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Author	渡部, 一郎(Watanabe, Ichiro) 渡辺, 剛男(Watanabe, Takeo) 安藤, 亨(Ando, Toru)
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Derivation of Formula for the Altitude Performance Characteristics of Aviation Piston Engine fitted with Mechanically driven Supercharger

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Itirô WATANABE*
Takeo WATANABE**
Tôru ANDÔ***

Abstract

It is a general practice for the aviation piston engines, to estimate the brake horsepower at high altitudes, by making a correction to the brake horsepower measured under the test condition, in which the intake pressure alone is kept to the pressure corresponding to the specified altitude, while the temperature of the entering air as well as the back pressure are left to those of the surrounding atmosphere. Various correction formulae have hitherto been published, but the results by them show some discrepancies.

The present authors propose, in this paper, a new formula. This formula serves, not only for the correction of the brake horsepower in the above-mentioned test condition, but also for the evaluation of the altitude performance curves for the whole range of altitudes. The comparison of the results by the present formula with the results by hitherto published theoretical considerations and practical data confirm us that the present formula is of practical use.

I Introduction

It is a general practice for the aviation piston engines, to estimate the brake horsepower at different altitudes, by the correction of the brake horsepower measured under the test condition, in which the intake pressure alone is kept to the pressure corresponding to the specified altitude, while the temperature of the entering air as well as the back pressure are left to those of the surrounding atmosphere. Various correction formulae for the above-mentioned purpose have hitherto been published,[1] but the results by them show some discrepancies. As to the theoretical surveys concerning the altitude performance characteristics for

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*渡部一郎 Dr. Eng., Professor at Keio University
**渡辺剛男 Assistant at Keio University
***安藤亨 Assistant at Keio University

[1] K. Tomizuka; Aero Engines, 1943, pp. 1133/1137

aviation piston engines, papers by K. Tanaka, O. Hirao[2] and S. Awano[3] have been published. The theoretical expressions by these authors, however, involve various efficiencies of engine itself and of supercharger, and so seem somewhat inadequate for the present purpose of application. Hence, the present authors have derived a simple formula semi-theoretically. The results by this new formula have been compared with the results by K. Tanaka, O. Hirao and S. Awano and the various data available, to find the present formula to be adequate for the practical use.

II Derivation of the Formula

The practical method of altitude testing for piston engine with mechanically driven supercharger is shown schematically in fig. 1, in which the pressure p_z at the intake reservoir alone is lowered to the prescribed pressure corresponding to the specified altitude by means of the intake throttle valve, while the air temperature at the intake reservoir and the back pressure of the engine are left to the temperature T_0 and the pressure p_0 of the surrounding atmosphere respectively. Thus the engine is operated at full throttle to obtain the brake horsepower, fuel consumptions and so on. In actual flight, however, the intake air temperature T_0 and the back pressure p_0 coincide with the atmospheric temperature T_z and the pressure p_z of the specified altitude respectively. Thus, the correction of the brake horsepower obtained by the test equipment shown in fig. 1, is necessary.

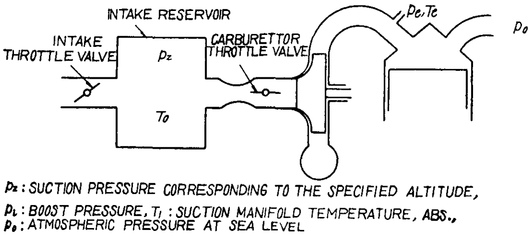

Fig. 1 Altitude Test Equipment for Aviation Engine

Let us denote the brake horsepower at full throttle by HP^0_{z0}, when the intake reservoir pressure, back pressure and the air temperature at the intake reservoir correspond to p_z, p_0 and T_0 respectively. In the above expression, the subscripts $z0$ denote that the intake pressure corresponds to the altitude z, while the back pressure corresponds to the sea level. The superscript, on the other hand, describes the temperature condition at the intake reservoir, and thus, in this case, means that the air temperature at the intake reservoir is equal to the atmospheric temperature at sea level. Denoting the driving horsepower of the supercharger under the same condition by HP^0_{s0}, and further, the brake horsepower developed on the

[2] K. Tanaka, O. Hirao; Rep. Aero. Res. Inst., Tokyo Univ., No. 267, Aug. 1943.

[3] Same as 1). pp. 224/239.

cylinder sides as $I\cdot HP^0_{z0}$ [4], we obtain the following expression.

$$HP^0_{z0}=I\cdot HP^0_{z0}-HP^0_{s0} \tag{1}$$

It is the well-known fact, that the weight flow of air sucked in by the engine increases as the induction manifold pressure or boost pressure p_l exceeds the back pressure p_z. The increment of weight flow of air, in this case, was derived theoretically by F. A. F. Schmidt [5], under the assumption that the residual gases in the cylinders are compressed adiabatically by new charges, while the new charges in turn are heated by the residual gases. The final result thus obtained is as follows.

$$\frac{G(p_l>p_a)}{G(p_l=p_a)}=1+\frac{1}{\varepsilon-1}\;\frac{1}{k}\left(1-\frac{p_z}{p_l}\right) \tag{2}$$

where ε: compression ratio of the engine, p_l: the induction manifold pressure abs. and k: the adiabatic index for the new charges. [6] The eq. (2) coincides with experimental results to some extent. [7] As to the horsepower correction, however, the eq. (2) is not applicable. In this case, it is further necessary to take into considerations the horsepower, developed by the pistons due to the differences of pressures p_l and p_z. The increment of horsepower due to these pressure differences may be expressed as linear function of p_l-p_z, and so the expression for the horsepower yields to

$$\frac{I\cdot HP^0_{zz}}{I\cdot HP^0_{z0}}=\frac{1+\dfrac{1}{\varepsilon-1}\;\dfrac{a}{k}\left(1-\dfrac{p_z}{p_l}\right)}{1+\dfrac{1}{\varepsilon-1}\;\dfrac{a}{k}\left(1-\dfrac{p_0}{p_l}\right)} \tag{3}$$

where a denotes a constant which may be determined experimentally. The following empirical formula [8], derived from the experimental results obtained in the altitude test chamber at Isotta Fraschini with Asso 750 engine, is available for the present purpose.

$$\frac{B\cdot HP_{0z}}{B\cdot HP_{00}}=1+\frac{1}{100}\;\frac{p_0-p_z}{40.8} \tag{4}$$

Substitution of $\varepsilon=6.4$ for Asso 750 engine and $p_l=p_0=760$mmHg into eq. (3),

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[4] In this case, $I\cdot HP^0_{z0}$ does not mean the indicated horsepower of the engine, but denotes the brake horsepower developed on the cylinder sides.

[5] F. A. F. Schmidt; Verbrennungsmotoren 1939, pp. 130/133

[6] Calculation of c_p and c_v of the new charge, for the case of the mixture ratio 15, using $c_p=0.240$, $c_v=0.172$ for air and $c_p=0.254$, $c_v=0.188$ for gasoline vapor, yields to $c_p=0.241$, $c_v=0.172$. Thus the adiabatic index $k=0.241/0.172=1.398$. The adoption of 1.4 for the value of k is, therefore, allowable.

