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# Resistive Forces Against the Motion of a Model Car Rolling on Sand (IV) (Inclination of the Supporting Fork of the Wheels Rolling on Sand)

(December 1, 1955)

Kiyoshi TAKAHASHI\*

## Abstract

It is desirable that the inclination of the supporting fork of the wheels rolling on sand agrees with the angle between the direction of the combined resistance and that of the traction resistance, both of which act upon the wheels. But since the direction of the combined resistance varies with the state of the sand, the shape of the wheels and the velocity, it becomes necessary to change the inclination of the fork according to these conditions.

## I Introduction

The resistance experienced by a wheel rolling on sand is directed towards the axle provided that the friction upon the axle is neglected. Consequently the fork is subjected to a torque around its connecting point to the body of the car, except in those cases when the fork is set to take the direction coinciding with that of the resistance. As a result, the fork is bent and in extreme cases it may be broken. If the direction of the fork coincides with that of the resistance, there is no torque. This state is very desirable in practice. The resistance varies with the shape and the velocity of the wheels as well as with the state of the sand and the load. The author has studied the influences of these factors upon the resistance, from the results of which the conditions to satisfy this requirement from mechanical viewpoints are described here. Further, it is also desirable that the torque around the fork, which appears when the direction of the progress of the car is changed, should be small. For this purpose it would be better to use spherical wheels instead of the ordinary ones. But here appears the problem of the resistance which is required to be as small as possible. We made experiments to compare the spherical wheels with the ordinary ones required in order to get information whether the former can possess the necessary conditions. Lastly the case of the air tired wheels were investigated, and the relation between the pressure in the tire and the traction resistance was determined.

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## II Apparatus and method

The method and the apparatus are almost the same as those of the previous experiment, except that a part of the apparatus was improved in order to measure the depth of the wheel sunk in the sand accurately and quickly: Identical gears were attached to make the rotations of  $A_1$  and  $A_2$ , shown in the figure of the foregoing paper, the same. In addition, similar worm gears were equipped to the rotation transmission shaft. For the measurement of the traction resistance, that the wire-rope should be extended horizontally is a necessary condition. The state of the wire-rope is observed by means of a leveller to attain the horizontal position in a short time. The one end of a long leveller was placed on a ring which can rotate almost without friction around the axle  $P$ , whereas the other end was on the front side of the car body. When the air bubble in the leveller moved from the central position by the change of the depth of the sinking of the wheel the supporting bar of  $P$  was moved vertically by hand through a cam mechanism so that the bubble comes to the original position. As for the wheels, spherical wheels made of "Rawan" and ordinary wheels of wood were used. The sand was in a state of natural drying, except in the cases of the changed humidity. Finally the case of the tired wheels was studied too.

## III Experimental results

### 3.1 Relation between the radius of the wheel( $r$ ) and the resistance ( $R$ )

For the case of the gradually increasing radius of the wheel under the same load, the behaviour of the change of the traction resistance( $H$ ), the combined resistance( $R$ ), the depth of the wheel sunk in sand( $h$ ) and the angle of the fork( $\theta$ ) have been determined. The results are shown in Fig. 2. In this figure are given the values of the resistance when the wheel diameter was changed from 15cm, through 20, 25, 28 to 35cm for either 5kg or 7.5kg of the load and under the other conditions written there. As shown in the figure it was found that the traction resistance decreases exponentially with the increase of the diameter. In addition the absolute value of the combined traction resistance directed through the axle is seen to decrease by degrees with the increase of the diameter. On the other hand it was found the angle  $\theta$  which

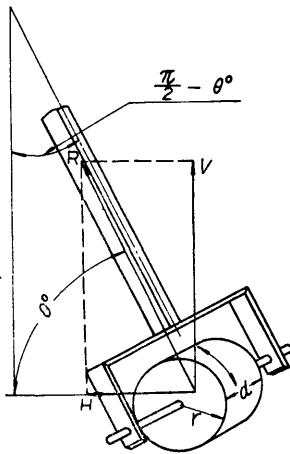


Fig. 1 Configuration of the wheel and an explanation of the notations.

- $d$ : thickness of the wheel
- $r$ : radius of the wheel
- $(\pi/2 - \theta)$ : inclination of the supporting fork
- $H$ : traction resistance
- $R$ : combined resistance

the combined resistance makes against the horizontal surface in the clockwise direction increases with the diameter slowly, provided other conditions are the same, as seen in the plot of  $\theta$  vs diameter  $2r$ . Consequently the angle of the fork should be increased according to the increase of the wheel radius as is given by the  $r$ - $\theta$  curve. Lastly the depth( $h$ ) of the wheels sunk in sand lessens with the diameter gradually.

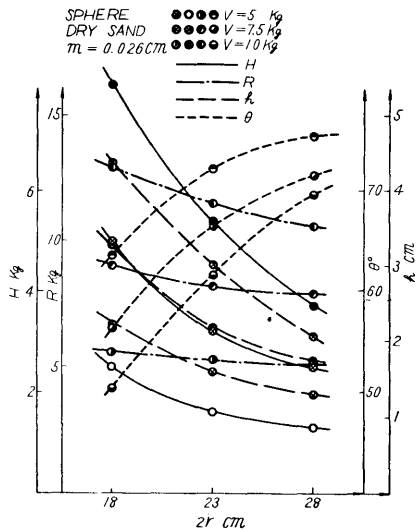


Fig. 2 Changes in the traction resistance  $H$ , the load  $V$ , the combined resistance  $R = \sqrt{H^2 + V^2}$  the angle between the horizontal and the direction of the combined resistance  $\theta$  and the depth of the sinking of the wheel in sand  $h$ , in relation to the wheel radius( $r$ ).

