpha/2}\right\}$$

$$\left.\begin{aligned}
&\mu_i=\sqrt{\{1-(1-K)\cdot(1+e_i)\}^2+\{(1-K)\cdot f_i\}^2}\\
&\delta_i=-\tan^{-1}\{(1-K)\cdot f_i\}/\{1-(1-K)\cdot(1+e_i)\}]\\
&D_i=\frac{(1-K)^2}{i\pi}\left(1+\frac{\sin\beta}{\sin\alpha/2}\right)\sqrt{\left(\frac{\sin\beta}{\sin\alpha/2}-e_i\right)^2 f_i^2}\\
&\varepsilon_i=\tan^{-1}\left[\left(\frac{\sin\beta}{\sin\alpha/2}-e_i\right)/f_i\right]
\end{aligned}\right\}\tag{18}$$

This approximately yields $r_1(\theta)$ expressing the profile of the cross section of the work after one rotation.

Next, if the profile curve of the workafter two rotations is called $r_2(\theta)$, considering the change of eq. (1) into eq. (17),

$$\begin{aligned}
r_2(\theta)=&[\{(a_0-a_m)\eta+a_m\}-a_m]\eta+a_m+\sum_{i=1}^{\infty}(C_i\mu_i)\mu_i\cos\{(i\theta+\varphi_i+\delta_i)+\delta_i\}\\
&+\{(a_0-a_m)D_i\}\mu_i\cos\{(i\theta-i\theta_0-\varepsilon_i)+\delta_i\}+\{(a_0-a_m)\eta+a_m-a_m\}D_i\\
&\times\cos(i\theta-i\theta_i-\varepsilon_i)\}=(a_0-a_m)\eta^2+a_m\sum_{i=1}^{\infty}\{C_i\mu_i^2\cos(i\theta+\varphi_i+2\delta_i)\\
&+(a_0-a_m)D_i\mu_i\cos(i\theta-i\theta_0-\varepsilon_i+\delta_i)+(a_0-a_m)\eta D_i\cos(i\theta-i\theta_0-\varepsilon_i)]
\end{aligned}$$

The same process may be repeated until, after N rotations, $r_N(\theta)$ expressing the profile of the work will be as follows:

$$\begin{aligned}
r_N(\theta)=&(a_0-a_m)\eta^N+a_m+\sum_{i=1}^{\infty}C_i\mu_i^N\cos(i\theta+\varphi_i+N\delta_i)\\
&+\sum_{i=1}^{\infty}(a_0-a_m)D_i\left[\begin{aligned}
&\mu_i^{N-1}\cos\{i\theta-i\theta_0-\varepsilon_i+(N-1)\delta_i\}+\eta_i\mu^{N-2}\cos(i\theta-i\theta_0\\
&-\varepsilon_i+(N-2)\delta_i\}+\eta^2\mu^{N-3}\cos\{i\theta-i\theta_0-\varepsilon_i+(N-3)\delta_i\}+\cdots\cdots\\
&+\eta^{N-3}\mu_i^2\cos\{i\theta-i\theta_0-\varepsilon_i+2\delta_i\}\\
&+\eta^{N-2}\mu_i\cos(i\theta-i\theta_0-\varepsilon_i+\delta_i)+\eta^{N-1}\cos(i\theta-i\theta_0-\varepsilon_i)
\end{aligned}\right]
\end{aligned}\tag{19}$$

