Title	Experimental study and rational planning of insulation coating to suppress the corona appearance on the coil surface
Sub Title	
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Publisher	慶応義塾大学藤原記念工学部
Publication year	1958
Jtitle	Proceedings of the Fujihara Memorial Faculty of Engineering Keio University Vol.11, No.41 (1958.) ,p.34(8)- 53(27)
JaLC DOI	
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Notes	
Genre	Departmental Bulletin Paper
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=KO50001004-00110041-0008

Experimental Study and Rational Planning of Insulation Coating to Suppress the Corona Appearance on the Coil Surface

(Received July 20, 1959)

Motokichi MORI*

Abstract

Corona developement in the coil insulation has become World's concern in order to suppress the insulation deterioration and to find appropriate methode for its measurement, and also according to them it has been extended to the problem of suppressing the corona appearance from the surface of the coil.

An answer to this problem up to date has been to coat the coil surface with the resistance paint. In this report, auther adopted tan δ- method to find whether the coating was effectual to prevent the corona bursting on the surface, and also tan δ may become a measure of the corona loss when the voltage higher than the critical one is applied, and also in this report, tan δ is used in experimentally analysing the locations according to the slot part and the end turn part.

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In reference to the surface contact between the slot and the iron core, we discuss the limitation of the surface resistance, i. e. if the resistance is too high, it will not be sufficient to suppress the corona and if the resistance is too low, it will enlarge the eddy current loss on the surface.

I. In General

The coating of the corona preventing paints are generally classified into two kinds by the location of application on the coil and also by their workings and efforts.

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One of their locations being on the end turns, situated on the part which is the joint between the straight part of the coil placed in the iron core and the curved part of the coil sticking out of the iron core into the air. There we will find the surface to be coated by the higher resistance paint. The other location is on the part which is the straight line of the coil inserted into the slot of the iron core. There we will find the lower resistance coating on the surface and always it keeps the contact between the peripheral surface of the coil and the inner wall of the slot.

We will call the former the end turn and the latter the slot part or the straight part of the coil for convenience of explanation in this report.

The end turns have almost all their surfaces in the free air and as it gets further away along the surfaces from the grounding point which is considered to be the sides of the iron core, certain potential is induced on every point on the coil surface through the insulation from the conductor which is situated in the center of the coil. We will assume that these potentials may become so high with removal from the grounding on the surface, that the extermly far away points may attain the same potential as the one of the conductor. While the electric field lies along the surface, the corona will be started from the point of the highest electric stress which may be near the grounding. The highest field is always located close to the grouning surface, therefore the corona initiation will be always restricted by this fact. If we assume the surface resistance to be uniform and make it lower than the original one by some artificial means, then we may be able to presume that the highest field becomes lower. However there will exist the fact that the highest field still lies at the near point of the grounding surface.

Moreover we should call attension to the following:— the surface potential will be distributed in far longer range on the end turns, because the surface potential is distributed on a slower form by making its resistance lower. Consequently it will give some threat on the insulation of the end turns by stressing the end turn insulation in far longer range, because this insulation is designed and constructed by the materials having weaker strength.

One of the objects of this treaties is to calculate how effectivelly to spread the resistance paint on the surface for the purpose of getting such results:— we make the potential gradient on the whole end turn surface to be uniform and we restrict the width of this distribution to be narrow if possible.

Next, refering to the straight part of the coil, the low resistance coating is recently being used in practice. This coating means to protect the surface from the corona attack which occurs there by the voltage rise in the air gap between the coil surface and the slot wall, because the uniform contacts usually cannot be expected in practice. The above fact is made more effective by using paint of low resistance on the surface of the coil. However, if the surface resistance is made too low, the magnetic flux advancing laterally toward the coil sides cause greater eddy current losses on their surfaces and accordingly raise the temperature of the coil. Therefore it should be important to limit the value of surface resistance.

At the beginning in this treatise, we are to compute how potential rises on the coil surface free from contact according to the distance from the point of contact, and also to calculate the possibility of the corona initation occuring in the gap between the non-contact point and the slot wall. From the results above mentioned, we must discuss the higher limit of the surface resistance in order to protect it from the corona initiation. Its lower limit may be determined by the temperature-rise owing to the eddy current losses.

Refering to the end turn, that is how to coat the end turn surface, since the corona preventing paint should be spread in such manner as to make the near point of the iron core lower and the farther points gradually higher by means of a certain formula. Then the question may be raised how in practice the paint should be spread on the surface. As for an instance, if we use the paint brushes, we should explain the working guide how the brush is to be used.

II. Making of the Corona Preventing Paint for Trial and the Surface Resistance

One of the purposes of coating with paint is to make the surface resistance lower, the other purpose is to control the surface resistance distribution, which is subjected to the voltage induced from the other side. At the outset of the study, we should consider the resistance materials. We began by taking the amorphous cardorn powder, the triangular graphite and the colloidal graphite as elementary material and dissolved or suspeneded them in the insulation varnish. And then, the spreading was done by using the paint brushes or the spray gun carefully adjusting their spreading thickness on the surface.

At the beginning, these paints were previously tried on the glass plate to check the fitness of spreading and the conditions of their sticking on the glass surface. After they were fully dried, we mesured the surface resistance on them by the usual method.

Now, the paints we tried in our experiments, were three kinds as follows:— C—8, C—75 and C—120. The surface resistance measured are shown in Table I. They were

Table I. Surface Resistivity of Resistance Coating on the glass.

Kind of Paint	Carbon	No. of spreading	Resistivity	
			Surf. Resist	Voltage
C-8	Graphite	1	3.5×10^{11}	D. C. 3, 500 v
C-8	Graphite	2	1.5×10^{11}	D. C. 3, 500 v
C-75	Graphite	1	7.5×10^{5}	500v megger
C-120	Graphite	1	3.2×10^{3}	Bridge

measured after they had been spread on the glass plate and then dried.

III. Dielectric Loss Angle :— tan δ

To make sure whether the painted coats were effective to prevent the corona appearance from the coil surface or not, it is best to take the method so finding the dielectric loss angle of the whole coil specimen which contains both the loss angle of coil itself and its surface losses, if we previously had known the loss angle of the coil itself. This method seems to be more suitable than to find the corona initiation voltage.

This is because we can find the quantity of power loss caused by both the slot and the end coil surfaces and the relation between the two losses and the applied voltages. The relation explains the increase of the corona energy according to the voltage.

For proceeding in this method, it is previously necessary that we measurs the loss angle in the coil own by Schering Bridge, before it had been spread wlth the paints and also to measure tan δ. after it had been furnished with the paint. Comparing those tan δ together, we may be able to consider the availability of the corona preventing paint by finding how the surface losses had been reduced by making the surface resistance lower.

