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# Selfevaporation of Steam from Saturated Water by Reducing Pressure 

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#### Abstract

The quantities of selfevaporated steam from saturated water under the diminution of pressure are obtained theoretically and convenient diagrams are constructed from them for the pressure range up to $50 \mathrm{~kg} / \mathrm{cm}^{2}$ ，and their values are considerably renewed from those of Knorf＇s dagram．＇


## I．Introduction

When the pressure of the saturated water in a vessel such as the steam accu－ mulator or boiler proper is reduced，the saturated temperature of the water will decrease and the surplus heat will spend to evaporate a part of the water．

These phenomena are called the selfevaporation of saturated waier．It is well－ known that the selfevaporation takes place when the boiler water is blown off from the boiler and also when hot water is blown off through the steam trap．

Formerly，a convenient diagram was constructed which gave the selfevaporated quantity of steam in $\mathrm{kg} / \mathrm{m}^{3}$ per $\mathrm{m}^{3}$ of saturated water for given initial and final pressure ${ }^{1)}$ ．But this diagram is constructed for initial pressures up to $28 \mathrm{~kg} / \mathrm{cm}^{2}$ ，and also it seems to give somewhat approximate values at present．So I have tried to get a similar diagram for initial pressures up to $50 \mathrm{~kg} / \mathrm{cm}^{2}$ by the use of the basic data of the Revised Steam Table of Japan Society＇of Mechanical Engineers．

## II．Basic Formulae

The basic formulae used here are the same as those published in my own paper ${ }^{2}$ ．
Fig． 1 shows a vessel such as the steam ac－ cumulator or boiler proper in which the sat－ urated water and steam coexist．

We consider the general case such as shown in Fig．1，i．e．the steam flows in from other source and the evaporated steam is taken out from it，and further，the heat loss


Fig．1．Vessel in which the sat－ urated water and steam coerist．

[^0]is taken place from the outer surface of the vessel. If we open the steam outlet valve, the steam will flow out from the vessel and the pressure in the vessel will reduce, and the selfevaporation will take place at the same time.

Let $A=$ thermal equivalent of work $=1 / 427[\mathrm{kcal} / \mathrm{kgm}]$,
$E=$ sum of energy stored in the vessel in [kcal], $G=$ sum of weight of water and steam in the vessel in [kg],
$G_{1}=$ rate of inflow of steam to the vessel in $[\mathrm{kg} / \mathrm{s}]$,
$G_{2}=$ rate of outflow of steam from the vessel in [kg/s],
$G_{i}=$ total weight of the vessel wall materials in $[\mathrm{kg}]$,
$P=$ pressure in the vessel in $\left[\mathrm{kg} / \mathrm{m}^{2}\right]$,
$p=$ ditto in $\left[\mathrm{kg} / \mathrm{cm}^{2}\right]$,
$Q=$ rate of heat loss derived from the surface of the vessel in [ $\mathrm{kcal} / \mathrm{s}]$,
$V^{\prime}=$ volume of water space in the vessel in $\left[\mathrm{m}^{3}\right]$,
$V^{\prime \prime}=$ ditto of steam space in $\left[\mathrm{m}^{3}\right]$,
$V_{0}=$ total volume of the vessel in $\left[\mathrm{m}^{3}\right]$,
$c_{i}=$ specfic heat of the vessel wall materials in $\left[\mathrm{kcal} / \mathrm{kg}{ }^{\circ} \mathrm{C} \mid\right.$,
$i^{\prime}=$ enthaipy of saturated water in $[\mathrm{kcal} / \mathrm{kg}]$,
$i^{\prime \prime}=$ ditto of saturated steam in $[\mathrm{kcal} / \mathrm{kg}]$,
$i_{1}=$ ditto of inflow steam in $[\mathrm{kcal} / \mathrm{kg}]$.
$i_{2}=$ ditto of outflow steam in $[\mathrm{kcal} / \mathrm{kg}]$,
$r=$ latent heat of vaporization in $[\mathrm{kcal} / \mathrm{kg}]$,
$t_{i}=$ temperature of the vessel wall in $\left[{ }^{\circ} \mathrm{C}\right]$,
$u^{\prime}=$ internal energy of saturated water in $[\mathrm{kcal} / \mathrm{kg}]$,
$u^{\prime \prime}=$ ditto of saturated steam in $[\mathrm{kcal} / \mathrm{kg}]$,
$v^{\prime}=$ specific volume of saturated water in $\left[\mathrm{m}^{3} / \mathrm{kg}\right]$,
$v^{\prime \prime}=$ ditto of saturated steam in $\left[\mathrm{m}^{3} / \mathrm{kg}\right]$,
$z=$ time in [s],
$\phi=$ contribution factor of the heat capacity of the vessel wall materials.
Then, we have the following relation from the condition of conservation of mass:

$$
\begin{align*}
& G=\frac{V^{\prime}}{v^{\prime}}+\frac{V^{\prime \prime}}{v^{\prime \prime}}  \tag{1}\\
& G_{1}=G_{2}+\frac{d G}{d z} \tag{2}
\end{align*}
$$