The members inside [ ] in the fourth term on the right side of the above equation may be added up by means of complex numbers. Eq. (19) is thus rewritten as follows:

$$\begin{aligned}
r_N(\theta)=&(a_0-a_m)\eta^N+a_m+\sum_{i=1}^{\infty}C_i\mu_i^N\cos(i\theta+\varphi_i+N\delta_i)\\
&+\sum_{i=1}^{\infty}(a_0-a_m)D_i\sqrt{\frac{\eta^{2N}-2\eta\mu_i^N\cos N\delta_i+\mu_i^{2N}}{\eta^2-2\eta\mu_i\cos\delta_i+\mu_i^2}}\cos\Big[i\theta-i\theta_0\\
&-\varepsilon_i-\tan^{-1}\left\{\frac{\eta^N\mu_i\sin\delta_i-\eta\mu_i^N\sin N\delta_i+\mu_i^{N+1}\sin(N-1)\delta_i}{\eta^{N+1}-\eta^N\mu_i\cos\delta_i-\eta\mu_i^N\cos N\delta_i+\mu_i^{N+1}\cos(N-1)\delta_i}\right\}\Big]
\end{aligned}\tag{20}$$

If, in eq. (1) and (20), the terms for $i=2$ and over alone are taken, they are exclusively related to the out-of-roundness, and so the out-of-roundness curves may be expressed respectively by the following:

$$u(\theta)=\sum_{i=2}^{\infty} C_i \cos(i\theta+\varphi_i) \tag{21}$$

and

$$u_N(\theta)=\sum_{i=2}^{\infty}\{C_i\,\mu_i^N \cos(i\theta+\varphi_i+N\delta_i)+D_i' \cos(i\theta+\varphi_i+N\delta_i-\Phi)\} \tag{22}$$

where

$$\left.\begin{aligned} D_i' &=(a_0-a_m)\,D_i\sqrt{\frac{\eta^{2N}-2\eta^N\mu_i^N\cos N\delta_i+\mu_i^{2N}}{\eta^2-2\eta\,\mu_i\cos\delta_i+\mu_i^2}} \\ \Phi &=i\theta_0+\varepsilon_i+\varphi_i+N\delta_i \\ &\quad+\tan^{-1}\left\{\frac{\eta^N\mu_i\sin\delta_i-\eta\,\mu_i^N\sin N\delta_i+\mu_i^{N+1}\sin(N-1)\delta_i}{\eta^{N+1}-\eta^N\mu_i\cos\delta_i-\eta\,\mu_i^N\cos N\delta_i+\mu_i^{N+1}\cos(N-1)\delta_i}\right\} \end{aligned}\right\} \tag{23}$$

Also, if eq. (22) is expressed in the following form:

$$u_N(\theta)=\sum_{i=2}^{\infty} C_i' \cos(i\theta+\varphi_i') \tag{22'}$$

there are the following relationships between C_i and C'_i. and φ_i and φ'_i in eq. (22) and (22'):

$$C_i'=\sqrt{(C_i\mu_i^N)^2+2(C_i\mu_i^N)\,D_i'\cos\Phi+D_i'^2} \tag{24}$$

$$\varphi_i'=-\tan^{-1}\left\{\frac{-C_i\mu_i^N\sin(\varphi_i+N\delta_i)+D_i'\sin(\Phi-\varphi_i-N\delta_i)}{C_i\mu_i^N\cos(\varphi_i+N\delta_i)+D_i'\cos(\Phi-\varphi_i-N\delta_i)}\right\} \tag{25}$$

From the above analysis results, following may be seen:

(i) The higher harmonics $C_i\cos(i\theta+\varphi_i)(i=2, 3, \cdots\cdots)$ of the pre-centerless grinding out-of-roundness curve $u(\theta)$ is the out-of-roundness curve $u_N(\theta)$ yielded by compounding the higher harmonics $C_i\,\mu_i^N\cos(i\theta+\varphi_i+N\delta_i)$ having amplitudes proportionate to C_i after the centerless grinding and the higher harmonics $D_i'\cos(i\theta+\varphi_i+N\delta_i-\Phi)$ having amplitudes D_i' independent of C_i.

(ii) Therefore, if $C_i\,\mu_i^N \gg D_i'$ and D_i' is negligible, there is a relationship of approximate proportionality between the pre-grinding and post-grinding higher harmonics.

