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On Vibration of an Elastic Bar which is immersed in a Cylindrical Water Tank*

(Received Oct. 20, 1958)

Fumiki KITO**

Abstract

The Author has previously made a study on vibration of a cylindrical shell which is immersed in water, and reported the result in this Proceedings.[1] Present report is the outcome of continuation of this study. Here the vibration of an elastic straight bar of circular cross section, which is placed in a water tank of cylindrical form, is studied. From the analytical expression for the vibratory motion of water particles thus set up, the amount of hydraulic force acting on the elastic bar is obtained. For the case in which there exist no end-walls (lids) of the cylindrical tank, it is shown that the frequency of vibration of the bar can be expressed by a simple formula wherein the effect of surrounding water is taken into account, the bar itself being supported at both ends. For the case in which end-walls of the cylindrical tank exist, it becomes a very complicated task to evaluate the frequency of the bar. But, by actual examination of numerical values, it was possible to point out that, as long as the length of the bar is more than about ten times its diameter, the same formula as when there exist no end-walls can be used for estimation of the frequency of vibration of the bar, at least for practical purposes.

1. Vibratory Motion of Water when there exist no End Wall

Let us consider, as shown in Fig. 1, a cylindrical tank of radius b, its center line being $X-X$, as shown in the figure. Inside this tank an elastic bar of radius a is placed along the same center line $X-X$. At first, we assume that the cylindrical tank and the bar are of infinite length. The space between the bar and the cylindrical tank wall is assumed to be filled up with water. The tank wall is assumed to be rigid. When the bar makes some vibratory motion, there will be set up, in water, also a vibratory motion. Assuming that vibratory motions are of infinitesimal magnitude, and that the water is an ideal fluid (inviscid, incompressible), the motion of

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[1] F. Kito; On Vibration of a Cylindrical Shell immersed in Water, This Proceedings, 5, 32 (1952)

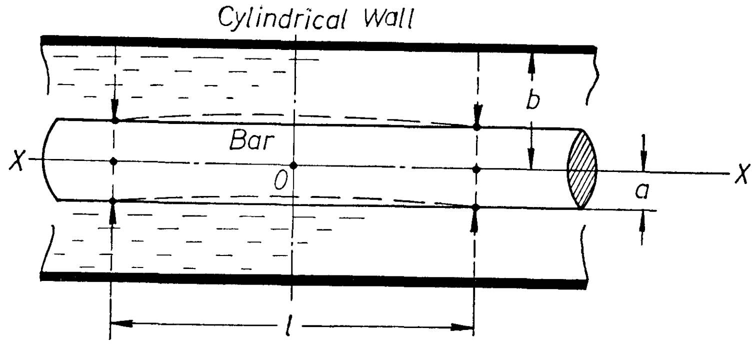

Fig. 1. Vibration of Continuous Bar immersed in Cylindrical Water Tank

water will be found by solving the potential equation $\nabla^2\phi=o$, under a suitable boundary conditions. ϕ is the velocity potential, and the hydraulic pressure p will be given by

$$p=-\rho\frac{\partial\phi}{\partial t} \tag{1}$$

In what follows, we shall use cylindrical coordinates (r,θ,x) to specify the position of a point in space, the center line $X-X$ of the cylindrical tank (Fig. 1) being taken as $x-$axis. The velocity components V_r, V_θ, V_x of water particles will be given by

$$V_r=\frac{\partial\phi}{\partial r},\quad V_\theta=\frac{\partial\phi}{r\partial\theta},\quad V_x=\frac{\partial\phi}{\partial x}$$

A solution ϕ_1 of the potential equation $\nabla^2\phi=0$, or the equation,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}+\frac{\partial^2\phi}{\partial x^2}=0,$$

which is suited to the present problem is found to be

$$\phi_1=AF(r)\cos kx\sin\theta\sin\omega t \tag{2}$$

where

$$F(r)=I_1'(kb)K_1(kr)-K_1'(kb)I_1(kr)$$

$$A=\frac{-\omega W}{k[K_1'(ka)I_1'(kb)-K_1'(kb)I_1'(ka)]}$$

$I_n(z)$, $K_n(z)$ means modified Bessel functions of first and second kind respectively, of order n. It satisfies the condition that along the rigid boundary wall $r=b$, $\partial\phi_1/\partial r=0$. The vibratory motion of the bar is expressed by