The total energy $E$ in the vessel is the sum of internal energy of the saturated water and vapour in the vessel:

$$
\begin{equation*}
E=\frac{u^{\prime}}{v^{\prime}} V^{\prime}+\frac{u^{\prime \prime}}{v^{\prime \prime}} V^{\prime \prime} \tag{3}
\end{equation*}
$$

Also, we have the following relation from the conservation of energy during the unit time :

$$
\begin{equation*}
G_{1} i_{1}=G_{2} i_{2}+Q+\frac{d\left(E+\varphi E_{i}\right)}{d z} \tag{4}
\end{equation*}
$$

where $E_{i}$ is the heat capacity of the vessel wall and it becomes as follows:

$$
\begin{equation*}
E_{i}=G_{i} \mathrm{c}_{i} t_{i} \tag{5}
\end{equation*}
$$

The volume of the vessel is the sum of the vloumes of steam space and water space:

$$
\begin{equation*}
V_{0}=V^{\prime}+V^{\prime \prime} \tag{6}
\end{equation*}
$$

Eliminating $E, G_{1}, G_{2}, V^{\prime}$ and $V^{\prime \prime}$ from the above equations, we have

$$
\begin{equation*}
\left(i_{2}-i^{\prime \prime}+X\right) \frac{d G}{d z}=G \frac{d\left(i^{\prime \prime}-X\right)}{d z}+V_{0} \frac{d}{d z}\left(\frac{u^{\prime \prime}-u^{\prime}}{v^{\prime \prime}-v^{\prime}}\right)+\phi \frac{d E_{i}}{d z}+G_{1}\left(i_{2}-i_{1}\right)+Q \tag{7}
\end{equation*}
$$

where

$$
\begin{equation*}
X \equiv \frac{v^{\prime \prime}}{v^{\prime \prime}-v^{\prime}} r=A P v^{\prime \prime}+\frac{v^{\prime \prime}\left(u^{\prime \prime}-u^{\prime}\right)}{v^{\prime \prime}-v^{\prime}} \tag{8}
\end{equation*}
$$

Eq. (7) is the most general equation for the determination of $G$ in a vessel such as shown in Fig. 1.

The enthalpy of the selfevaporated vapour can generally be written down as follows:

$$
\begin{equation*}
i_{2}=i^{\prime}+x r \tag{9}
\end{equation*}
$$

where $x$ is dryness of the vapour. But we may assume approximately as $x=1$, so that

$$
i_{2} \doteq i^{\prime \prime} .
$$

Also, if we take as

$$
\begin{equation*}
\varphi=G_{1}=Q=0 \tag{10}
\end{equation*}
$$

for the selfevaporation of saturated water in a vessel, the Eq. (7) becomes as follows :

$$
\begin{equation*}
\frac{d g}{d P}={ }_{X}^{1} \frac{d\left(i^{\prime \prime}-X\right)}{d P} g+{ }_{X}^{1} \frac{d}{d P}\left(\frac{u^{\prime \prime}-u^{\prime}}{v^{\prime \prime}-v^{\prime}}\right) \tag{11}
\end{equation*}
$$

where the independent variable is changed to the pressure $P$ from the time $z$, and $g$ is the weight of water and steam per unit volume of the vessel. That is

$$
g=\begin{align*}
& G  \tag{12}\\
& V_{0}
\end{align*}
$$

The coefficient of $g$ and the second term of the right hand side of Eq. (11) are the functions of the pressure $P$ only, so that Eq. (11) is a ordinal linear differential equation. Solving this equation, we get the selfevaporated steam quantity $\left(g_{0}-g\right)$, where $g_{0}$ is the initial value of $g$.

## III. Method of Calculation

The differential Equation (11) can be solved analytically if the coefficient $\left(1 / X^{\prime}\right) d\left(i^{\prime \prime}-X\right) / d P$ of $g$ and the second term $(1 / X) d\left[\left(u^{\prime \prime}-u^{\prime}\right) /\left(v^{\prime \prime}-v^{\prime}\right)\right] / d P$ of the right hand side of this equation is expressed as the functions of pressure, but as this is not the case we chose the method of numerical integration showing below.