(iii) And if, before the grinding, $C_i=0$, the post-grinding out-of-roundness curve will inevitably contain the number of order i higher harmonics, $D_i'\times\cos(i\theta+\varphi_i+N\delta_i+\Phi)$. Moreover, even if C_i is 0 for all values of $i\,(i=2, 3, 4, \cdots\cdots)$, i.e., even if the work is perfectly round, the post-grinding out-of-roundness curve will be $\sum_{i=2}^{\infty} D_i'\cos(i\theta+\varphi_i+N\delta_i-\Phi)$ and will not be perfectly round.

Hence it is supposed that μ_i^N and D_i' mentioned above are closely related to a_i and C_{i0}, respectively, which, as was stated in the last report, characterize the relationship between the pre-grinding and post-grinding higher harmonics. Let us look further into these by numerical calculations in the following sections.

3. Numerical calculations.

For comparison with the experiment results about a_i and C_{i0} in tbe last report, μ_i and D_i' were numerically calculated under the following experiment conditions:

Diameter of grinding wheel, $Dg=400$mm

Diameter of regulating wheel, $Dc=230$mm

Diameter of work, $d=30$mm

3. 1 Numerical calculations of μ_i.

It is seen from eq. (18) and (7) that μ_i is a function of i and K and the work supporting conditions (α, β), and from eq. (23), (18) aud (7) it is seen that D_i' is a function of i and K, work supporting conditions (α, β), average radius reduction (a_0-a_m) and the number of grinding rotations N. Moreover, D_i' is also a function having μ_i as one of its independent variables. Therefore, μ_i is considered an important variable determining the characteristics of the pre-grinding higher harmonic amplitudes.

Now, let us assume that K is 0.7(note 1), a value considered practically appropriate, and figure out the μ_i values against the supporting height H, with the vertical angle ϕ of the support set at four stages, $\phi=45°$, 60°, 75° and 90°. The μ_i values thus calculated are illustrated in Fig. 5.

In the above cases, μ_i was figured out on the assumption that $K=0.7$. Now, with

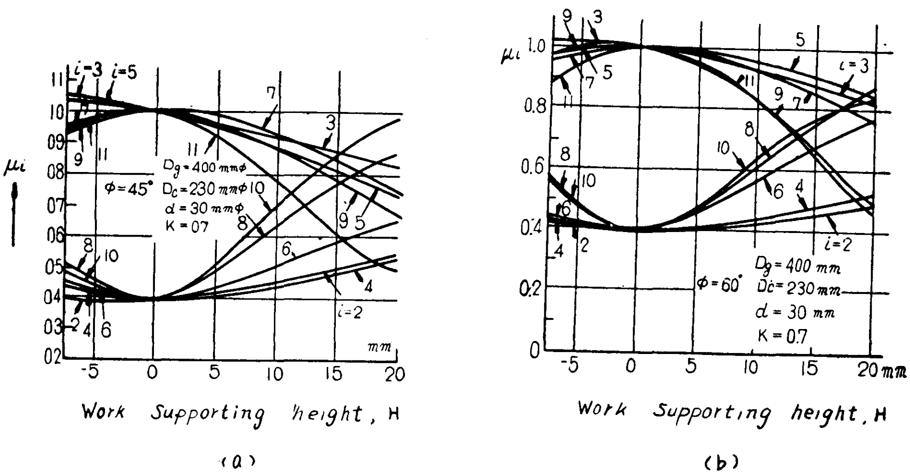


Fig. 5. (*a*), (*b*)

Note 1: In eq. (4) K was a constant representing consideration for the work supporting systyem's distortion and the cutting residue. If it is considered to represent the latter alone, the appropriate value of K under the experiment conditions of the last report is approximately 0.7. For calculations of (depth of cut)−(actual stock removal), see, for instance,

Shiozaki: Mechanical Society Papers, Vol. 18, No. 74, 1952, p. 10.

