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Compression of Sand Mass in a Cylindrical Vessel (A Method to Obtain Linear Relation between Applied Force and Depression)

(Received December 9, 1953)

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Abstract

This report shows an example of methods to obtain some comparatively great amounts of deformation in proportion to additional forces over a wide range of large forces. For this purpose an apparatus is devised which consists of a pressing rod and a cylindrical vessel in which sand mass is contained in the form of a column.

When the column of sand in the cylindrical vessel is compressed by the rod whose diameter is different from the vessel's, under some optimum conditions an applied force per unit cross sectional area of the rod is linearly proportion to an amount of deformation or depression of the column over a wide range of large forces. These optimum conditions are decided in section 8 in Results.

A methed to get a large amount of depression is devised and is named "Series Connection Method" or "Piling up Method" which is to pile up many apparatuses above mentioned one upon another.

I. Introduction

For certain practical purposes, it will be convenient if a device which is deformable to large extent in proprtion to the forces applied could be made simply and cheaply without using such devices as the spring.

There are some apparatuses using air or water pressure, but the force which can be applied on air or deformation of water is small. Moreover they can be only used in tight states. In fact, besides these, the wooden mine-pillar is used for the purpose above mentioned, but it breaks when the total amount of applied force reaches a certain large value, because its compression strength is small.

If many conditions are good, the above requirement can be satisfied by a method of compressing a column of sand, which is containd in a cylindrical vessel, by a pressing rod. In this case, however, a depression is substituted for the deformation.

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II. Method

Sand mass is filled in a cylindrical steel vessel to form a column of sand. A steel rod whose diameter is smaller than the vessel's is put upon the column of sand with the central axes of rod and vessl coinciding, and the upper end of this rod is pressed by 15 tons Olsen testing machine. Then the sand mass is depressed and the rod descends. This "Depression" or "Descent" is measured by a cathetometer at the upper end of the rod. The schema of the apparatus is shown in Fig. 1.

The applied load divided by the cross sectional area of the rod is named "Applied Force per Unit Cross Sectonal Area of Pressing Rod" and its abbreviation is "Force". The relation bewteen this "Force" and the "Deprseion" is investigated. This relation is influenced by diameters of rod and vessel, size of the sand grain, initial height and apparent density of the column of sand and so forth.

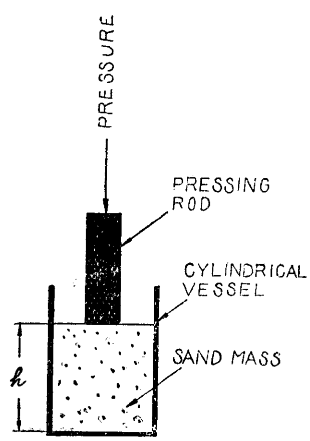

Fig. 1. The schema of the experimental apparatus

III. Results

1. Initial Apparent Denisty of Sand in the Vessel (Compacting Method of Sand Mass)

If intial apparent density of sand in the vessel is changed, "Force" — "Depression" curve changes as shown in Fig. 2.

Curve (2): Leaving the sand mass as it is poured in the vessel, but leveling the top of the sand column, it is compressed by the pressing rod. In this case the sand mass is loosely packed, so the initial apparent density is small and its value is 1.08 g/cc.

Curve (1): The same weight of the sand mass is poured into the vessel while the outer wall of the vessel is being tapped, and moreover after this, the outer wall is tapped many times.

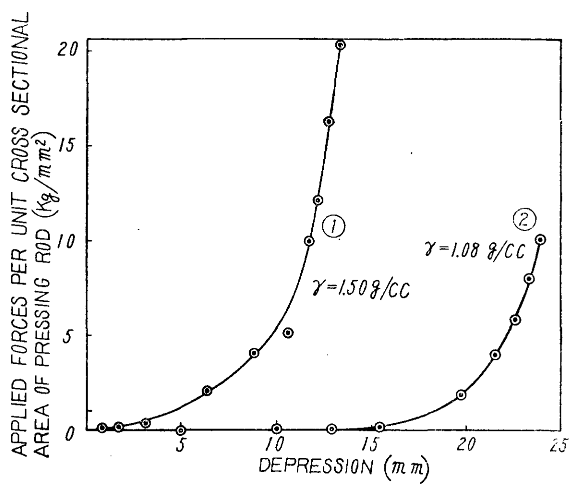

Fig. 2.