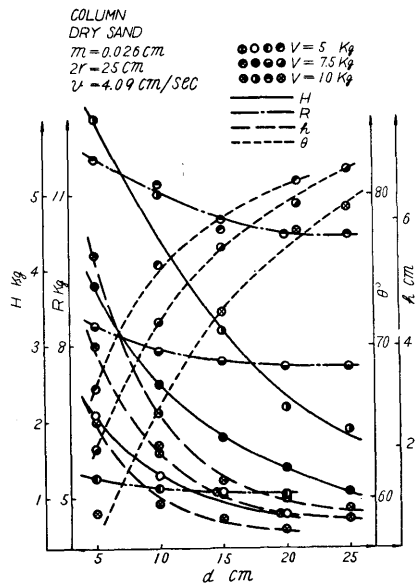


Fig. 3  $H$ ,  $V$ ,  $R$ ,  $\theta$  and  $h$  as function of the wheel thickness  $d$

3.2-A Relation between the thickness of the wheel ( $d$ ) and the resistance ( $R$ )

For the case of the change of the wheel thickness the variations of  $H$ ,  $R$ ,  $\theta$  and  $h$  were measured with the result as given in Fig. 3. As can be seen in the figure,  $H$  decreases exponentially with the thickness of the wheel  $d$ . This means that the traction resistance decrease very little beyond a critical value in the thickness of the wheel. From this fact it may be concluded that there is a limit in the wheel thickness for least resistance in the sand. Further, as shown in the figure,  $R$  decreases with  $d$  gradually, perhaps approaching some definite value. These relations seem to indicate that  $\theta$  approaches a nearly constant value with the increase of  $d$  under a constant load. This supposition coincides with the experimental results as proved in the figure. Accordingly, it is concluded that the angle of the fork should be increased gradually with the thickness as long as the latter

Table 1 Table of the Observed values for Figures from Fig. 2 to Fig. 11.