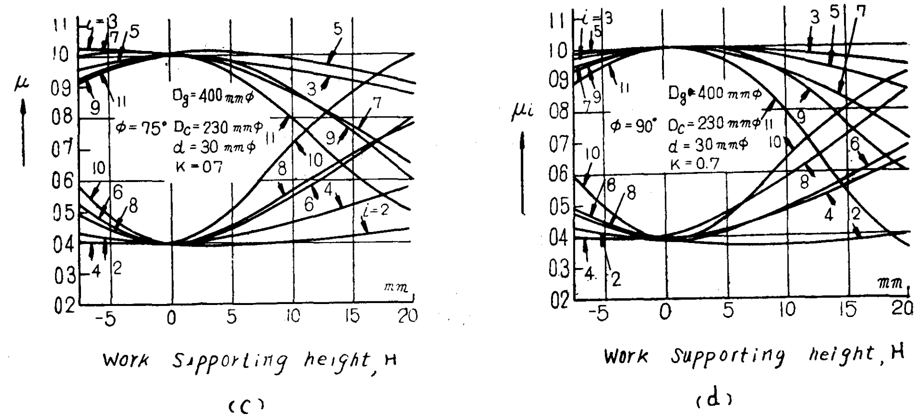


Fig.5. (*c*), (*d*)

H held constant at 7.5mm, and ϕ set at two stages, 60° and 75°, μ_i was figured out against K. The results are illustrated in Fig. 6.

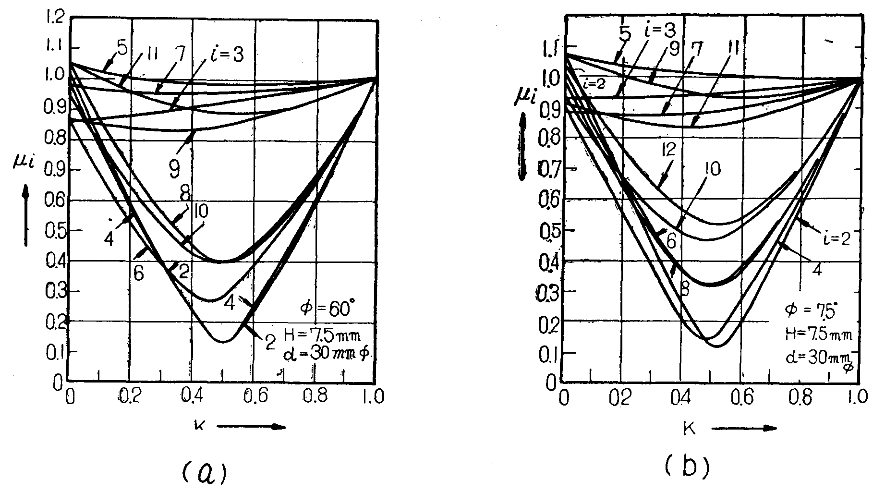


Fig. 6. (*a*), (*b*)

On the basis of the above numerical calculation results, let us see in the following conditions affect μ_i^N and D_i'.

3. 2 Influence of work supporting height.

In centerless grinding, a actual stock reooval equal to the depth of cut of the grinding wheel can be achieved only after the work has been rotated 10~20 times[7]. If the number of times it is ground $N=10$, μ_i^N shows the Fig. 7 curves against the supporting height H; and with the depth of cut measured in terme of radial length, D_i' may be illusrrcted against H as in Fig. 8. In both cases, $K=0.7$ and $\phi=60°$.