A). Coil Specimen [1]

We chose the coil specimens with 11 kv class insulation as shown in Fig 1. These coils are all the same as those of the diamond coils manufctured in the industry, having the inner conductors and 11 kv class insulation, execpt for the straight line shape. It goes without saying that they had been given insulation treatment same as industrial ones. The specimens are three as follow :— M–1, M–6 and M–7.

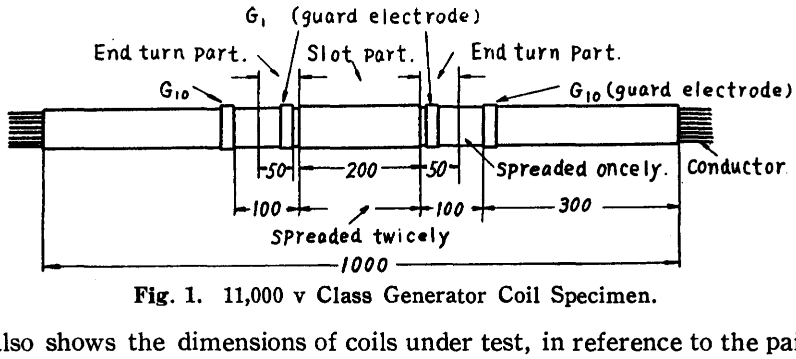

Fig. 1. 11,000 v Class Generator Coil Specimen.

Fig. 1. also shows the dimensions of coils under test, in reference to the paint spreading and the situation of the electrode. In the middle part of the specimen, there is the slot part of 200 m/m spread with special low resistance paint. The metalic electrode of 200 m/m having moderate thickness is applied at this location, fastening the coil up moderatelly with the bolts as if the coil were in the iron core of a machine.

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[1] M. Mori: Comittee of insulation conservation for power machine in Japan, April 1950.

In such a fastening situation, as one can not expect the contact between them to be perfect on the whole surface, we may consider that the corona occurs in the air gaps somewhere in their contact.

On the other hand, if we choose the tin-foil electrode and make their surface stick closely with glycerine as the binding medium, there will not be any discharge due to corona, because there is no air gaps between them.

Outward from the slot part, there are the end turn parts, each of which has a dimension of 100 m/m and has been furnished with the corona preventing paint. This coating on the end turn is subjected naturally to do the proper work different from that of the slot. It has extremely higher surface resistivity than that of the slot and full consideration must be given to their distribution of resistivity on their snrface.

Guard electrodes G_1 and G_{10} were installed as shown in Fig. 1. G_1 is set closely up near the main electrode. If $\tan\delta$ has been obtained by using both the main electrode and guard G_1 through the Schering Bridge methode, its value should merely correspond to the slot length, i. e. 200 m/m of the coil. On the contrary, if $\tan\delta$ is obtained by using both the main electrode and guard G_{10}, it is clear that we may get $\tan\delta$ corresponding to both the slot part and the end turn part of the specimen.

And then G_∞ means $\tan\delta$ in the case where the guards has been moved to infinite distance from the main electrode. Then, practically speaking, it means $\tan\delta$ without any guard.

B). Experimental Results [2)]

In Table II, III and IV, there are experimental results of $\tan\delta$ classifying two cases of the same coil specimen above mentioned, one with the corona preventing coating

Table II. Modification of Dielectric Loss Angle due to the Presence of Coating. (I)
Coil under test, M-1. Coating:-slot C-75, end turn C-8.

Slot part	Metal plate						Tin-foil					
Coating or not / Voltage in kV	G_1		G_{10}		G_∞		G_1		G_{10}		G_∞	
	Non	Exist	Non	Exist	Non	Exist	Non	Exist	Non	Exist	Non	Exist
2.5	13.0	12.0	14.8	17·6	20.7	18.9	5.7	5.95	11.8	11.7	12.5	12.85
5.0	14.7	13.0	19.1	19.4	23.1	20.0	6.0	6.44	12.3	12.1	12.7	13.15
7.5	16.6	13.5	23.5	20.6	26.3	22.5	6.6	6.84	12.8	12.55	13.6	13.65
5.0	15.0	13.0	19.3	19.4	23.3	21.5	6.0	6.54	12.4	12.15	13.0	13.25
2.5	12.5	12.0	15.3	17.6	21.0	19.4	5.9	6.0	12.0	11.85	12.2	12.95

Note:- G_1: Distance between main and guard electrones in 1~2 mm.
G_{10}: Distance between main and guard electrodes in 100 mm.
G^∞: Without any guard.

2) M. Mori, and Fukuda: Comittee of rotating machine in Japan, Oct. 1954.

Table III. Modification of Dielectric Loss Angle due to the Presence of Coating. (II)
Coil under test, M-6. Coating:– slot C-120, end turn C-75.

Slot part	Metal plate						Tin-foil					
Coating or not	G_1		G_{10}		G_∞		G_1		G_{10}		G_∞	
Voltage in kV	Non	Exist	Non	Exist	Non	Exist	Non	Exist	Non	Exist	Non	Exist
2.5	4.9	4.42	5.84	6.8	6.0	7.0	4.64	5.02	5.6	6.0	5.6	8.8
5.0	6.5	4.62	9.7	6.9	10.0	7.5	4.74	5.43	5.9	6.1	6.0	9.0
7.5	11.4	5.83	17.0	7.6	17.5	8.2	5.45	6.75	6.7	7.1	7.1	9.7
10.0	18.1	7.64	22.2	11.6	24.6	12.7	9.9	9.75	12.9	10.5	13.5	13.5
7.5	11.6	6.03	17.2	7.8	19.0	8.9	5.9	6.82	8.0	7.1	8.7	10.0
5.0	6.6	4.82	9.8	6.9	13.0	8.2	4.8	5.62	6.0	6.1	6.0	9.1
2.5	5.0	4.62	6.3	6.8	6.1	8.2	4.7	5.43	5.8	6.1	5.7	8.9

Note:– G_1: Distance between main and guard electrodcs in 1∼2 mm.
G_{10}: Distance between main and guard electrodes in 100 mm.
G_∞: Without any guard.

Table IV. Modification of Dielectric Loss Angle due to the Presence of Coating. (III)
Coil under test, M-7. Coating:– slot C-120, end turn C-8.

