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Electromagnetic Field on Multi-Ring Line

(Received Sept. 2, 1959)

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Abstract

The basic study on the propagation characteristics of microwave which is excited to the open transmission line of the *multi-ring line* is our subject. TE-wave and TM-wave propagate along the line, but the frequency range of propagation is narrow in each propagating mode. The propagation cuts off when circumference of the ring equals the integral multiple of half wave length. The wave length on the line is observed to be shorter than the wave length in free space. The contractivity of the wave length is a function of wave length per radii, and we get the contractivity from zero to fifty per-cent.

I. Introduction

We pilld many conductor rings with space to make a microwave transmission line. This is a reason for the name of the "*multi-ring line*". We investigated a propagation constant and patterns of TE-mode of electromagnetic wave. The transverse magnetic modes must be used in the traveling wave tubes. And the transverse electric mode will be used in circular wave guide, and these multi-ring lines are considered to prevent generation of TM_{11} which is the degenerative mode of TE_{01} in the low loss transmission system of circular wave guide in the region of millimetre wave.

Nowadays, the investigations to eliminate the electric losses produced by degenerations in circular wave guides are all in the closed system.[1] However, a distinct feature of our work is to make experiments and theories in the open system.

Applying an idea of the sheeth helix,[2] we made an idea of *sheeth multi-ring line* in which the electric current flowed only along the circumference. As the distribution of electromagnetic field into the free space is *Bessel function*, is needs infinite electric power at the initial point, and the calculation does not coincide qualitatively with the experimental results. That is to say, the phase constant in free space,

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1) Bunichi Oguchi, Midori Kato "The Journal of the Institute of Electrical Communication Engineers of Japan" p. 717 Aug. 1956

2) J. R. Pierce "Traveling Wave Tube"

but in experiment, the magnitude of phase constant becomes vice versa. From the experimental results, the attenuation on the multi-ring line for a special wave length is very small. And the cut off phenomenon occurs when the circumference of the rings equals the integral multiple of half wave length,

In this report, we show the experimental processes and the results mainly, but we are now preparing the theoretical analyses with considerations on the dimensions of the ring. These will be reported in the coming memoir.

II. Propagation in Open Systen

The figure of propagation of the actual helix is quite similar to that of the seeth helix which is considered for convenience of calculation (Fig. 1). However, sheeth multi-ring line is very different from the actual multi-ring line in propagation of electromagnetic wave. For example, the phase velocity on the sheeth multi-ring line is faster than light velocity, but in actual multi-ring line the phase velocity is slower than light velocity.

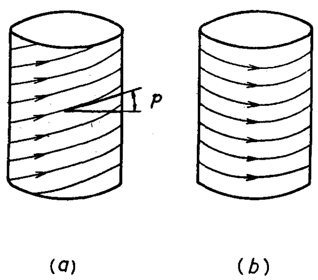

Fig. 1. (a) Sheeth helix having pitch angle p.
(b) Sheeth multi-ring line: electric current flows only along the circumference.

Accordingly, the meaning of "*to propagate*" may be different from the closed system. We are liable to cause misunderstanding that the propagation of the wave in sheeth multi-ring line is similar to the circular wave guide. If we integrate the z-component of *Poynting* vector from zero to infinite with r in the theory of sheeth multi-ring, the electromagnetic field which is given by *Bessel function* diverges. Now, we have to pay further consideration on the convergence of the field.

To make the field converge, the Bessel function must be a modified Bessel function, For the modified Bessel function we have to get a condition of $\beta_k^2 = \beta_K^2 + \beta_\gamma^2$ where β_k is phase constant in free space, β_K, β_γ are propagation constants of radial and axial direction. In this stimulation, β_K must be an imaginary number, accordingly $\beta_K^2 < 0$ and $\beta_\gamma > \beta_k$, namely, only the slower wave satisfies the physical phenomenon. We conclude there is no propagating mode in sheeth multi-ring line unlike sheeth helix.

Accordingly, we must give further consideration to the analyses which include thickness and space between rings. Accordingly, we are now on the way of calculation considering thickness and space, and the final results will be reported in near future.