Then this densely packed column of sand is compressed. In this case the sand is densely packed, so the initial apparent density is large and its value is 1.50 g/cc.

In both cases the diameters of the rod and the cylindrical vessel are 17.7 and 25.0mm. The ratio of the cross sectional area of them is 1/2. The average diameter of the used sand grains is 0.17mm., and the sand mass is the dried one in a furnace or is the dry sand. The drying method is explained in the author's report.[1]

The difierence in initial apparent density makes the initial height of the sand column difierent, which in this case is 9 mm. The shapes of both curves resemble each other, and the difference of the amount of depression between both curves is about 10-13 mm. over most "Forces". This value is in the same order with 9 mm. This shows that the loosely packed one soon assumes same state as the densely packed one in early compressing process. After assuming this same state, the processes of depression for both cases are entirely equal.

The difference of the compacting method or the initial apparent density does not affect the shape of the curve; in other words, in whatever way the sand mass may be compacted into the vessel, similar curves may be obtaind. So a little shock does not disturb these investigations. Through these experiments, the auther did not have to pay attention to the compacting method, and the experiments were performed only for the case of the tapped one because it shows good reappearance.

The mechanism of the deformation of the compressed sand mass is explained by the above curves. It consists of about two stages mainly. The first is mainly due to the filling of voids in the sand mass by freely movable grains or broken pieces of grains, and the next stage is mainly due to the crushing of grains into smaller pieces like powder. Also a stage of elastic deformation of grains will exsist, but as it is very small, so it need not be considered. By the way, the values of the ultimate compression strength of rocks are between 6 and 14kg/mm.^{2}, but at the edges of grains it will be less than these values.

Besides the above mentioned process, however, in this experiment there are two important processes which are seen throughout all stages. The one is that some grains are pushed out on the upper surface of the column of sand through the clearance between rod and vessel, and another is that some grains are pushed into the voids surrounding the immersed part of the rod. As these processes coexist with the above deformation process, "Force" can be linerly in proprtion to "Depression" over wide range of large "Force" under optimum conditions.

2. Initial Height of a Column of Sand

The higher the initial height of a column of sand is, the greater the amount of depression becomes under the same "Force". This shows that the bottom of a vessel influences the "Depression".

1) M.Sakata, Proc. Faculty of Eng., Keio Univ. 4, 88(1951)

If the initial height, however, is greater than a certain value, nearly equal curves of shape and amount are obtained. This shows that above certain height the bottom of the vessel can not influence the depression. This is explained too, that pressure does not act on a sand layer which is below a certain depth from the top of the column of sand. The value of "a certain depth" above mentioned is equal to "a certain height" which is already mentioned.

The curves (1), (2) and (3) corespond to 12, 36 and 150 mm. in initial height respectively in Fig. 3. The other conditions are the same as the curve (1) in Fig. 1. In this report, the above mentioned "a certain height" is between 100 and 150 mm. according to the experimental conditions. So the amount of depression divided by 100-150 mm. gives the rate of deppression.

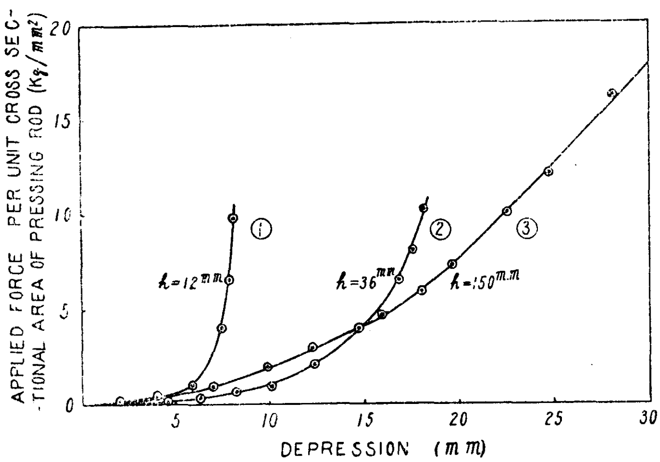

Fig. 3 Change of initial height of column of sand
'*h*' shows the initial height

In the case where the initial height is higher than a certain value, say 150 mm., why the "Depression" curves become all equal. This will be explained as follows:

Even if pressure acts on a column of sand in a vessel, as its power is absorbed by the sand grains, it grows weaker gradually, and it can not be transmitted to very deep layer of sand, and it stops at a ceatain depth, say 150 mm. or less. The voids among the range of sand layer over which pressure can be transmitted will be filled with the broken pieces of grains; the sand mass is coagulated and is hardened. The hardened block of sand near the inner surface of the vessel is stuck on its surface, and makes a concave rigid sand block together with the lowest bottom layer of sand to which pressure is transmitted. Under this layer on which pressure can not be transmitted, however, the sand mass is left as it is compacted at first.

As the rod is immersed in the column of sand, the frictional force between the inner surface of the vessel and sand mass or between the surface of the rod and sand mass becomes suddenly large.

By these two reasons, it is hard for the rod to be immeresed into lower layer of sand beyond a certain depth. However higher the initial height of the sand column is in comparion with the depth to which the pressure can be transmitted, the amount of "Descent" of a rod or "Depressoin" of sand mass is unchangeable under the same "Force". Then all the "Force"-"Depression" curves become equal.

3. Ratio of a Cross Sectional Area of Pressing Rod and Vessel

The three experiments using three different diameters of the rods and the same vessel are compared in Fig. 4. The curves (1), (2) and (3) show the cases in which the ratio of cross sectional area of rod and vessel are 0.1, 0.25 and 0.5 in turn. Under the same "Force" and for a unit cross sectional area of the rods, the resistances of the sand mass which act upon the under surfaces of the rods, the fricton-al forces between sand mass and the rods, and the numbers of the voids which exist in the sand mass under the rods are equal in all cases. However, the total numbers of the voids contained in the clearance be-tween the part of the immersed rods and the vessels, and the space between the pressed rods and the vessels to which the pushed sand mass will be transfered are not equal in these three cases.

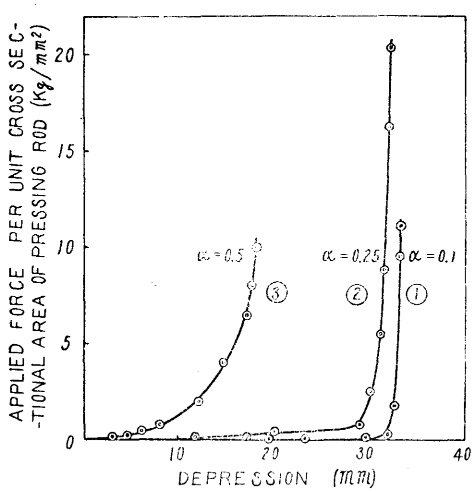

Fig. 4. Change of the ratio of the cross sectional area of the pressing rod and the vessel. 'α' shows the ratio

If the ratio is small, the space or the clearance between the rod and the vessel is large, so a great deal of sand mass is pushed out on the upper surface of the column, or much broken pieces of grains are pushed into the voids which exist in the clearance, then the rod is deeply immersed in the column and the amount of "Depression" becomes large. If the ratio is large, the space or the clearance is small. Then the rod can not be deeply immersed and the amount of "Depression" becomes small. Of course, besides these processes, the processes which are explaind in section 1 coexist.

If the ratio becomes near one, the clearance or the space is very small and the amount of "Depression" becomes very small. This case is not interesting for the purpose of this experiment.

4. Sand Grain Size

As an average diameter of sand grains becomes large, edges of a grain take obtuse angles. Then the grain has the compression strength of the rock itself.

At the same time, the size of the void in sand mass becomes large and it acts in the role of a cushion, moreover the chance of the changing of the position of grain is increased.