Slot part	Metal plate						Tin-foil					
Coating or not	G_1		G_{10}		G_∞		G_1		G_{10}		G_∞	
Voltage in kV	Non	Exist	Non	Exist	Non	Exist	Non	Exist	Non	Exist	Non	Exist
2.5	5.8	5.0	7.4	7.4	6.0	7.0	4.85	5.24	6.23	7.3	6.53	7.1
5.0	6.1	5.2	10.8	7.68	11.0	7.3	4.94	5.34	6.43	7.4	6.63	7.3
7.5	12.0	6.6	18.8	8.2	19.4	8.4	5.63	6.82	7.71	9.48	8.01	8.4
10.0	19.0	11.0	25.2	12.9	25.3	13.1	10.28	10.9	12.35	13.9	12.4	13.0
7.5	12.1	6.9	19.0	8.7	20.0	8.6	5.93	6.9	8.11	10.4	8.1	8.5
5.0	6.2	5.7	11.0	7.7	11.3	7.5	5.05	5.5	6.72	8.78	6.9	7.4
2.5	5.8	5.1	7.9	7.5	7.1	7.1	4.94	5.4	6.53	8.68	6.9	7.2

Note:– G_1: Distance between main and guard electrodes in 1∼2 mm.
G_{10}: Distance between main and guard electrodes in 100 mm.

and the other without it. Table II shows the case where the coil specimen is $M-1$, the slot part coat $C-75$ and the end turn coat $C-8$. Table III phows the cases where the coil spœcimen is $M-6$, the slot part coat $C-120$ and end turn part coat $C-75$. Table IV shows the case where the coil specimen is $M-7$, the slot part coat $C-20$ and the end turn part $C-80$. Fig. 2 shows the graphical representation for Table II. In Fig. 2, G_1— curve (lowest) denotes the tan δ of the coil with the surface losses removed, G_1—curve (upper most) denotes tan δ including both the coil and the surface losses when no paint was applied and the contact with the metal electrode was kept, and G_1—curve (middle) denotes the case where the slot surface is spread with the paint.

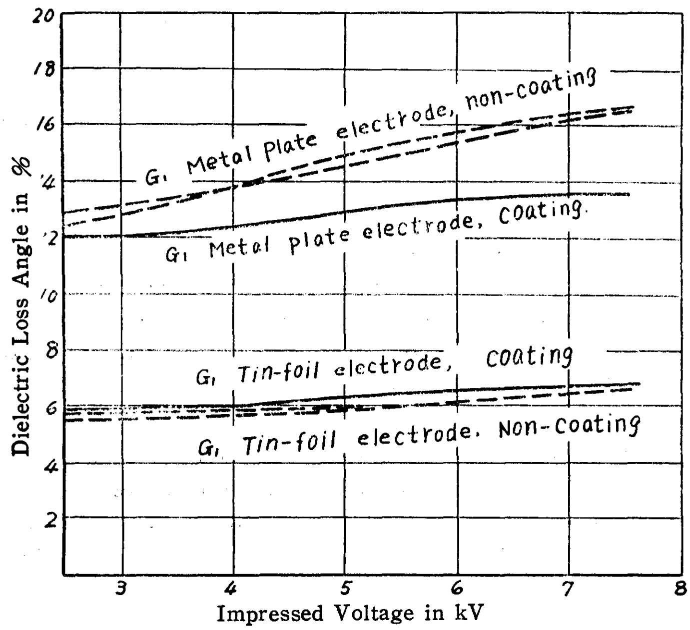

Fig. 2. Modification of Dielectric Loss Angle Due to The Coating of Corona Preventing Paint. (I).
Coil under test, M-1. Coating:- Slot part C-75, End turn part C-8.

The paint spreading in this case was not sufficient to make the surface contact losses lower, because the resistance of paint was high.

Fig. 3 and 4, it will be seen that the corona preventing paint is more effective. It will be clear that the losses due to the imcomplete contact with the coil surface because of the use of the metalic electrode were reduced, because tan δ of the metalic electrode is more approaching to that of the tin-foil electrode.

In Fig. 4, tan δ without paint at 10kV application is 19%. On the contrary, tan δ with paint is 11.0% which is nearer to 10.9% of the tin-foil electrobe with paint. Tan δ of the tin-foil electrode with paint is 10.9% and the same without paint is 10.28%. The difference between the two is due to the existence of the paint layer and it will be always negligibly small.

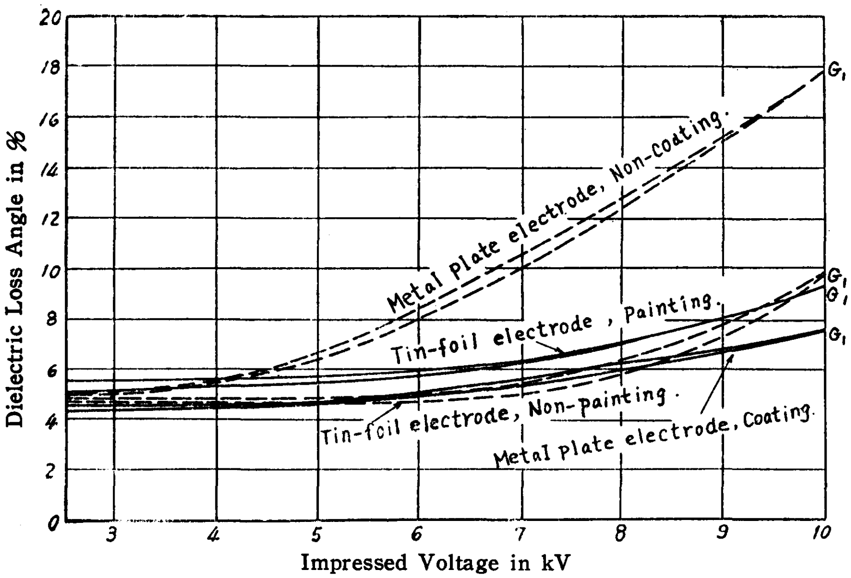

Fig. 3. Modification of Dielectric Loss Angle Due to The Coating of Corona Preventing Paint. (II). Coil under test M-6. Coating:- Slot part C-120, End turn part C-75.