For convenience, we put as follows :

$$
\begin{align*}
& P \equiv x, g \equiv y, q \equiv \frac{d y}{d x}  \tag{13}\\
& 1 \quad \frac{d\left(i^{\prime \prime}-X\right)}{d x} \equiv X_{1}, \quad \stackrel{1}{X} d x\left(\frac{u^{\prime \prime}-u^{\prime}}{v^{\prime \prime}-v^{\prime}}\right) \equiv X_{2}  \tag{14}\\
& X \quad
\end{align*}
$$

Then Eq. (11) becomes

$$
\begin{equation*}
q=\frac{d y}{d x}=X_{1} y+X_{2}=f(x, y) \tag{15}
\end{equation*}
$$

where $f(x, y)$ indicates $X_{1} y+X_{2}$ as the function of $x$ and $y$.
We assume $y_{0}, y_{1}, y_{2}, y_{3} \cdots$ as the values of $y$ satisfying Eq. (15) corresponding to $x=x_{0}, x_{1}=x_{0}+h, x_{2}=x_{0}+2 h, x_{3}=x_{0}+3 h \cdots$ respectively, and also let us assume that any adjacent three such points $\left(x_{n-1}, y_{n-1}\right),\left(x_{n}, y_{n}\right)$ and $\left(x_{n+1}, y_{n+1}\right)$ belong to one parabola and that the equation of the parabola is

$$
\begin{equation*}
y=y_{n}+a\left(x-x_{n}\right)+b\left(x-x_{n}\right)^{2} \tag{16}
\end{equation*}
$$

whose constants $a$ and $b$ can be determined from the above conditions such as

$$
\begin{equation*}
a=q_{n}, \quad b=\frac{q_{n}-q_{n-1}}{2 h} \tag{17}
\end{equation*}
$$

Therefore the value of $y$ at the point of $\left(x_{n+1}, y_{n+1}\right)$ may be expressed as

$$
\begin{equation*}
y_{n+1}=y_{n}+\left(q_{n}+\frac{q_{n}-q_{n-1}}{2}\right) h \tag{18}
\end{equation*}
$$

By this equation, we can cetermine the values of $y_{2}, y_{3} \cdots$ successively if the values of $y_{0}, q_{0}$ and $y_{1}, q_{1}$ are known initially. The values of $x_{0}, y_{0}$ are given for the initial state, and the value of $q_{0}$ may be calculated by Eq. (15). Also, the values of $y_{1}$ and $q_{1}$ corresponding to $x_{1}=x_{0}+h$ cai be determined as follows; by
expanding the function $y=y(x)$ by the Taylor's series near the point of $x_{n}$, we have

$$
\begin{equation*}
y=y_{0}+\left(\frac{d y}{d x}\right)_{0}\left(x-x_{0}\right)+\frac{1}{2!}\left(\frac{d^{2} y}{d x^{2}}\right)_{0}\left(x-x_{0}\right)^{2}+\frac{1}{3!}\left(\frac{d^{3} y}{d x^{3}}\right)_{0}\left(x-x_{0}\right)^{3}+\cdots \tag{19}
\end{equation*}
$$

The differential coefficients $(d y / d x)_{0},\left(d^{2} y / d x^{2}\right)_{0},\left(d^{3} y / d x^{3}\right)_{0} \cdots$ can be determined by Eq. (15) as follows:

$$
\begin{aligned}
\left(\frac{d y}{d x}\right)_{0} & =f(x, y)=\left(X_{1} y+X_{2}\right)_{0} \\
\left(\frac{d^{2} y}{d x^{2}}\right)_{0} & =\left(\frac{\partial f}{\partial x}+\frac{\partial f}{\partial y} \frac{d y}{d x}\right)_{0}=\left(\frac{d X_{1}}{d x} y+\frac{d X_{2}}{d x}-\right)_{0}+\left(X_{1} f\right)_{0} \\
\left(\frac{d^{3} y}{d x^{3}}\right)_{0} & =\left[\frac{\partial^{2} f}{\partial x^{2}}+2 f \frac{\partial^{2} f}{\partial x \partial y}+\left(\frac{\partial f}{\partial y}\right)^{2} f+\frac{\partial^{2} f}{\partial y^{2}} f^{2}+\frac{\partial f}{\partial x} \frac{\partial f}{\partial y}\right]_{0} \\
& =\left(\frac{d^{2} X_{1}}{d x^{2}} y+\frac{d^{2} X_{2}}{d x^{2}}\right)_{0}+2\left(f \frac{d X_{1}}{d x}\right)_{0}+\left[X_{1}\left(\frac{d X_{1}}{d x} y+\frac{d X_{3}}{d x}+X_{1} f\right)\right]_{0}
\end{aligned}
$$



Fig. 2. The values functions $X_{1}, X_{2}$ and their derivatives at lower pressure range.