By the former reason the linear portion in the the "Depression" curve is lengthened. By the latter reason the inclination of the linear portion is increased.

It is not suitable for this experiment when the grain size becomes too large in comparison with the size of the rod and the vessel.

In Fig. 5 these curves are shown.

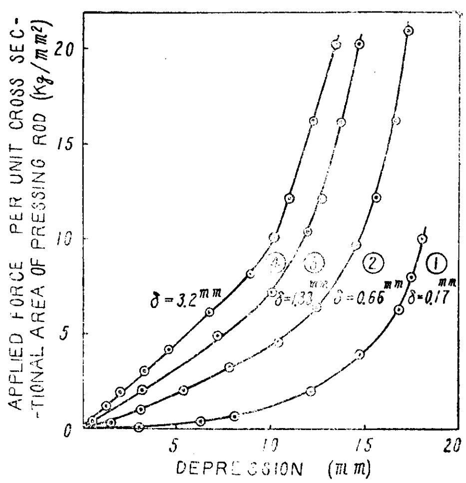

Fig. 5. Change of the sand grain size, 'δ' shows the average diameter of grains

5. Mixing of Different Sizes of Sand Grain

Even though the mixed state is changed with many different average diameters of grains, the "Depression" curves do not change much. This result is based on the experiments in Fig. 6. The curve (1) shows the case in which the lower half of the column of sand consists of small grains (1.33 mm. in dia.) and the upper half consists of large grains (3.20 mm. in dia.). The curve (2) shows the case in which the above mentioned small and large grains are uniformly mixed. The curve (3) and (4) are shown for comparison. In (3), the small grains, and in (4), the large ones are compacted alone respectively. Of course, the heights of the columns are equal.

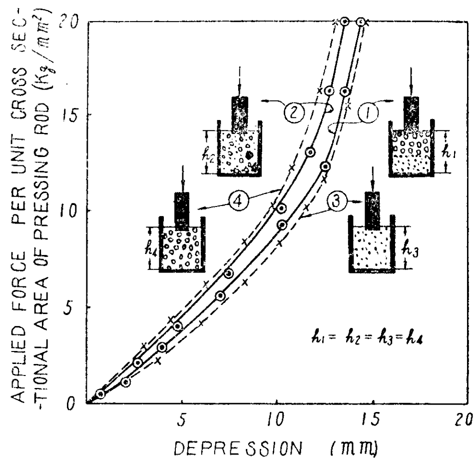

Fig. 6. Change of the mixed state of sand grains having different average diameter

By these comparisons, it is conjecturd that all curves which may be obtaind by various ways of mixing using the grains of sizes between the above mentioned ones will lie in the space between the curves (3) and (4).

6. Wet Sand

The above data are obtaind by using dry sand. The data for wet sand are shown in this section and in Fig. 7.

The more the percentage of moisture content, the more the amount of "Depression" is increased under the same "Force". The conditions of the curves in Fig.

7 are showns follows:

No.	grain size (mm.)	moisture content(%)
(1)	0.17	28 (saturated)*
(2)	″	20
(3)	″	0 (dried)
(4)	0.66	23 (saturated)*
(5)	″	0 (dried)
(6)	1.33	21 (saturated)*
(7)	″	0 (dried)

* These show the saturation values for those sand grains respectively.

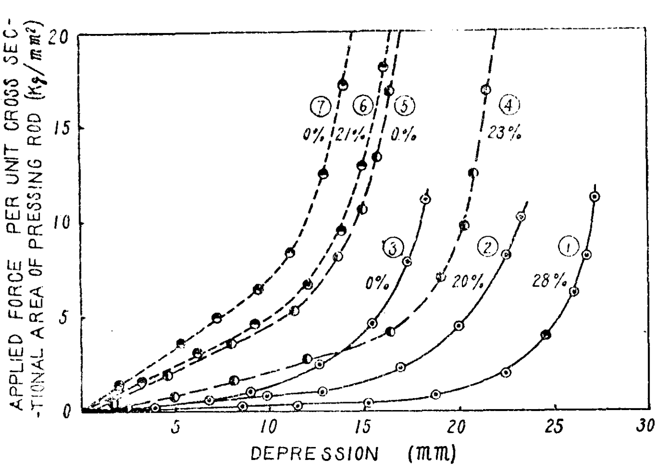

Fig. 7 Change of the percentage of moisture content. The percentage is written near the curve. The average diameter of the sand grain is distinguished by the mark:
⊙ ; 0.17mm. ◐ ; 0.66mm ◓; 1.33mm

Comparing the curve for the same grains size, they are very similar excepting the difference of the absolute value. As the diameter of grain becomes large, the difference between the dry and the wet sand becomes small, and with some large grain the difference will become negligibly small.