[7] Jour. Aero. Res. Inst., Tokyo Univ., No. 195, Nov. 1940, pp. 331/392

[8] L'Aerotecnica, Vol. 15 No. 5, May 1935, pp. 456/482

and the comparison of the resulting equation with eq. (4), determine the value a as follows.

$$a=1.409 \tag{5}$$

The formula

$$\frac{B\cdot HP_{0z}}{B\cdot HP_{00}}=1+\frac{1}{100}\frac{p_0-p_z}{35.0} \tag{6}$$

had been widely used, before the formula (4) was established, for the horsepower correction due to the difference of back pressure from the induction manifold pressure. Although the engines used for the establishment of eq. (6) are not certain, the application of eq. (5) into eq. (3) and comparison with eq. (6) render to $\varepsilon=5.64$, which is conceivable to be appropriate value for the engines of the date. Thus, the value $a=1.409$ may be available for the present purposes. Application of this value into eq. (2) yields to the following relation.

$$\frac{I\cdot HP^0_{zz}}{I\cdot HP^0_{z0}}=\frac{1+\frac{1.006}{\varepsilon-1}\left(1-\frac{p_z}{p_l}\right)}{1+\frac{1.006}{\varepsilon-1}\left(1-\frac{p_0}{p_l}\right)} \tag{7}$$

It is clear that the following relation also holds in the present case.

$$HP^0_{zz}=I\cdot HP^0_{zz}-HP^0_{sz} \tag{8}$$

Putting eq. (7) into eq. (8), we have

$$\begin{aligned} HP^0_{zz}&=I\cdot HP^0_{z0}\frac{1+\frac{1.006}{\varepsilon-1}\left(1-\frac{p_z}{p_l}\right)}{1+\frac{1.006}{\varepsilon-1}\left(1-\frac{p_0}{p_l}\right)}-HP^0_{sz} \\ &=\left(HP^0_{z0}+HP^0_{s0}\right)\frac{1+\frac{1.006}{\varepsilon-1}\left(1-\frac{p_z}{p_l}\right)}{1+\frac{1.006}{\varepsilon-1}\left(1-\frac{p_0}{p_l}\right)}-HP^0_{sz} \end{aligned} \tag{7.1}$$

Now, for the driving horsepower of the supercharger, we have

$$HP^0_{s0}=\frac{1}{75\eta_0}\frac{k}{k-1}GRT_0\left\{\left(\frac{p_l}{p_z}\right)^{\frac{k-1}{k}}-1\right\} \tag{9}$$

where η_0: overall adiabatic efficiency of the supercharger and G: weight flow of air per second. Further, we have for the four cycle engine,

$$G=\frac{z\frac{\pi}{4}d^2ln\gamma_l\eta_{v0}}{2\times 60}=\frac{z\frac{\pi}{4}d^2lnp_l\eta_{v0}}{2\times 60\ RT_l} \tag{10}$$

where η_{v0}: volumetric efficiency of the engine when the induction manifold pressure and the back pressure amount to p_l and p_0 respectively, d: cylinder bore, l:

stroke, z: number of cylinders and n: number of revolutions of the crankshaft per minute.

Denoting the adiabatic temperature efficiency of the supercharger as η_t, we have

$$\eta_t=\frac{\left(\frac{p_l}{p_z}\right)^{\frac{k-1}{k}}-1}{\left(\frac{p_l}{p_z}\right)^{\frac{m-1}{m}}-1}$$

where m means the polytropic index. Further, if we denote the polytropic efficiency of the supercharger as η_p, we have $\eta_p=\frac{k-1}{k}\cdot\frac{m}{m-1}$.

Thus,

$$T_l=T_0(p_l/p_z)^{\frac{m-1}{m}}=T_0(p_l/p_z)^{\frac{1}{\eta_p}\frac{k-1}{k}} \tag{11}$$

Substituting eq. (11) into eq. (10), we get

$$G=\frac{z\frac{\pi}{4}d^2lnp_l\eta_{v0}}{2\times 60RT_0(p_l/p_z)^{\frac{1}{\eta_p}\frac{k-1}{k}}} \tag{10.1}$$

Substitution of eq. (10.1) into eq. (8) yields to

$$HP^0_{s0}=\frac{1}{9000}\frac{1}{\eta_0}\frac{k}{k-1}\frac{z\frac{\pi}{4}d^2lnp_l\eta_{v0}}{(p_l/p_z)^{\frac{1}{\eta_p}\frac{k-1}{k}}}\left\{\left(\frac{p_l}{p_z}\right)^{\frac{k-1}{k}}-1\right\} \tag{9.1}$$

Representing the volumetric efficiency of the engine under the condition of induction manifold pressure p_l and back pressure p_z as η_{vz}, we have further the following relation.

$$\frac{\eta_{vz}}{\eta_{v0}}=\frac{1+\frac{1}{\varepsilon-1}\frac{1}{k}\left(1-\frac{p_z}{p_l}\right)}{1+\frac{1}{\varepsilon-1}\frac{1}{k}\left(1-\frac{p_0}{p_l}\right)}$$

Hence, we obtain

$$HP^0_{sz}=\frac{1}{9000}\frac{1}{\eta_0}\frac{k}{k-1}\frac{z\frac{\pi}{4}d^2lnp_l\eta_{v0}}{(p_l/p_z)^{\frac{1}{\eta_p}\frac{k-1}{k}}}\left\{\left(\frac{p_l}{p_z}\right)^{\frac{k-1}{k}}-1\right\}\frac{1+\frac{1}{\varepsilon-1}\frac{1}{k}\left(1-\frac{p_z}{p_l}\right)}{1+\frac{1}{\varepsilon-1}\frac{1}{k}\left(1-\frac{p_0}{p_l}\right)}$$

$$=HP^0_{s0}\frac{1+\frac{1}{\varepsilon-1}\frac{1}{k}\left(1-\frac{p_z}{p_l}\right)}{1+\frac{1}{\varepsilon-1}\frac{1}{k}\left(1-\frac{p_0}{p_l}\right)} \tag{9.2}$$

Therefore, eq. (7.1) yields to the following expression.

$$HP^0_{zz}=HP^0_{z0}\frac{1+\frac{1.006}{\varepsilon-1}\left(1-\frac{p_z}{p_l}\right)}{1+\frac{1.006}{\varepsilon-1}\left(1-\frac{p_0}{p_l}\right)}+HP^0_{s0}\left\{\frac{1+\frac{1.006}{\varepsilon-1}\left(1-\frac{p_z}{p_l}\right)}{1+\frac{1.006}{\varepsilon-1}\left(1-\frac{p_0}{p_l}\right)}-\frac{1+\frac{0.714}{\varepsilon-1}\left(1-\frac{p_z}{p_l}\right)}{1+\frac{0.714}{\varepsilon-1}\left(1-\frac{p_0}{p_l}\right)}\right\} \tag{12}$$