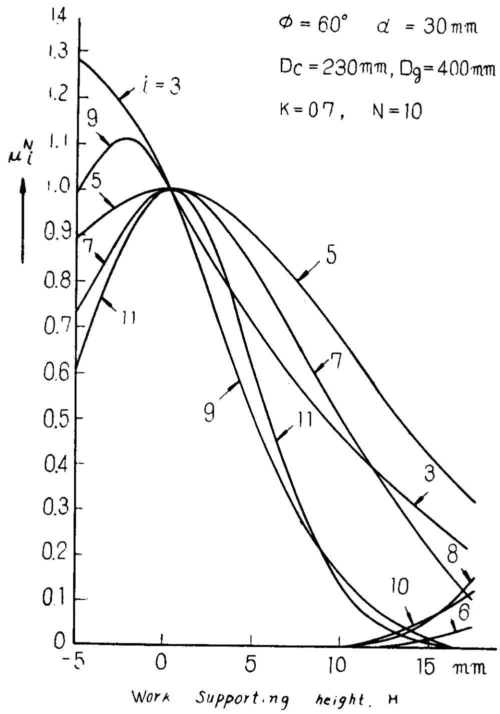
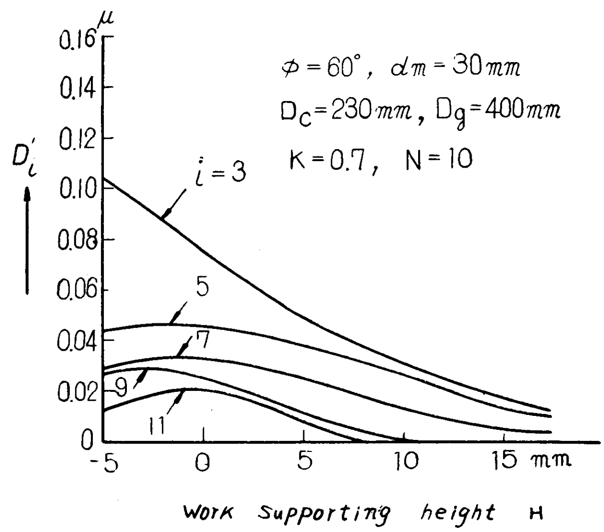

Φ = 60° α = 30 mm
Dc = 230 mm, Dg = 400 mm
K = 0.7, N = 10
μ_i^N
i = 3
Work Supporting height H
mm

Fig. 7.


Φ = 60°, dm = 30 mm
Dc = 230 mm, Dg = 400 mm
K = 0.7, N = 10
D'_i
μ
i = 3
Work Supporting height H
mm

(a)

Fig. 8. (a)

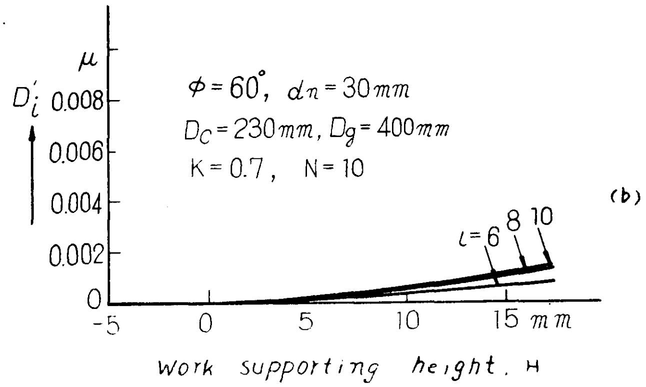


Fig. 8. (*b*)

From Fig. 7 it is seen that μ_i^N has the following characteristics against

(i) When $H=0$mm, μ_i^N values for odd numbers of order are all 1, and for even numbers of order μ_i^N is close to 0.

(ii) When H is within the range of 0∼15mm, μ_i^N for odd numbers of order tends to decrease as H increases whereas for even numbers of order μ_i^N tends to increase, thongh at a different rate for each number of order.

(iii) When H is within the range of 0∼−5mm, μ_i^N for odd numbers of order except 3 and 9 decreases below 1 as H becomes smaller.

Also, from Fig. 8 the following may be seen:

(i) When H is within the range of −5∼15mm, D_i' for odd numbers of order except 3 and 9 reaches the maximum at $H=0$mm.