$$w=W\cos kx\sin\theta\cos\omega t \tag{3}$$

where ω=angular frequency of vibration, $k=\pi/l$, l being length of one span of the bar, W=vibration amplitude of the bar. w denotes the radial displacement of any point on the surface of the bar. The constant A in the solution (2) is so chosen as to satisfy the condition that at $r=a$, $\partial\phi_1/\partial r=\partial w/\partial t$. In the above chosen state of vibration, the bar is kept supported at a row of supports which are arranged along the bar with the distance l. In the present paper, this state of vibraiton will be called, merely for convenience' sake, the case of continuous bar. The hydraulic pressure p is, by Equ. (1); ——

$$p=-\rho\frac{\partial\phi_1}{\partial t}$$

$$=-\rho\omega AF(r)\cos kx\sin\theta\cos\omega t$$

So that the resultant force P per unit length of the bar, which is caused by the pressure p is:——

$$P_1=-\int_0^{2\pi}p\sin\theta\,d\theta$$

$$=\rho\omega\pi aAF(r)\cos kx\cos\omega t$$

or,

$$P_1=\Pi_1\cos\omega t \qquad (4)$$

where

$$\Pi_1=\rho\omega^2\pi a^2\,G.W.\cos kx \qquad (5)$$

$$G=\frac{-[K_1(ka)I_1'(kb)-K_1'(kb)I_1(ka)]}{ka[K_1'(ka)I_1'(kb)-K_1'(kb)I_1'(ka)]} \qquad (6)$$

II. Frequency of Vibration of an Elastic Continuous Bar immersed in Water

The fundamental equation of the transverse vibration of an elastic bar (straight uniform section) is given by;

$$EI\frac{\partial^4u}{\partial x^4}+\rho_0A_0\frac{\partial^2u}{\partial t^2}=P, \qquad (7)$$

where u is the transverse displacement of the bar, which is here assumed to be of a very small magnitude. EI is the flexural rigidity of the cross section of the bar, A_0 being its cross-sectional area. ρ_0 is the density of the material composing the bar. P is the value of an external force per unit length acting on the bar. Taking the case of sustained vibration with angular frequency ω, let us put

$$u=\eta\cos\omega t,\ P=\Pi\cos\omega t$$

into the Equ. (7), where η and Π are functions of the variable x. Then we have

$$\frac{d^4\eta}{dx^4}-m^4\eta=\frac{\Pi}{EI} \qquad (8)$$

putting, for shortness

$$m^4=\frac{\rho_0A_0\omega^2}{EI}$$

When we confine ourselves to the case of "continuous bar" which is vibrating in a water region as considered in the previous section, we can obtain the solution of Equ. (8) in the form

$$\eta = W\cos kx, \quad \Pi = W\Lambda\cos kx \tag{9}$$

The external force P acting on the bar being taken to be equal to that given by Equ. (4), we have,

$$\Lambda = \rho\omega^2 a^2 \pi G$$

Putting Equ. (9) into Equ. (8) we obtain the relation: ——

$$k^4 - m^4 = \frac{\Lambda}{EI} = \frac{1}{EI}\rho\omega^2 a^2 \pi G$$

from which we have

$$k^4 = \frac{\rho_0 \pi a^2}{EI}\left[1 + \frac{\rho}{\rho_0} G\right]\omega^2 \tag{10}$$

where, as mentioned above, $k = \pi/l$. Thus, we see that, so long as we confine ourselves to the case of vibration of "continuous bar", the natural frequency $f(=\omega/2\pi)$ of vibration can be obtained by a simple Equ. (10), wherein the effect of surrounding water is taken into account. From Equ. (10) we see that the effect of surrounding water is to reduce the frequency by the ratio

$$\frac{1}{\sqrt{1+(\rho/\rho_0)G}}$$

In this factor, the ratio of density of water ρ to that of bar-material ρ_0 is concerned. It also depends on a constant G (of no dimension). The values of constant G, for various values of the ratio b/a and for the values of $l/a=19.6$ and $l/a=39.3$, were calculated, and the result is shown in Fig. 2

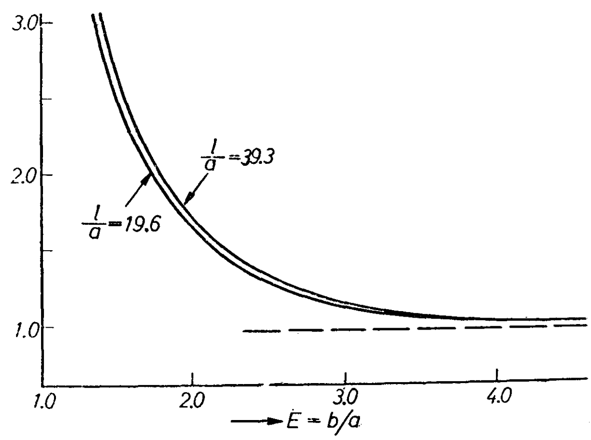

Fig. 2. Values of G ($ka = 0.16$ & 0.08)

III. Vibration of Water when there exist End Walls

As the next problem, let us consider the case in which there exist end-wall at both ends of the cylindrical tank. Fig. 3 shows the sketch of this case.