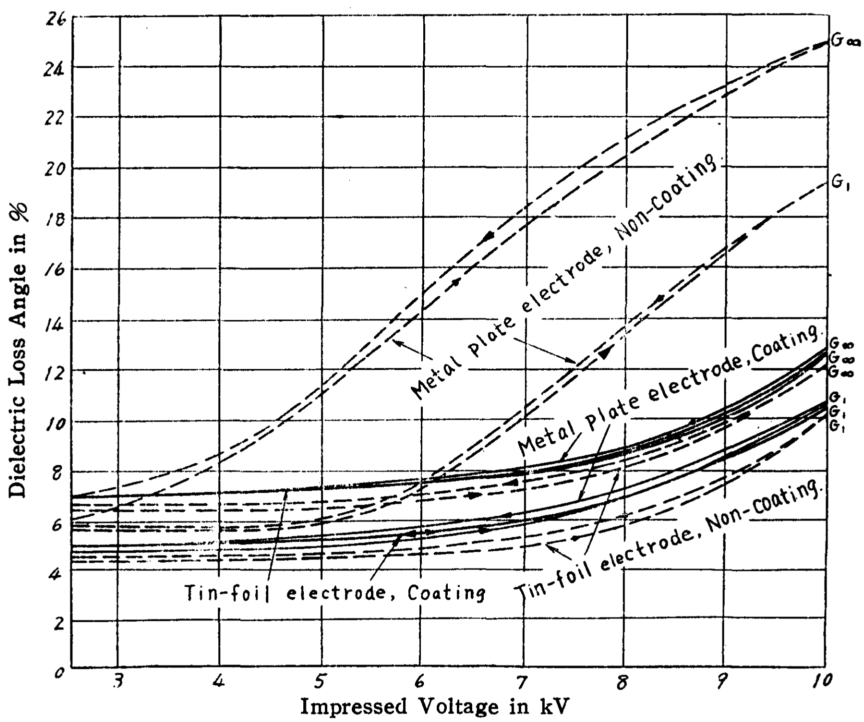

Fig. 4. Modefication of Dielectric Loss Angle Due to the Coating of Corona Preventing Paint. (III). Coil under test M-7. Coating:- Slot part C-120, End turn part C-8.

The fact above mentioned is only refering to the contact loss of the slot part of the coil. Next we should explaine the corona loss of the end turn surface and let us consider tan δ in the case of 10 kV only in order to simplify the explanation. Tan δ of G_1 in the case of the metal electrode with paint is 11.0 % and that of G_{10} under the same condition is 12.9 % Therefore, it ought to be considered that this difference is due to the end turn surface and it will possible to say that the corona on the surface of the end turn was efficiently suppressed.

The values of tan δ of G_{10} and G_∞ may be said to contain about the same surface losses in the end turn but in G_∞ it is slightly than in G_{10}. Therefare, it will be possible to neglect this difference among them.

In practical synthetic view, from the fact in 10 kV application, tan δ without paint is 25.3 % and tan δ with paint is 13.1 %, we may be able to conclude that the painted coating has an efficient effect in preventing the corona occurrence.

Fig. 5 shows the reduction of losses by the paint spreading, which is obtained from the difference between G_∞ with paint and G_∞ without paint in the case of the metalic electrode on the boil. Fig. 6 shows contact losses between slot and coil surface.

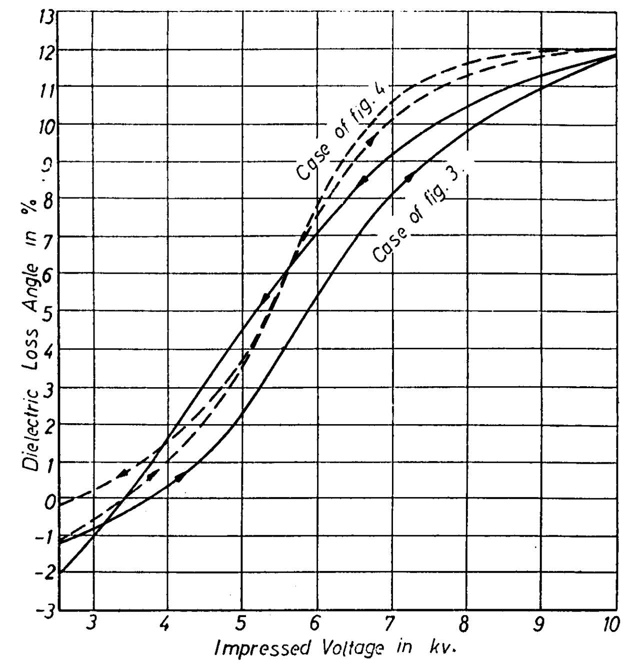

Fig. 5. Reduction of Coil Surface losses Due to Coating. With the metal plate electrode. Without Guard Electrode.

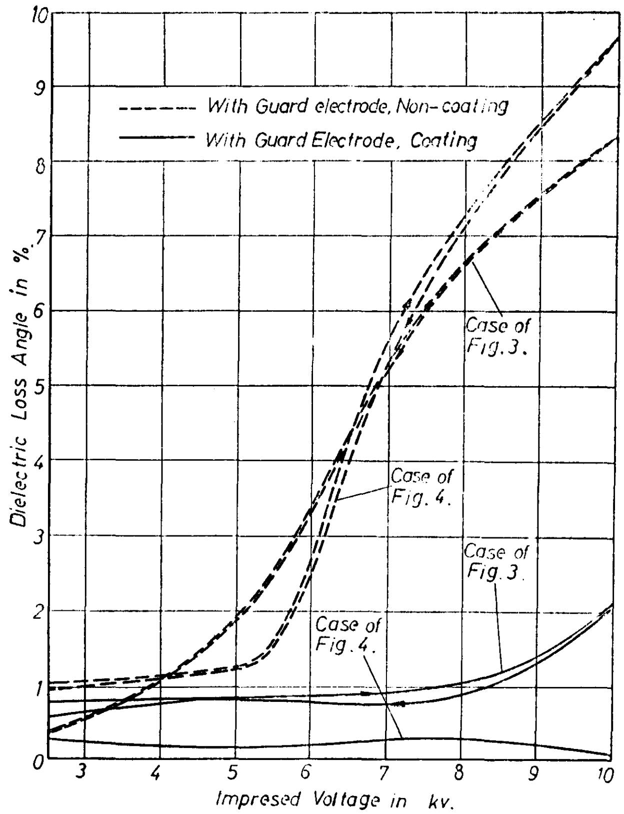

Fig. 6. Contact Losses Between Slot and Coil Surfaces.

IV. The Preventation of the Corona Discharges from the End Turns[3)]

IV.-A. The case where the end turn has uniform surface resistance

Fig. 7 shows the diagrams of the end turns of the coil, where their surface has uniform surface resistance. Fig. 7–a shows that the electrode 1 indicates the core end at zero potential and the electrode 2 indicates the copper conductor. Fig. 7–b shows the equivalent network diagram having the uniform surface resistance R per unit length, the distributed capacitance C per unit length meaning the insulation of the coil, and the distributed leakage conductance G per unit length meaning the insulation leakage. Fig. 7–c shows the simplified diagram for computation.

The potential V at P_1 means the potential difference between point P_1 and conductor at distance x from the grounding point, and the potential $V+\frac{dV}{dx}\cdot dx$ at P_2 means the same at $x+dx$ distance.