III. Experimental Apparatus

Actual Multi-ring line

Construction of actual multi-ring line is as follows: mean diameters of rings are 52.5 mm, 29.2 mm and 17.3 mm, and cross sections are squares of 3.5 mm, 2.0 mm and 1.15 mm, respectively. They were piled to the length of about 1.5 metres, with spaces equal to the thikness of a ring. The spacing gap is made by air, and suspensions are by fishing lines and musical strings.

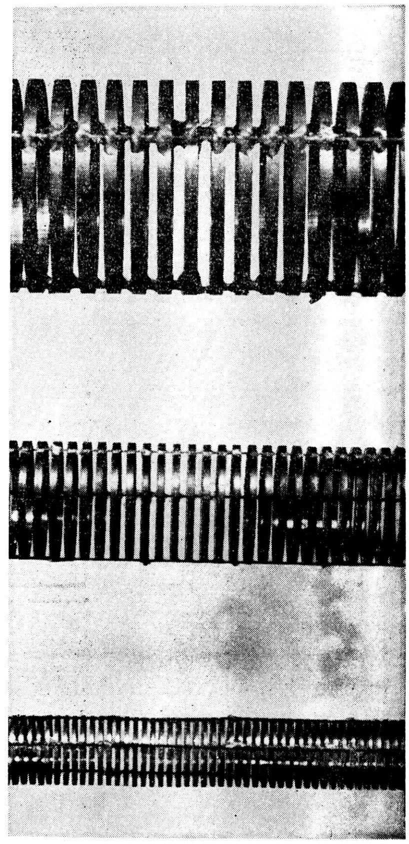

Fig. 2. Construction of the actual multi-ring line.

Excitation Method

For the excitation of transverse electric mode we cut a portion of a ring, and attach a coaxial cable to the ring (Fig.3). For TM-mode, as we see in Fig. 4, we use the same method to G-line. We used horns of large or small diameter. But by this method, we could not make the wave propagate on the multi-ring line We don't think that the method of excitation is inadequate, for, as we see in Fig. 5, the wave propagates in the some region of multi-ring line whree the G-line is inserted. From this fact, we conclude that excitation of TM-mode is almost impossible.

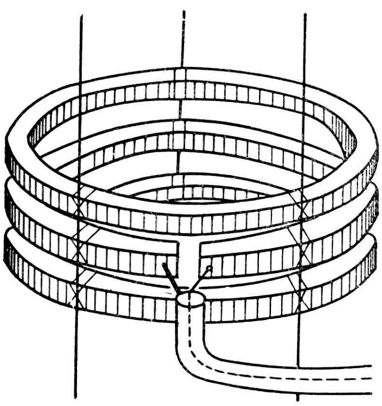

Fig. 3. Exitation of TE-mode.

Recording Equipments

Block diagrams to record electric fields are as in Fig. 6.

Microwave is oscillated from 2700 MC to 5400 MC by 6BL6, passes through a coaxial cable, and excites the multi-ring line. Excited wave propagating upward reflects by the reflector, and makes a standing wave. Small test dipole detects electric field on the multi-ring line without effecting the original field. And the very small electric current which is modulated to rectangular wave is guided to synchronous detector, and is amplified. This electric current

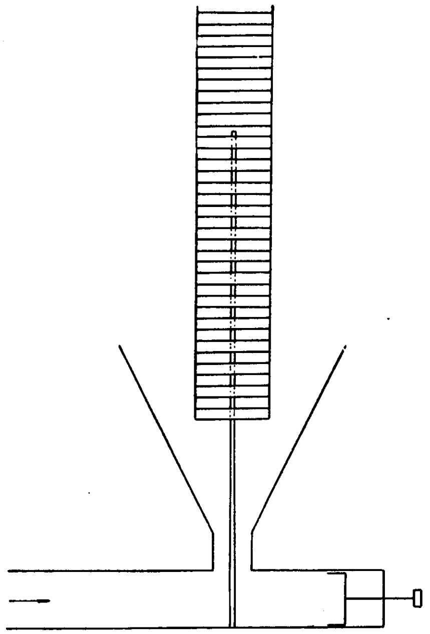

Fig. 4. Excitation of TM-mode.