Sand				Speed	Wheel			Observed value			Calculated value		
Apparent specific gravity	Ropose angle	Mean diameter of particle (cm)	Dry, Wet	$v$ (cm/sec)	Diameter $2r$ (cm)	Thickness $d$ (cm)	Shape	$V$ (kg)	$H$ (kg)	$h$ (cm)	$\theta$	$R$ (kg)	
1.27~1.47	29°30'	0.026	Dry	4.09	15 20	5 5	Column	5	3.1	4.2	58.2	5.9	
								5	2.4	3.4	64.4	5.6	
								7.5	5.3	5.4	54.6	9.2	
								5	1.8	1.3	70.2	5.3	
								7.5	4.4	2.5	59.5	8.7	
								10	7.0	3.9	54.6	12.2	
								5	1.3	0.8	75.4	5.2	
								7.5	2.4	1.2	72.3	7.8	
								10	5.0	1.9	63.5	11.2	
								12.5	8.4	2.9	56.3	15.1	
								15	11.2	3.8	53.4	18.7	
								17.5	13.7	4.7	52.0	22.2	
								5	1.0	0.5	78.8	5.1	
								7.5	1.6	0.8	78.0	7.7	
								10	2.7	1.1	74.8	10.4	
								12.5	6.1	1.7	64.0	13.9	
								15	9.2	2.2	58.3	16.4	
								17.5	12.3	3.0	54.9	21.4	
								20	15.7	3.8	52.0	25.4	
								5	0.8	0.4	80.9	5.1	
7.5	1.5	0.7	78.8	7.7									
10	2.5	0.9	76.0	10.3									
12.5	4.6	1.2	69.9	13.3									
15	8.1	1.7	61.6	19.8									
17.5	11.4	2.3	56.9	20.9									
20	15.2	2.7	52.6	25.1									
5	2.1	2.5	67.2	5.4									
7.5	3.8	4.0	63.2	8.4									
10	6.0	5.3	59.0	11.7									
15	12.0	7.4	51.3	19.2									
1.55~1.60	39°00'	0.430						5	2.1	1.4	67.2	5.4	
								7.5	3.7	2.3	63.6	8.4	
								10	5.8	3.3	59.9	11.6	
								15	11.3	5.2	53.0	18.8	
20	18.4	7.9	47.5	27.2									
1.50~1.58	38°30'	0.310						5	1.9	1.2	69.2	5.3	
								7.5	3.6	2.2	64.3	8.3	
								10	5.5	3.2	61.2	11.4	
								15	10.3	5.1	55.6	18.2	
20	18.2	7.6	47.6	27.0									
1.46~1.57	37°00'	0.170						5	1.7	1.1	71.3	5.3	
								7.5	3.5	2.1	65.0	8.3	
								10	5.4	3.1	61.7	11.4	
								15	10.1	5.0	56.0	18.1	
20	17.1	7.5	49.5	26.3									
1.40~1.55	35°30'	0.070						5	1.5	1.0	73.3	5.2	
								7.5	3.2	2.0	66.9	8.2	
								10	5.2	2.8	62.5	11.3	
								15	9.7	4.7	57.2	17.9	
20	16.3	6.7	50.9	25.8									
1.27~1.47	29°30'	0.026						5	1.3	0.9	75.4	5.2	
								7.5	2.5	1.9	71.6	7.9	
								10	5.0	2.7	63.5	11.2	
								15	9.5	4.4	57.6	17.8	
								20	15.0	5.9	53.1	25.0	
								15	5	1.1	0.6	77.6	5.1
									7.5	1.8	1.1	76.5	7.7
									10	3.2	1.3	72.2	10.5
									15	7.0	2.3	65.0	16.6
									20	12.3	3.7	58.5	23.5
									22.5	15.3	4.3	55.8	27.2
									25	18.3	4.9	53.9	31.0
									27.5	20.5	5.4	53.3	34.3
								20	30	23.5	6.1	51.9	38.0
									5	0.8	0.4	80.9	5.1
									7.5	1.4	0.7	79.4	7.6
									10	2.2	1.0	77.6	10.2
									15	4.2	1.5	74.4	15.6
									20	9.5	2.2	64.7	22.1
									22.5	12.4	2.7	61.2	25.7
25	14.6	3.1	59.7	29.0									
25	27.5	17.6	3.4	57.4	32.7								
	30	20.5	3.9	55.6	36.3								
	35	25.9	4.8	53.4	43.5								
	7.5	1.1	0.6	81.7	7.6								
	10	1.9	0.8	79.2	10.2								
	15	3.4	1.1	77.3	15.4								
	20	6.2	1.5	72.8	21.0								
	22.5	8.7	1.8	68.9	24.1								
28	25	12.2	2.1	64.0	27.8								
	27.5	15.4	2.4	60.8	31.5								
	30	18.5	2.8	58.3	35.3								
	35	24.0	3.6	55.6	42.4								
	5	2.0	2.4	68.2	5.4								
	7.5	3.3	3.5	66.2	8.2								
	10	5.2	4.8	62.5	11.3								
	15	8.3	7.1	61.3	17.1								
10	20	13.1	9.3	56.8	23.9								
	22.5	15.5	10.5	55.5	27.3								
	5	0.9	1.0	79.7	5.1								
	7.5	2.3	1.5	72.9	7.8								
	10	3.6	2.1	70.1	10.6								
	15	6.9	3.6	65.4	16.5								
	20	11.3	5.1	60.5	22.9								
	22.5	14.5	5.9	57.2	26.8								
15	25	16.5	6.7	56.7	30.0								
	30	22.4	8.0	53.4	37.4								
	35	27.5	9.2	51.9	44.5								
	5	0.8	0.6	80.9	5.1								
	7.5	1.5	0.9	78.8	7.6								
	10	2.2	1.1	77.6	10.2								
	15	4.7	2.0	72.7	15.7								
	20	9.7	3.1	64.2	22.2								
	22.5	11.7	3.5	62.5	25.4								
	25	14.6	4.1	59.7	29.0								
	30	19.7	5.0	56.7	35.9								
	35	24.8	5.9	54.7	42.9								
	7.5	0.8	0.6	83.9	7.5								
	10	1.6	0.8	80.9	10.1								
	15	3.1	1.3	78.4	15.3								
	20	6.5	2.0	72.1	21.0								
22.5	9.0	2.4	68.2	24.2									
25	25	11.6	2.8	65.2	27.6								
	30	17.3	3.6	60.0	34.6								
	35	23.0	4.1	56.7	41.9								
	7.5	0.7	0.4	84.7	7.5								
	10	1.4	0.5	82.1	10.1								
	15	2.5	1.0	80.7	15.2								
	20	5.0	1.4	76.0	20.6								
	22.5	6.3	1.6	74.4	23.4								
35	25	8.3	1.8	71.7	26.3								
	30	13.6	2.2	65.7	33.0								
	35	18.8	2.5	61.8	39.7								
	5	1.7	1.9	71.2	5.3								
	7.5	2.7	2.9	70.2	8.0								
	10	4.0	3.8	68.2	10.8								
	12.5	5.1	4.8	67.8	13.5								
	15	6.5	6.0	66.6	16.3								
1.40~1.55	35°30'	0.070	Wet	0.50	25	10		5	1.7	1.9	71.2	5.3	
								7.5	2.7	2.9	70.2	8.0	
								10	4.0	3.8	68.2	10.8	
								12.5	5.1	4.8	67.8	13.5	
								15	6.5	6.0	66.6	16.3	
								20	10.1	8.1	63.2	22.4	
								7.5	3.0	3.0	68.2	8.1	
								10	5.3	3.5	62.1	11.3	
4.09								15	10.8	6.1	54.3	18.5	
								5	1.8	2.5	70.3	5.3	
								7.5	3.0	2.7	68.2	8.1	
								10	5.3	3.5	62.1	11.3	
								15	10.8	6.1	54.3	18.5	
								7.5	3.0	2.7	68.2	8.1	
								10	5.3	3.5	62.1	11.3	
								15	10.8	6.1	54.3	18.5	
18.07								5	1.8	2.5	70.3	5.3	
								7.5	3.0	2.7	68.2	8.1	
								10	5.3	3.5	62.1	11.3	
								15	10.8	6.1	54.3	18.5	
								7.5	3.0	2.7	68.2	8.1	
								10	5.3	3.5	62.1	11.3	
								15	10.8	6.1	54.3	18.5	
								15	10.8	6.1	54.3	18.5	