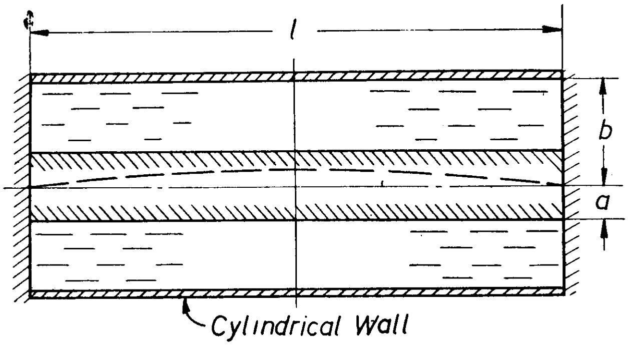

Fig. 3. Vibration of Elastic Bar immersed in Cylindrical Water Tank with End Walls

There is a cylindrical tank of length l and radius b, inside of which water is filled up. Both end walls and cylindrical side-wall of the tank is assumed to be rigid. The bar of uniform circular section (of radius a) is placed concentric with the tank, the bar being kept supported at both ends. When the bar is making transverse vibration, we put

$$w = W \cos kx \sin\theta \cos\omega t$$

$$u = W \cos kx \cos\omega t$$

where u is the transverse displacement at a point x of the center line of the bar, w being the radial velocity at a point on the surface of the bar. Due to this motion of the bar, there will also be set up a vibratory motion of water. This vibratory motion of water has been studied by the author in the previous paper as mentioned above.[1] The result may be summarized as follows:—

The solution is represented by the velocity potential $\phi_s = \phi_1 + \phi_2$, where ϕ_1 is given, as before, by Equ. (2), with the same values of the function $F(r)$ and constant A. The another potential function ϕ_2 is to be chosen so that

(i) $\partial\phi_2/\partial r = 0$ at $r = a$ and $r = b$,

(ii) $\partial(\phi_1 + \phi_2)/\partial x = 0$ at $x = \pm l/2$.

This functicn ϕ_2 is taken in the form

$$\phi_2 = \sum_{i=1}^{\infty} B_i \cosh m_i x \, f_i(r) \sin\theta \sin\omega t \tag{11}$$

where

$$f_i(r)=S_1(m_i,r)$$
$$=J_1(m_ir)Y_1'(m_ia)-J_1'(m_ia)Y_1(m_ir) \tag{12}$$

$J_n(Z)$ means the Bessel function of first kind of order n. $Y_n(Z)$ is the Bessel function of second kind (in Weber's form) of order n, as defined by

$$Y_\nu(Z)=\frac{J_\nu(Z)\cos\nu\pi-J_{-\nu}(Z)}{\sin\nu\pi}$$

According to the condition (i) we must have

$$J_1'(m_ib)Y_1'(m_ia)-J_1'(m_ia)Y_1'(m_ib)=0$$

So that the values of parameter $\lambda_i=m_ia$ must be so chosen as to satisfy the equation

$$\frac{J_1'(\lambda_i)}{J_1'(\lambda_iE)}=\frac{Y_1'(\lambda_i)}{Y_1'(\lambda_iE)} \tag{13}$$

where $E=b/a$. Next, according to the condition (ii) we must have

$$\sum_{i=1}^{\infty}B_im_i\sinh\frac{m_il}{2}S_1(m_i,r)=A[I_1'(kb)K_1(kr)-I_1(kr)K_1'(kb)] \tag{14}$$