I and $I+\frac{dI}{dx}\cdot dx$ express the currents coresponding to P_1 and P_2 respectively. Therefore

[3)] M, Mori, and Nakakuma: Comittee of insulation deterioration of machines for power service in Japan, March 1959.

the next formula may be established as follows:—

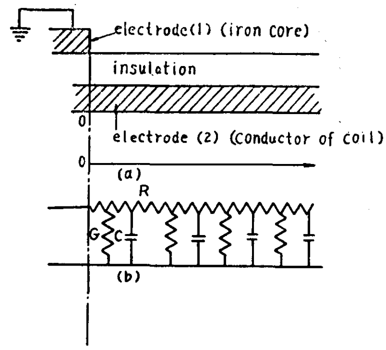

Fig. 7. Schematic diagram of coil insulation.
R: distributed surface resistance.
C: distributed static capacity.
G: distributed conductance.
Y: distributed admittance.
$Y=G+j\omega C$.

$$\frac{dV}{dx}=-ZI \qquad (1)$$

$$\frac{dI}{dx}=-YV \qquad (2)$$

where $Y=G+j\omega C$.

Then we will get the next formula

$$\frac{d^2V}{dx^2}=R.Y.V. \qquad (3)$$

From Eq. (3), we will get the solution of V:—

$$V=V_0\, e^{-(\alpha+j\beta)x} \qquad (4)$$

where V_0 indicates the potential difference between the conductor and the grounding one and is known as the phase voltage of the generator.

$$\alpha=\sqrt{\frac{1}{2}R\,(G+\sqrt{G^2+\omega^2C^2})}, \qquad \beta=\frac{\omega C R}{2\alpha} \qquad (5)$$

If we let V' be the surface potential on the arbitrary point of the coil then we will get the next vector equation

$$\dot{V}'=\dot{V}_0-\dot{V} \qquad (6)$$

therefore,

$$|V'|=V_0\sqrt{(1-e^{-\alpha x}\cos\beta x)^2+(e^{-\alpha x}\sin\beta x)^2}. \qquad (7)$$

If we take the generator coils of Numakura Hydraulic power Station as an example, whose coil has such data and dimensions summarized as follow, then we will get C

name of power station	rated volt in kv	tan δ %	static cap. in $\mu\mu$F	area of elec. in cm^2
numakura	11.0	3.8	381	1215
thickness of insulalion: 5.7 mm, length of electrode on the coil: 81 cm. peripheral length of coil: 15 cm. $\therefore C=3.58\times10^{-13}$ (F/cm^2), $G=4.27\times10^{-12}$ (℧/cm^2)				

and G from tan δ=3.8 % by using tan δ$=G/\omega C$ as follows:—

$$C=3.58\times10^{-13}\,\text{F/cm}^2$$
$$G=4.27\times10^{-12}\,\text{℧/cm}^2. \qquad (8)$$

Fig. 8 shows the potential distrbutions on the surface of the end turn of Numakura Power Station in the case where their surface resistance have various values from $10^{11}\,\Omega$ to $10^8\,\Omega$. From this Fig. 8, we will find that the potential curve grows more flat with the lower resistances.

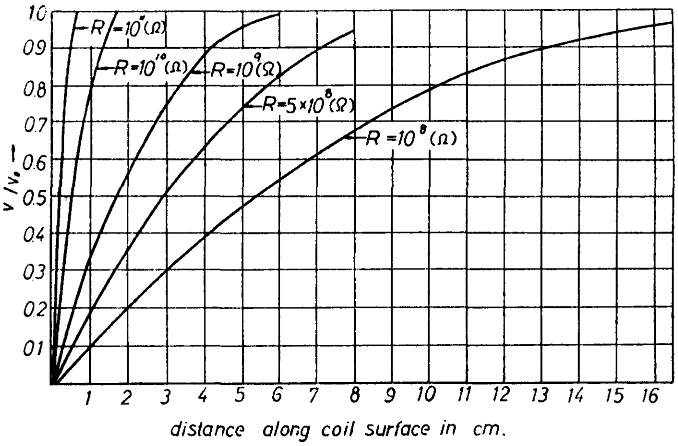

Fig. 8. Voltage distribution on the coil surface. (R – constant)

The potential gradient along the surface $\frac{dV'}{dx}$ may attain its crest value at $x=0$ and therefore the corona will occur here. We assume the critical value of the potential gradient for starting the corona discharge along the dielectric surface by the following formula,

$$V_{p0} \doteqdot 3.0/C \text{ kV} \tag{9}$$

where C is the capacity per unit area of dielectrics toward ground which forms the bypass to the corona starting path. Fig. 9 shows the maximum potential gradient

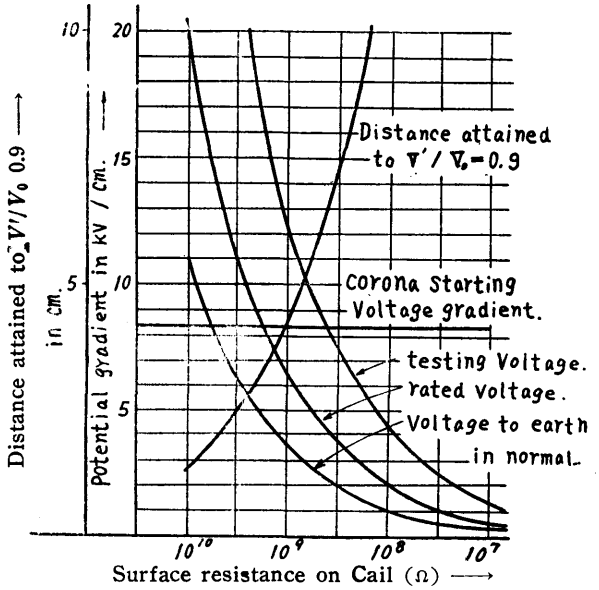

Fig. 9. Relation between surface resistance on coil and max. pot. gradient on it.

appearing at $x=0$ in reference to various surface resistance where the corona starting potential gradient is calculated as follows:

$$V_{p0} \doteqdot 8.1\ \mathrm{kV/cm}.$$

From these diagrams above mentioned, it will be deserable to keep the surface resistance under $5\times10^8\,\Omega$ with a view to non-corona appearance even in commercial testing. The distance from $x=0$ to the corresponding point of $V'/V_0=0.9$ is expressed in this Fig. 9. The surface resistance $5\times10^8\,\Omega$ may be also favourable from this point of view.

