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On the Maneuver of a Controllable Pitch Propeller

(Received Nov. 20, 1959)

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Abstract

In the case of a ship equipped with controllable pitch propeller, when the pitch of propeller is changed, by maneuver of pitch-changing device, the speed of ship will also change. But, due to inertia of ship-body, the speed-change does not take place immediately, The author has made a theoretical study about this phenomenon, by solving an equation of motion corresponding to it.

In constructing the equation of motion, the towing resistance is firstly assumed to be proportional to the square of ship-speed. Also, a simple relation between the propeller-thrust, ship-speed and pitch is assumed. Thus, it was possible to show the timely change of propeller-slip, during the pitch-changing operation. The theoretical formula obtained in this way is illustrated by numerical examples. Also a supplementary consideration about the case wherein the towing resistance is proportional to V^3, instead of V^2, is made.

I. Introduction

Let us consider a sea-going vessel which is advancing in a straight line with uniform speed V. When the pitch of propeller is, by maneuver of pitch-changing device, suddenly altered, the speed of ship will also suffer a change. However, since the actual ship has a considerable amount of mass, the change in its speed will not immediately follow the change of pitch, and there will be some time-lag between these two matters. Thus, the value of propeller-slip at each instant, during the maneuver, will considerably be affected by the amount of mass of

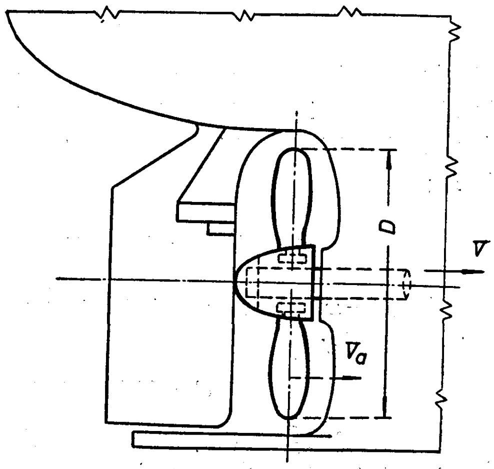

Fig. 1. A sketch of controllable pitch propeller.

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the vessel. The author has made a dynamical calculation about the relation of pitch-change to speed-change, wherein the ship resistance, propeller-thrust, etc., are given by simple expressions. Thus, we could present an approximate theory about the behavior of propeller-slip, while the propeller-pitch is undergoing an alteration. The theoretical formula obtained in this way is explained numerically by some numerical examples.

II. Notations

In the present paper, we shall use the following notations:

(A) About the propeller; D=diameter (m), P=effective pitch (m), N=no. of revolutions per sec. T=thrust (kg), Q=torque (mkg), V_a=propeller advance (m/sec), w=wake factor (in the mean), $1-E_0$=effective slip ratio at the steady run.

(B) About the ship-body; V=its speed (m/sec), R=towing resistance (kg), M =total mass of the ship-body (including the virtual mass of water (kg/(m/sec²)), τ=thrust deduction factor, η=hull efficiency.

The values of the quantities P, N, ... at steady run will be denoted by P_0, N_0, ... For the values during pitch-changing operation, we shall put $P=P_0p$, $N=N_0n$, $V=V_0v$. Calling the time t, and assuming that the initial instant from which the pitch-changing operation begins is $t=0$, we have $p=1$, $n=1$, $v=1$ at $t=0$. K, f and C are numerical constants whose actual values need not necessarily to be known.