This equation means that the right hand side of this equation is to be expressed as a modified form of Fourier-Bessel series.[2] We have shown the relation

$$\int_a^b S_n(\lambda,u)S_n(\mu,u)udu=0$$

for $\lambda=m_i,\ \mu=m_j,\ (i\neq j)$

$(i,\ j=1,2,3,\cdots\cdots\cdots\cdots)$

where

$$S_n(m_i,r)=J_n(m_ir)Y_n'(m_ia)-J_n'(m_ia)Y_n(m_ir)$$

n being an integer, and $m_1, m_2, m_3, \cdots\cdots\cdots$ are different roots of the equation

$$J_n'(m_ib)Y_n'(m_ia)-J_n'(m_ia)Y_n'(m_ib)=0 \tag{15}$$

arranged in their order of magnitude. Also we have (with $\lambda=m_i$)

$$\int_a^b [S_n(\lambda,u)]^2udu=M_i$$

$$=\left(1-\frac{n^2}{\lambda^2b^2}\right)[Y_n'(\lambda a)J_n(\lambda b)-J_n'(\lambda a)Y_n(\lambda b)]^2\left(\frac{b^2}{2}\right)-\left(\frac{a^2}{2}\right)\left(1-\frac{n^2}{\lambda^2a^2}\right)\left(\frac{1}{\lambda a}\right)^2 \tag{16}$$

When we write, for convenience

$$g(u)=I_n'(kb)K_n(kr)-I_n(kr)K_n'(kb),$$

we have, by some lengthy but easy calculation,

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[2] F. Kito: On a Fourier-Bessel expansion of special kind, This Proceedings, 5, 41 (1952)

$$\int_a^b ug(u)S_n(\lambda,u)\,du$$

$$=\frac{\lambda}{k}\frac{1}{k^2+\lambda^2}[Y_n'(\lambda a)J_n'(\lambda b)-J_n'(\lambda a)Y_n'(\lambda b)]$$

$$-\frac{k}{\lambda}\frac{1}{k^2+\lambda^2}[I_n'(kb)K_n'(ka)-K_n'(kb)I_n'(ka)]$$

so that we have, referring to Equ. (15),

$$\int_a^b ug(u)S_n(m_i,u)du$$

$$=\frac{k}{m_i}\frac{1}{k^2+m_i^2}[I_n'(kb)K_n'(ka)-K_n'(kb)I_n'(ka)] \tag{17}$$

Using these relations, we have the following value of the coefficient B_i which satisfies the Equ. (14):——

$$B_i=\frac{-Ak}{m_i^2M_i\sinh\frac{m_il}{2}}\cdot\frac{k}{k^2+m_i^2}\cdot[I_1'(kb)K_n'(ka)-K_n'(kb)I_n'(ka)] \tag{18}$$

Thus the solution ϕ_2 as expressed by Equ. (11) is fully determined. Corresponding value of the hydraulic pressure p is given by

$$p_2=-\rho\frac{\partial\phi_2}{\partial t}=-\rho\omega\ \sin\theta\ \cos\omega t\cdot\sum_{i=1}^{\infty}B_i\ \text{sosh}\ m_ix.\ f_i(r)$$

and the resultant force per unit length acting on the bar is

$$P_2=\rho\omega\pi a\ \cos\omega t\sum_{i=1}^{\infty}B_i\cosh m_ix\ f_i(r)=\Pi_2\cos\omega t \tag{19}$$

where

$$\Pi_2=W\ \rho\omega^2\pi a^2\sum_{i=1}^{\infty}C_i\cosh m_ix \tag{20}$$

with

$$C_i=\frac{B_if_i(a)}{\omega aW.}$$

IV. Examination of Numerical Values and Conclusion deduced from It

In order to examine the numerical values of the analytical expressions obtained in the previous section (§3), let us first modify expressions for B_i, M_i into more convenient forms for actual estimation. First, the value of coefficient M_i can be written, by virtue of the relation (15) (where $n=1$), in the following form ;——

$$M_i=\frac{1}{2m_i^2}\left(1-\frac{1}{m_i^2b^2}\right)\left\{\frac{J_1'(m_ia)}{J_1'(m_ib)}\right\}^2-\frac{1}{2m_i^2}\left(1-\frac{1}{m_i^2a^2}\right)$$

$$=\frac{1}{2m_i^2}\left[\left(1-\frac{1}{m_i^2b^2}\right)\left\{\frac{J_1'(m_ia)}{J_1'(m_ib)}\right\}^2-\left(1-\frac{1}{m_i^2a^2}\right)\right]$$

Also we have, for C_i ;

$$C_i=-\frac{k}{m_ia}\cdot\frac{1}{m_i(k^2+m_i^2)\sinh(m_il/2)}$$

$$\times\frac{2m_i^2}{\left[\left(1-\frac{1}{m_i^2b^2}\right)\left\{\frac{J_1'(m_ia)}{J_1'(m_ib)}\right\}^2-\left(1-\frac{1}{m_i^2a^2}\right)\right]}$$

$$=-\frac{2(ka)}{(ka)^2+(m_ia)^2}\times\frac{1}{\sinh(m_il/2)}$$

$$\div\left[\left(1-\frac{1}{m_i^2b^2}\right)\left\{\frac{J_1'(m_ia)}{J_1'(m_ib)}\right\}^2-\left(1-\frac{1}{m_i^2a^2}\right)\right]$$