Table 2 Curves on Fig.2

Sand			Speed $v$ (cm/sec)	Sphere Diameter $2r$ (cm)	Observed value			Calculated value			
Apparent specific gravity	Repose angle	Mean diameter of particle (cm)			Dry, Wet	$V$ (kg)	$H$ (kg)	$h$ (cm)	$\theta$	$R$ (kg)	
1.27~1.47	29°30'	0.026	Dry	4.09	18	5	2.5	2.2	63.5	5.6	
						7.5	5.0	3.3	56.3	9.0	
						9	7.0	3.9	52.5	11.4	
						10	8.1	4.4	50.3	12.9	
						23	5	1.6	1.6	72.2	5.3
						7.5	3.2	2.1	66.5	8.2	
					9	4.6	2.6	62.3	10.1		
					10	5.4	3.0	61.6	11.5		
					15	10	4.1	56.3	18.0		
					17.5	13.6	4.8	52.5	22.2		
					28	5	1.3	1.3	75.4	5.1	
						7.5	2.5	1.7	71.5	7.9	
						9	3.2	1.9	70.4	9.5	
						10	3.7	2.1	69.6	10.6	
						15	6.6	2.7	66.2	16.4	
						17.5	8.8	3.1	63.4	19.5	
						20	11.4	3.7	60.3	23.0	
						30	20.2	5.1	55.1	36.1	

quantity remains below a certain value, and above which the angle of the fork make be kept constant.

3.2-B Relation between the pneumatic tire pressure ( $P$ ) and the resistance ( $R$ )

Since the contact area of the tire with the sand becomes wider when the pressure in the tire is decreased, we are going to discuss the connection between this fact and the above experimental results, according to which the increase of the

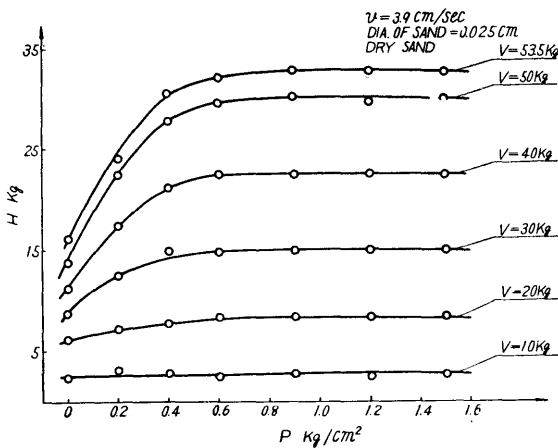


Fig. 4  $H$  and  $V$  as function of the tire pressure  $P$

wheel thickness results in the decrease of the traction resistance. The decrease of the inner pressure of the tire corresponds to the increase of the thickness.

Therefore in order to lessen the traction resistance it is required to decrease the pressure until the contact area of the tire becomes maximum. This is in good agreement with the experimental results.

Table 3 Table of the Observed Values for Figures from Fig. 4 to Fig. 5

Sand				Speed	Tire			Observed Value				Calculated Value	
Apparent specific gravity	Repose angle	Mean diameter of particle (cm)	Dry, Wet	$v$ (cm/sec)	Diameter $2r$ (cm)	Thick-ness of wheel (cm)	Thick-ness of tire wall (cm)	$P$ (kg/cm <sup>2</sup> )	$V$ (kg)	$H$ (kg)	$h$ (cm)	$\theta$	$R$ (kg)
1.27~1.47	29°30'	0.026	Dry	3.9	40	7	0.8	0	10	2.5	2.8	75.8	10.3
									20	4.0	4.0	78.7	20.4
									30	7.5	4.7	75.8	30.9
									40	11.5	5.8	73.9	41.6
									50	15.5	6.7	72.7	52.4
									53.5	16.3	6.6	73.1	55.9
								0.2	10	3.2	2.9	72.2	10.5
									20	7.6	4.4	69.2	21.4
									30	12.7	5.6	67.1	32.6
									40	17.8	6.3	66.0	43.6
									50	22.5	7.1	65.7	55.0
									53.5	24.0	7.4	65.7	58.5
								0.4	10	3.0	2.8	73.2	10.4
									20	8.1	4.7	68.0	21.6
									30	15.2	6.3	63.5	33.6
									40	21.5	7.4	61.6	45.4
									50	28.2	8.6	60.6	57.4
									53.5	31.0	8.9	60.3	61.9
								0.6	10	2.8	2.8	74.4	10.4
									20	8.5	4.7	67.5	21.7
									30	15.0	6.2	63.3	33.5
									40	22.5	7.8	60.6	45.8
									50	29.5	8.8	59.5	58.0
									53.5	32.0	9.3	59.0	62.3
								0.9	10	3.2	2.8	72.2	10.5
									20	8.5	5.0	67.0	21.7
									30	15.2	6.3	63.1	33.3
									40	22.7	7.6	70.5	45.9
									50	30.3	9.0	58.7	58.7
									53.5	33.0	9.4	58.5	62.9
								1.2	10	3.0	2.8	73.2	10.4
									20	8.3	4.5	67.4	22.8
									30	15.0	6.3	63.3	33.5
									40	23.0	7.8	60.1	46.1
									50	29.9	8.9	59.0	49.0
									53.5	32.8	9.3	58.3	61.6
								1.5	10	3.2	2.7	72.3	10.5
									20	8.6	4.7	66.8	29.4
									30	15.5	6.3	62.7	34.6
									40	23.0	7.7	60.6	64.2
									50	30.8	9.0	58.3	58.2
									53.5	33.5	9.1	57.9	62.3