Now, we may proceed to the case of T_z instead of T_0. Thus, we have

$$HP^z_{zz}=I\cdot HP^0_{zz}\sqrt{\frac{T_0}{T_z}}\frac{r_z}{r_0}-HP^z_{sz}=\left(HP^0_{zz}+HP^0_{sz}\right)\sqrt{\frac{T_0}{T_z}}\frac{r_z}{r_0}-HP^z_{sz}$$

$$=HP^0_{zz}\sqrt{\frac{T_0}{T_z}}\frac{r_z}{r_0}+HP^0_{sz}\sqrt{\frac{T_0}{T_z}}\frac{r_z}{r_0}-HP^z_{sz}$$

or

$$HP^z_{zz}=\left[HP^0_{z0}\frac{1+\frac{1.006}{\varepsilon-1}\left(1-\frac{p_z}{p_l}\right)}{1+\frac{1.006}{\varepsilon-1}\left(1-\frac{p_0}{p_l}\right)}+HP^0_{s0}\left\{\frac{1+\frac{1.006}{\varepsilon-1}\left(1-\frac{p_z}{p_l}\right)}{1+\frac{1.006}{\varepsilon-1}\left(1-\frac{p_0}{p_l}\right)}-\frac{1+\frac{0.714}{\varepsilon-1}\left(1-\frac{p_z}{p_l}\right)}{1+\frac{0.714}{\varepsilon-1}\left(1-\frac{p_0}{p_l}\right)}\right\}\right]$$

$$\times\sqrt{\frac{T_0}{T_z}}\frac{r_z}{r_0}+HP^0_{sz}\sqrt{\frac{T_0}{T_z}}\frac{r_z}{r_0}-HP^z_{sz} \tag{13}$$

The pressure ratios of the supercharger p_l/p_z and p_l/p_0 increase as the intake temperature T_0 decreases to T_z. This variation of pressure ratios due to the variation of intake temperatures may be well expressed by the following expressions.

$$r_z-1=(r_0-1)\frac{T_0}{T_z} \tag{14}$$

and

$$T_z\left(r_z^{\frac{k-1}{k}}-1\right)=T_0\left(r_0^{\frac{k-1}{k}}-1\right) \tag{15}$$

where r_0 and r_z denote the pressure ratios when the intake temperatures amount to T_0 and T_z respectively. The eq. (14) coincides with experimental results more closely than eq. (15), and so is more preferable. In the following derivation of the formula, however, either of the above two eqs. (14) and (15) was used for the sake of simplicity of the deduction, and the coefficient a in eq. (3) or (5) was corrected finally, to allow thus obtained formula collaborate with eq. (14).

From eq. (14), we have

$$\left(\frac{p_l}{p_z}\right)_z-1=\left\{\left(\frac{p_l}{p_z}\right)_0-1\right\}\frac{T_0}{T_z}$$

p_z/p_l in eq. (13) corresponds to $(p_z/p_l)_z$ in the above expression. Rearrangement of the above equation yields to

$$1-\left(\frac{p_z}{p_l}\right)_z=\frac{\left(\frac{p_l}{p_z}\right)_0-1}{\frac{T_z}{T_0}+\left\{\left(\frac{p_l}{p_z}\right)_0-1\right\}}$$

$1-(p_0/p_l)$ may be expressed in a similar manner. Substitution of these expressions into eq. (13) yields to

$$HP^z_{zz}=\left[HP^0_{z0}\frac{1+\dfrac{1.006}{\varepsilon-1}\dfrac{\left(\frac{p_l}{p_z}\right)_0-1}{\frac{T_z}{T_0}+\left(\frac{p_l}{p_z}\right)_0-1}}{1+\dfrac{1.006}{\varepsilon-1}\dfrac{\left(\frac{p_l}{p_0}\right)_0-1}{\frac{T_z}{T_0}+\left(\frac{p_l}{p_0}\right)_0-1}}+HP^0_{s0}\left\{\frac{1+\dfrac{1.006}{\varepsilon-1}\dfrac{\left(\frac{p_l}{p_z}\right)_0-1}{\frac{T_z}{T_0}+\left(\frac{p_l}{p_z}\right)_0-1}}{1+\dfrac{1.006}{\varepsilon-1}\dfrac{\left(\frac{p_l}{p_0}\right)_0-1}{\frac{T_z}{T_0}+\left(\frac{p_l}{p_0}\right)_0-1}}-\frac{1+\dfrac{0.714}{\varepsilon-1}\dfrac{\left(\frac{p_l}{p_z}\right)_0-1}{\frac{T_z}{T_0}+\left(\frac{p_l}{p_z}\right)_0-1}}{1+\dfrac{0.714}{\varepsilon-1}\dfrac{\left(\frac{p_l}{p_0}\right)_0-1}{\frac{T_z}{T_0}+\left(\frac{p_l}{p_0}\right)_0-1}}\right\}\right]$$

$$\times\sqrt{\frac{T_0}{T_z}}\,\frac{r_z}{r_0}+HP^0_{sz}\sqrt{\frac{T_0}{T_z}}\,\frac{r_z}{r_0}-HP^z_{sz} \tag{13.1}$$

Now, HP^0_{sz} and HP^z_{sz} in eq. (13.1) are expressed as follows.

$$HP^0_{sz}=\frac{1}{75\eta_0}\,\frac{k}{k-1}G_0RT_0\left\{\left(\frac{p_l}{p_z}\right)_0^{\frac{k-1}{k}}-1\right\} \tag{16}$$

$$HP^z_{sz}=\frac{1}{75\eta_0}\,\frac{k}{k-1}G_zRT_z\left\{\left(\frac{p_l}{p_z}\right)_z^{\frac{k-1}{k}}-1\right\} \tag{17}$$

Further, as we have $G_z=G_0\sqrt{T_0/T_z}\cdot r_z/r_0$ and

$$T_0\left\{(p_l/p_z)_0^{\frac{k-1}{k}}-1\right\}=T_z\left\{(p_l/p_z)_z^{\frac{k-1}{k}}-1\right\}$$

by eq. (15), it is obvious that the following relation exists.

$$HP^0_{sz}\sqrt{\frac{T_0}{T_z}}\,\frac{r_z}{r_0}=HP^z_{sz}$$

Therefore, we obtain finally the following expression.