Now, putting, as mentioned above, $m_ia=\lambda_i$, $b/a=E$, we have

$$m_ib=\lambda_iE, \qquad m_il/2=\lambda_il/(2a)$$

$$m_ix=\lambda_i(x/a)$$

so that the expression for Π_2 may be modified into as follows : ——

$$\Pi_2=W\rho\pi\omega^2a^2\sum_{i=1}^{\infty}D_i\frac{\cosh m_ix}{\sinh[\lambda_il/(2a)]}$$

$$=W\rho\pi\omega^2a^2\sum_{i=1}^{\infty}D_i\frac{\cosh(\lambda_ix/a)}{\sinh[\lambda_il/(2a)]} \tag{21}$$

where

$$D_i=-\frac{2ka}{(ka)^2+\lambda_i^2}\div\left[\left\{1-\frac{1}{(\lambda_iE)^2}\right\}\left\{\frac{J_1'(\lambda_i)}{J_1'(\lambda_iE)}\right\}^2-\left(1-\frac{1}{\lambda_i^2}\right)\right] \tag{22}$$

In order to obtain some idea about numerical values, we have chosen three typical cases as shown in Fig. 4. In each of them value of l/a is taken equal to *19.6* which means that the length of the bar is about ten times its diameter. The ratio of radii $b/a=E$ are taken as *1.5*, *2.0* and *2.5* respectively. In Fig. 5 graph of the function

$$y=\frac{J_1'(x)}{Y_1'(x)}=\frac{xJ_0(x)-J_1(x)}{xY_0(x)-Y_1(x)}$$

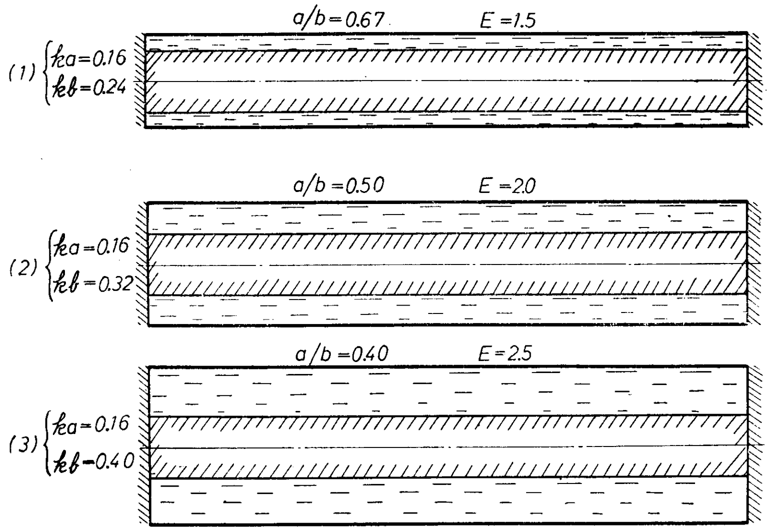

Fig. 4. Proportions of Bar and Tank used in Calculation

is shown. Let a horizontal line drawn parallel to x-axis cut the curves of Fig. 5 at points whose abscissa are ξ_1 and ξ_2 respectively. Then we shall have

$$\frac{J_1'(\xi_1)}{Y_1'(\xi_1)}=\frac{J_1'(\xi_2)}{Y_1'(\xi_2)}$$

or, writing $\xi_2/\xi_1=E$, we shall have

$$\frac{J_1'(\xi_1)}{J_1'(\xi_1 E)}=\frac{Y_1'(\xi_1)}{Y_1'(\xi_1 E)}$$

Thus, if drawing suitable horizontal lines, we sought out those points ξ_1, ξ_2 which just satisfy the relation $\xi_2/\xi_1=E$ (for a previously given value of E), it may be said that a root $\lambda_i=\xi_1$ of the Equ. (13) was found out. In this way the following values of roots of the Equ. (13) were obtained. Putting these values

Table 1.

$E=1.5$	$\lambda_1=0.82$	$\lambda_2=6.20$,	$\lambda_3=12.35$,	....
$E=2.0$	$\lambda_1=0.68$	$\lambda_2=3.35$,	$\lambda_3=6.20$,	$\lambda_4=9.50$,
$E=2.5$	$\lambda_1=0.58$	$\lambda_2=2.20$,	$\lambda_3=4.30$,	....