IV.-B. The Case Where the End-Turn Has Certain Surface Resistance Distributed as $R=R_0\,e^{ax}$

In the case of constant surface resistance above mentioned, we explained that the potential gradient along the surface is expressed as the attenuation of certain logarithmic curve. Meanwhile, if they had the resistance with the distribution of the logarithmic increasing charactor as $R_0\,e^{ax}$, we might expect the potential gradient to be uniform along the distance x, that is to say, we might make it to be the rational distribution of the potential gradient on the surface of the end-turn.

If we put the formula (1) into the formula (10), then we have,

$$\frac{dV}{dx} - R(x)\cdot I \tag{10}$$

$$\frac{dI}{dx} = -Y\cdot V \tag{2}$$

where $R(x)=R_0\,e^{ax}$.

From (10) and (2), we can obtain the following equation

$$\frac{dV^2}{d^2x} - a\frac{dV}{dx} - R_0\,Y e^{ax}\,V=0 \tag{11}$$

If we transform the equation (11) by putting $p=-\frac{R_0 Y}{a^2}e^{ax}$, we will get

$$\frac{d^2V}{dp^2} + \frac{1}{p}V=0 \tag{12}$$

And again if we transform the Eq. (12) by putting $V=p^{\gamma}\cdot u$ and $w=p^h$ we will get

$$\frac{d^2u}{dz^2} + \frac{1}{z}\frac{du}{dz} + \left(1-\frac{1}{z^2}\right)u=0 \tag{13}$$

where $2w=z$.

As the Eq. (13) is Bessel's differential equation, the solution may be expresse as follows:—

$$u=AJ_1(z)+BJ_{-1}(z) \tag{14}$$

Then we have

$$V=e^{\frac{a}{2}x}\left[A\,J_1\left(j\frac{2\sqrt{R_0Y}}{a}e^{\frac{a}{2}x}\right)+BY_1\left(j\frac{2\sqrt{R_0Y}}{a}e^{\frac{a}{2}x}\right)\right]. \tag{15}$$

Also, Eq. (15) is expressed by the Hankel's function

$$V=e^{\frac{ax}{2}}\left[AH_{1}^{(1)}\left(j\frac{2\sqrt{R_0Y}}{a}e^{\frac{ax}{2}}\right)+BH_{1}^{(2)}\left(j\frac{2\sqrt{R_0Y}}{a}e^{\frac{ax}{2}}\right)\right] \tag{16}$$

where $H_1^{(1)}$ and $H_1^{(2)}$ are called the 1 st kind and the 2 nd kind of Hankel's functions respectively. A and B are arbitrary constants to be determined by the boundary conditions.

We must take the boundary condititions to fix the constants A and B as follows:—

$$\begin{aligned} &\text{at } x=0, \quad V=V_0, \\ &\text{at } x=\infty, \quad V=0. \end{aligned} \tag{17}$$

So we have

$$V=\frac{e^{\frac{a}{2}x}\cdot V_0}{H_1^{(2)}\left(\frac{2\sqrt{2}}{a}\alpha\, e^{-j\frac{\pi}{4}}\right)}H_1^{(2)}\left(\frac{2\sqrt{2}}{a}\alpha\, e^{\frac{a}{2}x}\cdot e^{-j\frac{\pi}{4}}\right) \tag{18}$$

where $\alpha=\sqrt{\frac{1}{2}R_0\,\omega\, C}$

From (18), we can get V'.

$$|V'|=V_0\left|1-\frac{e^{\frac{a}{2}x}}{H_1^{(2)}\left(\frac{2\sqrt{2}\,\alpha}{a}\,e^{-j\frac{\pi}{4}}\right)}H_1^{(2)}\left(\frac{2\sqrt{2}}{a}\alpha\, e^{\frac{a}{2}x}\cdot e^{-j\frac{\pi}{4}}\right)\right|. \tag{19}$$

Fig. 10 shows the resistance distributions along the surface of the end turn with the various exponential indexes. Fig. 11 shows the potential distribution along the surface of the end-turn with both the various surface resistance and also the various exponential indexes.

It will be acknowledged from the Fig. 11 that the higher the exponential index is the more the potential rises on the surface. Fig. 12. 13 shows the relation between the

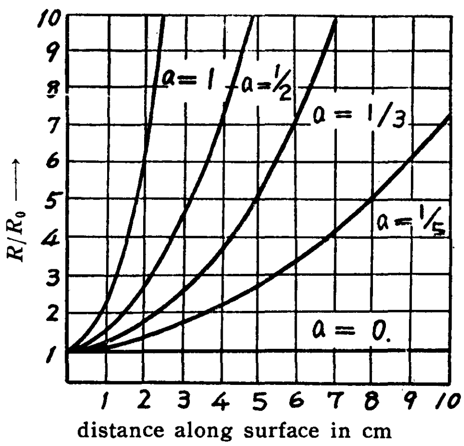

Fig. 10. resistance distribution $R=R_0\,e^{ax}$.

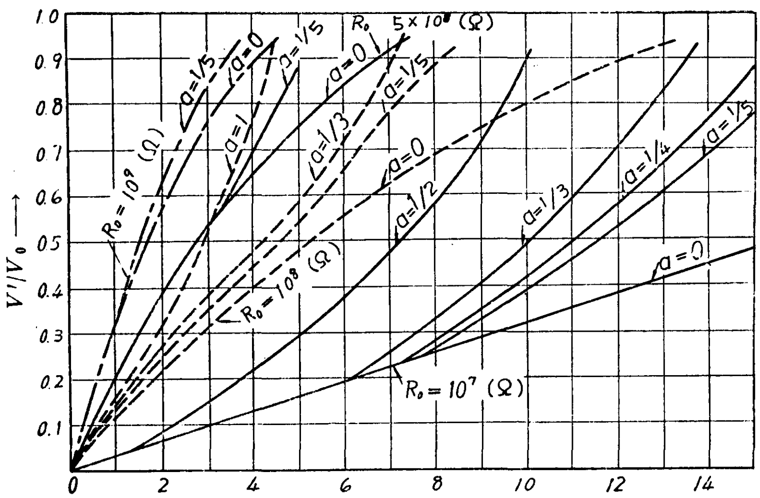

Fig. 11. Potential gradient in the case of $R = R_0 e^{ax}$.