Fig. 4 shows that the traction resistance does not depend on the pressure in the tire when the load is 10kg. But when the load is increased the resistance becomes proportional to the pressure in the tire, whereas for the pressure above some critical value the resistance becomes constant. It was found that the critical value of the pressure increases with the load. This situation was found to hold for each case of 20, 30, 40, 50 and 53.4kg of the load. As for the kind of the tire tested, the wheel of the 400-8, 4PLY tire of the "Rabbit" was used. It is desirable that the tire is easily deformed with the increase of the pressure. So an old tire with thin wall was selected.

In Fig. 5 are shown the dependencies of  $R$ ,  $H$ ,  $\theta$  and  $h$  upon the pressure in the tire ( $P$ ). As seen in the figure the angle of the fork should be decreased with the pressure until a certain critical pressure is reached and should be kept constant above this pressure.

### 3.3 Relation between the resistance ( $R$ ) and the size of the sand particles ( $m$ )

For the case when the size of the sand particles was changed, the accompanying changes of  $H$ ,  $R$ ,  $\theta$  and  $h$  were determined as given in Fig. 6. In the case of the sand dried naturally the result was that  $H$ ,  $R$  and  $\theta$  had the completely reverse dependencies upon the change of the particle size of the sand when compared with those cases of the change of the diameter and the thickness of the wheel, where the load was either 5, 7.5 or 10kg. Namely the traction resistance increases with the particle size of the sand slowly, whereas above a given value of the size it seemed to become nearly constant. On the other hand it is likely that  $R$  is not affected appreciably by the particle size as revealed by the relation of  $R$  and  $m$  and that of  $H$  and  $m$ , coinciding with the experimental results of  $\theta$ - $m$  curves. Therefore the angle of the fork should be decreased with the increase of the size of the sand particles as given in the  $\theta$ - $m$  curve.

### 3.4 Relation between the resistance ( $R$ ) and the velocity of the wheel ( $v$ )

The results of the dependencies of  $R$ ,  $H$ , and  $h$  on the velocity of the wheel is given in Fig. 7. For simplicity the sand tank was moved instead of the wheel with a determined velocity.

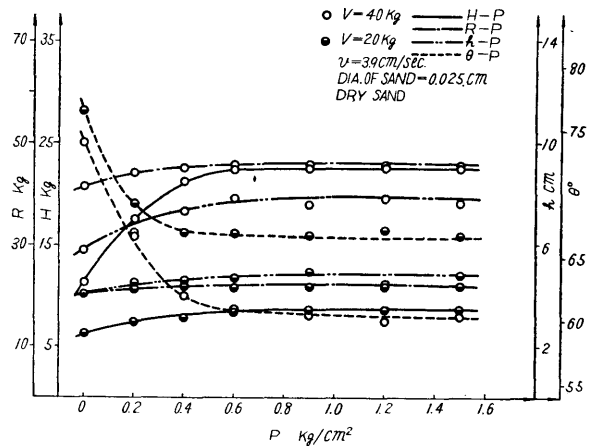


Fig. 5  $H$ ,  $V$ ,  $R$ ,  $\theta$  and  $h$  as function of the tire pressure  $p$



$H$ ,  $R$  and  $h$  showed almost no changes due to the change of the velocity. It is concluded therefore that the angle of the fork need not be varied under the change of the velocity, at least in the present range of the velocity.

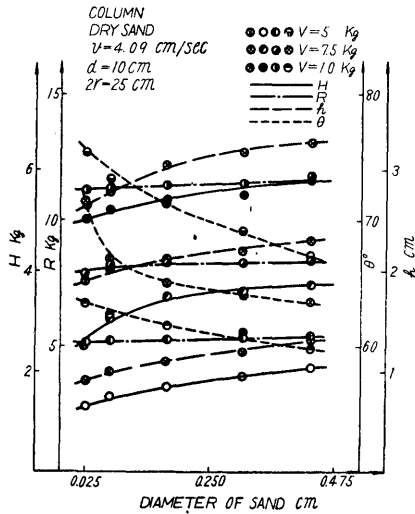


Fig. 6  $H$ ,  $V$ ,  $R$ ,  $\theta$  and  $h$  as function of the size of sand particles  $m$

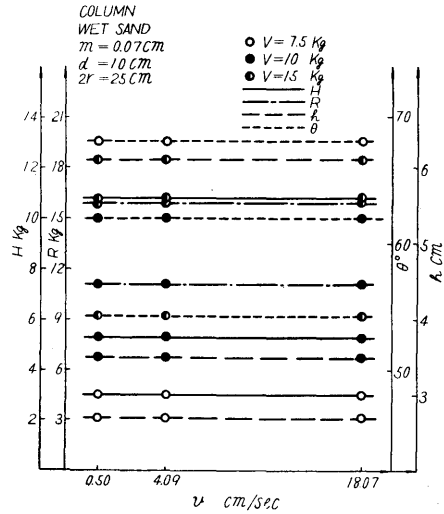


Fig. 7  $H$ ,  $V$ ,  $R$ ,  $\theta$  and  $h$  as function of the velocity of the wheel

3.5 Relation between the resistance ( $R$ ) and the humidity of the sand

Fig. 8 shows the dependency of  $H$ ,  $R$ ,  $\theta$  and  $h$  on the change of the humidity of the sand. The former quantities did not show any appreciable difference between the case of the sand in a naturally dried state and the case when the volume of the water to that of the sand was 1:7.