Table 2.

$E=1.5$	$D_1=0.350$	$D_2=-0.0105$	....	....
$E=2.0$	$D_1=0.177$	$D_2=-0.0284$	$D_3=-0.0047$	$D_4=-0.0032$
$E=2.5$	$D_1=0.161$	$D_2=-0.0471$	....	....

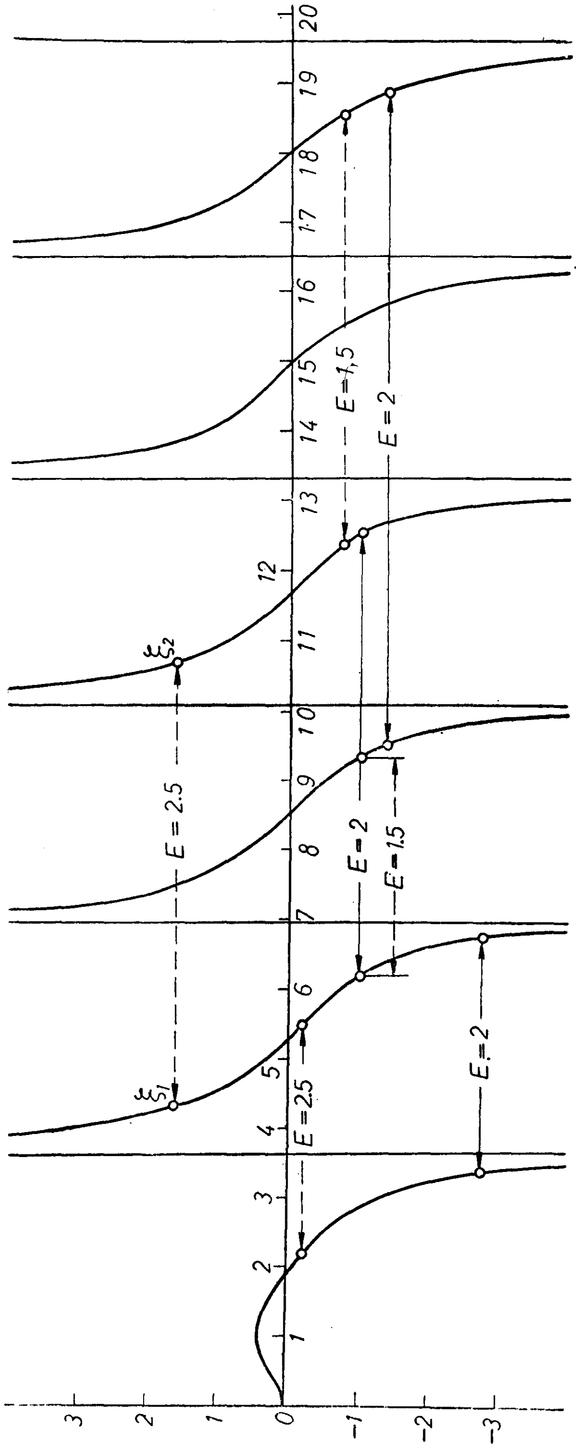

Fig. 5 *a*. Graphical Solution of Equation $J_1'(\xi)/J_1'(\xi E)=Y_1'(\xi)/Y_1'(\xi E)$

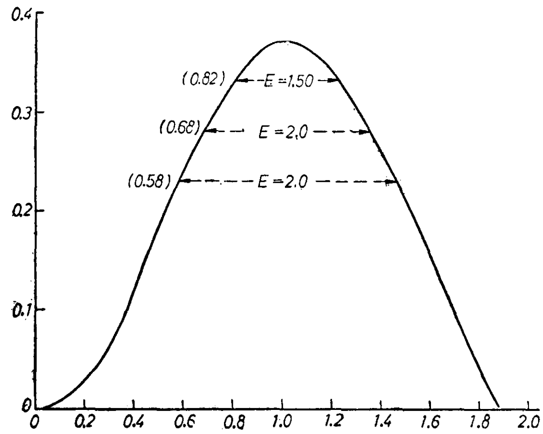

Fig. 5 *b*.

into Equ. (22) we calculated numerical values of coefficients D_i, and obtained values as shown in Table 2. Knowing the values of D_i, the value of Π_2 can easily be obtained by Equ. (12). The comparative values of Π_1 (as given by Equ. (5)) and Π_2 are shown in a rough sketch in Fig. 6. From the Figure it will be seen that the effect of end-