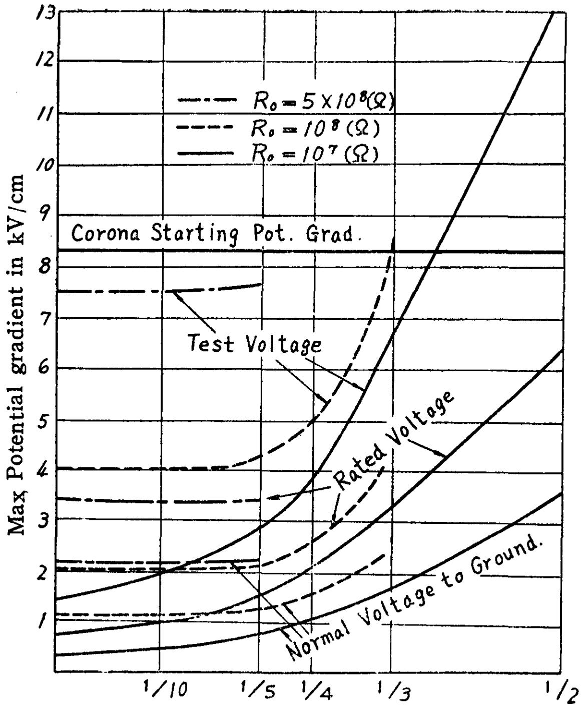

Fig. 12. Changes of Max. Pot. Grad. due to variation of *a*.

maximum potential gradient and the exponential index of the surface resistance. We will find that it is deserable to select the surface resistance to be the order of $10^7\,\Omega$, if the potential index is put under 1/3, and the ratio between the mean potential gradient and the maximum become flatter if we put $R_0=10^7\Omega$.

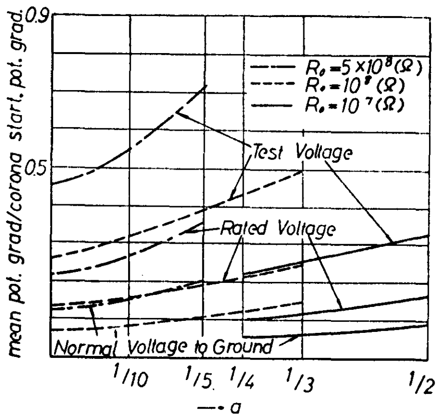

Fig. 13. Changes of Mean Pot. Grad. due to variation of a.

V. Preventatation of Corona Appearance in The Spaces between The Slot Wall and The Coil Surface

The straight part of the coil is inserted in the slot of the iron core and beacuse of the insufficient contact with the slot wall, there always are air spaces between them. When the potential appears between them sufficiently, the corona will break out in the air gaps. This corona, called "micro-discharge", will cause the failure of the coil insulation by perforating them from the surface after a long run of many years.

Because of such a perforation, we have to expect the insulation failure to prevent continuous service of the power supply.

Let us consider the following schematic diagram with electrodes as shown in Fig. 14. Fig. 14 shows the surrounding of some part of the coil with insufficient contact with the slot wall. Electrode A is a circular disc analogous to the grounded iron core with a small contact area and the electrod B is one of the coil conductors. If we take the surface resistance R_x and admittance Y_x at arbitrary point x apart from the origin 0, and then $R_{(x+dx)}$ and $Y_{(x+dx)}$ as at the point $(x+dx)$ correspondingly, we will denote R_x Y_x $R_{(x+dx)}$ and $Y_{(x+dx)}$ as follows:

$$R_x=R_0/2\pi x\,,\qquad Y_x=2\pi xY_0,$$

$$R_{(x+dx)}=R_0/2\pi\,(x+dx)\,,$$

$$Y_{(x+dx)}=2\pi\,(x+dx)\,Y_0$$

where R_0 and Y_0 are the surface resistance and the admittance of the insulating material per unit area respectively.

And also $\quad Y_0=G+j\omega C_0\,.$

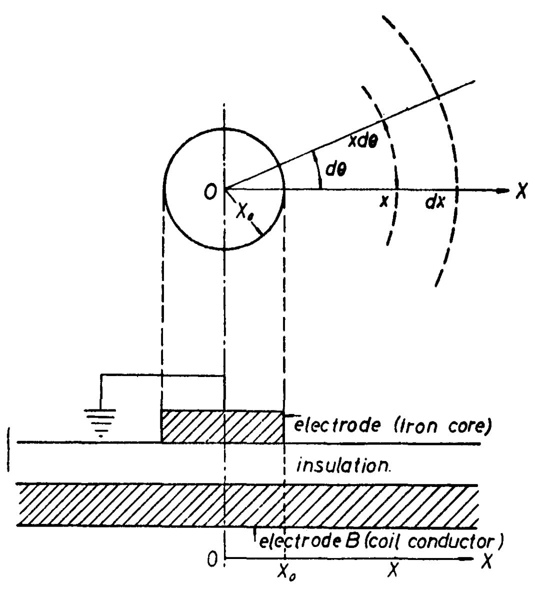

Fig. 14. schematic diagram for circulaa disd electrode.

From the above formula, we will obtain such an expression as follows:—

$$\frac{dV_x}{dx} = -\frac{R_0}{2\pi x^2} \cdot I_x, \qquad \frac{dI_x}{dx} = 2\pi Y_0 \cdot V_x. \tag{20}$$

Then from (20), the next differential equotion is detived.

$$\frac{d^2V}{dx} + \frac{2}{x}\frac{dV}{dx} + \frac{1}{x^2}R_0 Y_0 V = 0 \tag{21}$$

The solution of (21) is expressed as (22) in general,

$$V = Ax^{\lambda_1} + Bx^{\lambda_2}, \tag{22}$$

where

$$\lambda_1 = -\frac{1}{2} + \sqrt{\frac{1}{4} - j\omega C_0 R_0}, \qquad \lambda_2 = -\frac{1}{2} - \sqrt{\frac{1}{4} - j\omega C_0 R_0}. \tag{23}$$

Constants A and B may be determined from the boundary conditions (24)

$$\begin{aligned} &\text{at } x = x_0, && V = V_0, \\ &\text{at } x = D/2, && \left(\frac{dV}{dx}\right) = 0. \end{aligned} \tag{25}$$

V_0 is the potential between electrodes A and B at x_0 where x_0 is the radius of the upper grounded contact. D is the distance between two grouned electrodes on the surface of the coil as we may assume that there are many contact points on the surface in practice.

We will determine A and B from (24) as follows:—

$$A = V_0 \Big/ \left[x_0^{-j\gamma} + \frac{j\gamma\left(\frac{D}{2}\right)^{-1-j\gamma} \cdot x^{+j\gamma-1}}{(-1+j\gamma)\left(\frac{D}{2}\right)^{-2+j\gamma}} \right] \tag{25}$$

$$B = \frac{\left(\frac{D}{2}\right)^{1-j2\gamma} \cdot A}{\left(1 + j\frac{1}{\gamma}\right)} \tag{25}$$

The surface potential on the coil V' may be expressed from (26).

$$\dot{V}' = \dot{V}_0 - \dot{V} \tag{26}$$

We applied the above formula to the 11kV generator coil of Numakura power station and got Fig. 15 (A) and (B). Fig. (A) shows the potential distribution of the area concerned which is free from the contact with the slot wall, and the point of grouned contact with is somewhere apart from this area and has the radius of $x=0.01$cm and the distances between the two grounded points are $D=2$ cm and also $D=4$ cm. Fig. 15 (B) shows the case where $x=0.001$ cm. In Fig. 15 (A) and (B), we find that the surface potentials rise quickly according to the distance from the grounded electrode, but we may not be able to clearly assertain what potential initiates the corona discharge.