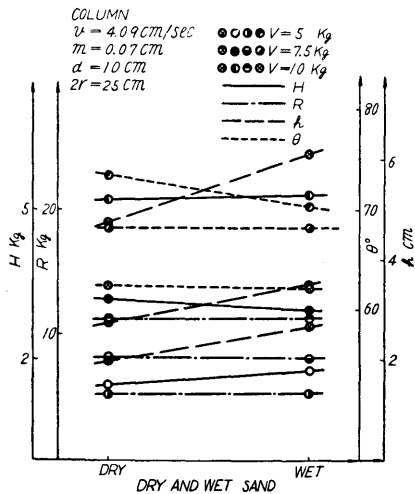


Fig. 8  $H$ ,  $V$ ,  $R$ ,  $\theta$  and  $h$  as function of the water content of the sand

Accordingly if we assume that these facts hold in the whole range of the water content extending from the state of the naturally dried to that with the content below 1:7, the angle of the fork may be kept constant in every case.

3.6 On the characteristics of the ordinary wheel and the spherical wheel

As described before the inclination of the fork supporting the axle should be changed properly according to the different conditions in rolling on sand. In addition to this, that the traction resistance should be as small as possible under a determined load

(*V*) and that the torque needed for the change of direction should also be small constitute the three conditions, under which the design of the wheels should be made to solve the problem of the traffic on sand from mechanical viewpoints.

It is apparent that the torque needed to change the direction is smaller for the spherical wheel than the ordinary wheels with the same diameter.

Therefore if the traction resistance *H* in case of spherical wheels is smaller than in case of ordinary wheels under the same load, the former can be said as being the very wheels suitable for these requirements. In Fig. 9<sub>A</sub> are shown the results of the comparison of the traction resistances for the ordinary wheels (thickness 15cm and 20cm) and of the spherical wheel with the same diameter. As shown in the figure the common diameter for these two kinds of wheels was 28cm. From these measurements it was found that the traction resistance is smaller for the ordinary wheels than for the spherical wheels in every case, Although the latter possess the good property that the torque needed for the direction change is small, it can not avoid the inconvenience that the traction resistance is higher than the ordinary wheel. Consequently it is necessary to design other forms of wheels to lessen this bad property.

Fig. 9<sub>B</sub> shows the relation among the radius of the spherical wheels (*r*), the combined resistancne (*R*), the traction resistance (*H*) the angle between the horizontal (*θ*) and the depth of the spherical wheel sunk in sand (*h*).

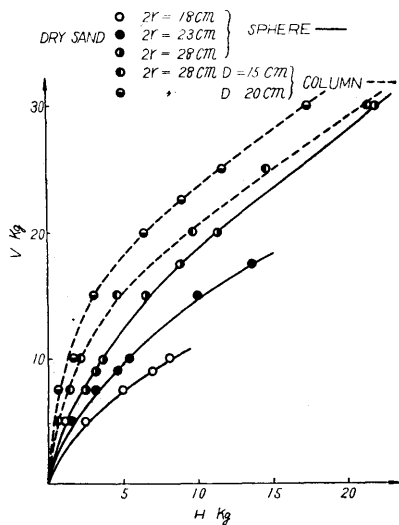


Fig. 9<sub>A</sub> *H* and *V* as function of the shape of the wheel, spherical or ordinary

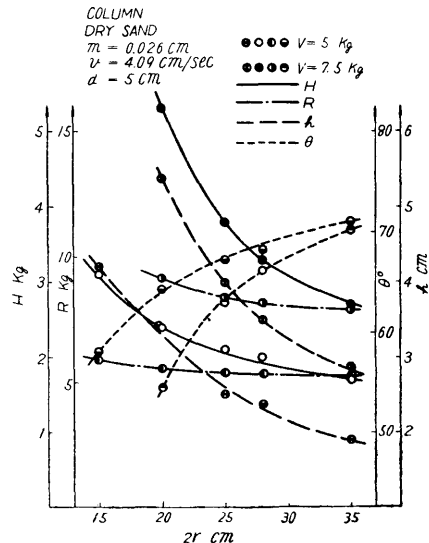


Fig. 9<sub>B</sub> *H*, *V*, *R*, *θ* and *h* as function of the radius of the spherical wheel

3.7 Sand stream lines of the spherical and ordinary wheels

(a) Measurements were made to see whether there are special relations among the radius ( $r_1, r_2$ ), the load ( $V_1, V_2$ ), the traction resistance ( $H_1, H_2$ ) and the depth ( $h_1, h_2$ ), for two spherical wheels differing in diameter.

i) It was found in the case of virgin state of the sand that  $H_1 = H_2$  and  $h_1 = h_2$  when the load is light and the equation  $r_1/r_2 = V_1/V_2$  holds. These relations are shown in Fig. 10.

ii) But if the load is increased,  $H_1 \neq H_2$  and  $h_1 \neq h_2$  even in the case of  $r_1/r_2 = V_1/V_2$ .

Fig. 11 is the graph showing this fact.