(ii) With H within the range of −5∼15mm, D_i' for even numbers of order increases slightly as H becomes larger.

From the above findings it is seen that the characteristics of μ_i^N and D_i' against H qualitatively agree well with the characteristics of a_i and C_{i0} shown in the last chapter is taken into consideration, it appears that a_i and C_{i0} are closely related to the work supporting conditions, particularly the work supporting height.

3. 3 Influence of top angle of blade.

Next, with H held constant at 7.5, the $D_i'-\phi\,(45°\sim90°)$ relationship is as shown in Fig. 9. Here, again D_i' for even numbers of order is close to 0. The illustration shows that, as ϕ increases from 45° to 90°, D_i' varies differently for each number of order; but it does not seem to have any simple tendency in its relationship to ϕ.

Also, with H held constant at 7.5mm, ϕ was set at 60° and 90°, and μ_i^N values under these conditions were calculated to be as shown in Table 1.

From the above it is seen that, with the supporting height fixed; μ_i^N and D_i' vary according to the top angle of blade, but its influence is not so marked as that of the work supporting height, and it shows a tendendy roughtly in agreement with the characteristics of a_i and C_{io} in the experiments.

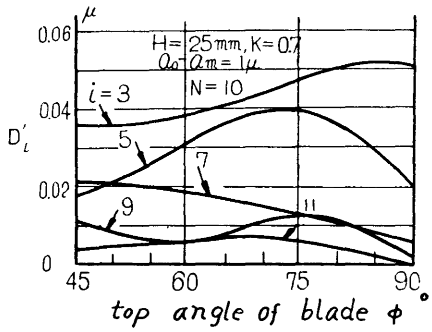


Fig. 9.

Table 1

φ	60°	90°
μ_3^{10}	1·23	0·96
μ_5^{10}	0·94	0·90
μ_7^{10}	0·90	0·78
μ_9^{10}	1·06	0·74
μ_{11}^{10}	0·77	0·59

3. 4 Influence of depth of cut.

It is well known that, in grinding, the roughness of the finished surface of the work generally tends to increase, though slightly, as the depth of cut of the grinding wheel becomes larger[(8)].

Here, for the sake of simplification, let us assume that an increase in the roughness may be represented by a slight decrease in the value of K in eq. (4), and figure out K on the assumption. When the depth of cut as expressed in terms of the diameter reduction is 20μ and 40μ, K is 0.75 and 0.70, respectively, showing a slight difference between them. The vaues of μ_i^N numerically calculated in these cases are as shown in Table 2, indicating that μ_i^N for odd number of order decreases slightly as the depth of cut of the grinding wheel increase from 20μ to 40μ. On the other hand, from the D_i'-vs-K curves in Fig. 10, it is seen that as the depth of cut increases and K decreases, D_i' increases slightly, but in the experiments C_{io} does not increases so much as to be proportionate to the cut-in depth. The reason is considered to be as follows:

Table 2

Diameter reduction	20μ	40μ
K	0.75	0.70
μ_3^{10}	0.63	0.57
μ_5^{10}	0.82	0.79
μ_7^{10}	0.68	0.67
μ_9^{10}	0.35	0.29
μ_{11}^{10}	0.39	0.33

In the theoretical analysis in the last chapter, the depth of cut of the grinding wheel was determined geometrically on the assumption that there was no distortion of the work supporting system or evasion of the work. Consequently, D_i' in eq. (23) is proportionate to the average depth of cut in terms of radial length (a_0-a_m). In the actual centerless grinding in the experiments, however, it is difficult to consider that there was no distortion of the work supporting system or evasion of the work at all. This must be the reason why the numerical calculation results do not agree with the experiment results.