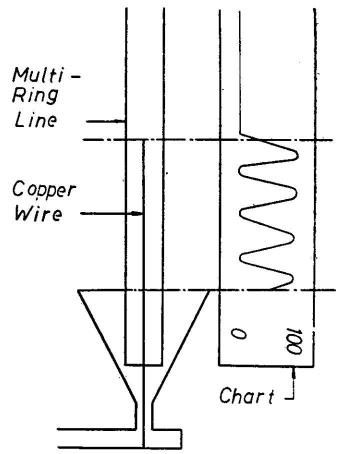

Fig. 5. The wave travels until the end of copper wire in TM-mode excitation.

is sent to magnetic amplifier, and then it drives recorder of 3 mA full scale. Recording system is convenient to know the whole phenomena in the long range. Test dipole moves 2.8 milli metres per second and this is suitable for a response time of equipments.

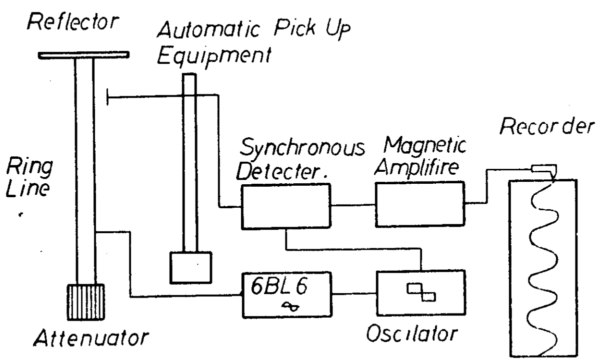

Fig. 6. Recording system.

Test Dipole for Measuring Electric field

The measuring method of electric field in the closed system has been well studied not to disturb the electric field like standing wave detector. In some fields of open system such as measurement of pattern of antenna have been studied.

However in transmission of power in open system, the experimental data are very few. Having no special measuring aparatus, we obtained a satisfactory test dipole after many investigations. We must consider the causes of error in measuring the electromagnetic field by the test dipole, such as (1) getting mean value of wider sphere, because of excessive length of antenna, (2) disturbance of electric field at the test dipole coming from the lead wire. The test dipole has a silicon diode stuck directly at the center of very short and slender doublet antenna (Fig 7).

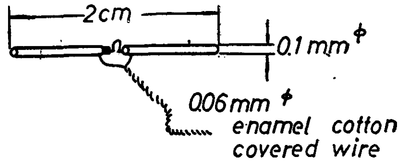

Fig. 7. Construction of test dipole.

Disturbing effect of lead wire is made very small by using wire as slender as possible. Reflection from the lead wire becomes smaller in proportion to the slenderness of the wire, as we can see in Fig. 8. We use 0.06 mmϕ enamel silk covered copper wire. When we make the lead wire slender, influence on electric field which comes perpendicular to the lead wire becomes very small. But when the electric field comes parallel to the lead wire, TM-mode is excited just like surface wave line of Gaubau, and forces the flow of the electric current at the test dipole. To keep away from the electric current of surface wave, it was expected that if the length of antenna of the test dipole is symmetric, the current will not influence the detected electric current because of the cancelling each other of the surface waves at the silicon diode. We cut and tried the length of antenna making the detected electric current of surface wave to be zero.

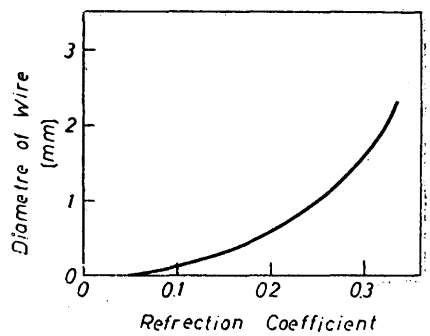

Fig. 8. Reaction of wire which is put near the transmission line.

After careful preparation as stated above, influence on the lead wire became negligible small. We coated aquadac on the lead wire to suppress reflection of and to attenuate the surface wave. We dicided the thickness of aquadac the wave after many experiments. These lead wires consisting of two conductors are twisted tightly, and thus choked the induction of magnetic field on the lead wire. As we decide TE-mode or TM-mode by this test dipole, it is very important to know a relationship between directions of electric field and the test dipole. To decide the characteristics of thst dipole, we put it in parallel electric field which is produced in rectangular wave guide, From the experimental result, we can decide that direction of electric field is parallel to the test dipole when it detects maximum current.