III. Fundamental Relations

The ship-resistance is assumed to be given by

$$R=KV^{\nu} \tag{1}$$

The index ν is, in the approximate theory given below, is taken to be equal to 2, but it may be given other suitable value to represent the actual variation of ship-resistance R with the ship-speed V. The equation of motion of ship-body can be written;

$$M\frac{dV}{dt}=fT-KV^{\nu} \tag{2}$$

where $f=\eta(1-w)$. Here, we shall adopt the thrust coefficient of the working propeller, defined by

$$C_T=T/[\rho N^2P^2D^2].$$

In the usual case of the screw propeller, the relation between the thrust coefficient and the advance coefficient is given by a curve which is nearly a straight line. Also, by examination of propeller chart, the value of C_T at $V_a=0$ (100% slip ratio), is nearly inversely proportional to the pitch ratio of the propeller (see Appendix). From these considerations, we see that we may put approximately

$$C_T=\frac{C}{P}\frac{PN-V_a}{PN} \tag{3}$$

Hence, the corresponding value of thrust T will be given, also approximately, by

$$T=\rho N^2D^2PC\frac{PN-V_a}{PN} \tag{4}$$

Inserting this value of Eq. (4) into Eq. (2), and noting that at a steady run we have $dV/dt=0$, $P=P_0$, $N=N_0$, $V=V_0$, the equation of motion can be modified into the form;

$$\frac{dv}{dt}+Gv^2-Bn^2p\left[1-E_0\frac{v}{pn}\right]=0 \tag{5}$$

where we put $V_a=(1-w)V$, $E_0=(1-w)V_0/(P_0N_0)$. Putting, in Eq. (5), $dv/dt=0$, $v=1$, $n=1$, $p=1$, which is in accordance with the above notice, we have

$$G-B(1-E_0)=0$$

Thus, the equation of motion is finally put into the following form, where we take $\xi=Gt$ as a new independent variable

$$\frac{dv}{d\xi}=\frac{n^2p}{1-E_0}-\left(\frac{E_0}{1-E_0}\right)nv-v^2 \tag{6}$$

The value of the constant G is given by

$$G=KV_0^2/(MV_0)=R_0/(MV_0)$$

and the effective slip is given by

$$s=P_0N_0(pn-E_0v)$$

When the values of n and p are given as functions of time t (and consequently, of the variable ξ), Eq. (6) becomes an ordinary differential equation with respect to v, which is called one of *Riccati's type*. It may conveniently be classified into two cases, namely;

(A) No. of revolutions is kept constant ($n=1$), while the pitch varies linearly with time t, $p=(1-k\xi)$, as shown in Fig. 2. In this figure t_0 is the duration of operation, and k is a constant given by $k=\alpha/(Gt_0)$ This is a case in which the pitch control is made.

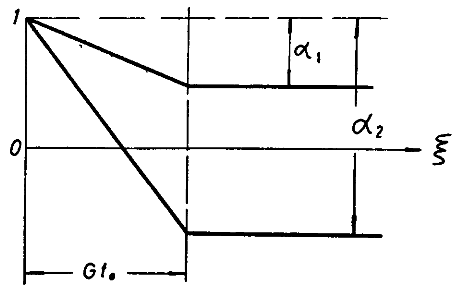

Fig. 2. Graph of $p=(1-k\xi)$

(B) Pitch is kept constant ($p=1$), while the no. of revolutions varies with time t in the manner given by $n=(1-k\xi)$. This is a case in which the revolution is controlled. Of course, there may arise more complicate cases. In what follows we shall mainly be concerned with the case (A), in which case Eq. (6) may be written

$$\frac{dv}{d\xi}=\frac{p}{1-E_0}-\frac{E_0}{1-E_0}v-v^2 \tag{6'}$$

IV. The Solution of the Problem

We must solve the ordinary differential equation (6), of Riccati's type, under the initial condition that at $\xi=0$ we have $v=1$, $dv/dt=0$. For that purpose, let us put $v=d(\log U)/dx$ into Eq. (6), then we see that, U must satisfy the linear ordinary differential equation

$$\frac{d^2U}{d\xi^2}+\left(\frac{E_0}{1-E_0}\right)n\frac{dU}{d\xi}-\frac{n^2p}{1-E_0}U=0 \tag{7}$$