$$HP^z_{zz}=HP^0_{z0}\frac{1+\dfrac{1.006}{\varepsilon-1}\dfrac{\left(\frac{p_l}{p_z}\right)_0-1}{\frac{T_z}{T_0}+\left(\frac{p_l}{p_z}\right)_0-1}}{1+\dfrac{1.006}{\varepsilon-1}\dfrac{\left(\frac{p_l}{p_0}\right)_0-1}{\frac{T_z}{T_0}+\left(\frac{p_l}{p_0}\right)_0-1}}\sqrt{\frac{T_0}{T_z}}\,\frac{r_z}{r_0}$$

$$+I\!P^0_{s0}\left\{\frac{1+\dfrac{1.006}{\varepsilon-1}\dfrac{\left(\dfrac{p_l}{p_z}\right)_0-1}{\dfrac{T_z}{T_0}+\left(\dfrac{p_l}{p_z}\right)_0-1}}{1+\dfrac{1.006}{\varepsilon-1}\dfrac{\left(\dfrac{p_l}{p_0}\right)_0-1}{\dfrac{T_z}{T_0}+\left(\dfrac{p_l}{p_0}\right)_0-1}}-\frac{1+\dfrac{0.714}{\varepsilon-1}\dfrac{\left(\dfrac{p_l}{p_z}\right)_0-1}{\dfrac{T_z}{T_0}+\left(\dfrac{p_l}{p_z}\right)_0-1}}{1+\dfrac{0.714}{\varepsilon-1}\dfrac{\left(\dfrac{p_l}{p_0}\right)_0-1}{\dfrac{T_z}{T_0}+\left(\dfrac{p_l}{p_0}\right)_0-1}}\right\}\sqrt{\frac{T_0}{T_z}}\frac{r_z}{r_0} \qquad (18)$$

The pressure ratio r_z/r_0 should be evaluated by eq. (15) in this case. Further, $I\!P^0_{s0}$ in eq. (18) may be written as follows.

$$I\!P^0_{s0}=\frac{1}{9000}\frac{1}{\eta_0}\frac{k}{k-1}\frac{z\dfrac{\pi}{4}d^2 lnp_l\eta_{v0}}{\left(\dfrac{p_l}{p_z}\right)^{\frac{1}{\eta_p}\frac{k-1}{k}}}\left\{\left(\frac{p_l}{p_z}\right)_0^{\frac{k}{k-1}}-1\right\} \qquad (9.1)$$

Fortunately, the second term of the right hand side of the eq. (18) is of the order of about 0.2% compared with the first term, and so the second term is negligible. To ascertain this, the following example is explanatory. If we put, for example, $\eta_{v0}=0.85$, $\eta_0=0.75$, $\eta_p=0.85$, $p_{me}=10\text{kg/cm}^2$, $p_l=1.3\text{kg/cm}^2$, we have $k/(k-1)=3.5$, $\{(k-1)/k\}(1/\eta_p)=0.336$. As the relation between $I\!P^0_{z0}$ and the brake mean effective pressure p_{me} may be expressed as

$$I\!P^0_{z0}=\frac{z\dfrac{\pi}{4}d^2 lnp_{me}}{2\times60\times75}=\frac{z\dfrac{\pi}{4}d^2 lnp_{me}}{9000},$$

it follows from eq. (9.1) that

$$I\!P^0_{s0}=I\!P^0_{z0}\frac{p_l}{p_{me}}\frac{\eta_{v0}}{\eta_0}\frac{k}{k-1}\frac{\left(\dfrac{p_l}{p_z}\right)_0^{\frac{k-1}{k}}-1}{\left(\dfrac{p_l}{p_z}\right)^{\frac{1}{\eta_p}\frac{k-1}{k}}} \qquad (19)$$

Putting the above-mentioned values in eq. (19), we have

$$I\!P^0_{s0}=I\!P^0_{z0}0.516\frac{\left(\dfrac{p_l}{p_z}\right)_0^{0.2857}-1}{\left(\dfrac{p_l}{p_z}\right)_0^{0.336}} \qquad (19.1)$$

If we put $(p_l/p_z)_0=2.224$ in eq. (19.1), we obtain $I\!P^0_{s0}=0.1012I\!P^0_{z0}$. Hence, if we put further $p_z=353.9\text{mmHg}$ ($z=6\text{km}$), the second term in eq. (18) amounts to $0.00213I\!P^0_{z0}\sqrt{T_0/T_z}\cdot(r_z/r_0)$. As the first term in eq. (18) in this case turns to $1.077I\!P^0_{z0}\sqrt{T_0/T_z}\,(r_z/r_0)$, we are able to find that the second term in question is of the magnitudes of about 0.2% compared with the first term.

Therefore, it is obvious that the following equation, instead of eq. (18), is practically available.

$$IP_{zz}^{z}=IP_{z0}^{0}\frac{1+\dfrac{1.006}{\varepsilon-1}\dfrac{\left(\dfrac{p_l}{p_z}\right)_0-1}{\dfrac{T_z}{T_0}+\left(\dfrac{p_l}{p_z}\right)_0-1}}{1+\dfrac{1.006}{\varepsilon-1}\dfrac{\left(\dfrac{p_l}{p_0}\right)_0-1}{\dfrac{T_z}{T_0}+\left(\dfrac{p_l}{p_0}\right)_0-1}}\sqrt{\frac{T_0}{T_z}}\frac{r_z}{r_0} \tag{20}$$

As already cited, eq. (20) must be applied in combination with pressure ratio eq. (15). In order to use the more preferable pressure ratio eq. (14), it is necessary to adjust the value of the coefficient 1.006. The present authors have found that the value 1.5 instead of 1.006 is necessary for the adoption of the eq. (14). Thus, the final equation yields to

$$IP_{zz}^{z}=IP_{z0}^{0}\frac{1+\dfrac{1.5}{\varepsilon-1}\dfrac{\left(\dfrac{p_l}{p_z}\right)_0-1}{\dfrac{T_z}{T_0}+\left(\dfrac{p_l}{p_z}\right)_0-1}}{1+\dfrac{1.5}{\varepsilon-1}\dfrac{\left(\dfrac{p_l}{p_0}\right)_0-1}{\dfrac{T_z}{T_0}+\left(\dfrac{p_l}{p_0}\right)_0-1}}\sqrt{\frac{T_0}{T_z}}\frac{r_z}{r_0} \tag{21}$$

III Methods of Application of the Formula

The altitude performance curves of a piston engine fitted with mechanically driven supercharger may thus be evaluated by means of eq. (21) and eq. (14). The method of application of the formula, however, differs slightly, whether the altitude concerned is below or above the rated altitude of the supercharged engine. The practical methods of application of the present formula are explained below.

3.1 Altitude Performance at Altitudes below the Rated Altitude.

To obtain the altitude performance curve of a piston engine operating under constant boost pressure and r. p. m. from sea level to the rated altitude, it is necessary to evaluate the brake horsepower IP_{zz}^{z} from the following procedure. Then, the straight line joining IP_{z0}^{0} at sea level with IP_{zz}^{z} at the rated altitude z, represents the altitude performance of the engine.

(*a*) The pressure ratio $(p_l/p_z)_0$, measured under the test equipment shown in fig. 1, may be put directly into eqs. (14) and (21). To determine the rated altitude, first calculate the value $T_0(r_0-1)$. Then, the rated altitude z may be determined in such a way, that the equation $T_z\{(p_l/p_z)_z-1\}=T_0(r_0-1)$ may be satisfied using the same value p_l in $(p_l/p_z)_z$ as that in $(p_l/p_z)_0$.