IV. Experimental Results

General method to investigate a characteristics of travelling electromagnetic wave is to show a relationship between the phase constant in free space and the phase constant in the medium. And to make a comparison with a theory, we have to know the propagation constant in the medium, patterns of circumferencial field and radial directions by the experiments. Still more, we take here experimental results of TE-mode and TM-mode. They are not directly necessary in order to compare with the theory, but these are very distinct facts. That is, there is phase difference of 90° between TE-mode and TM-mode at all conditions, and if we put a disturbance on one mode, the other mode is effected by the disturbance.

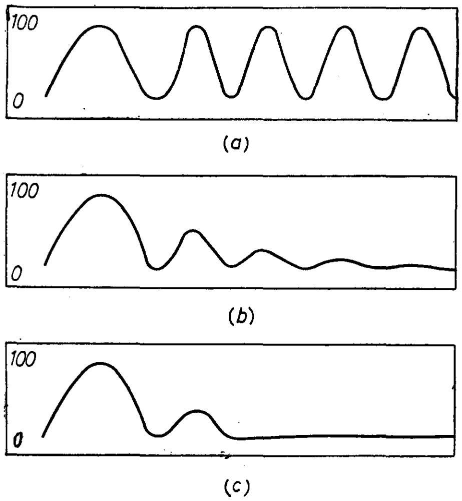

Fig. 9. Pattern of axial direction.
(a) z-direction pattern in propagating region.
(b) Intermediate region between propagation and non-propagation.
(c) z-direction pattern in non-propagating region.

Pattern of z-Direction

We have to investigate smoothly a long distance along the multi-ring line to judge its propagation. For that purpose, we used automatic pick up equipment, and record the electric field along z-direction automatically, and then we could see the circumstances far from feed point.

At rather large values of contractivity, propagating status is very clear as seen in Fig. 9 (a), and the vicinity of non-propagating region, in other words, near the zero point of contractivity becomes as Fig. 9 (b). In non-propagating range, a pattern becomes like Fig. 9 (c). From the figures, we can observe a propagation of sinusoidal wave which include no higher harmonics.

Pattern of θ-Direction

Circumferential pattern which is rectangular to z-direction, is very important for knowing the number of higher mode of the wave.

Here is instituted a question whether the influence upon θ-direction pattern is dependent only on frequency, or also on any other causes. Among these other influences considered, the first is the position of neighbouring structures and the second is the method of placing the reflector, the third is the position of feeding point and coaxial cable.

We explained the influences experimentally. Since the distance from the neighbouring structures, excluding the test dipole, are at least fifty centimeters, the effect is negligibly small. Even if we change the angle of reflector to the multi-ring line from 90° to 60°, the inclining does not effect the mode-number. We can conclude that there is no influence by small change of position and sphere of reflector. The method of TE feed, that is, feeding position and symmetry or unsymmetry are effectless on mode numbers. Accordingly, we may decide that the pattern of θ-direction is only effected by frequency.

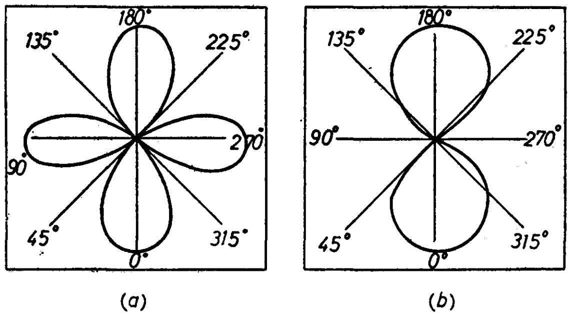

Fig. 10. Pattern of θ-direction in propagating region.
(a) Propagating region of $\lambda_0/2a=1.6\sim1.9$.
(b) Propagating region of $\lambda_0/3a=3.1\sim5.1$.

Propagating Region and Wave Contractivity

Propagating region and mode number are closely related to the diametre of ring and the wave length. Present experimental range of $\lambda_0/2a$ is from 1.2 to 6.4, and non-propagating regions are $\lambda_0/2a=1.2$ to 1.6, 1.92 to 3.12, 5.12 to 6.38 and propagating regions are $\lambda_0/2a$ 1.62 to 1.90, 3.14 to 5.12. Maximum contractivity investigated is about 50%, that is, propagative wave length becomes the half length of the wave in free space.

Patterns which are drawn in Fig. 11 mean θ-direction pattern in the region. We rewrite the Fig. 11 to Fig. 12 which shows a relation between phase constant β_γ on the multi-ring line and phase constant β_k in free space.