Moreover, denoting by $U_1(\xi)$, $U_2(\xi)$ a set of independent solutions of Eq. (7), the general solution of the original equation (6) will be given by;

$$v=\frac{C_1U_1'(\xi)+C_2U_2'(\xi)}{C_1U_1(\xi)+C_2U_2(\xi)} \tag{8}$$

where C_1 and C_2 are integration constants. If we put for convenience, $\eta=1-k\xi$, and take η as a new independent variable, Eq. (7) may, for two cases (A) and (B) cited above, may be written;

$$\frac{d^2U}{d\eta^2}-A\frac{dU}{d\eta}-B\eta U=0 \tag{A}$$

$$\frac{d^2U}{d\eta^2}-A\eta\frac{dU}{d\eta}-B\eta^2U=0 \tag{B}$$

where we put

$$A=\frac{E_0}{(1-E_0)k},\quad B=\frac{1}{(1-E_0)k^2}$$

In Eq. (8), the differentiation $U'(\xi)$ is to be made with respect to the independent variable ξ. If we take η as the independent variable, we should write, instead of Eq. (8):——

$$v=(-k)\frac{C_1U_1'(\eta)+C_2U_2'(\eta)}{C_1U_1(\eta)+C_2U_2(\eta)} \tag{9}$$

where the differentiation $U'(\eta)$ is to be made with respect to the independent variable η.

Case (A) We modify again the differential equation (A) given above into the form;

$$\frac{d^2W}{d\eta^2}-\left(\frac{1}{4}A^2+B\eta\right)W=0 \tag{10}$$

where we put

$$U=\exp\left(\frac{1}{2}A\eta\right)\cdot W$$

This equation (10) can be solved in terms of Bessel functions, but for a practical use, it is more convenient to give the solution in form of a power series in η.

Equations (A), (B) and (10) are those ones whose solution can be expanded into a power series having radius of convergence infinitely large. Thus, a pair of independent solutions of Eq. (10) is found to be

$$\left.\begin{aligned} W_1(\eta)&=1+a_2\eta^2+a_3\eta^3+\cdots\cdots \\ W_2(\eta)&=\eta+b_3\eta^3+b_4\eta^4+\cdots\cdots \end{aligned}\right\} \tag{11}$$

where the coefficients a_2, a_3, etc., have following values

$$a_2=\frac{1}{8}A^2,\ a_3=\frac{1}{6}B,\ a_4=\frac{1}{384}A^4,\ a_5=\frac{1}{120}A^2B,\ \cdots\cdots$$

$$b_3=\frac{1}{24}A^2,\ b_4=\frac{1}{12}B,\ b_5=\frac{1}{1920}A^4,\ b_6=\frac{1}{480}A^2B,\ \cdots\cdots$$

The solution $v(\eta)$ becomes;

$$v(\eta)=(-k)\left[\frac{W_1'(\eta)+\lambda W_2'(\eta)}{W_1(\eta)+\lambda W_2(\eta)}+\frac{1}{2}A\right] \tag{12}$$

where

$$\lambda=-\frac{W_1'(1)+\mu W_1(1)}{W_2'(1)+\mu W_2(1)} \tag{13}$$

$$\mu=\frac{1}{k}+\frac{1}{2}A$$

Case (B) For this case, treating the differential equation (B) in the same manner as for the Case (A), a pair of independent solutions is given by

$$U_1=1+a_2\eta^2+a_4\eta^4+\cdots\cdots$$
$$U_2=\eta+b_3\eta^3+b_5\eta^5+\cdots\cdots$$

where the coefficients a_2, a_4, etc., are given by

$$a_2=0,\ a_4=\frac{1}{12}B,\ a_6=\frac{1}{90}B,\ a_8=\frac{1}{56}\left[\frac{1}{15}AB+\frac{1}{12}B^2\right],\ \cdots\cdots$$

$$b_3=\frac{1}{6}A,\ b_5=\frac{1}{20}\left[\frac{1}{2}A^2+B\right],\ b_7=\frac{1}{168}\left[\frac{1}{2}A^3+\frac{5}{3}AB\right],\ \cdots\cdots$$