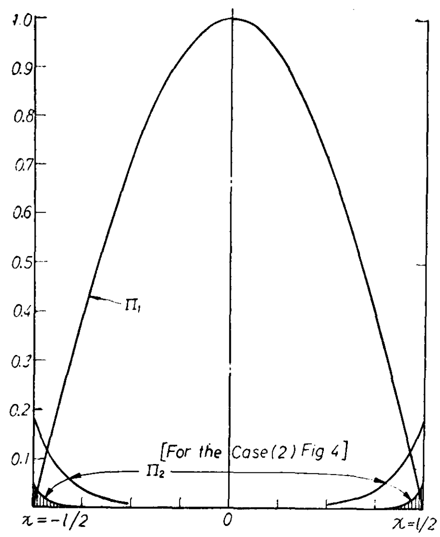

Fig. 6. Comparison of Values of Π_1 and Π_2

walls upon the force (Π_2) is very small in comparison to the force (Π_1) when there exist no end-walls. Even if the term Π_2 has some influence, it is only restricted to the close neighbourhood of both ends of the bar ($x=\pm l/2$). Now, for a bar whose both ends are supported, forces which are restricted to close neighbourhood of both ends will give a very small effect upon transverse vibration of the bar. Thus we see that the Equ. (10) for frequency of vibration of bar immersed in water may, at least for practical purpose, be used also for the case in which there exist end-walls. This inference is, of course, only applicable to long bars as shown in Fig. 4. Theoretically, we should make following evaluations: Starting from the assumption that the bar is making a transverse vibration expressed by

$$u = W \cos kx \cos \omega t,$$

we have found that the bar will be subject to transverse force $P = P_1 + P_2$, which is caused by the action of hydraulic pressure of surrounding water. When we put the value of P thus found into right-hand side of Equ. (7), and solve it under boundary conditions for supported ends, we shall find better approach to correct value of u.

Repeating this procedure, we could obtaion closer and closer approximation to correct value. But, in actual work, it is seen that this requires much complicated task of calculation. The above inference shows us that, if the length of the bar is equal to or more than ten times its diameter, the mode and frequency of vibration will only very slightly be affected by the presence of end-walls, and consequently the Equ. (10) can be used at least for practical purposes. Therefore the author stopped pursuing the above-mentioned task of successive approximations.

Note: On Roots of the Equation $J_n'(\lambda)/J_n'(\lambda E) = Y_n'(\lambda)/Y_n'(\lambda E)$

If we draw a graph of the function

$$y = \frac{J_n'(x)}{Y_n'(x)} \tag{23}$$

it will be of the form as shown in Fig. 7. Drawing a straight line AA parallel to x-axis, let abscissa of points of intersection of this straight line with the given curves be x_1, x_2, x_3, ...

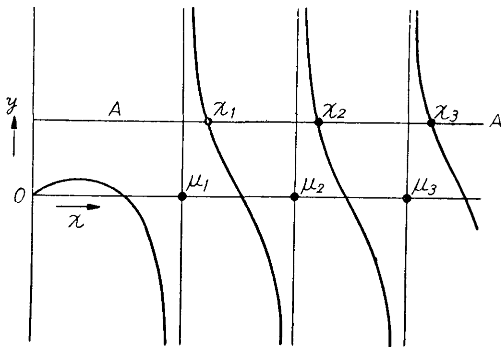

Fig. 7. Graph of $y = J_n'(x)/Y_n'(x)$

There are infinite number of such abscissa for any given straight line AA. The expression (23) can also be written

$$y = \frac{nJ_n(x) - xJ_{n+1}(x)}{nY_n(x) - xY_{n+1}(x)}$$

Now, for a very large real positive value of x, we have, asymptotically,

$$J_n(x) = \sqrt{\frac{2}{\pi x}} \left\{ P_n(x) \cos\left(x - \frac{2n+1}{4}\pi\right) - Q_n(x) \sin\left(x - \frac{2n+1}{4}\pi\right) \right\}$$

$$Y_n(x) = \sqrt{\frac{2}{\pi x}} \left\{ P_n(x) \sin\left(x - \frac{2n+1}{4}\pi\right) + Q_n(x) \cos\left(x - \frac{2n+1}{4}\pi\right) \right\}$$

where

$$P_n(x)=1-\frac{(4n^2-1)(4n^2-3^2)}{2!(8x)^2}+\cdots\cdots\cdots$$

$$Q_n(x)=\frac{4n^2-1}{8x}-\frac{(4n^2-1)(4n^2-3^2)(4n^2-5^2)}{3!(8x)^3}+\cdots\cdots\cdots$$