When the constant boost pressure p_l and the rated altitude z or the pressure at the rated altitude p_z are known, and the value $(p_l/p_z)_0$ is unavailable, the enumerated value $(p_l/p_z)_z=r_z$ may be substituted into eq. (14) to determine the pressure ratio $r_0=(p_l/p_z)_0$

(*b*) IP^z_{zz} is determined from the observed brake horsepower IP^0_{z0} by means of eq. (21), if $(p_l/p_z)_0$ obtained by the method explained under (*a*) and T_z of the rated altitude are employed. In the above procedure, r_z/r_0 in eq. (21) becomes unity if the constant boost pressure is maintained, because of the fact that $r_z/r_0=\frac{(p_l/p_z)_z}{(p_l/p_z)_0}=1$.

3.2 Altitude Performance at Altitudes above the Rated Altitude

In this case, we denote $(p_l/p_z)_z$ at the rated altitude as $(p_l/p_z)_{cr}$ and substitute it into $(p_l/p_z)_0$ of eq. (21). T_0 in eq. (21), in this case, must be written as T_{cr} which is equal to T_z at the rated altitude. Also, we denote r_{cr} instead of r_z. In this case, r_z/r_0 in eq. (20) turns to $p_zr_z/(p_l)_{cr}=(p_l)_z/(p_l)_{cr}$. Thus, eq. (21) yields to

$$IP_z=IP_{cr}\frac{\left\{1+\frac{1.5}{\varepsilon-1}\frac{\left(\frac{p_l}{p_z}\right)_{cr}-1}{\frac{T_z}{T_{cr}}+\left(\frac{p_l}{p_z}\right)_{cr}-1}\right\}}{\left\{1+\frac{1.5}{\varepsilon-1}\frac{\left(\frac{p_l}{p_z}\right)_{cr}-1}{\left(\frac{p_l}{p_z}\right)_{cr}}\right\}}\sqrt{\frac{T_{cr}}{T_z}}\;\frac{p_zr_z}{(p_l)_{cr}} \tag{21.1}$$

If we denote the absolute temperature of the standard atmosphere of any altitude z above the rated altitude by T_z, then eq. (14) yields to

$$T_z(r_z-1)=T_{cr}(r_{cr}-1)$$

From this equation, $r_z=(p_l/p_z)_z$ may be evaluated at any altitude z. As the induction manifold pressure $(p_l)_z$ at the altitude concerned is calculable by means of the expression $(p_l)_z=p_z(p_l/p_z)_z=p_zr_z$, division of $(p_l)_z$ by the induction manifold pressure $(p_l)_{cr}$ gives the ratio $p_zr_z/(p_l)_{cr}$ in eq. (21.1).

IV Comparisons of the Present Formula with the Results of previously published Theoretical Considerations and the Actual Data.

As already cited, the theoretical considerations concerning the altitude performance of a piston engine with mechanically driven supercharger were developed by K. Tanaka, O. Hirao and S. Awano. In addition, the practical data from several sources are available. Thus, the comparisons of the present formula with the results of the above—mentioned theories and data are shown below. In these, the brake horsepowers at sea level HP^0_{z0} were identified both for the present formula and the results to be compared.

4.1 Comparisons with the Numerical Results by K. Tanaka and O. Hirao.

(*a*) Compression ratio $\varepsilon=7$, induction manifold pressure $p_l=760$mmHg abs. (boost pressure 0 mmHg), rated altitude $z=6$km.

The comparison is shown in fig.2 to find the difference at the rated altitude to be 1.6%.

(*b*) Compression ratio $\varepsilon=7$, induction manifold pressure $p_l=1360$mmHg abs. (boost pressure +600mmHg), rated altitude $z=6$km.

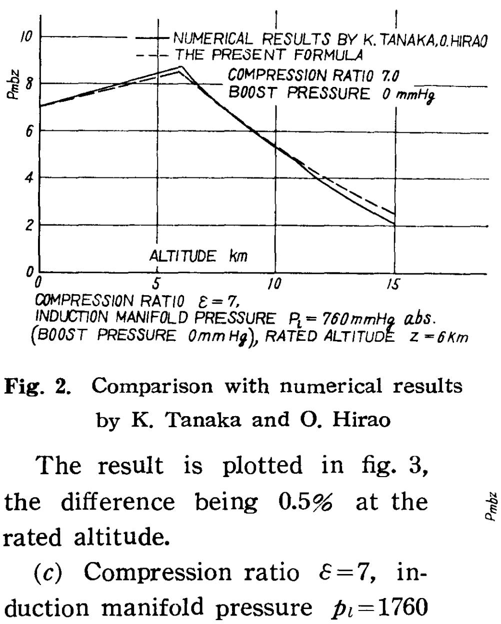

Fig. 2. Comparison with numerical results by K. Tanaka and O. Hirao

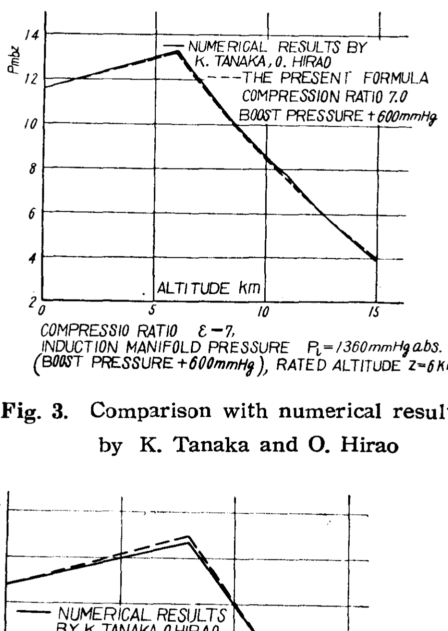

Fig. 3. Comparison with numerical results by K. Tanaka and O. Hirao

The result is plotted in fig. 3, the difference being 0.5% at the rated altitude.

(*c*) Compression ratio $\varepsilon=7$, induction manifold pressure $p_l=1760$ mmHg abs. (boost pressure +1000 mmHg), rated altitude $z=8$km.

This case is shown in fig. 4. The deviation at the rated altitude amounts to 1.5%.

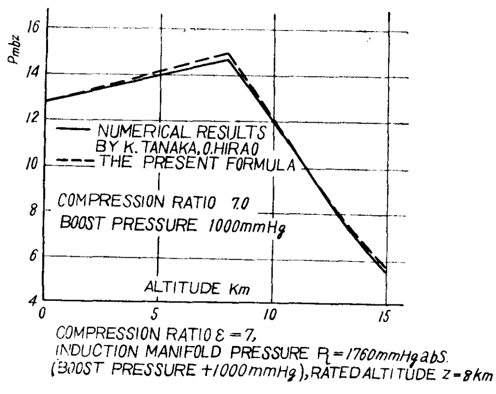

Fig. 4. Comparison with numerical results by K. Tanaka and O. Hirao

4.2 Comparison with the Numerical Results by S. Awano.

Compression ratio $\varepsilon=6.8$, induction manifold pressure $p_l=860$mmHg abs. (boost pressure +100mmHg), rated altitude $z=6.2$km.

The result of comparison is shown in fig.5. The difference between the numerical result by S. Awano and the present formula amounts to 0.6% at the rated altitude, the value by the present formula being lower in this case.