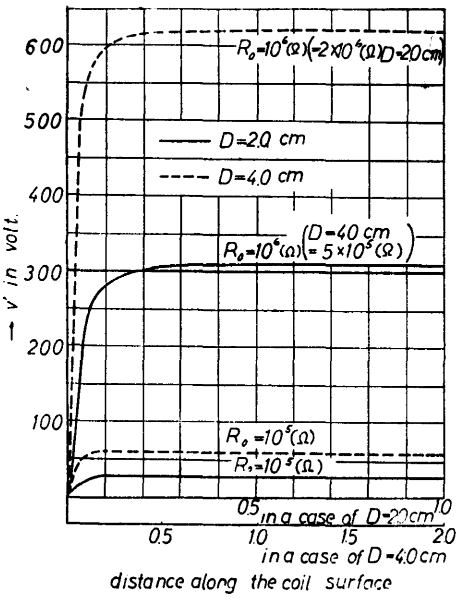

Fig. 15. (A) Pot. grad. on the coil in the case of $x=0.01$ cm. (A)

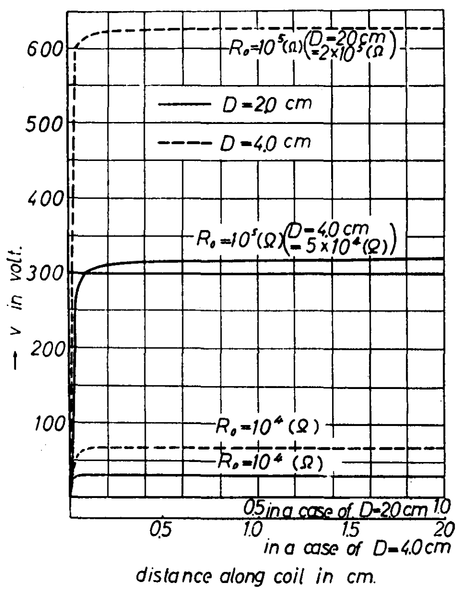

Fig. 15. (B) Pot. grad. on the coil in the case of $x=0.01$ cm. (B)

But as an instance, if we assume the critical voltage of corona to be more than 300 V as in the micro-discharge, then the value of surface resistance may be placed at less than $10^4\Omega$.

We can consider that it is desirable to put the surface resistance of the coil at lower value, but its value may be restricted by the eddy current loss due to the magnetic flux perpendicular to the coil surface.

The eddy current will cause the increase of the surface loss by which the temperature

Table V. Relation between eddy current losses and surface resistivity.

FLUX DENSITY in gauss	SURFACE RESISTIVITY in ohm	EDDY CURRENT LOSS in watt/cm^2
5000	10	2.45×10^{-1}
	10^2	2.45×10^{-2}
	10^3	2.45×10^{-3}
	10^4	2.45×10^{-4}
6000	10	3.50×10^{-1}
	10^2	3.50×10^{-2}
	10^3	3.50×10^{-3}
	10^4	3.50×10^{-4}

of the coil surface is raised. Therefore, we must refrain from lowering the surface resistance too much.

The eddy current loss may be expressed by $\omega^2 B_0^2/\rho_s$[W/cm²] per unit area of the coil surface, where B_0[Wb/cm²] is the magnetic induction and ρ_s is the surface resistance per unit area. The table V shows the relation between the eddy current losses and the surface resistances.

In general, since we take the current density at 280 A/cm, the density of the leakage magnetic flux lateraly passing through the slot side may be estimated at about 5,000 gauss. Therefore, it will be more favourable to put the surface resistance to the amount higher than $10^2\Omega$, because the eddy current loss on the surface is lower than the copper loss of the conductor, in which its loss is about 1.4×10^{-1}[W/cm²].

VII. Summary

The characteristics of the corona preventing paint are preferred according to different parts of the coil as follows:—

The surface resistance on the end turn is best at $5\times10^8\Omega$ for uniform painting and also $10^4\Omega$ for the ununiform painting where the logarithmic index is 1/5 as the most favourable surface distribution.

The surface resistance on the straight part of the coil is in the order of $10^3\Omega$ for practical application.

Supplement.

Is it possible in practice to distribute the surface resistance according to $R=R_0e^{ax}$? This question may be answered simply by the consideration that the surface resistance grows less in proportion to the number of paint application as shown in Fig. 16, where

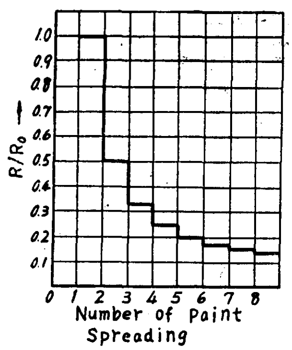

Fig. 16. Relation between changes of resistance and number of paint spreading.

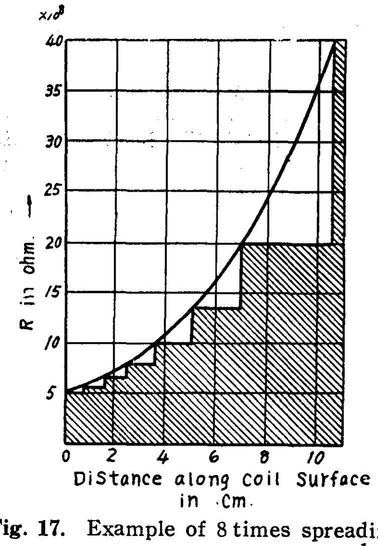

Fig. 17. Example of 8 times spreading, in the case of $R=5\times10^8\varepsilon^{\frac{1}{5}x}$.

R is the reduction of the surface resistance by one application of the paint.

Fig. 17 shows one example in which $R=5\times10^8\times e^{\frac{1}{5}x}$ is realized by 8 applications of the paint. We took a paint with the value of 4×10^9 and spread it over the coil several times, each time reducing the area of application as shown in Fig. 17.

This distribution of the resistance may the expressed by the stepping grades approximately $e^{\frac{1}{5}x}$. However, if we increase the number of the steps or eliminate the corners of steps by scrumbling the spreading, then we will be able to make continuous distribution of the surface resistance.