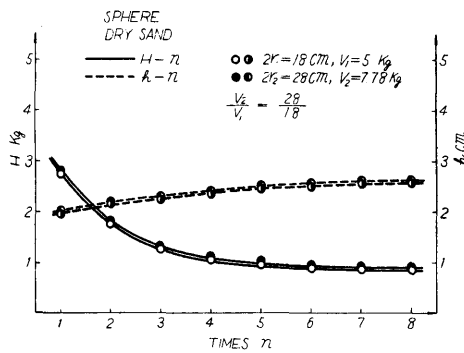


Fig. 10  $H$  and  $V$  as function of the number of runnings  $n$  when the load  $V_1$  and  $V_2$  are added to the ration of the radii ( $r_1, r_2$ ) of spherical wheels under light load

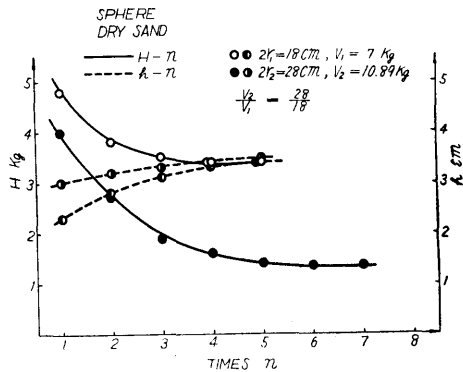


Fig. 11  $H$  and  $h$  as function of the number of runnings when the load  $V_1$  and  $V_2$  are added to the ration between the radii ( $r_1, r_2$ ) of spherical under light load

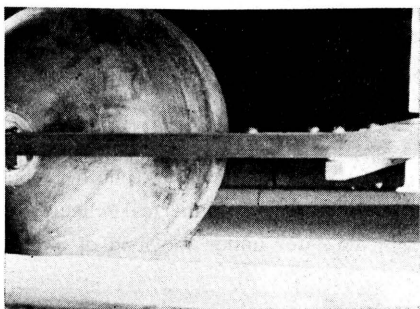
iii) Even in the case of the running of the wheel over the trace which was made by the previous runnings of the wheel, the relations i) and ii) hold, being independent on the number of the running, as shown in Fig. 10 and Fig. 11.

(b) On the other hand we can make clear the difference between the cases i) and ii) from the photographs of the sand stream lines.

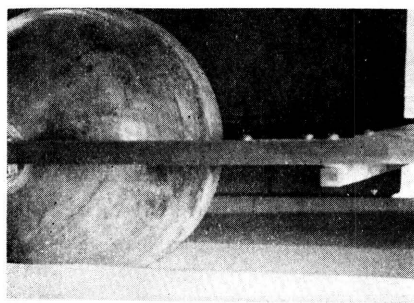
The conditions under which the case i) takes place are as follows in Fig. 12 and 13: that there do not appear that portion of the sand on the front of the wheel which is to be pushed away during the progress, that the ratio of the load upon one wheel to that upon another wheel is equal to the ratio between the two values of the radii, and that the number of running is common for both, being more than 2.

When the load is increased, there can not be seen the appearance of the portion of the sand to be pushed apart for the spherical wheel shown in Fig. 14, while it is seen for the case shown in Fig. 15. This corresponds to the case ii).

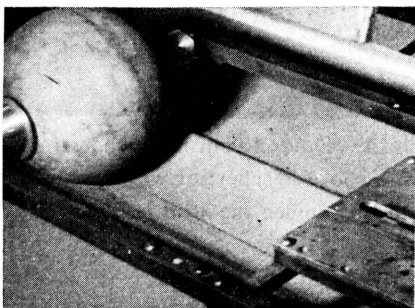
In short, it is presumed that the difference between 1) and 2) appears according to whether the maximum inclination of the sand surface touching the sphere is more than or less than the angle of repose of the sand. And because the inclination of the sand surface depends on the load and the radius of the spherical wheel, a case like 2) may be considered to appear.



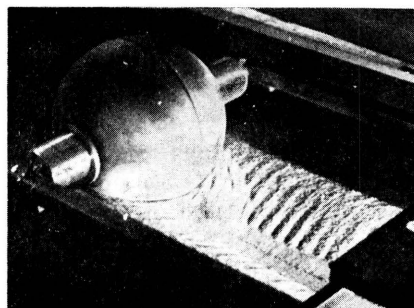
**Fig. 12** Sand stream lines after the second running of a spherical wheel with 28cm diameter under the load of 7.75Kg



**Fig. 13** Sand stream lines after the second running of a spherical wheel with 28cm diameter under the load of 10.85Kg



**Fig. 14** Sand stream lines after the second running of a spherical wheel with 18cm diameter under the load of 5Kg



**Fig. 15** Sand stream lines after the second running of a spherical wheel with 18cm diameter under the load of 7Kg

(c) Fig. 16 and 17 are for the sand stream lines photographed from the side for the case of the loading of the same weight on each of an ordinary wheel and a spherical wheel, all having the same diameter. Fig. 18 and 19 show the sand stream lines photographed from the front.

(d) Fig. 20 shows the way of the change experienced by the layers in sand beneath the wheel during the progress. Here the white lines which are placed at equal distances within the sand are chalk powder. This photo reveals that the sand layer is raised after the running of the wheel as can be seen from the observation of these white lines. This is one of the phenomena showing that the

sand could not be regarded as a perfect elastic matter from mechanical viewpoints.