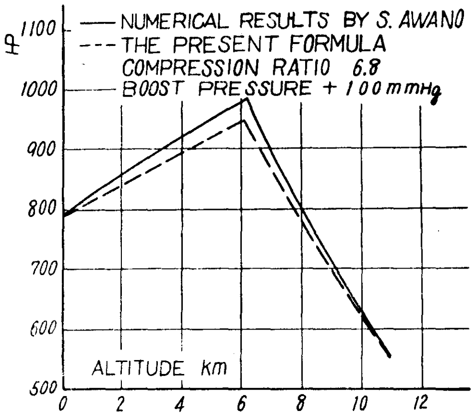

Fig. 5 Comparison with numerical results by S. Awano

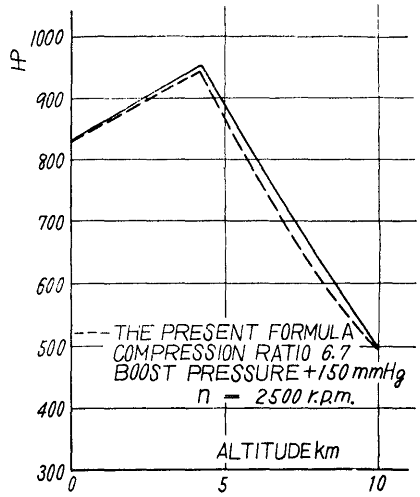

Fig. 6 Comparison with altitude performance characteristics of "Sakae" type I-2 engine

4.3 Comparison with the Altitude Performance Characteristics of "Sakae" I-2 Engine.

Compression ratio $\varepsilon=6.7$, induction manifold pressure $p_i=910$ mmHg abs. (boost pressur e+150 mmHg), rated altitude $z=4.2$km, $n=2500$r.p.m. (fig.6).

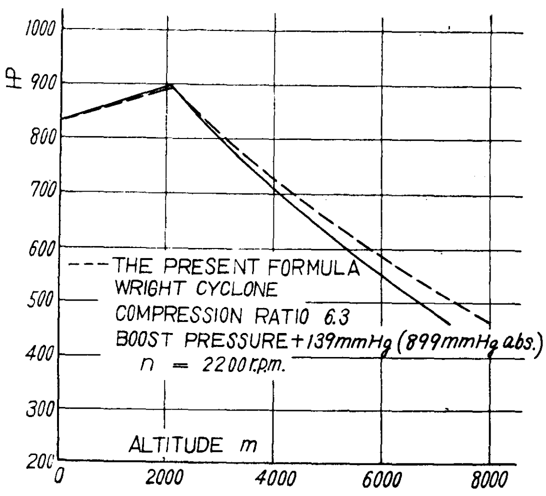

Fig. 7 Comparison with altitude performance characteristics of Wright Cyclone engine

4.4 Comparison with the Altitude Performance Characteristics of Wright Cyclone Engine [9]

Compression ratio $\varepsilon=6.3$, induction manifold pressure $p_i=$ 899mmHg abs. (boost pressure +139mmHg), rated altitude $z=$ 2.073km, $n=2200$r.p.m. (fig.7).

[9] Marks; Mech. Engineers Handook 4th Ed., p. 1267

4.5 Comparison with the Altitude Performance Characteristics of Bristol Centaurus 660, 661 Sleeve Valve Engines.[10]

Compression ratio $\varepsilon=7.2$, induction manifold pressure $p_l=1219$mmHg abs. (boost pressure $+459$mmHg), rated altitude $z=3.66$km (12,000ft), $n=2500$r.p.m. (fig.8).

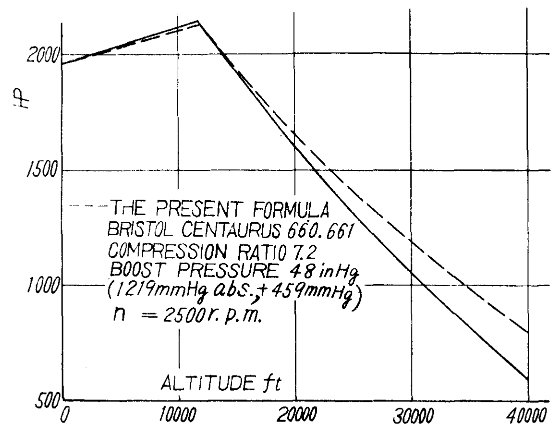

Fig. 8 Comparison with the altitude performance characteristics of Bristol Centaurus 660, 661 sleeve valve engine

4.6 Comparison with the Altitude Performance Characteristics of "Kinsei" Type IV-O Engine[11]

Compression ratio $\varepsilon=6.6$, induction manifold pressure $p_l=910$mmHg abs. (boost pressure $+150$mmHg), $n=2500$r.p.m. (fig.9).

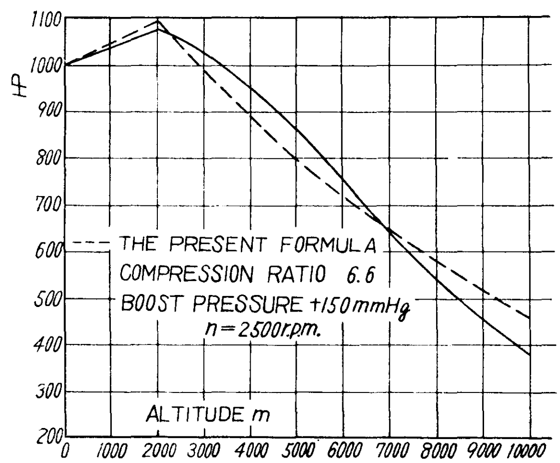

Fig. 9 Comparison with altitude performance characteristics of "Kinsei" type IV-O engine

4.7 Comparison with the Results by Altitude Performance Calculation Method by O. Nagano[12]

Compression ratio $\varepsilon=6$, induction manifold pressure 760mmHg abs. (boost pres-

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10) Manual for Centaurus 660 series sleeve-valve engine.

11) Manual for "Kinsei" type IV-O engine, 1941.

12) O. Nagano; J. Japan Soc. Aero. Eng. Vol. 6 No. 53, 1939, pp 1009/1027.

sure 0 mmHg), rated altitude $z=2.68$km, $n=2100$r.p.m. (fig. 10).

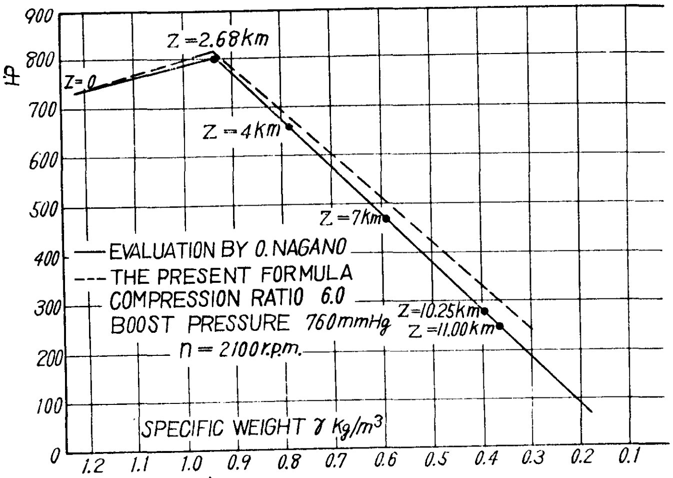

Fig. 10 Comparison with numerical results by O. Nagano.