And the solution $v(\eta)$ becomes

$$v(\eta)=(-k)\frac{U_1'(\eta)+\lambda U_2'(\eta)}{U_1(\eta)+\lambda U_2(\eta)} \tag{14}$$

where

$$\lambda=-\frac{U_1(1)+kU_1'(1)}{U_2(1)+kU_2'(1)} \tag{15}$$

V. Numerical Example for the Solution of Case (A) of previous Section

In order to explain numerically the nature of solution of Case (A) given in previous section, let us take the case of a fishing-boat whose main items are as follows:

Displacement = 110 ton, $V_0 = 4.8$ m/sec (about 9.3 knots), towing resistance = 2150 kg, $E_0 = 0.70$ (which means that the effective slip ratio is 0.30, at a steady run).

Assume that the pitch of the propeller of this tug-boat is controlled with a value of $\alpha = 1.5$ (Fig. 2). This means that, the pitch of the propeller, which at steady run is equal to P_0, is changed into $-0.50\ P_0$ during the operating duration of t_0 sec. From the above-mentioned relation it is found that (a) for $t_0 = 5$ sec, $k = 6.0$ and (b) for $t_0 = 10$ sec, $k = 3.0$.

By numerical evaluation of expressions (11) and (12), for two cases (a) and (b), the values of the solution $v(\eta)$ was obtained, and are shown as graphs in Fig. 3 and Fig. 4. As we see from these figures, when the propeller-pitch is made to decrease, linearly with respect to time t, the ship speed V gradually decreases due to this maneuver of pitch-change. But, owing to inertia of ship-body, the reduction of ship velocity occur considerably later than the reduction of pitch. At any instant during the maneuver of pitch-change, the value of so-called propeller speed is $N_0 P_0 p$, while the propeller advance is (assuming the wake factor w to be of constant value), given by $(1-w)V_0 v$. Hence, the value of slip is given by

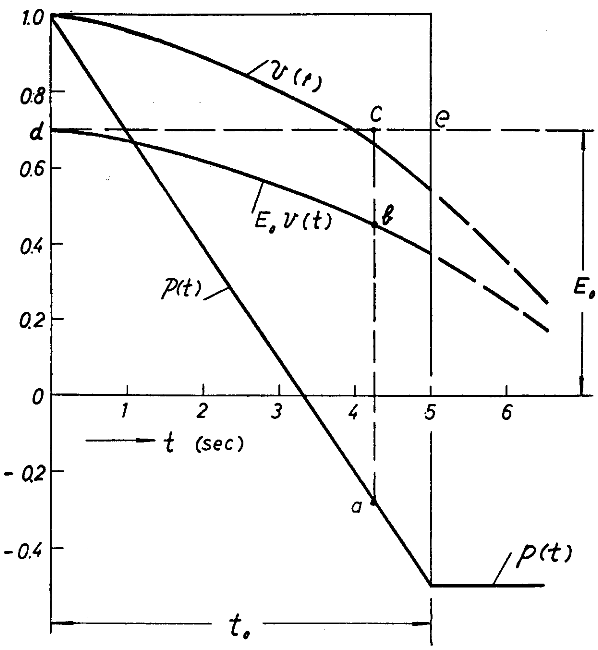

Fig. 3. Solution $v(t)$ for the case (a).