From Fig. 12, we see that propgating wave on multi-ring line is always slower than the wave in free space. In the vicinity of integer of β_k, β_γ becomes infinite and enters in cutoff region. Chages

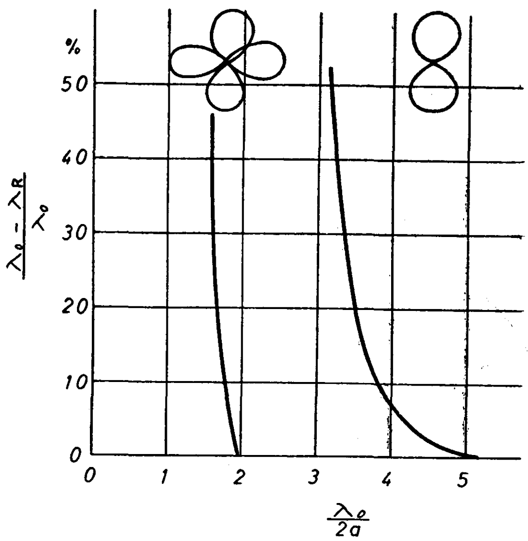

Fig. 11. Relation between $\lambda_0/2a$ and the contractivity of propagating wave. Pattern of θ-direction is different for each propagating region.

along circumference of a ring satisfies $\cos n\pi$ qualitatively. Then, 8-type region in Fig. 11 is in $n=1$ mode, and the cross type region is in $n=2$ mode. Therefore, we expect $n=0$ mode in lower region, and $n=3, 4, \cdots\cdots$ modes in upper region.

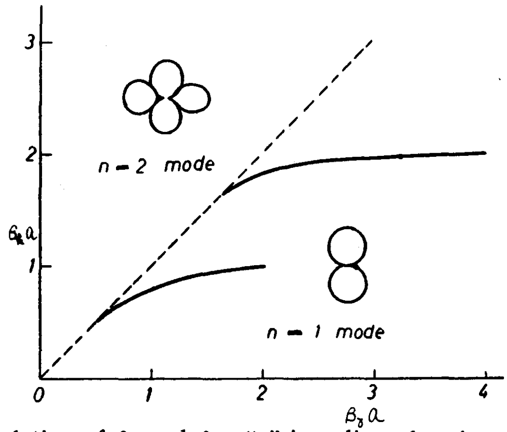

Fig. 12. Relation of β_γ and β_k. "a" is radius of a ring, dashed line means that the velocity of the wave on multi-ring line and the velocity in free space are the same. Above the dashed line the velocity on the multi-ring line is faster than in free space, and under the line vice versa.

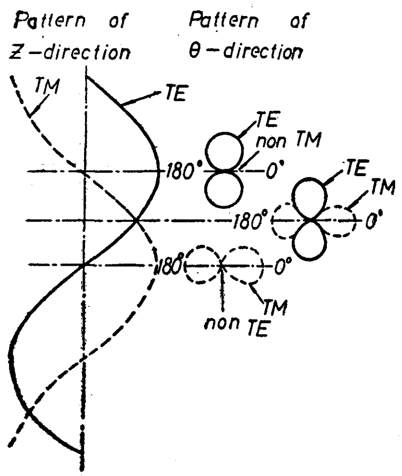

Fig. 13. TE-wave and TM-wave exist at the same time and have 90° phase difference in z-direction and θ-direction.

Relations of TE- and TM-mode

We couldn't make the TM excitation by G-line and electromagnetic horn. However we observe transverse magnetic wave at the same time when we excite the transverse electric wave. Furthermore, they 90° phase difference with each other in the pattern of z-direction and θdirection. Separation of TE-wave and TMwave or to observe any other phase differences have not been successful so far.

V. Conclusion

Multi-ring type transmission lines are treated as a portion of circular wave guide or delay circuit, but they are all under a boundary condition in which the electromagnetic field does not exist outside the line. However, here the electromagnetic field outside the line is allowed to exist as it is in helical transmission line. We cannot apply the idea of sheeth helix to our multi-ring line, therefore we conclude to take a model of which the dimensions are considered. Comparing with TE-mode, TM-mode is considered to be hard to propagate on the multi-ring line, for the electric current is hard to flow in z-direction because of its construction.