By these curves, it is explained that moisture in sand mass can not influence the shape of the "Depression" curve, and with large grain the influence of moisture is entirely lost, or the amount of "Depression" for the wet sand will entirely agree with that for the dry sand.

7. Size of Pressing Rod and Vessel

In this experiment an important factor is the ratio of the cross sectional area of the rod and the vessel as already shown in section 3. If the size, however, is changed leaving the ratio equal, the amount of "Depression" will be changed, for the clearance between the rod and the vessel is changed. For example, if the diameter of rod and vessel are increased by n times, then the area of clearance is increased by n^2 times. In this case the amount of "Depression" increases in proportion to the area of clearance but does not become so large as n^2 times, for the penetrating resistance increses too. Then it is hard to decide the increment of the amount of "Depresson" only by the increased figure as n times.

8. Optimum Conditions to Obtain Long Linear Portion Under Large "Force" in "Force"-"Depression" Curve

Considering the above experiments, the optimum conditions to get a long linear portion in "Depression" curve under large "Force" is decided. By the way, the sizes of the rod, the vessel and sand grain used in this experiment are shown as follows;

The average diameter of sand grains:	0.1 – 8mm.
The diameter of the rod:	10 – 30mm.
The diameter of the vessel:	20 – 40mm.

The optimum conditions are shown as follows:

1. The average diameter of sand grains must be 3-10 mm.
2. The initial height of the column of the mass must be high enough so that the bottom of the vessel does not influence the "Deprression" of the sand mass. In this experiment, it is 100-150mm.
3. The ratio of the cross sectional area of the pressing rod and the vessel must be 0.4-0.75.
4. The ratio of the average diameter of the sand grains and the diameter of the pressing rod must be 0.1-0.35.
5. The ratio of the differnce between the radius of the rod and the vessel and the average diameter of grains must be 0.5-1.5.

The 4th and 5th conditions are not explained, but is evident that these are important factors.

The curves that satisfy these above mentioned optimum conditions are shown in Fig. 8. The practical data are as follows:

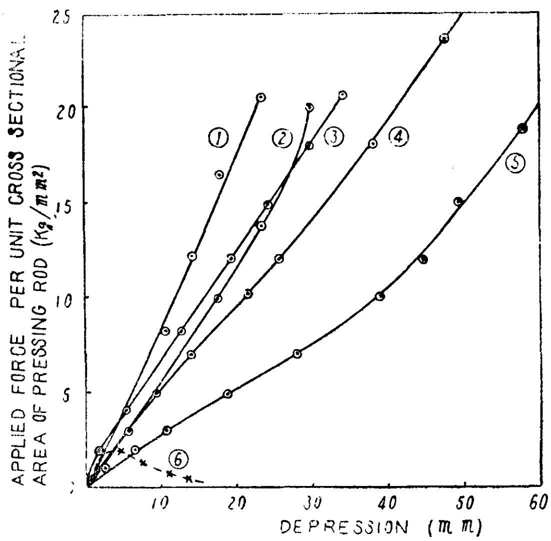

Fig. 8. The exampls that satisfy the optimum conditions to get the long liner portion at large depression in 'Force'-'Depression' curve excluding the curve (6)

conditions No.	1. Dia. of grain(*mm.*)	2. Hight (*mm.*)	2. Apparent density g·/cc.	3. Area of rod / Area of vessel	4. Dia. of grain / Dia. of rod	5. (Rad. of vessel)-(Rad. of rod) / Dia. of grain
1	3.2	190	1.65	0.74	0.18	0.47
2.	〃	185	〃	0.62	0.15	0.94
3.	〃	〃	〃	0.5	0.18	1.1
4.	5.9	300	1.66	0.42	0.33	0.8
5.	8.1	180	1.78	0.54	0.28	0.63

The curve (6) is the compression test of the pine wood for comparison with the above experiment, and its size is 57×57×600 mm. The ultimate compression strength of pine wood is about 2-3 kg./mm.^{2}.