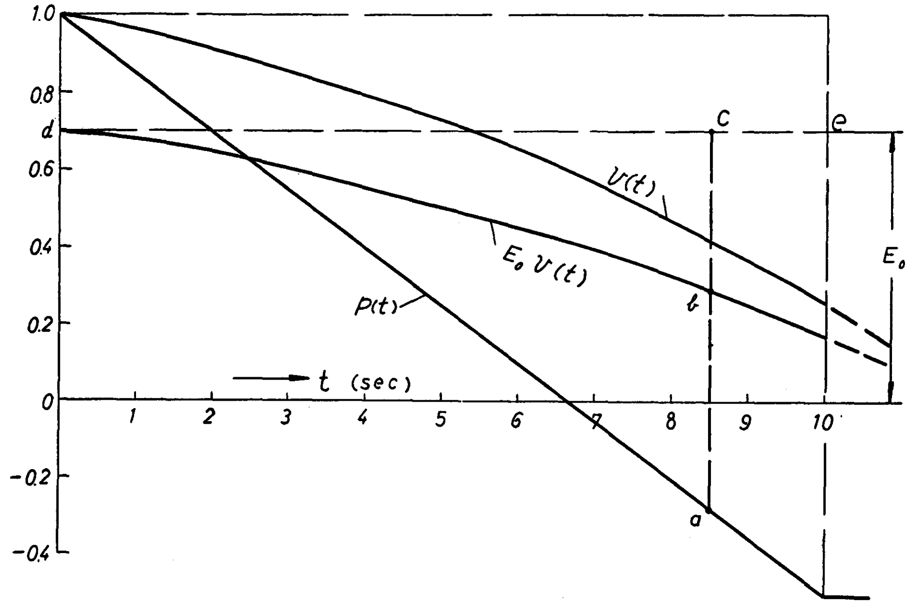

Fig. 4. Solution $v(t)$ for the case (b).

$$s=N_0P_0[p-E_0v]$$

If the mass of the ship were infinitely large, the value of slip at any instant would have been *ac* of Fig. 2 and Fig. 3. But, for a ship of finite mass the ship speed has already dropped at this instant, and the value of slip would become *ab* of these Figures. (In the configuration of these figures, the slip *ab* have negative values.)

VI. The Case in which the Value of Index ν is not Equal to 2.

In the above discussion, it was tentatively assumed that $\nu=2$. But, in an actual ship, the law of variation of towing resistance of the ship does not necessarily follow the formula $R=KV^2$. When the wave resistance play considerable part in the total towing resistance, the index ν in the formula $R=KV^\nu$ would be larger than 2. One way to take into account this fact is; first, we make the calculation as mentioned in previous section assuming that $\nu=2$, and then, we make correction for ν not being equal to 2, by applying Picard's method of successive approximation. Thus, for tne case (A), in which we have $n=1$, let us assume that the law of resistance is given by $R=KV^3$. In this case, we must solve the differential equation

$$\frac{dv}{d\xi}=\frac{p}{1-E_0}-\frac{E_0}{1-E_0}v-v^3 \tag{16}$$

where $p=1-k\xi$, instead of Eqs. (6) or (6'), under the initial condition that for $\xi=0$, $v=1$, $dv/d\xi$(or dv/dt)$=0$. This equation (16) can be written, returning to original independent variable $t(=\xi/G)$;

$$\frac{dv}{dt}=G\Big[\frac{p}{1-E_0}-\Big(\frac{E_0}{1-E_0}\Big)v-v^3\Big] \tag{17}$$

According to Picard's method of successive approximation, we take as starting function a suitable one, say v_1, and find the second function v_2 by solving the differential equation

$$\frac{dv_2}{dt}=G\Big[\frac{p}{1-E_0}-\Big(\frac{E_0}{1-E_0}\Big)v_1-v_1^3\Big] \tag{18}$$

under the same initial condition as before, and so on for v_2, v_3, ······ .

Let us choose, as the starting function v_1, the solution of the differential equation

$$\frac{dv_1}{dt}=G\Big[\frac{p}{1-E_0}-\Big(\frac{E_0}{1-E_0}\Big)v_1-v_1^2\Big] \tag{19}$$

under the same initial condition. This latter one v_1 is what was found in the previous section. Subtracting both sides of Eqs. (18) and (19) we obtain ;——

$$\frac{d(v_2-v_1)}{dt}=G[v_1^2-v_1^3]$$

or,

$$v_2-v_1=G\int_0^t[v_1^2-v_1^3]dt \tag{20}$$

In order to estimate the approximate value of the left hand side of Eq. (20), we look at the curves of Fig. 3 and Fig. 4, which shows us the values of $v_1(t)$. From these figures we observe that the curve $v_1(t)$ is one resembling an inverted parabola which may be represented approximately by