However, from an experimental result we observe that both of them propagate well, the propagation losses are so small that we cannot observe the losses in our range of one metre long. If we define that a circumferencial pattern of $\partial E/\partial\theta=0$ is the zeroth mode, we got the first and the second mode. If we make the frequency range wide, we will obtain the zeroth and up to third pattern. Since the n-th mode can bear a close resemblance to $\cos n\theta$ ($\theta=0$ to 2π), the zeroth mode becomes a circle, the first mode becomes 8-type and the second becomes cross type. And the experimental results on the circumferencial pattern coincide with the theory. By our experimental results, the wave length on the multi-ring line is shorter than the wave length in free space, namely, the contraction of the wave takes positive sign on every occasion.

At the contraction of 0% and 100% the cut-off phenamena occurs. In other words, the cut-off phenomena arises when the wave length in free space shifts to the vicinity of even number/circumference of ring. From these experiments, we are able to know some characteristics of the multi-ring line.

Acknowlegement

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Appendix

Electromagnetic Field on Sheeth Multi-Ring Line

At the outset when we got a solution of electromagnetic field on a helical transmission line, we took so called "sheeth helix" on which current flows only in the direction of pitch angle p (Fig. 1(a)). And the solution of electromagnetic field on the sheeth helix bears a resemblance to experimental measurement. Accordingly, we tried to entertain an idea of 'sheeth multi-ring line' which is very similar to the seeth helix and is constructed by infinitely slender rings with infinitely thin space, this is a kind of cylindrical wave guide which has no thickness and in which electric current flows only in the direction of the circumference.

Assumptions under the theorem of sheeth multi-ring line are as follows; electromagnetic field satisfies the Maxwellian electromagnetic equations, conductivity of the ring is infinite, propagation constant of propagating wave must be pure imaginary number, then electromagnetic field strength along θ-direction is identical and no higher modes exist.

The Case of Transverse Electric Wave Excitation

As we consider H-wave, we substitute $H_z e^{j\omega t-\gamma z}$ into the wave equation using cylindrical coordinates,

$$\frac{1}{r}\frac{\partial H_z}{\partial r}+\frac{\partial^2 H_z}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 H_z}{\partial \theta^2}=-K^2 H_z \tag{1}$$

$$K^2=\gamma^2-k^2 \tag{2}$$

where K, γ means respectively the propagation constant of radial, axial direction and k means popagation constant in free space.
From the assumption $\partial/\partial\theta=0$, we take

$$\frac{\partial^2 H_z}{\partial r^2}+\frac{1}{r}\frac{\partial H_z}{\partial r}+K^2 H_z=0 \tag{3}$$

And we get from the rotational equation as

$$H_r=-\frac{r}{K^2}\frac{\partial H_z}{\partial r} \tag{4}$$

A solution of Eq. (3) is

$$H_z=AJ_0(Kr)+BN_0(Kr) \tag{5}$$

As we have to decide A and B considering boundary conditions, we get $B=0$ then

$$H_z=AJ_0(Kr) \tag{6}$$

Outside the seeth multi-ring line, we get

$$H_z=A'J_0(Kr)+B'N_0(Kr) \tag{7}$$

$$H_r=\frac{\gamma}{K}\left\{A'J_1(Kr)+B'N_1(Kr)\right\} \tag{8}$$

$$E_\theta=-\frac{j\omega\mu}{K}\left\{A'J_1(Kr)+B'N_1(Kr)\right\} \tag{9}$$

At $r=a$, E_θ must vanish, and $J_1(Ka)=0$ then $B'=0$ may be settled.
Accordingly, the type of the solutions of the inside and outside of the sheeth multi-ring line is quite the same. A and A' will be settled as follows; the electric current which flows at the inside surface is

$$|\boldsymbol{J}_\theta|=|\boldsymbol{n}\times\boldsymbol{H}_{z\,in}|=AJ_0(Ka) \tag{10}$$

and to the outside is

$$|\boldsymbol{J}_\theta|=|\boldsymbol{n}\times\boldsymbol{H}_{z\,out}|=A'J_0(Ka) \tag{11}$$

Namely

$$A=A'$$

Electromagnetic field becomes as in Fig. 14. According to the propagation constant, we take k *and* γ pure imaginary and K has to be a real number, and in the case of

$$K^2=\gamma^2-k^2 \tag{13}$$

it becomes $|k|<|\gamma|$. From the above results, we conclude the wave length of propagating wave becomes longer than the wave length in free space.