We get the value of $y_{1}$ by putting these expressions into the Taylor's series (19) and also the value of $q_{1}$ from Eq. (15). Then, we get the values of $y_{2}, y_{3} \ldots$ using the Eq. (15) and (18) successively.

The functions $X_{1}$ and $X_{2}$ were obtained from the Revised Steam Tables and Diagrams of Japan Society of Mechanical Engineers. The functions $X_{1}, X_{2}$ and the differential coefficients thereof are indicated by the curves in Fig. 2 and 3.


Fig. 3. The values of functions $X_{1}, X_{2}$ and their derivatives at higher pressure range.

## IV. Results of Calculations

Calculations were carried out for intial conditions:

$$
\begin{aligned}
& y_{0} \equiv g_{0}=950,900,850\left[\mathrm{~kg} / \mathrm{m}^{3}\right] \\
& x_{0} \equiv p_{0}=50,45,40,35,30,25,20,15,10,5,4,3,2\left[\mathrm{~kg} / \mathrm{cm}^{2}\right]
\end{aligned}
$$

The results of calculations are shown in Fig. 4 ( $g_{0}=950 \mathrm{~kg} / \mathrm{m}^{3}$ ), Fig. 5 ( $g_{0}=900$ $\mathrm{kg} / \mathrm{m}^{3}$ ) and Fig. $6\left(g_{0}=850 \mathrm{~kg} / \mathrm{m}^{3}\right)$. In these diagrams, the self-evaporated steam


Fig. 4. Steam generated per unit volume for given initial and final pressures. ( $g_{0}=950 \mathrm{~kg} \cdot \mathrm{~m}^{3}$ )


Fig. 5. Steam generated per unit volume for given initial and final pressures ( $g_{0}=000 \mathrm{~kg} / \mathrm{m}^{3}$ )


Fig. 6. Steam generated per unit volume for given initial and final pressures ( $g_{0}=850 \mathrm{~kg} / \mathrm{m}^{3}$ )


Fig. 7. Comparison of the curves in Figs. 4~6 and the similar curves obtained by Knopf.
quantity $\left(g-g_{0}\right)\left[\mathrm{kg} / \mathrm{m}^{3}\right]$ per unit volume of the vessel is taken as the ordinate, and the initial pressure $x_{0}$ as the abscissa and the final pressure as the parameter.

Therefore, if we draw straight line from the point on the abscissa coresponding to the initial pressure upward parallel to the ordinate axis until to a point on the final pressure curve, then the ordinate of this point gives the selfevaporated steam quantity.

Fig. 7 indicates the comparison of the curves in Figs. 4~6. We see that the effects of initial value of $g_{0}$ (i.e. the effect of the volume of steam space) are considerably large so that they can not be neglected as used to be.

## V. Comparison with the Results of Knopf

The similar results obtained by Knopf are indicated in Fig. 7 by chain lines.
The discrepancies are fairly large, especially at higher pressure range. I can't see whether the cause of these discrepancies lies on the basic formula or on the steam table used.

## VI. Conclusion

A general basic Eq. (7) is obtained for a definite quantity of saturated water and vapour in a vessel such as the steam accumulator or boiler drum. By solving the Eq. (11), the special case of Eq. (7), the selfevaporation quantity of the steam accumulator is obtained as shown in Figs. 4~6 using the basic data of the Revised Steam Table of Japan Society of Mechanical Engineers. And these diagrams are compared with the diagrams of Knopf in Fig. 7.


[^0]:    ＊谷下市烃 ：Dr．Eng．，Prof．at Keio University，Tokyo，Japan．
    1）Knopf；Dissertation，Dresden，1925；cr Wärmetechnische Arbeitsmappe，Abb．39，1939，VDI．
    ${ }^{2)}$ I．Tanishita；Heat Storing Caracity of Boiler，Proc．of the Fac．cf Eng．，Keio Uni．，p． 54. Vol．6，No．22， 1953.