V Conclusions

The present authors have derived a formula for estimating the altitude performance characteristics of aviation piston engine fitted with mechanically driven supercharger. The results of comparison with the numerical results by several theoretical considerations and practical data, reveal us that the present formula eq. (21), together with eq. (14), is of practical use. The present formula is adopted in the Japanese Industrial Standard (JIS) W 4101, " The Testing of Aero-engines".

Appendix I. Comparison of the Rated Altitude calculated by Eq. (14) with the Results by other Determination Methods

As to the method of rated altitude determination, methods proposed by Nakajima Aeronautical Corporation [13] and by O. Nagano [14] were available in this country. The method by Nakajima Corporation gives the rated altitude lower than that by the Nagano's method, because of the fact that the constant pressure ratios of the supercharger are assumed for all altitudes. The comparison of the results by eq. (14) with the results by the above-mentioned methods were made for the case of compression ratio $\varepsilon=7$, induction manifold pressure $p_i=910$mmHg abs. and $HP^0_{z0}=1000$ HP, to find the rated altitude by eq. (14) lying between the other

[13] Research Rep., Ogikubo Works, Nakajima Corp., Aug. 1944

[14] loc. cit. in 12)

two as shown in fig. 11.

Appendix II. Calculated Results by the Present Formula Eq. (21)

Calculated results by the present formula for the case of compression ratio $\varepsilon=8$, 7.5 and 7 are shown in figs. 12, 13 and 14.

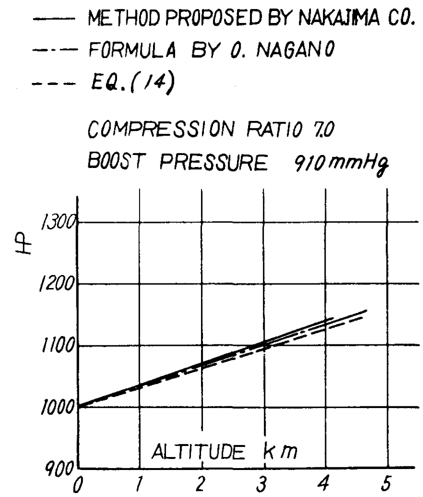

Fig 11 Comparison of rated altitudes by several methods.

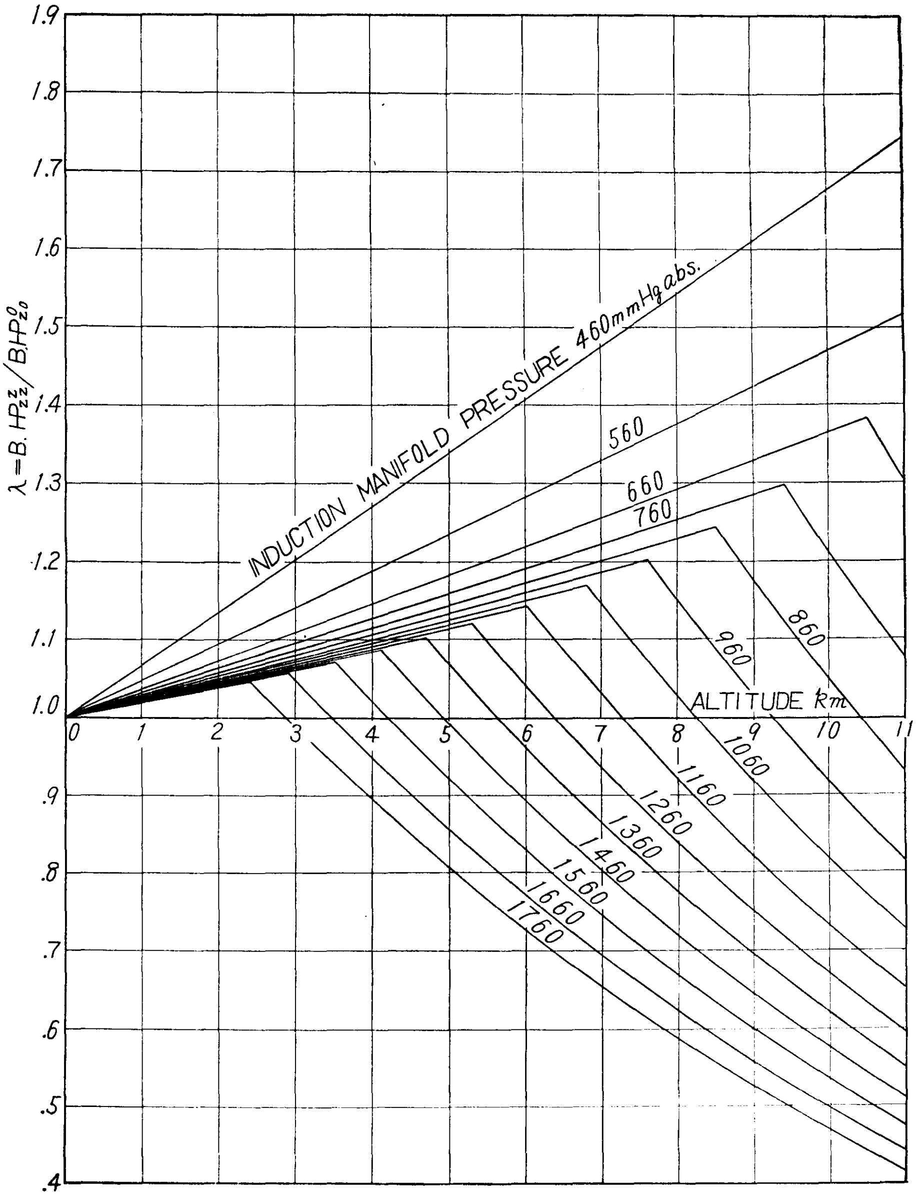

Fig. 12 Altitude Performance Curves for Aero-engine fitted with Mechanically driven Supercharger (Compression Ratio $\varepsilon=8.0$)