Whence we have, also for a large real positive value of x ;

$$[nJ_n(x)-x\,J_{n+1}(x)]\frac{1}{x}=\frac{1}{x}\cos\left(x-\frac{2n+1}{4}\pi\right)\left(-\frac{n^2}{2}-\frac{3}{8}\right)-\sin\left(x-\frac{2n+1}{4}\pi\right)$$

$$[nY_n(x)-x\,Y_{n+1}(x)]\frac{1}{x}=\frac{1}{x}\sin\left(x-\frac{2n+1}{4}\pi\right)\left(-\frac{n^2}{2}-\frac{3}{8}\right)-\cos\left(x-\frac{2n+1}{4}\pi\right)$$

Therefore

$$y=\frac{\sin\left(x-\frac{2n+1}{4}\pi\right)+\left(\frac{n^2}{2}+\frac{3}{8}\right)\frac{1}{x}\cos\left(x-\frac{2n+1}{4}\pi\right)}{\cos\left(x-\frac{2n+1}{4}\pi\right)+\left(\frac{n^2}{2}+\frac{3}{8}\right)\frac{1}{x}\sin\left(x-\frac{2n+1}{4}\pi\right)}$$

from which it may easily be seen that values x_1, x_2, $\cdots\cdots\cdots$ for any straight line AA are unlimited in number. Fig. 8 shows the curves of which values of ratios

(i) $\frac{x_2}{x_1}$, $\frac{x_3}{x_1}$, $\cdots\cdots\cdots$

(ii) $\frac{x_3}{x_2}$, $\frac{x_4}{x_2}$, $\cdots\cdots\cdots$

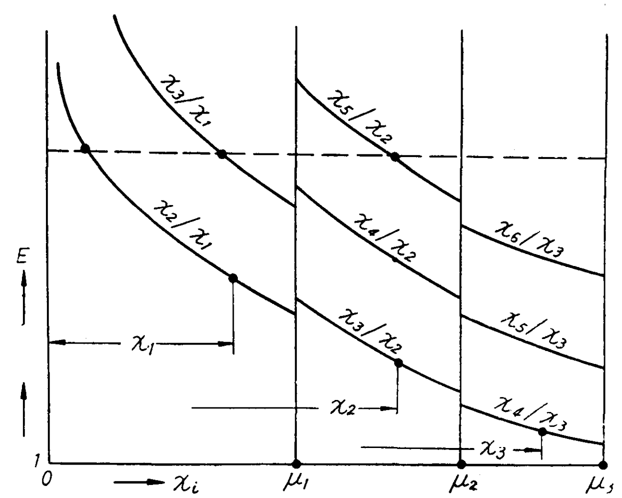

Fig. 8

are taken as ordinates, abscissa being x_1, x_2, $\cdots\cdots\cdots$ respectively. For a given value of E, let us draw a straight line $y=E$. Then the points of intersection of this straight line with above-mentioned curves will give us roots of the equation

$$\frac{J_n'(\lambda)}{Y_n'(\lambda)} = \frac{J_n'(\lambda E)}{Y_n'(\lambda E)}$$

Thus we see that there are infinite number of roots of the Equ. (13).

Corrigenda

to " On Vibration of a Cylindrical Shell immersed in Water, by F. KITO (This Proceedings, Vol. 5, No. 17, 1952)

p. 35 : 7th line from the bottom, instead of $T_n(m_i, r)$, read $S_n(m_i, r)$

p. 36 : 9th line from the top, instead of M_i, read $(1/M_i)$; 7th line from the bottom, instead of M_i, read A/M_i ; 7th line from the bottom, instead of m_i, read m_i^2.

p. 37 : 7th, 8th and 12th line from the top, instead of Q_n, read S_n.

p. 37 : 9th line from the top, read

$$=\left(1-\frac{n^2}{\lambda^2 b^2}\right)[Y_n'(\lambda a)J_n(\lambda b)-J_n'(\lambda a)Y_n(\lambda b)]^2\left(\frac{b^2}{2}\right)$$

$$-\left(\frac{a^2}{2}\right)\left(1-\frac{n^2}{\lambda^2 a^2}\right)\left(\frac{1}{\lambda a}\right)^2=M_i$$

Equ. (14) : instead of $\cos kx$ read $\cosh sx$

p. 38 : top line, instead of $Q_n(s,r)$, read $s \sinh\frac{sl}{2} S_n(s,r)$