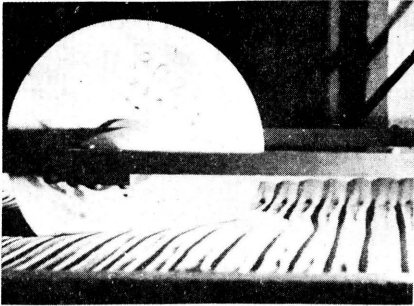


Fig. 16 Sand stream lines photographed from the side of a wheel ( $d=5\text{cm}$ ) with 25cm diameter under the load of 5kg.

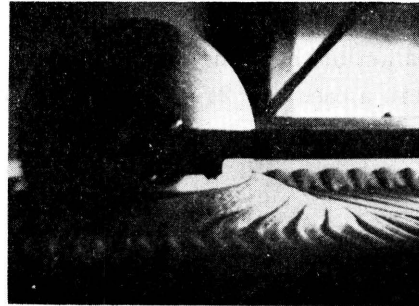


Fig. 17 Sand stream lines photographed from the side of a spherical wheel with 25cm diameter under the load of 5kg.

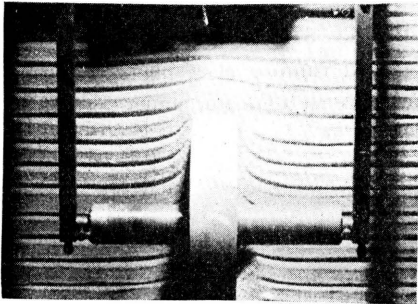


Fig. 18 Sand stream lines photographed from above a wheel ( $d=5\text{cm}$ ) with 25cm diameter under the load of 5kg.

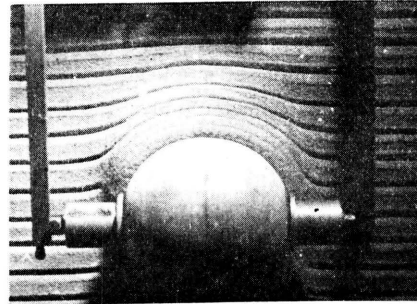


Fig. 19 Sand stream lines photographed from above a spherical wheel with 25cm diameter under the load of 5kg.

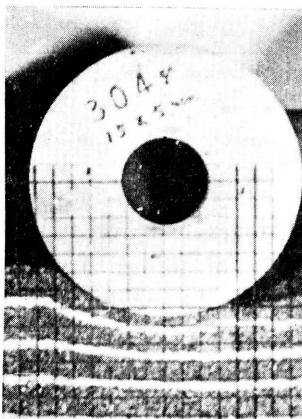


Fig. 20 Sand stream lines for a wheel ( $d=5\text{cm}$ ,  $2r=15\text{cm}$ ) rolling on sand.

#### IV. Conclusions

1. The exponential increase of the fork angle is necessary according as the radius and the thickness of the wheels and also the particle size of sand is increased.
2. The same thing becomes necessary again if the radius of the spherical wheel is increased.
3. Below the limit at which the contact area of a tire becomes maximum, the fork angle should be inversely proportional to the pressure in the tire.
4. In the above experimental ranges of the change of the wheel velocity and the humidity of sand, the fork angle may be kept nearly constant.
5. Between an ordinary wheel and a spherical

wheel of the same radius, the traction resistance of the former is always smaller

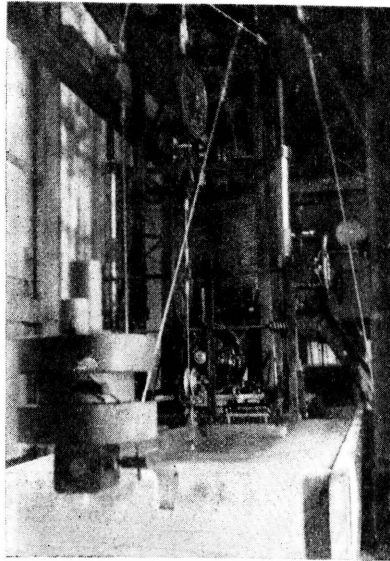


Fig. 21A Apparatus

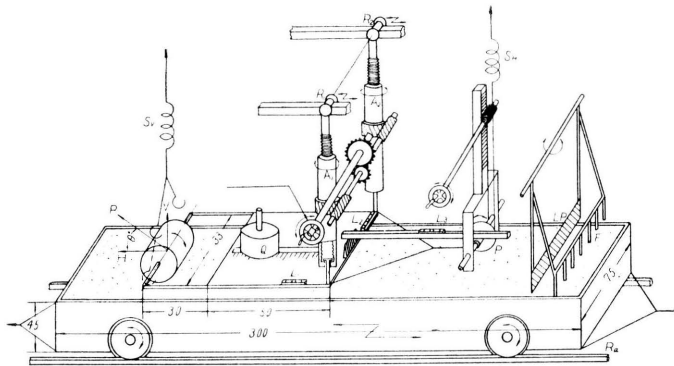


Fig. 21B Apparatus.

than that of the latter in the experimental range if the load is the same.

6. If the load is light, two spherical wheels with different diameters have equal traction resistance when the ratio of the loads is equal to that of the radii.

7. Judging from the photos in Fig. 20 it is impossible to regard the sand as being a perfectly elastic body.

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