9. Series Connection of Apparatuses (Piling-up Method)

One pair of a pressing rod and a vessel is used in the former experiments. There is, however, a method to use many pairs of rods and vessels, but their pairs are piled up in series one above another. This method is named "Series Connection Method" or "Piling-up Method". If n equal pairs are piled up, the amount of "Depression" is increased by n times in comparison with one pair system, but the inclination of "Depression" curve becomes gentle.

In this method, the pairs which make up a system need not be equal to each other. When the series are constructed by many different pairs, the amount of

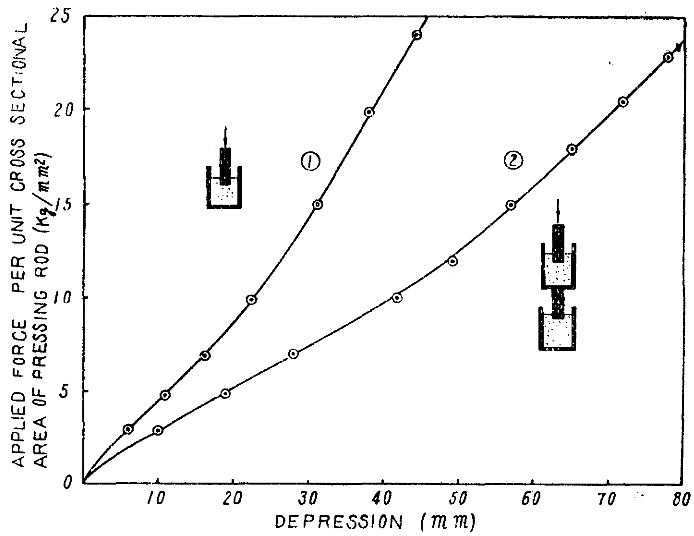

Fig. 9. One pair system (one rod and one vessel) is compared with two pairs system (two rods and two vessels) which is named 'Series Connection Method' or 'Piling-up Method'

"Depression" will be the sum of all pairs' "Depression" values.

In Fig. 9, the series constructed by two equal pairs is shown by the curve (2), and this is compared with the original one pair, which is shown by the curve (1).

IV. Conclusion

When a column of sand in a cylindrical steel vessel is pressed by a pressing steel rod with diameter smaller than the vessel's, a part of the upper surface of the column is depressed and the rod descends. This "Descent" of the rod, or the deformation of sand mass is named "Depression" of the column of sand. The relation between the amount of "Depression" and "Applied Force per Unit Cross Sectional Area of Pressing Rod" (this is abbreviated as "Force") is investigated.

The shape of "Force"-"Depression" curve is not influenced by the under mentioned conditions, though the absolute "Depression" value is:

1. Changing of the compacted state of sand mass, in other words, changing of the way of pouring the grains.
2. Changing of the initial height of the column, excepting within the range in which the bottom of vessel influences the " Depression".
3. Changing of the way of mixing the different grain sizes.
4. Changing of the moisture content.

The curve is influenced by the under mentiond:

1. Changing of the ratio of the cross sectional area of rod and vessel.
2. Changing of the sizes of rod and vessel, leaving their ratio equal.
3. Changing of the size of the sand grain.

If the optimum sizes of the rod, the vessel and the sand grain are selected, for a wide range of large "Force", long linear portion is obtained in "Depression" curve. These optimum conditions are explained in section 8 in Results.

If large amount of "Depression" is wanted, there is "Series Connection Method" or"Piling-up Method" which is shown in section 9 in Results.

Acknowledgments

The author wishes to express his deep thanks to Dr. M. Masima, Professor of the Keio University for suggesting the problem and for many valuable advices.