$$v_1(t)=1-\delta t^2+\varepsilon t^3 \tag{21}$$

In that case, we have by Eq. (20) ;——

$$v_2(t)-v_1(t)=G\int_0^t(\delta t^2-\varepsilon t^3)[1-\delta t^2+\varepsilon t^3]^2dt$$

$$=Gt^3\Big[\frac{1}{3}\delta-\frac{1}{4}\varepsilon t-\frac{5}{2}\delta^2t^2+\frac{2}{3}\varepsilon\delta t^3$$

$$+\frac{1}{7}(\delta^3-2\varepsilon^2)t^4-\frac{3}{8}\delta^2\varepsilon t^5+\frac{1}{3}\varepsilon^2\delta-\frac{1}{10}\varepsilon^3t^{10}\Big] \tag{22}$$

By checking the curves of Fig. 3 and Fig. 4 we see that the constants δ and ε in Eq. (21) have following values.

For case (a) (Fig. 3): $\delta=0.66/t_0^2$, $\varepsilon=0.20/t_0^3$; For case (b) (Fig. 4): $\delta=1.43/t_0^2$, $\varepsilon=0.68/t_0^3$. Putting these values of δ and ε into Eq. (22), we find, by numerical calculation, the values of (v_2-v_1) as tabulated below:

Thus we see that the values of v_2-v_1 are fairly small, at least in our examples, showing us that we may take approximately $v=v_1+(v_2-v_1)$ for the case in which the law of towing resistance is given by $R=KV^3$.

Table. 1. Values of (v_2-v_1)

	$t=\frac{1}{2}t_0$	$t=t_0$
Case (a)	0.0050	0.0217
Case (b)	0.0167	0.0442

VII. The Case in which the Value Gt_0 is small in Comparison with Unity

When the value of Gt_0 is small in comparison with unity, so also is the value of Gt so long as $t\leqq t_0$, that is, during the operation of pitch-change. In that case, we can express the solution v in form of a power series in $\xi=Gt$ as follows;

$$v(t)=1+a_2\xi^2+a_3\xi^3+a_4\xi^4+a_5\xi^5+\cdots\cdots \tag{23}$$

In fact, putting this expression for $v(t)$ into Eq. (6′) (wherein we write v^ν instead of v^2), and comparing coefficients of each power of ξ, we obtain,

$$a_2=-\frac{k}{2(1-E_0)},\quad a_3=-\frac{1}{3}\beta\, a_2,\quad a_4=-\frac{1}{\beta}a_3,$$

$$a_5=-\frac{1}{5}\left[\beta a_4+\frac{\nu(\nu-1)}{2}a_2^2\right]$$

etc., etc..

where we write

$$\beta=\nu+\frac{E_0}{1-E_0},\quad k=\frac{\alpha}{Gt_0},\quad \xi=Gt$$

VIII. Concluding Remarks

The author has given in the present paper, an approximate theory by which we can estimate the variation of ship-speed and propeller-slip during the maneuver of pitch-change, in a vessel equipped with controllable pitch propeller. Further, following remarks may be made:

(1) In the above discussion, the state of affairs during the operation of pitch-change was considered. As to the behavior after the operation, at which the ship has not yet attained the final steady-state, the similar analysis as in the text can be made. Only difference is that the parameter p has a constant value.

(2) In the above discussion, the wake factor w was assumed to have constant value irrespective of the ship speed. This may be admissible for an approximate evaluation as in the text. But, if the ship-speed is reversed (astern-motion), the value of w will, of course, undergo considerable change.

(3) In the above discussion, the ship-speed V (and, consequently the value of v)

is assumed to be always positive. After an elapse of time, the ship speed may become negative. In that case, we muat write $+K|V|^{\nu}$ instead of $-KV^{\nu}$, in Eq. (2). The subsequent calculation can be made quite similarly as in the text.

(4) In the above discussion, the equations (3) and (4) are assumed to hold even when the slip is negative. Of course, this is an approximate expression. If more accurate formula between thrust and slip is given, the formula (3) or (4) may be replaced by it.