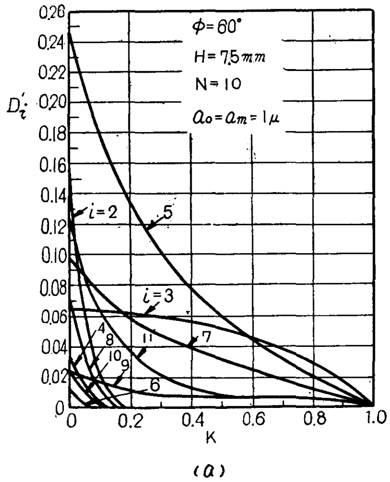
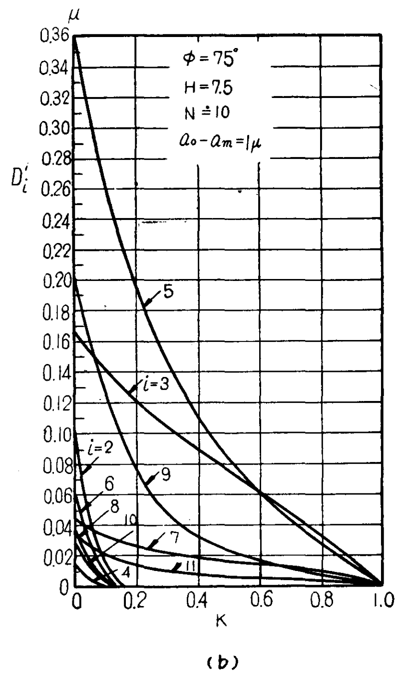


Fig. 10. (a), (b)



3. 5 Influence of circumferential velocity of work.

Generally, the smaller the circumferential velocity of the work, the larger the stock removal per rotation of the work, and the smaller the degree of roughness of the work. Hence it is possible to calculate K in the same manner as in the last section.

Table 3 shows the values of K and μ_i^N for three different circumferential velocity: Vc=13.3, 21 and 28.8m/min. According to the table, μ_i^N for odd numbers of order tends to decrease slightly as the circumferential velocity of the work decreases.

Table 3

Vcm/min	12.3	21	28.9
K	0.55	0.68	0.75
μ_3^{10}	0.43	0.55	0.63
μ_5^{10}	0.78	0.78	0.82
μ_7^{10}	0.59	0.64	0.68
μ_9^{10}	0.19	0.28	0.35
μ_{11}^{10}	0.18	0.31	0.39

Also, Fig. 9 shown previously indicates that as the circumferential velocity of the work decreses and K becomes smaller, D_i' for odd numbers of order tends to grow larger for the range of K shown in Table 3.

Consequently, it is seen that both μ_i^N and D_i' vary according to the circumferential velocity of the work roughly in the same manner as a_i and C_{io} vary in the experiments.

4. Conclusion.

From the facts in the preceding chapters, we find the following:

(i) The post-centerless grinding out-of-roundness curve equals higher hasmonics with amplitudes ($\mu_i^N C_i$) proportionate to the amplitudes of the pre-grinding out-of-roundness curve plus higher harmonics with entirely different amplitudes (D_i').

(ii) The factors μ_i^N and D_i' correspond to a_i and C_{io} in the last report[10], respectively.

(iii) μ_i^N and D_i' were numerically calculated in cases where the grinding conditions were held constant (K constant) and the supporting conditions varied, and in other cases where the supporting conditions were held constant and the grinding conditions varied (K varied). Comparison of these results with the experiment results in the last report indicates that both show amplitude characteristics of the same leaning.

Thus, it has been shown that the out-of-roundness curve expression (eq. 22 or 22′) in Chapter 2 is an appropriate one.

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(6) ibid, (4)

(7) ibid, (5)

(8) e. g., E. Saljé, Werkst. u. Betrieb, 86, (1953), Nr. 2, p. 45.

(9) Shiozaki: Mechanical Society Papers, Vol. 18, No. 47, 1952, p. 10.

(10) ibid, (4)