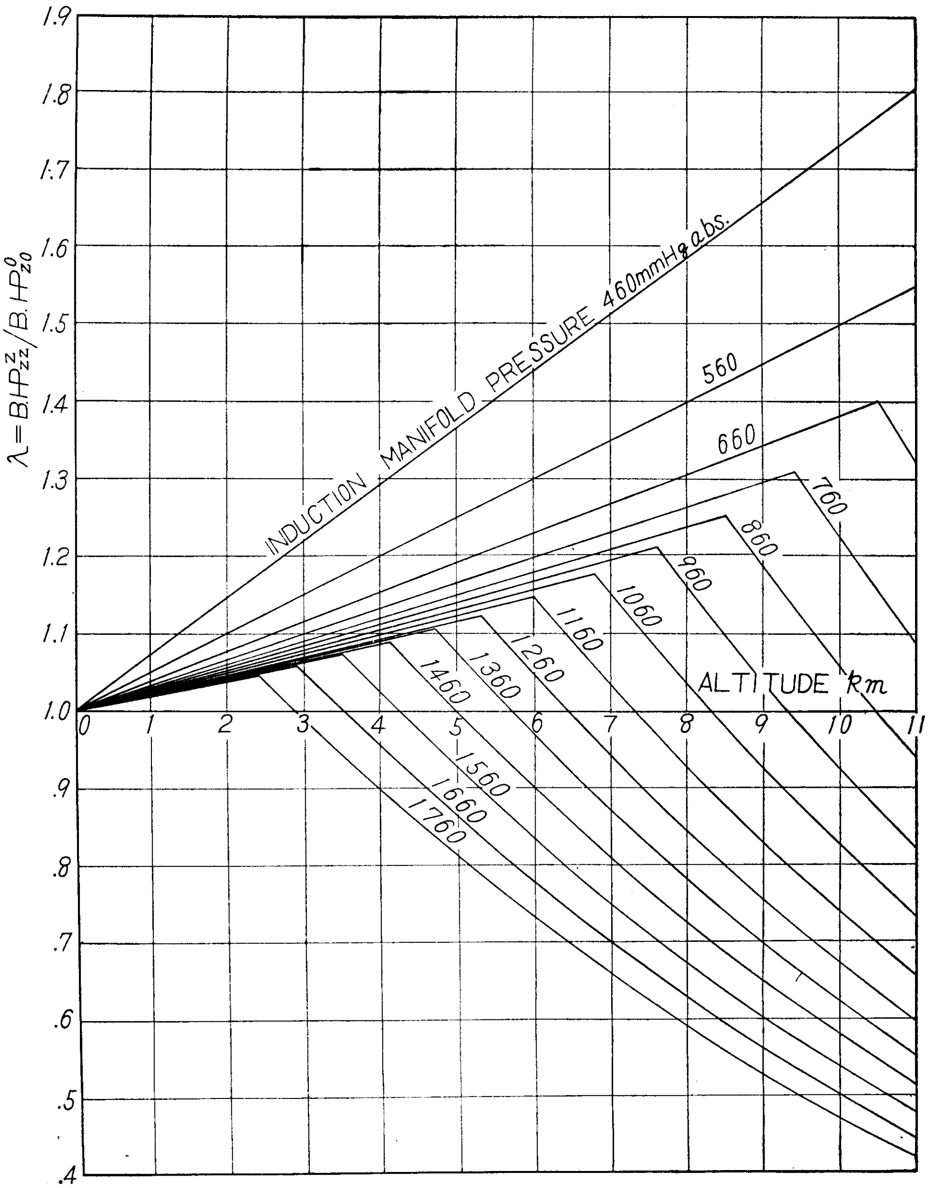

Fig. 13 Altitude Performance Curves for Aero-engine fitted with Mechanically driven Supercharger (Compression Ratio $\varepsilon = 7.5$)

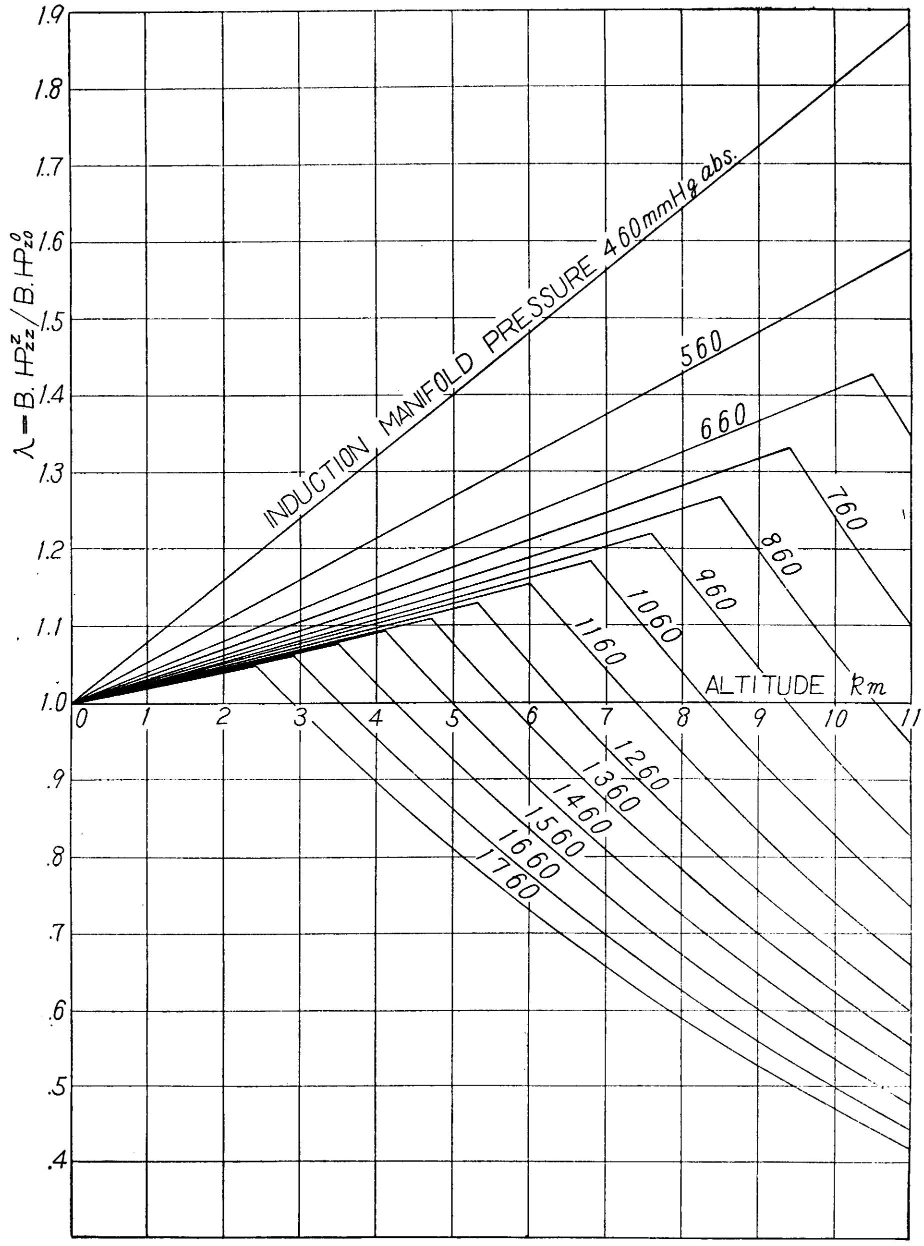

Fig. 14 Altitude Performance Curves for Aero-engine fitted with Mechanically driven Supercharger (Compression Ratio $\varepsilon=7.0$)