APPENDIX

(A) In usual form of screw propeller, the thrust T exerted by the propeller varies with slip ratio s, in a manner shown by the curve *ob* of Fig. 5. This curve *ob*

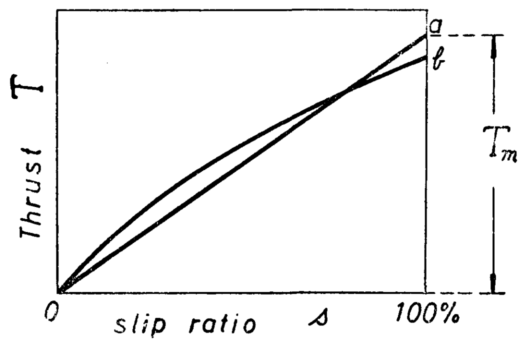

Fig. 5. Relation between thrust and slip ratio

can nearly be represented by a straight line *oa*. The maximum value T_m of the thrust T is attained at $s=100\%$. This fact is true for a propeller having fixed value of the pitch P. In connection with the maneuver of controllable pitch propeller, there arises a question as to how the matter stands if the pitch is changed.

In order to see this, it is sufficient if we know how the value of maximum thrust T_m varies with variation of the pitch P. As one way of doing this, let us examine the propeller chart of *Troost*. For example, for 3 bladed propeller with area ratio $F_a/F=0.50$, the value of thrust coefficient $K_s=S/(\rho D^4 n^2)$ for 100% slip ratio is as shown in Table 2, according to Troost's Chart. (S=thrust, D= diameter, n=no.

Table. 2.

(1) H/D	(2) K_s(for $s=100\%$)	(2) ÷ (1)
1.4	0.60	0.43
1.2	0.53	0.44
1.0	0.45	0.45
0.8	0.35	0.44
0.6	0.25	0.42
0.5	0.20	0.40

of rev. per sec., H=face-pitch) Here the value of K_s for $s=100\%$ is not taken at the point b on the curve, but for the point a of approximating straight line oa. From this table we see that the value of K_s (for $s=100\%$) may be regarded to be approximately proportional to pitch H. In Troost's notation, H means the face-pitch. But for a rough estimate, we may regard that the same proportionality relation hold for effective pitch P instead of face-pitch H.

Table 2 refer to the case of fixed blade propellers. As to the case of moveable blade propellers, there are few data available. Here, we shall quote the case of YTB 502 (a tug-boat belonging to U S Navy)[1] For the case of propeller of this vessel YTB 502, checking the performance curve in the same way as before, we obtain the following Table 3. In this table, pitch means the pitch at 0.70×propeller radius. Also, the thrust coefficient $C_T=T/(N^2P^2D^2)$ is used.

Table. 3.

(1) pitch (ft)	(2) C_T (for $s=100\%$)	(1)×(2)
(3.96)	(1.43)	(5.65)
6.7	1.04	7.00
7.6	0.95	7.20
8.5	0.90	7.65

From these examinations, we may conclude that, at least for a rough estimation, we can put

$$T=\text{const}\times N^2P\times\frac{PN-V_a}{PN}$$

which is the equation (3).

(**B**) *The value of the constant G.*

The constant G was defined to be $R_0/(MV_0)$, that is, the towing resistance divided by the momentum of the moving ship. From the above discussion, we saw that, this constant G greatly affects the performance during maneuver, of controllable pitch propeller. Some examples of actual values of this constant G is given in Table 4.

Table. 4.

kind of ship	displacement (ton)	ship speed (kts)	towing resistance (ton)	G (sec^{-1})
tugboat	330	12	4.75	0.024
fishing boat	100	9.3	2.15	0.049
medium freighter	11,000	14	24.7	0.00314
supertanker	42,700	16	93.8	0.0027
destroyer	1,600	36	150.	0.050

1) L. A. Rupp; Controllable Pitch Propellers, Soc. Nav. Architects and Marine Engineers, 1948.