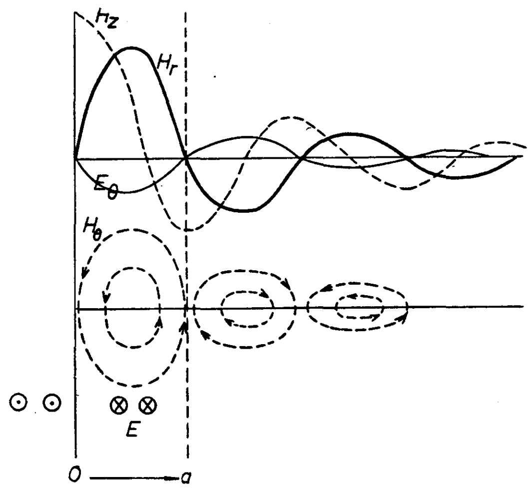

Fig. 14. Electromagnetic field of TE-wave in sheeth multi-ring line.

The Case of Transverse Magnetic Wave Excitation

We are able to get a wave equation according to E_z assuming $\partial/\partial\theta=0$

$$\frac{1}{r}\frac{\partial E_z}{\partial r}+\frac{\partial^2 E_z}{\partial r^2}=-K^2E_z \tag{14}$$

$$K^2=\gamma^2-k^2 \tag{15}$$

The equations exist inside the sheeth multi-ring line are

$$E_z=AJ_0(Kr) \tag{16}$$

$$E_r=A\cdot\frac{\gamma}{K}J_1(Kr) \tag{17}$$

$$H_\theta=A\cdot\frac{j\omega\varepsilon}{K}J_1(Kr) \tag{18}$$

$$H_r=H_\theta=E_\theta=0 \tag{19}$$

On the other hand, the equations outside the sheeth multi-ring line are

$$E_z=A'J_0(Kr)+B'N_0(Kr) \tag{20}$$

$$E_r=\frac{\gamma}{K}[A'J_1(Kr)+B'N_1(Kr)] \tag{21}$$

$$H_\theta = j\frac{\omega\varepsilon}{K}[A'J_1(Kr)+B'N_1(Kr)] \tag{22}$$

Considering a continuity of E_z and H_θ at $r=a$, we cancel the constants A, A', B, B', then we take

$$\frac{J_0(Ka)}{N_0(Ka)} = \frac{J_1(Ka)}{N_1(Ka)} \tag{23}$$

We cannot find out K satisfying the Eq. (23) in real numbers.
If it is the imaginary number, we put $K=jP$, then we get a modified bessel function, that is

$$E_z = BI_0(Pr) \tag{24}$$

$$E_r = -B\frac{\gamma}{P}I_1(Pr) \tag{25}$$

$$H_\theta = -Bj\frac{\omega\varepsilon}{P}I_1(Pr) \tag{26}$$

Considering the continuity at $r=a$, we can cancel out the constants A, A', B, B'.

$$\frac{I_0(Pa)}{I_1(Pa)} = \frac{K_0(Pa)}{K_1(Pa)} \tag{27}$$

However P which satisfies the Eq. (27) also does not exist. Electric current must flow in z-direction for the transverse magnetic mode on the surface of the sheeth multi-ring line.

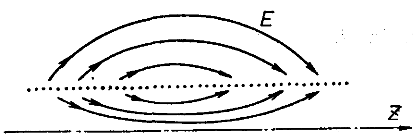

Fig. 15. Electric current cannot be flowed along z-direction.

However, electric current flow in the z-direction because of the stractual reason as we see in Fig. 15.

Here, we get a conclusion about the theorem on the sheeth multi-ring line, that is; transverse electric mode will propagate in the same way as TE_{01} mode in a circular wave guide. But outside the sheeth multi-ring line, the integration of Poynting vector from zero to infinite along the radial direction diverges as

$$\int_0^\infty \boldsymbol{E}\times\boldsymbol{H}\,dr = r\cdot P\int_0^\infty J_1(Kr)J_0(Kr)\,dr + ZQ\int_0^\infty \{J_1(Kr)\}^2 \to \infty$$

The propagation of transverse electric mode cannot be in existence as the transverse magnetic mode.